

**LS-DYNA<sup>®</sup>**  
**KEYWORD USER'S MANUAL**

**VOLUME II**  
**Material Models**

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LS-DYNA Dev

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This file contains the code for implementing the key schedule for AES (Rijndael) for block and key sizes of 16, 24, and 32 bytes.

# TABLE OF CONTENTS

---

## TABLE OF CONTENTS

TABLE OF CONTENTS .....	0-1
*EOS .....	1-10
*EOS_LINEAR_POLYNOMIAL .....	1-16
*EOS_JWL_{OPTION} .....	1-18
*EOS_SACK_TUESDAY .....	1-22
*EOS_GRUNEISEN .....	1-23
*EOS_RATIO_OF_POLYNOMIALS .....	1-25
*EOS_LINEAR_POLYNOMIAL_WITH_ENERGY_LEAK .....	1-28
*EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE .....	1-30
*EOS_TABULATED_COMPACTION .....	1-34
*EOS_TABULATED .....	1-37
*EOS_PROPELLANT_DEFLAGRATION .....	1-39
*EOS_TENSOR_PORE_COLLAPSE .....	1-44
*EOS_IDEAL_GAS .....	1-47
*EOS_PHASE_CHANGE .....	1-49
*EOS_JWLB .....	1-50
*EOS_GASKET .....	1-53
*EOS_MIE_GRUNEISEN .....	1-56
*EOS_MURNAGHAN .....	1-58
*EOS_USER_DEFINED .....	1-59
*MAT .....	2-1
MATERIAL MODEL REFERENCE TABLES .....	2-9
Alphabetized Materials List .....	2-20
*MAT_ADD_AIRBAG_POROSITY_LEAKAGE .....	2-31
*MAT_ADD_COHESIVE .....	2-34
*MAT_ADD_EROSION .....	2-37
GISSMO: .....	2-50
DIEM (Damage Initiation and Evolution Models): .....	2-53
*MAT_ADD_FATIGUE .....	2-59
*MAT_ADD_GENERALIZED_DAMAGE .....	2-62
*MAT_ADD_PERMEABILITY .....	2-73
*MAT_ADD_PORE_AIR .....	2-74
*MAT_ADD_THERMAL_EXPANSION .....	2-76

# TABLE OF CONTENTS

---

*MAT_NONLOCAL.....	2-78
*MAT_ELASTIC.....	2-82
*MAT_OPTIONTROPIC_ELASTIC.....	2-85
*MAT_PLASTIC_KINEMATIC.....	2-96
*MAT_ELASTIC_PLASTIC_THERMAL.....	2-99
*MAT_SOIL_AND_FOAM.....	2-102
*MAT_VISCOELASTIC.....	2-106
*MAT_BLATZ-KO_RUBBER.....	2-108
*MAT_HIGH_EXPLOSIVE_BURN.....	2-110
*MAT_NULL.....	2-113
*MAT_ELASTIC_PLASTIC_HYDRO.....	2-115
*MAT_STEINBERG.....	2-120
*MAT_STEINBERG_LUND.....	2-124
*MAT_ISOTROPIC_ELASTIC_PLASTIC.....	2-128
*MAT_ISOTROPIC_ELASTIC_FAILURE.....	2-129
*MAT_SOIL_AND_FOAM_FAILURE.....	2-131
*MAT_JOHNSON_COOK.....	2-132
*MAT_PSEUDO_TENSOR.....	2-139
*MAT_ORIENTED_CRACK.....	2-147
*MAT_POWER_LAW_PLASTICITY.....	2-150
*MAT_STRAIN_RATE_DEPENDENT_PLASTICITY.....	2-153
*MAT_RIGID.....	2-156
*MAT_ORTHOTROPIC_THERMAL.....	2-162
*MAT_COMPOSITE_DAMAGE.....	2-169
*MAT_TEMPERATURE_DEPENDENT_ORTHOTROPIC.....	2-175
*MAT_PIECEWISE_LINEAR_PLASTICITY.....	2-180
*MAT_GEOLOGIC_CAP_MODEL.....	2-186
*MAT_HONEYCOMB.....	2-192
*MAT_MOONEY-RIVLIN_RUBBER.....	2-200
*MAT_RESULTANT_PLASTICITY.....	2-203
*MAT_FORCE_LIMITED.....	2-204
*MAT_SHAPE_MEMORY.....	2-211
*MAT_FRAZER_NASH_RUBBER_MODEL.....	2-216
*MAT_LAMINATED_GLASS.....	2-219
*MAT_BARLAT_ANISOTROPIC_PLASTICITY.....	2-221
*MAT_BARLAT_YLD96.....	2-225
*MAT_FABRIC.....	2-229

# TABLE OF CONTENTS

---

*MAT_FABRIC_MAP .....	2-246
*MAT_PLASTIC_GREEN-NAGHDI_RATE .....	2-252
*MAT_3-PARAMETER_BARLAT .....	2-253
*MAT_EXTENDED_3-PARAMETER_BARLAT .....	2-271
*MAT_TRANSVERSELY_ANISOTROPIC_ELASTIC_PLASTIC .....	2-275
*MAT_BLATZ-KO_FOAM .....	2-286
*MAT_FLD_TRANSVERSELY_ANISOTROPIC .....	2-288
*MAT_NONLINEAR_ORTHOTROPIC .....	2-290
*MAT_USER_DEFINED_MATERIAL_MODELS .....	2-295
*MAT_BAMMAN .....	2-301
*MAT_BAMMAN_DAMAGE .....	2-306
*MAT_CLOSED_CELL_FOAM .....	2-309
*MAT_ENHANCED_COMPOSITE_DAMAGE .....	2-312
*MAT_LOW_DENSITY_FOAM .....	2-324
*MAT_LAMINATED_COMPOSITE_FABRIC .....	2-329
*MAT_COMPOSITE_FAILURE .....	2-340
*MAT_ELASTIC_WITH_VISCOSITY .....	2-345
*MAT_ELASTIC_WITH_VISCOSITY_CURVE .....	2-348
*MAT_KELVIN-MAXWELL_VISCOELASTIC .....	2-351
*MAT_VISCOUS_FOAM .....	2-353
*MAT_CRUSHABLE_FOAM .....	2-355
*MAT_RATE_SENSITIVE_POWERLAW_PLASTICITY .....	2-357
*MAT_MODIFIED_ZERILLI_ARMSTRONG .....	2-359
*MAT_LINEAR_ELASTIC_DISCRETE_BEAM .....	2-362
*MAT_NONLINEAR_ELASTIC_DISCRETE_BEAM .....	2-365
*MAT_NONLINEAR_PLASTIC_DISCRETE_BEAM .....	2-370
*MAT_SID_DAMPER_DISCRETE_BEAM .....	2-376
*MAT_HYDRAULIC_GAS_DAMPER_DISCRETE_BEAM .....	2-380
*MAT_CABLE_DISCRETE_BEAM .....	2-382
*MAT_CONCRETE_DAMAGE .....	2-385
*MAT_CONCRETE_DAMAGE_REL3 .....	2-389
*MAT_LOW_DENSITY_VISCOUS_FOAM .....	2-398
*MAT_ELASTIC_SPRING_DISCRETE_BEAM .....	2-403
*MAT_BILKHU/DUBOIS_FOAM .....	2-405
*MAT_GENERAL_VISCOELASTIC .....	2-408
*MAT_HYPERELASTIC_RUBBER .....	2-413
*MAT_OGDEN_RUBBER .....	2-420

# TABLE OF CONTENTS

---

*MAT_SOIL_CONCRETE .....	2-426
*MAT_HYSTERETIC_SOIL .....	2-430
*MAT_RAMBERG-OSGOOD .....	2-438
*MAT_PLASTICITY_WITH_DAMAGE.....	2-440
*MAT_FU_CHANG_FOAM_{OPTION} .....	2-449
*MAT_WINFRITH_CONCRETE .....	2-460
*MAT_WINFRITH_CONCRETE_REINFORCEMENT .....	2-465
*MAT_ORTHOTROPIC_VISCOELASTIC.....	2-467
*MAT_CELLULAR_RUBBER.....	2-470
*MAT_MTS .....	2-474
*MAT_PLASTICITY_POLYMER.....	2-478
*MAT_ACOUSTIC .....	2-482
*MAT_SOFT_TISSUE .....	2-484
*MAT_ELASTIC_6DOF_SPRING_DISCRETE_BEAM .....	2-489
*MAT_INELASTIC_SPRING_DISCRETE_BEAM .....	2-491
*MAT_INELASTIC_6DOF_SPRING_DISCRETE_BEAM.....	2-494
*MAT_BRITTLE_DAMAGE .....	2-496
*MAT_GENERAL_JOINT_DISCRETE_BEAM.....	2-499
*MAT_SIMPLIFIED_JOHNSON_COOK .....	2-501
*MAT_SIMPLIFIED_JOHNSON_COOK_ORTHOTROPIC_DAMAGE.....	2-504
*MAT_SPOTWELD .....	2-506
*MAT_SPOTWELD_DAIMLERCHRYSLER .....	2-524
*MAT_GEPLASTIC_SRATE_2000a.....	2-527
*MAT_INV_HYPERBOLIC_SIN_{OPTION}.....	2-529
*MAT_ANISOTROPIC_VISCOPLASTIC .....	2-532
*MAT_ANISOTROPIC_PLASTIC .....	2-539
*MAT_DAMAGE_1.....	2-543
*MAT_DAMAGE_2.....	2-550
*MAT_ELASTIC_VISCOPLASTIC_THERMAL .....	2-554
*MAT_MODIFIED_JOHNSON_COOK.....	2-558
*MAT_ORTHO_ELASTIC_PLASTIC .....	2-566
*MAT_JOHNSON_HOLMQUIST_CERAMICS.....	2-570
*MAT_JOHNSON_HOLMQUIST_CONCRETE.....	2-574
*MAT_FINITE_ELASTIC_STRAIN_PLASTICITY .....	2-577
*MAT_TRIP .....	2-580
*MAT_LAYERED_LINEAR_PLASTICITY .....	2-584
*MAT_UNIFIED_CREEP .....	2-588

# TABLE OF CONTENTS

---

*MAT_UNIFIED_CREEP_ORTHO.....	2-590
*MAT_COMPOSITE_LAYUP.....	2-595
*MAT_COMPOSITE_MATRIX.....	2-598
*MAT_COMPOSITE_DIRECT.....	2-601
*MAT_GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM.....	2-603
*MAT_GURSON.....	2-611
*MAT_GURSON_JC.....	2-617
*MAT_GURSON_RCDC.....	2-625
*MAT_GENERAL_NONLINEAR_1DOF_DISCRETE_BEAM.....	2-631
*MAT_HILL_3R.....	2-634
*MAT_HILL_3R_3D.....	2-637
*MAT_HILL_3R_TABULATED.....	2-645
*MAT_MODIFIED_PIECEWISE_LINEAR_PLASTICITY.....	2-648
*MAT_PLASTICITY_COMPRESSION_TENSION.....	2-654
*MAT_KINEMATIC_HARDENING_TRANSVERSELY_ANISOTROPIC_{OPTION}.....	2-659
*MAT_MODIFIED_HONEYCOMB.....	2-672
*MAT_ARRUDA_BOYCE_RUBBER.....	2-684
*MAT_HEART_TISSUE.....	2-687
*MAT_LUNG_TISSUE.....	2-692
*MAT_SPECIAL_ORTHOTROPIC.....	2-695
*MAT_ISOTROPIC_SMEARED_CRACK.....	2-699
*MAT_ORTHOTROPIC_SMEARED_CRACK.....	2-702
*MAT_BARLAT_YLD2000.....	2-707
*MAT_VISCOELASTIC_FABRIC.....	2-720
*MAT_WTM_STM.....	2-723
*MAT_WTM_STM_PLC.....	2-733
*MAT_CORUS_VEGTER.....	2-738
*MAT_COHESIVE_MIXED_MODE.....	2-744
*MAT_MODIFIED_FORCE_LIMITED.....	2-748
*MAT_VACUUM.....	2-759
*MAT_RATE_SENSITIVE_POLYMER.....	2-760
*MAT_TRANSVERSELY_ISOTROPIC_CRUSHABLE_FOAM.....	2-762
*MAT_WOOD.....	2-767
*MAT_PITZER_CRUSHABLE_FOAM.....	2-774
*MAT_SCHWER_MURRAY_CAP_MODEL.....	2-776
*MAT_1DOF_GENERALIZED_SPRING.....	2-784
*MAT_FHWA_SOIL.....	2-786

# TABLE OF CONTENTS

---

*MAT_FHWA_SOIL_NEBRASKA .....	2-789
*MAT_GAS_MIXTURE.....	2-790
*MAT_EMMI .....	2-797
*MAT_DAMAGE_3.....	2-804
*MAT_DESHPANDE_FLECK_FOAM.....	2-810
*MAT_PLASTICITY_COMPRESSION_TENSION_EOS.....	2-813
*MAT_MUSCLE.....	2-818
*MAT_ANISOTROPIC_ELASTIC_PLASTIC.....	2-822
*MAT_RATE_SENSITIVE_COMPOSITE_FABRIC .....	2-833
*MAT_CSCM.....	2-839
*MAT_ALE_INCOMPRESSIBLE.....	2-851
*MAT_COMPOSITE_MSC.....	2-853
*MAT_MODIFIED_CRUSHABLE_FOAM.....	2-864
*MAT_BRAIN_LINEAR_VISCOELASTIC .....	2-867
*MAT_PLASTIC_NONLINEAR_KINEMATIC .....	2-869
*MAT_PLASTIC_NONLINEAR_KINEMATIC_B .....	2-871
*MAT_MOMENT_CURVATURE_BEAM .....	2-874
*MAT_MCCORMICK.....	2-878
*MAT_POLYMER .....	2-881
*MAT_ARUP_ADHESIVE.....	2-886
*MAT_RESULTANT_ANISOTROPIC.....	2-894
*MAT_STEEL_CONCENTRIC_BRACE.....	2-898
*MAT_CONCRETE_EC2 .....	2-903
*MAT_MOHR_COULOMB.....	2-923
*MAT_RC_BEAM.....	2-928
*MAT_VISCOELASTIC_THERMAL.....	2-934
*MAT_QUASILINEAR_VISCOELASTIC.....	2-939
*MAT_HILL_FOAM .....	2-944
*MAT_VISCOELASTIC_HILL_FOAM.....	2-947
*MAT_LOW_DENSITY_SYNTHETIC_FOAM.....	2-951
*MAT_SIMPLIFIED_RUBBER/FOAM.....	2-957
*MAT_SIMPLIFIED_RUBBER_WITH_DAMAGE.....	2-965
*MAT_COHESIVE_ELASTIC .....	2-968
*MAT_COHESIVE_TH .....	2-969
*MAT_COHESIVE_GENERAL .....	2-972
*MAT_SAMP-1 .....	2-977
*MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP.....	2-986



# TABLE OF CONTENTS

---

*MAT_ANISOTROPIC_THERMOELASTIC.....	2-991
*MAT_FLD_3-PARAMETER_BARLAT.....	2-994
*MAT_SEISMIC_BEAM.....	2-1000
*MAT_SOIL_BRICK.....	2-1009
*MAT_DRUCKER_PRAGER.....	2-1013
*MAT_RC_SHEAR_WALL.....	2-1016
*MAT_CONCRETE_BEAM.....	2-1024
*MAT_GENERAL_SPRING_DISCRETE_BEAM.....	2-1027
*MAT_SEISMIC_ISOLATOR.....	2-1031
*MAT_JOINTED_ROCK.....	2-1037
*MAT_STEEL_EC3.....	2-1042
*MAT_HYSTERETIC_REINFORCEMENT.....	2-1045
*MAT_BOLT_BEAM.....	2-1052
*MAT_SPR_JLR.....	2-1055
*MAT_DRY_FABRIC.....	2-1063
*MAT_4A_MICROMECH.....	2-1070
*MAT_ELASTIC_PHASE_CHANGE.....	2-1079
*MAT_OPTIONTROPIC_ELASTIC_PHASE_CHANGE.....	2-1081
*MAT_MOONEY-RIVLIN_PHASE_CHANGE.....	2-1090
*MAT_CODAM2.....	2-1095
*MAT_RIGID_DISCRETE.....	2-1107
*MAT_ORTHOTROPIC_SIMPLIFIED_DAMAGE.....	2-1108
*MAT_TABULATED_JOHNSON_COOK.....	2-1117
*MAT_TABULATED_JOHNSON_COOK_GYS.....	2-1124
*MAT_VISCOPLASTIC_MIXED_HARDENING.....	2-1128
*MAT_KINEMATIC_HARDENING_BARLAT89.....	2-1130
*MAT_PML_ELASTIC.....	2-1139
*MAT_PML_ELASTIC_FLUID.....	2-1141
*MAT_PML_ACOUSTIC.....	2-1142
*MAT_BIOT_HYSTERETIC.....	2-1144
*MAT_CAZACU_BARLAT.....	2-1146
*MAT_VISCOELASTIC_LOOSE_FABRIC.....	2-1160
*MAT_MICROMECHANICS_DRY_FABRIC.....	2-1166
*MAT_SCC_ON_RCC.....	2-1172
*MAT_PML_HYSTERETIC.....	2-1176
*MAT PERT_PIECEWISE_LINEAR_PLASTICITY.....	2-1177
*MAT_COHESIVE_MIXED_MODE_ELASTOPLASTIC_RATE.....	2-1178

# TABLE OF CONTENTS

---

*MAT_JOHNSON_HOLMQUIST_JH1 .....	2-1186
*MAT_KINEMATIC_HARDENING_BARLAT2000 .....	2-1190
*MAT_HILL_90 .....	2-1195
*MAT_UHS_STEEL .....	2-1204
*MAT_PML_OPTIONTROPIC_ELASTIC .....	2-1224
*MAT_PML_NULL .....	2-1227
*MAT_PHS_BMW .....	2-1229
*MAT_REINFORCED_THERMOPLASTIC .....	2-1247
*MAT_REINFORCED_THERMOPLASTIC_UDFIBER .....	2-1255
*MAT_TAILORED_PROPERTIES .....	2-1261
*MAT_TOUGHENED_ADHESIVE_POLYMER .....	2-1264
*MAT_GENERALIZED_PHASE_CHANGE .....	2-1271
*MAT_PIECEWISE_LINEAR_PLASTIC_THERMAL .....	2-1277
*MAT_AMORPHOUS_SOLIDS_FINITE_STRAIN .....	2-1281
*MAT_STOUGHTON_NON_ASSOCIATED_FLOW_{OPTION} .....	2-1284
*MAT_MOHR_NON_ASSOCIATED_FLOW_{OPTION} .....	2-1294
*MAT_LAMINATED_FRACTURE_DAIMLER_PINHO .....	2-1308
*MAT_LAMINATED_FRACTURE_DAIMLER_CAMANHO .....	2-1320
*MAT_TABULATED_JOHNSON_COOK_ORTHO_PLASTICITY .....	2-1331
*MAT_TISSUE_DISPERSED .....	2-1338
*MAT_EIGHT_CHAIN_RUBBER .....	2-1345
*MAT_BERGSTROM_BOYCE_RUBBER .....	2-1354
*MAT_CWM .....	2-1357
*MAT_POWDER .....	2-1361
*MAT_RHT .....	2-1368
*MAT_CONCRETE_DAMAGE_PLASTIC_MODEL .....	2-1376
*MAT_PAPER .....	2-1384
*MAT_SMOOTH_VISCOELASTIC_VISCOPLASTIC .....	2-1393
*MAT_CHRONOLOGICAL_VISCOELASTIC .....	2-1402
*MAT_ADHESIVE_CURING_VISCOELASTIC .....	2-1406
*MAT_COHESIVE_PAPER .....	2-1410
*MAT_GLASS .....	2-1415
*MAT_COMPRF .....	2-1420
*MAT_ALE_VACUUM .....	2-1426
*MAT_ALE_GAS_MIXTURE .....	2-1427
*MAT_ALE_VISCOUS .....	2-1434
*MAT_ALE_MIXING_LENGTH .....	2-1437

# TABLE OF CONTENTS

---

*MAT_ALE_INCOMPRESSIBLE.....	2-1440
*MAT_ALE_HERSHEL.....	2-1441
*MAT_SPH_VISCOUS.....	2-1444
*MAT_SPRING_ELASTIC.....	2-1448
*MAT_DAMPER_VISCOUS.....	2-1449
*MAT_SPRING_ELASTOPLASTIC.....	2-1450
*MAT_SPRING_NONLINEAR_ELASTIC.....	2-1451
*MAT_DAMPER_NONLINEAR_VISCOUS.....	2-1452
*MAT_SPRING_GENERAL_NONLINEAR.....	2-1453
*MAT_SPRING_MAXWELL.....	2-1455
*MAT_SPRING_INELASTIC.....	2-1457
*MAT_SPRING_TRILINEAR_DEGRADING.....	2-1458
*MAT_SPRING_SQUAT_SHEARWALL.....	2-1459
*MAT_SPRING_MUSCLE.....	2-1460
*MAT_SEATBELT_{OPTION}.....	2-1465
*MAT_THERMAL.....	3-1
*MAT_THERMAL_ISOTROPIC.....	3-2
*MAT_THERMAL_ORTHOTROPIC.....	3-3
*MAT_THERMAL_ISOTROPIC_TD.....	3-5
*MAT_THERMAL_ORTHOTROPIC_TD.....	3-7
*MAT_THERMAL_DISCRETE_BEAM.....	3-10
*MAT_THERMAL_CHEMICAL_REACTION.....	3-11
*MAT_THERMAL_CWM.....	3-19
*MAT_THERMAL_ORTHOTROPIC_TD_LC.....	3-22
*MAT_THERMAL_ISOTROPIC_PHASE_CHANGE.....	3-25
*MAT_THERMAL_ISOTROPIC_TD_LC.....	3-28
*MAT_THERMAL_USER_DEFINED.....	3-29

# \*EOS

When defining an equation of state, the type of equation of state is specified by a corresponding 3-digit number in the command name, e.g., \*EOS\_004, or equivalently, by its more descriptive designation, e.g., \*EOS\_GRUNEISEN. The equations of state can be used with a subset of the materials that are available for solid elements; see the MATERIAL MODEL REFERENCE TABLES in the beginning of the \*MAT section of this Manual. \*EOS\_015 is linked to the type 2 thick shell element and can be used to model engine gaskets.

The meaning associated with particular extra history variables for a subset of material models and equations of state are tabulated at <http://www.dynasupport.com/howto-/material/history-variables>. The first three extra history variables when using an equation of state are (1) internal energy, (2) pressure due to bulk viscosity, and (3) the element volume from the previous time step.

TYPE 1:	*EOS_LINEAR_POLYNOMIAL
TYPE 2:	*EOS_JWL
TYPE 3:	*EOS_SACK_TUESDAY
TYPE 4:	*EOS_GRUNEISEN
TYPE 5:	*EOS_RATIO_OF_POLYNOMIALS
TYPE 6:	*EOS_LINEAR_POLYNOMIAL_WITH_ENERGY_LEAK
TYPE 7:	*EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE
TYPE 8:	*EOS_TABULATED_COMPACTION
TYPE 9:	*EOS_TABULATED
TYPE 10:	*EOS_PROPELLANT_DEFLAGRATION
TYPE 11:	*EOS_TENSOR_PORE_COLLAPSE
TYPE 12:	*EOS_IDEAL_GAS
TYPE 13:	*EOS_PHASE_CHANGE
TYPE 14:	*EOS_JWLB

TYPE 15:       \*EOS\_GASKET  
TYPE 16:       \*EOS\_MIE\_GRUNEISEN  
TYPE 19:       \*EOS\_MURNAGHAN  
TYPE 21-30:   \*EOS\_USER\_DEFINED

An additional option **TITLE** may be appended to all the **\*EOS** keywords. If this option is used then an additional line is read for each section in 80a format which can be used to describe the equation of state. At present LS-DYNA does not make use of the title. Inclusion of title simply gives greater clarity to input decks.

### Definitions and Conventions

In order to prescribe the boundary and/or initial thermodynamic condition, manual computations are often necessary. Conventions or definitions must be established to simplify this process. Some basic variables are defined in the following. Since many of these variables have already been denoted by different symbols, the notations used here are unique in this section only! They are presented to only clarify their usage. A corresponding SI unit set is also presented as an example.

First consider a few volumetric parameters since they are a measure of compression (or expansion).

Volume:

$$V \approx (\text{m}^3)$$

Mass:

$$M \approx (\text{Kg})$$

Current specific volume (per mass):

$$v = \frac{V}{M} = \frac{1}{\rho} \approx \left( \frac{\text{m}^3}{\text{Kg}} \right)$$

Reference specific volume:

$$v_0 = \frac{V_0}{M} = \frac{1}{\rho_0} \approx \left( \frac{\text{m}^3}{\text{Kg}} \right)$$

Relative volume:

$$v_r = \frac{V}{V_0} = \frac{(V/M)}{(V_0/M)} = \frac{v}{v_0} = \frac{\rho_0}{\rho}$$

Current normalized volume increment:

## \*EOS

$$\frac{dv}{v} = \frac{v - v_0}{v} = 1 - \frac{1}{v_r} = 1 - \frac{\rho}{\rho_0}$$

A frequently used volumetric parameter is:

$$\mu = \frac{1}{v_r} - 1 = \frac{v_0 - v}{v} = -\frac{dv}{v} = \frac{\rho}{\rho_0} - 1$$

Sometimes another volumetric parameter is used:

$$\eta = \frac{v_0}{v} = \frac{\rho}{\rho_0}$$

Thus, the relation between  $\mu$  and  $\eta$  is,

$$\mu = \frac{v_0 - v}{v} = \eta - 1$$

The following table summarizes these volumetric parameters.

VARIABLES	COMPRESSION	NO LOAD	EXPANSION
$v_r = \frac{v}{v_0} = \frac{\rho_0}{\rho}$	< 1	1	> 1
$\eta = \frac{1}{v_r} = \frac{v_0}{v} = \frac{\rho}{\rho_0}$	> 1	1	< 1
$\mu = \frac{1}{v_r} - 1 = \eta - 1$	> 0	0	< 0

### V0 – Initial Relative Volume

There are 3 definitions of density that must be distinguished from each other:

$$\rho_0 = \rho_{\text{ref}}$$

= Density at nominal/reference state, usually non-stress or non-deformed state.

$$\rho|_{t=0} = \text{Density at time 0}$$

$$\rho = \text{Current density}$$

Recalling the current relative volume

$$v_r = \frac{\rho_0}{\rho} = \frac{v}{v_0},$$

at time = 0 the relative volume is

$$v_{r0} = v_r|_{t=0} = \frac{\rho_0}{\rho|_{t=0}} = \frac{v|_{t=0}}{v_0}.$$

Generally, the  $V_0$  input parameter in an \*EOS card refers to this  $v_{r0}$ .  $\rho_0$  is generally the density defined in the \*MAT card. Hence, if a material is mechanically compressed at  $t = 0$ ,  $V_0$ , or  $v_{r0}$ , the initial relative volume, may be computed and input accordingly ( $v_0 \neq V_0$ ).

The “reference” state is a unique state with respect to which the material stress tensor is computed. Therefore  $v_0$  is very critical in computing the pressure level in a material. Incorrect choice of  $v_0$  would lead to incorrect pressure computed. In general,  $v_0$  is chosen such that at zero compression or expansion, the material should be in equilibrium with its ambient surrounding. In many of the equations shown in the EOS section,  $\mu$  is frequently used as a measure of compression (or expansion). However, the users must clearly distinguish between  $\mu$  and  $v_{r0}$ .

## E0 – Internal Energy

Internal energy represents the thermal energy state (temperature dependent component) of a system. One definition for internal energy is

$$E = MC_v T \approx (\text{Joule})$$

Note that the capital “E” here is the absolute internal energy. It is not the same as that used in the subsequent \*EOS keyword input, or some equations shown for each \*EOS card. This internal energy is often defined with respect to a mass or volume unit.

Internal energy per unit mass (also called specific internal energy):

$$e = \frac{E}{M} = C_v T \approx \left( \frac{\text{Joule}}{\text{Kg}} \right)$$

Internal energy per unit current volume:

$$e_V = \frac{M}{V} C_v T = \rho C_v T = \frac{C_v T}{v} \approx \left( \frac{\text{Joule}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2} \right)$$

Internal energy per unit reference volume:

$$e_{V_0} = \frac{M}{V_0} C_v T = \rho_0 C_v T = \frac{C_v T}{v_0} \approx \left( \frac{\text{Joule}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2} \right)$$

$e_{V_0}$  typically refers to the capital “E” shown in some equations under this “EOS” section. Hence the initial “*internal energy per unit reference volume*”, E0, a keyword input parameter in the \*EOS section can be computed from

$$e_{V_0}|_{t=0} = \rho_0 C_v T|_{t=0}$$

To convert from  $e_{V_0}$  to  $e_V$ , simply divide  $e_{V_0}$  by  $v_r$

$$e_V = \rho C_v T = [\rho_0 C_v T] \frac{\rho}{\rho_0} = \frac{e_{V_0}}{v_r}$$

# \*EOS

---

## Equations of States (EOS)

A thermodynamic state of a homogeneous material, not undergoing any chemical reactions or phase changes, may be defined by two state variables. This relation is generally called an equation of state. For example, a few possible forms relating pressure to two other state variables are

$$P = P(\rho, T) = P(v, e) = P(v_r, e_V) = P(\mu, e_{V_0})$$

The last equation form is frequently used to compute pressure. The EOS for solid phase materials is sometimes partitioned into 2 terms, a cold pressure and a thermal pressure

$$P = P_c(\mu) + P_T(\mu, e_{V_0})$$

$P_c(\mu)$  is the cold pressure hypothetically evaluated along a 0-degree-Kelvin isotherm. This is sometimes called a 0-K pressure-volume relation or cold compression curve.  $P_T(\mu, e_{V_0})$  is the thermal pressure component that depends on both volumetric compression and thermal state of the material.

Different forms of the EOS describe different types of materials and how their volumetric compression (or expansion) behaviors. The coefficients for each EOS model come from data-fitting, phenomenological descriptions, or derivations based on classical thermodynamics, etc.

## Linear Compression

In low pressure processes, pressure is not significantly affected by temperature. When volumetric compression is within an elastic linear deformation range, a linear bulk modulus may be used to relate volume changes to pressure changes. Recalling the definition of an isotropic bulk modulus is [Fung 1965],

$$\frac{\Delta v}{v} = -\frac{P}{K}$$

This may be rewritten as

$$P = K \left[ -\frac{\Delta v}{v} \right] = K\mu.$$

The bulk modulus,  $K$ , thus is equivalent to  $C_1$  in \*EOS\_LINEAR\_POLYNOMIAL when all other coefficients are zero. This is a simplest form of an EOS. To initialize a pressure for such a material, only  $v_{r0}$  must be defined.

## Initial Conditions

In general, a thermodynamic state must be defined by two state variables. The need to specify  $v_{r0}$  and/or  $e_{V_0}|_{t=0}$  depends on the form of the EOS chosen. The user should review the equation term-by-term to establish what parameters to be initialized.



For many of the EOS available, pressure is specified (given), and the user must make an assumption on either  $e_{V_0}|_{t=0}$  or  $v_{r_0}$ . Consider two possibilities (1)  $T|_{t=0}$  is defined or assumed from which  $e_{V_0}|_{t=0}$  may be computed, or (2)  $\rho|_{t=0}$  is defined or assumed from which  $v_{r_0}$  may be obtained.

### When to Use EOS

For small strains considerations, a total stress tensor may be partitioned into a deviatoric stress component and a mechanical pressure.

$$\sigma_{ij} = \sigma'_{ij} + \frac{\sigma_{kk}}{3} \delta_{ij} = \sigma'_{ij} - P \delta_{ij}$$

$$P = -\frac{\sigma_{kk}}{3}$$

The pressure component may be written from the diagonal stress components.

Note that  $\frac{\sigma_{kk}}{3} = \frac{[\sigma_{11} + \sigma_{22} + \sigma_{33}]}{3}$  is positive in tension while P is positive in compression.

Similarly, the total strain tensor may be partitioned into a deviatoric strain component (volume-preserving deformation) and a volumetric deformation.

$$\varepsilon_{ij} = \varepsilon'_{ij} + \frac{\varepsilon_{kk}}{3} \delta_{ij}$$

where  $\frac{\varepsilon_{kk}}{3}$  is called the mean normal strain, and  $\varepsilon_{kk}$  is called the dilatation or volume strain (change in volume per unit initial volume)

$$\varepsilon_{kk} = \frac{V - V_0}{V_0}$$

Roughly speaking, a typical convention may refer to the relation  $\sigma'_{ij} = f(\varepsilon'_{ij})$  as a “constitutive equation”, and  $P = f(\mu, e_{V_0})$  as an EOS. The use of an EOS may be omitted only when volumetric deformation is very small, and  $|P| \ll |\sigma'_{ij}|$ .

### A Note About Contact When Using an Equation of State

When a part includes an equation of state, it is important that the initial geometry of that part not be perturbed by the contact algorithm. Such perturbation can arise due to initial penetrations in the contact surfaces but can usually be avoided by setting the variable IGNORE to 1 or 2 in the \*CONTACT input or by using a segment based contact (SOFT = 2).

# \*EOS

# \*EOS\_LINEAR\_POLYNOMIAL

## \*EOS\_LINEAR\_POLYNOMIAL

This is Equation of state Form 1.

Purpose: Define coefficients for a linear polynomial EOS, and initialize the thermodynamic state of the material by defining E0 and V0 below.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	C0	C1	C2	C3	C4	C5	C6
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	E0	V0						
Type	F	F						

### VARIABLE

### DESCRIPTION

EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
C0	The 0 <sup>th</sup> polynomial equation coefficient.
C1	The 1 <sup>st</sup> polynomial equation coefficient (when used by itself, this is the <b>elastic</b> bulk modulus, i.e. it cannot be used for deformation that is beyond the elastic regime).
⋮	⋮
C6	The 6 <sup>th</sup> polynomial equation coefficient.
E0	Initial internal energy per unit reference volume (see the beginning of the *EOS section).
V0	Initial relative volume (see the beginning of the *EOS section).

**Remarks:**

1. The linear polynomial equation of state is linear in internal energy. The pressure is given by:

$$P = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 + (C_4 + C_5\mu + C_6\mu^2)E.$$

where terms  $C_2\mu^2$  and  $C_6\mu^2$  are set to zero if  $\mu < 0$ ,  $\mu = \frac{\rho}{\rho_0} - 1$ , and  $\frac{\rho}{\rho_0}$  is the ratio of current density to reference density.  $\rho$  is a nominal or reference density defined in the \*MAT\_NULL card.

The linear polynomial equation of state may be used to model gas with the gamma law equation of state. This may be achieved by setting:

$$C_0 = C_1 = C_2 = C_3 = C_6 = 0$$

and

$$C_4 = C_5 = \gamma - 1$$

where

$$\gamma = \frac{C_p}{C_v}$$

is the ratio of specific heats. Pressure for a perfect gas is then given by:

$$p = (\gamma - 1) \frac{\rho}{\rho_0} E$$

E has the unit of pressure (where  $\rho$  and  $\rho_0$ )

2. When  $C_0 = C_1 = C_2 = C_3 = C_6 = 0$ , it does not necessarily mean that the initial pressure is zero,  $P_0 \neq C_0$ ! The initial pressure depends the values of all the coefficients and on  $\mu|_{t=0}$  and  $E|_{t=0}$ . The pressure in a material is computed from the whole equation above,  $P = P(\mu, E)$ . It is always preferable to initialize the initial condition based on  $\mu|_{t=0}$  and  $E|_{t=0}$ . The use of  $C_0 = C_1 = C_2 = C_3 = C_6 = 0$  must be done with caution as it may change the form and behavior of the material. The safest way is to use the whole EOS equation to manually check for the pressure value. For example, for ideal gas, for ideal gas, only  $C_4$  and  $C_5$  are nonzero,  $C_4 = C_5 = \gamma - 1$  and all other coefficients  $C_0 = C_1 = C_2 = C_3 = C_6 = 0$  to satisfy the perfect gas equation form.
3. V0 and E0 defined in this card must be the same as the time-zero ordinates for the 2 load curves defined in the \*BOUNDARY\_AMBIENT\_EOS card, if it is used. This is so that they would both consistently define the same initial state for a material.

**\*EOS\_JWL\_{OPTION}**

This is Equation of state Form 2.

Available options are:

<BLANK>

AFTERBURN

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A	B	R1	R2	OMEG	E0	V0
Type	A8	F	F	F	F	F	F	F

**Afterburn card.** Additional card for afterburn option with OPT = 1 or 2.

Card 2	1	2	3	4	5	6	7	8
Variable	OPT	QT	T1	T2				
Type	F	F	F	F				

**Afterburn card.** Additional card for afterburn option with OPT = 3.

Card 2	1	2	3	4	5	6	7	8
Variable	OPT	Q0	QA	QM	QN	CONM	CONL	CONT
Type	F	F	F	F	F	F	F	F
Default	none	none	none	0.5	1/6	1.	1.	1.

**VARIABLE**

**DESCRIPTION**

EOSID

Equation of state ID, a unique number or label not exceeding 8 characters must be specified.

VARIABLE	DESCRIPTION
A	$A$ , See equation in Remarks.
B	$B$ , See equation in Remarks.
R1	$R_1$ , See equation in Remarks.
R2	$R_2$ , See equation in Remarks.
OMEG	$\omega$ , See equation in Remarks.
E0	Detonation energy per unit volume and initial value for $E$ . See equation in Remarks.
V0	Initial relative volume, which gives the initial value for $V$ . See equation in Remarks.
OPT	Afterburn option EQ.0.0: No afterburn energy (Standard EOS_JWL) EQ.1.0: Constant rate of afterburn energy added between times T1 and T2 EQ.2.0: Linearly-increasing rate of afterburn energy added between times T1 and T2 EQ.3.0: Miller's extension for afterburn energy
QT	Afterburn energy per unit volume for simple afterburn (OPT = 1,2)
T1	Start time of energy addition for simple afterburn
T2	End time of energy addition for simple afterburn
Q0	Afterburn energy per unit volume for Miller's extension (OPT = 3)
QA	Energy release constant $a$ for Miller's extension
QM	Energy release exponent $m$ for Miller's extension
QN	Pressure exponent $n$ for Miller's extension

<b>VARIABLE</b>	<b>DESCRIPTION</b>
CONM	GT.0.0: Mass conversion factor from model units to calibration units for Miller's extension LT.0.0: Use predefined factors to convert model units to published calibration units of g, cm, $\mu$ s. Choices for model units are: EQ.-1.0: g, mm, ms EQ.-2.0: g, cm, ms EQ.-3.0: kg, m, s EQ.-4.0: kg, mm, ms EQ.-5.0: metric ton, mm, s EQ.-6.0: lbf-s <sup>2</sup> /in, in, s EQ.-7.0: slug, ft, s
CONL	CONM.GT.0.0: Length conversion factor from model units to calibration units for Miller's extension CONM.LT.0.0: Ignored
CONT	CONM.GT.0.0: Time conversion factor from model units to calibration units for Miller's extension CONM.LT.0.0: Ignored

**Remarks:**

The JWL equation of state defines the pressure as

$$p = A \left( 1 - \frac{\omega}{R_1 V} \right) e^{-R_1 V} + B \left( 1 - \frac{\omega}{R_2 V} \right) e^{-R_2 V} + \frac{\omega E}{V},$$

and is usually used for detonation products of high explosives.

A, B, and E0 have units of pressure. R1, R2, OMEG, and V0 are dimensionless. It is recommended that a unit system of gram, centimeter, microsecond be used when a model includes high explosive(s). In this consistent unit system, pressure is in Mbar.

When this equation of state is used with \*MAT\_HIGH\_EXPLOSIVE\_BURN in which the variable BETA is set to 0 or 2, the absolute value of the history variable labeled as "effective plastic strain" is the explosive lighting time. This lighting time takes into account shadowing if invoked (see \*CONTROL\_EXPLOSIVE\_SHADOW).

There are four additional history variables for the JWL equation of state. Those history variables are internal energy, bulk viscosity in units of pressure, volume, and burn fraction, respectively. To output the history variables, set the variable NEIPH in \*DATABASE\_EXTENT\_BINARY.

The AFTERBURN option allows the addition of afterburn energy  $Q$  to the calculation of pressure by replacing  $E$  in the above equation with  $(E + Q)$ , i.e. the last term on the right-hand side becomes

$$\frac{\omega(E + Q)}{V}$$

The simple afterburn option adds the energy at a constant rate (OPT = 1) or a linearly-increasing rate (OPT = 2) between times T1 and T2 such that the total energy added per unit volume at time T2 is the specified energy QT.

For the Miller's extension model (OPT = 3), the afterburn energy is added via a time-dependent growth term

$$\frac{d\lambda}{dt} = a(1 - \lambda)^m p^n, \quad Q = \lambda Q_0$$

Here,  $m$ ,  $n$ , and  $\lambda$  are dimensionless, with  $\lambda$  a positive fraction less than 1.0. The parameter  $a$  has units consistent with this growth equation, and  $Q_0$  has units of pressure.

The values for  $Q_0$ ,  $a$ ,  $m$ ,  $n$  published by Miller and Guirguis (1993) are calibrated in the units of g, cm,  $\mu$ s, with the consistent pressure unit of Mbar, though in principle any consistent set of units may be used for calibration. The factors CONM, CONL, and CONT convert the unit system of the model being analyzed to the calibration unit system in which the Miller's extension parameters are specified, e.g. a mass value in model units may be multiplied by CONM to obtain the corresponding value in calibration units. These conversion factors allow consistent evaluation of the growth equation in the calibrated units. For user convenience, predefined conversion factors are provided for converting various choices for the model units system to the calibration unit system used by Miller and Guirguis.

The AFTERBURN option introduces an additional 5th history variable that records the added afterburn energy  $Q$  for simple afterburn (OPT = 1,2), but contains the growth term  $\lambda$  when using the Miller's extension model (OPT = 3).

**\*EOS\_SACK\_TUESDAY**

This is Equation of state Form 3.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A1	A2	A3	B1	B2	E0	V0
Type	A8	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
$A_i, B_i$	Constants in the equation of state
E0	Initial internal energy
V0	Initial relative volume

**Remarks:**

The Sack equation of state defines pressure as

$$p = \frac{A_3}{V^{A_1}} e^{-A_2 V} \left( 1 - \frac{B_1}{V} \right) + \frac{B_2}{V} E$$

and is used for detonation products of high explosives.



## \*EOS\_GRUNEISEN

This is Equation of state Form 4.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	C	S1	S2	S3	GAMMA0	A	E0
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	V0							
Type	F							

VARIABLE	DESCRIPTION
EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
C, Si, GAMMA0	Constants in the equation of state
A	First order volume correction coefficient
E0	Initial internal energy
V0	Initial relative volume

**Remarks:**

The Gruneisen equation of state with cubic shock-velocity as a function of particle-velocity  $v_s(v_p)$  defines pressure for compressed materials as

$$p = \frac{\rho_0 C^2 \mu \left[ 1 + \left( 1 - \frac{\gamma_0}{2} \right) \mu - \frac{a}{2} \mu^2 \right]}{\left[ 1 - (S_1 - 1) \mu - S_2 \frac{\mu^2}{\mu + 1} - S_3 \frac{\mu^3}{(\mu + 1)^2} \right]^2} + (\gamma_0 + a\mu)E.$$

and for expanded materials as

$$p = \rho_0 C^2 \mu + (\gamma_0 + a\mu)E.$$

where  $C$  is the intercept of the  $v_s(v_p)$  curve (in velocity units);  $S_1$ ,  $S_2$ , and  $S_3$  are the unitless coefficients of the slope of the  $v_s(v_p)$  curve;  $\gamma_0$  is the unitless Gruneisen gamma;  $a$  is the unitless, first order volume correction to  $\gamma_0$ ; and

$$\mu = \frac{\rho}{\rho_0} - 1.$$

\*EOS\_RATIO\_OF\_POLYNOMIALS

This is Equation of state Form 5.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID							
Type	A8							

Card 2	1	2	3	4	5	6	7	8
Variable	A10		A11		A12		A13	
Type	F		F		F		F	

Card 3	1	2	3	4	5	6	7	8
Variable	A20		A21		A22		A23	
Type	F		F		F		F	

Card 4	1	2	3	4	5	6	7	8
Variable	A30		A31		A32		A33	
Type	F		F		F		F	

Card 5	1	2	3	4	5	6	7	8
Variable	A40		A41		A42		A43	
Type	F		F		F		F	

**\*EOS****\*EOS\_RATIO\_OF\_POLYNOMIALS**

Card 6	1	2	3	4	5	6	7	8
Variable	A50		A51		A52		A53	
Type	F		F		F		F	

Card 7	1	2	3	4	5	6	7	8
Variable	A60		A61		A62		A63	
Type	F		F		F		F	

Card 8	1	2	3	4	5	6	7	8
Variable	A70		A71		A72		A73	
Type	F		F		F		F	

Card 9	1	2	3	4	5	6	7	8
Variable	A14		A24					
Type	F		F					

Card 10	1	2	3	4	5	6	7	8
Variable	ALPH		BETA		E0		V0	
Type	F		F		F		F	

**VARIABLE****DESCRIPTION**

EOSID

Equation of state ID, a unique number or label not exceeding 8 characters must be specified.

 $A_{ij}$ 

Polynomial coefficients

VARIABLE	DESCRIPTION
ALPHA	$\alpha$
BETA	$\beta$
E0	Initial internal energy
V0	Initial relative volume

**Remarks:**

The ratio of polynomials equation of state defines the pressure as

$$p = \frac{F_1 + F_2E + F_3E^2 + F_4E^3}{F_5 + F_6E + F_7E^2} (1 + \alpha\mu)$$

where

$$F_i = \sum_{j=0}^n A_{ij} \mu^j, \quad n = \begin{cases} 4 & i < 3 \\ 3 & i \geq 3 \end{cases}$$

$$\mu = \frac{\rho}{\rho_0} - 1$$

In expanded elements  $F_1$  is replaced by  $F'_1 = F_1 + \beta\mu^2$ . By setting coefficient  $A_{10} = 1.0$ , the delta-phase pressure modeling for this material will be initiated. The code will reset it to 0.0 after setting flags.

# \*EOS

## \*EOS\_LINEAR\_POLYNOMIAL\_WITH\_ENERGY\_LEAK

### \*EOS\_LINEAR\_POLYNOMIAL\_WITH\_ENERGY\_LEAK

This is Equation of state Form 6.

Purpose: Define coefficients for a linear polynomial EOS, and initialize the thermodynamic state of the material by defining E0 and V0 below. Energy deposition is prescribed via a curve.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	C0	C1	C2	C3	C4	C5	C6
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	E0	V0	LCID					
Type	F	F	I					

#### VARIABLE

#### DESCRIPTION

EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
$C_i$	Constants in the equation of state
E0	Initial internal energy
V0	Initial relative volume
LCID	Load curve ID defining the energy deposition rate.

#### Remarks:

This polynomial equation of state, linear in the internal energy per initial volume,  $E$ , is given by

$$p = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 + (C_4 + C_5\mu + C_6\mu^2)E$$

in which  $C_1, C_2, C_3, C_4, C_5,$  and  $C_6$  are user defined constants and

$$\mu = \frac{1}{V} - 1 .$$

where  $V$  is the relative volume. In expanded elements, we set the coefficients of  $\mu^2$  to zero, i.e.,

$$C_2 = C_6 = 0$$

Internal energy,  $E$ , is increased according to an energy deposition rate versus time curve whose ID is defined in the input.

**\*EOS****\*EOS\_IGNITION\_AND\_GROWTH\_OF\_REACTION\_IN\_HE****\*EOS\_IGNITION\_AND\_GROWTH\_OF\_REACTION\_IN\_HE**

This is Equation of state Form 7.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A	B	XP1	XP2	FRER	G	R1
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	R2	R3	R5	R6	FMXIG	FREQ	GROW1	EM
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	AR1	ES1	CVP	CVR	EETAL	CCRIT	ENQ	TMP0
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	GROW2	AR2	ES2	EN	FMXGR	FMNGR		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
A	Product JWL constant (see second equation in Remarks)
B	Product JWL constant (see second equation in Remarks)
XP1	Product JWL constant (see second equation in Remarks)



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<b>VARIABLE</b>	<b>DESCRIPTION</b>
XP2	Product JWL constant (see second equation in Remarks)
FRER	Constant in ignition term of reaction equation
G	$\omega C_v$ of product
R1	Unreacted JWL constant (see first equation in Remarks)
R2	Unreacted JWL constant (see first equation in Remarks)
R3	$\omega C_v$ of unreacted explosive
R5	Unreacted JWL constant (see first equation in Remarks)
R6	Unreacted JWL constant (see first equation in Remarks)
FMXIG	Maximum F for ignition term
FREQ	Constant in ignition term of reaction equation
GROW1	Constant in growth term of reaction equation
EM	Constant in growth term of reaction equation
AR1	Constant in growth term of reaction equation
ES1	Constant in growth term of reaction equation
CVP	Heat capacity of reaction products
CVR	Heat capacity of unreacted HE
EETAL	Constant in ignition term of reaction equation
CCRIT	Constant in ignition term of reaction equation
ENQ	Heat of reaction
TMP0	Initial temperature (°K)
GROW2	Constant in completion term of reaction equation
AR2	Constant in completion term of reaction equation
ES2	Constant in completion term of reaction equation
EN	Constant in completion term of reaction equation

VARIABLE	DESCRIPTION
FMXGR	Maximum F for growth term
FMNGR	Maximum F for completion term

**Remarks:**

Equation of State Form 7 is used to calculate the shock initiation (or failure to initiate) and detonation wave propagation of solid high explosives. It should be used instead of the ideal HE burn options whenever there is a question whether the HE will react, there is a finite time required for a shock wave to build up to detonation, and/or there is a finite thickness of the chemical reaction zone in a detonation wave. At relatively low initial pressures (<2-3 GPa), this equation of state should be used with material type 10 for accurate calculations of the unreacted HE behavior. At higher initial pressures, material type 9 can be used. A JWL equation of state defines the pressure in the unreacted explosive as

$$P_e = r_1 e^{-r_5 V_e} + r_2 e^{-r_6 V_e} + r_3 \frac{T_e}{V_e}, \quad (r_3 = \omega_e C_{v_r})$$

where  $V_e$  and  $T_e$  are the relative volume and temperature, respectively, of the unreacted explosive. Another JWL equation of state defines the pressure in the reaction products as

$$P_p = a e^{-x p_1 V_p} + b e^{-x p_2 V_p} + \frac{g T_p}{V_p}, \quad (g = \omega_p C_{v_p})$$

where  $V_p$  and  $T_p$  are the relative volume and temperature, respectively, of the reaction products. As the chemical reaction converts unreacted explosive to reaction products, these JWL equations of state are used to calculate the mixture of unreacted explosive and reaction products defined by the fraction reacted  $F$  ( $F = 0$  implies no reaction,  $F = 1$  implies complete reaction). The temperatures and pressures are assumed to be equal ( $T_e = T_p$ ,  $p_e = p_p$ ) and the relative volumes are additive, i.e.,

$$V = (1 - F)V_e + FV_p$$

The chemical reaction rate for conversion of unreacted explosive to reaction products consists of three physically realistic terms: an ignition term in which a small amount of explosive reacts soon after the shock wave compresses it; a slow growth of reaction as this initial reaction spreads; and a rapid completion of reaction at high pressure and temperature. The form of the reaction rate equation is

$$\frac{\partial F}{\partial t} = \underbrace{\text{FREQ} \times (1 - F)^{\text{FRER}} (V_e^{-1} - 1 - \text{CCRIT})^{\text{EETAL}}}_{\text{Ignition}} + \underbrace{\text{GROW1} \times (1 - F)^{\text{ES1}} F^{\text{AR1}} p^{\text{EM}}}_{\text{Growth}} + \underbrace{\text{GROW2} \times (1 - F)^{\text{ES2}} f^{\text{AR2}} p^{\text{EN}}}_{\text{Completion}}$$

The ignition rate is set equal to zero when  $F \geq \text{FMXIG}$ , the growth rate is set equal to zero when  $F \geq \text{FMXGR}$ , and the completion rate is set equal to zero when  $F \leq \text{FMNGR}$ .

Details of the computational methods and many examples of one and two dimensional shock initiation and detonation wave calculation can be found in the references (Cochran and Chan [1979], Lee and Tarver [1980]). Unfortunately, sufficient experimental data has been obtained for only two solid explosives to develop very reliable shock initiation models: PBX-9504 (and the related HMX-based explosives LX-14,LX-10,LX-04, etc.) and LX-17 (the insensitive TATB-based explosive). Reactive flow models have been developed for other explosives (TNT, PETN, Composition B, propellants, etc.) but are based on very limited experimental data.

When this EOS is used with \*MAT\_009, history variables 4, 7, 9, and 10 are temperature, burn fraction,  $1/V_e$ , and  $1/V_p$ , respectively. When used with \*MAT\_010, those history variables are incremented by 1, i.e., history variables 5, 8, 10, and 11 are temperature, burn fraction,  $1/V_e$ , and  $1/V_p$ , respectively. See NEIPH in \*DATABASE\_EXTENT\_BINARY if these output variables are desired in the databases for post-processing.

**\*EOS\_TABULATED\_COMPACTION**

This is Equation of state Form 8.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	GAMA	E0	V0	LCC	LCT	LCK	
Type	A8	F	F	F	I	I	I	

**Parameter Card Pairs.** Include one pair of the following two cards for each of  $VAR = \varepsilon_{v_i}$ ,  $C_i$ ,  $T_i$ , and  $K_i$ . These cards consist of four additional pairs for a total of 8 additional cards.

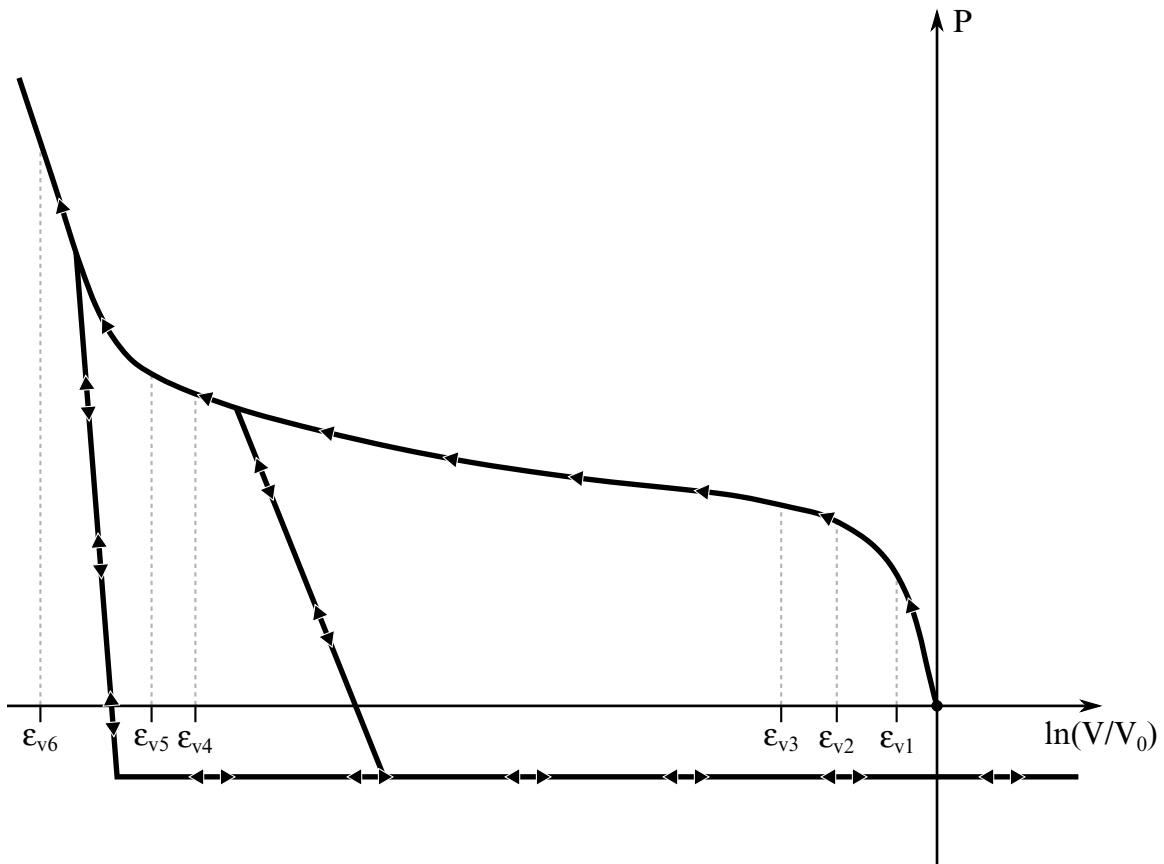
Card 3	1	2	3	4	5	6	7	8	9	10
Variable	[VAR]1		[VAR]2		[VAR]3		[VAR]4		[VAR]5	
Type	F		F		F		F		F	

Card 4	1	2	3	4	5	6	7	8	9	10
Variable	[VAR]6		[VAR]7		[VAR]8		[VAR]9		[VAR]10	
Type	F		F		F		F		F	

**VARIABLE****DESCRIPTION**

EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
GAMA	$\gamma$ , (unitless), see equation in Remarks.
E0	Initial internal energy per unit reference volume (force per unit area).
V0	Initial relative volume (unitless).

VARIABLE	DESCRIPTION
LCC	Load curve defining tabulated function $C$ . See equation in Remarks. The abscissa values of LCC, LCT and LCK must be negative of the volumetric strain in monotonically <i>increasing</i> order, in contrast to the convention in EOS_9. The definition can extend into the tensile regime.
LCT	Load curve defining tabulated function $T$ . See equation in Remarks.
LCK	Load curve defining tabulated bulk modulus.
$\varepsilon_{v1}, \varepsilon_{v2}, \dots, \varepsilon_{vN}$	Volumetric strain, $\ln(V)$ . The first abscissa point, EV1, must be 0.0 or positive if the curve extends into the tensile regime with subsequent points <i>decreasing</i> monotonically.
$C_1, C_2, \dots, C_N$	$C(\varepsilon_V)$ , (units = force per unit area), see equation in Remarks.
$T_1, T_2, \dots, T_N$	$T(\varepsilon_V)$ , (unitless), see equation in Remarks.
$K_1, K_2, \dots, K_N$	Bulk unloading modulus (units = force per unit area).



**Figure EOS8-1.** Pressure versus volumetric strain curve for Equation of state Form 8 with compaction. In the compacted states the bulk unloading modulus depends on the peak volumetric strain. Volumetric strain values should be input with correct sign (negative in compression) and in descending order. Pressure is positive in compression.

VARIABLE	DESCRIPTION
----------	-------------

**Remarks:**

The tabulated compaction model is linear in internal energy. Pressure is defined by

$$p = C(\epsilon_V) + \gamma T(\epsilon_V)E$$

in the loading phase. The volumetric strain,  $\epsilon_V$  is given by the natural logarithm of the relative volume  $V$ . Unloading occurs along the unloading bulk modulus to the pressure cutoff. The pressure cutoff, a tension limit, is defined in the material model definition. Reloading always follows the unloading path to the point where unloading began, and continues on the loading path, see [Figure EOS8-1](#). Up to 10 points and as few as 2 may be used when defining the tabulated functions. LS-DYNA will extrapolate to find the pressure if necessary.

## \*EOS\_TABULATED

This is Equation of state Form 9.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	GAMA	E0	V0	LCC	LCT		
Type	A8	F	F	F	I	I		

**Parameter Card Pairs.** Include one pair of the following two cards for each of  $VAR = \varepsilon_{V_i}, C_i, T_i$ . These cards consist of three additional pairs for a total of 6 additional cards. These cards are not required if LCC and LCT are specified.

Card 2	1	2	3	4	5	6	7	8	9	10
Variable	[VAR]1		[VAR]2		[VAR]3		[VAR]4		[VAR]5	
Type	F		F		F		F		F	

Card 3	1	2	3	4	5	6	7	8	9	10
Variable	[VAR]6		[VAR]7		[VAR]8		[VAR]9		[VAR]10	
Type	F		F		F		F		F	

**VARIABLE****DESCRIPTION**

EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
GAMA	$\gamma$ , (unitless) see equation in Remarks.
E0	Initial internal energy per unit reference volume (force per unit area).
V0	Initial relative volume (unitless).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCC	Load curve defining tabulated function $C$ . See equation in Remarks. The abscissa values of LCC and LCT must <i>increase</i> monotonically. The definition can extend into the tensile regime.
LCT	Load curve defining tabulated function $T$ . See equation in Remarks.
$\varepsilon_{V_1}, \varepsilon_{V_2}, \dots, \varepsilon_{V_N}$	Volumetric strain, $\ln(V)$ , where $V$ is the relative volume. The first abscissa point, EV1, must be 0.0 or positive if the curve extends into the tensile regime with subsequent points <i>decreasing</i> monotonically.
$C_1, C_2, \dots, C_N$	Tabulated points for function $C$ (force per unit area).
$T_1, T_2, \dots, T_N$	Tabulated points for function $T$ (unitless).

**Remarks:**

The tabulated equation of state model is linear in internal energy. Pressure is defined by

$$P = C(\varepsilon_V) + \gamma T(\varepsilon_V)E$$

The volumetric strain,  $\varepsilon_V$  is given by the natural logarithm of the relative volume  $V$ . Up to 10 points and as few as 2 may be used when defining the tabulated functions. LS-DYNA will extrapolate to find the pressure if necessary.



\*EOS\_PROPELLANT\_DEFLAGRATION

This Equation of state (10) has been added to model airbag propellants.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A	B	XP1	XP2	FRER		
Type	A8	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	G	R1	R2	R3	R5			
Type	F	F	F	F	F			

Card 3	1	2	3	4	5	6	7	8
Variable	R6	FMXIG	FREQ	GROW1	EM			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable	AR1	ES1	CVP	CVR	EETAL	CCRIT	ENQ	TMP0
Type	F	F	F	F	F			

Card 5	1	2	3	4	5	6	7	8
Variable	GROW2	AR2	ES2	EN	FMXGR	FMNGR		
Type	F	F	F	F	F	F		

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
A	Product JWL coefficient
B	Product JWL coefficient
XP1	Product JWL coefficient
XP2	Product JWL coefficient
FRER	Unreacted Co-volume
G	Product $\omega C_v$
R1	Unreacted JWL coefficient
R2	Unreacted JWL coefficient
R3	Unreacted $\omega C_v$
R5	Unreacted JWL coefficient
R6	Unreacted JWL coefficient
FMXIG	Initial Fraction Reacted $F_0$
FREQ	Initial Pressure $P_0$
GROW1	First burn rate coefficient
EM	Pressure Exponent (1 <sup>st</sup> term)
AR1	Exponent on $F$ (1 <sup>st</sup> term)
ES1	Exponent on $(1 - F)$ (1 <sup>st</sup> term)
CVP	Heat capacity $C_v$ for products
CVR	Heat capacity $C_v$ for unreacted material
EETAL	Extra, not presently used
CCRIT	Product co-volume
ENQ	Heat of Reaction

VARIABLE	DESCRIPTION
TMP0	Initial Temperature (298°K)
GROW2	Second burn rate coefficient
AR2	Exponent on $F$ (2 <sup>nd</sup> term)
ES2	Exponent on $(1 - F)$ (2 <sup>nd</sup> term)
EN	Pressure Exponent (2 <sup>nd</sup> term)
FMXGR	Maximum $F$ for 1 <sup>st</sup> term
FMNGR	Minimum $F$ for 2 <sup>nd</sup> term

### Remarks:

A deflagration (burn rate) reactive flow model requires an unreacted solid equation of state, a reaction product equation of state, a reaction rate law and a mixture rule for the two (or more) species. The mixture rule for the standard ignition and growth model [Lee and Tarver 1980] assumes that both pressures and temperatures are completely equilibrated as the reaction proceeds. However, the mixture rule can be modified to allow no thermal conduction or partial heating of the solid by the reaction product gases. For this relatively slow process of airbag propellant burn, the thermal and pressure equilibrium assumptions are valid. The equations of state currently used in the burn model are the JWL, Gruneisen, the van der Waals co-volume, and the perfect gas law, but other equations of state can be easily implemented. In this propellant burn, the gaseous nitrogen produced by the burning sodium azide obeys the perfect gas law as it fills the airbag but may have to be modeled as a van der Waal's gas at the high pressures and temperatures produced in the propellant chamber. The chemical reaction rate law is pressure, particle geometry and surface area dependent, as are most high-pressure burn processes. When the temperature profile of the reacting system is well known, temperature dependent Arrhenius chemical kinetics can be used.

Since the airbag propellant composition and performance data are company private information, it is very difficult to obtain the required information for burn rate modeling. However, Imperial Chemical Industries (ICI) Corporation supplied pressure exponent, particle geometry, packing density, heat of reaction, and atmospheric pressure burn rate data which allowed us to develop the numerical model presented here for their  $\text{NaN}_3 + \text{Fe}_2\text{O}_3$  driver airbag propellant. The deflagration model, its implementation, and the results for the ICI propellant are presented in [Hallquist, et.al., 1990].

The unreacted propellant and the reaction product equations of state are both of the form:

$$p = Ae^{-R_1V} + Be^{-R_2V} + \frac{\omega C_v T}{V - d}$$

where  $p$  is pressure (in Mbars),  $V$  is the relative specific volume (inverse of relative density),  $\omega$  is the Gruneisen coefficient,  $C_v$  is heat capacity (in Mbars -cc/cc°K),  $T$  is temperature in °K,  $d$  is the co-volume, and  $A, B, R_1$  and  $R_2$  are constants. Setting  $A = B = 0$  yields the van der Waal's co-volume equation of state. The JWL equation of state is generally useful at pressures above several kilobars, while the van der Waal's is useful at pressures below that range and above the range for which the perfect gas law holds. Additionally, setting  $A = B = d = 0$  yields the perfect gas law. If accurate values of  $\omega$  and  $C_v$  plus the correct distribution between "cold" compression and internal energies are used, the calculated temperatures are very reasonable and thus can be used to check propellant performance.

The reaction rate used for the propellant deflagration process is of the form:

$$\frac{\partial F}{\partial t} = \underbrace{Z(1 - F)^y F^x p^w}_{0 < F < F_{\text{limit1}}} + \underbrace{V(1 - F)^u F^r p^s}_{F_{\text{limit2}} < F < 1}$$

where  $F$  is the fraction reacted ( $F = 0$  implies no reaction,  $F = 1$  is complete reaction),  $t$  is time, and  $p$  is pressure (in Mbars),  $r, s, u, w, x, y, F_{\text{limit1}}$  and  $F_{\text{limit2}}$  are constants used to describe the pressure dependence and surface area dependence of the reaction rates. Two (or more) pressure dependant reaction rates are included in case the propellant is a mixture or exhibited a sharp change in reaction rate at some pressure or temperature. Burning surface area dependencies can be approximated using the  $(1 - F)^y F^x$  terms. Other forms of the reaction rate law, such as Arrhenius temperature dependent  $e^{-E/RT}$  type rates, can be used, but these require very accurate temperatures calculations. Although the theoretical justification of pressure dependent burn rates at kilobar type pressures is not complete, a vast amount of experimental burn rate versus pressure data does demonstrate this effect and hydrodynamic calculations using pressure dependent burn accurately simulate such experiments.

The deflagration reactive flow model is activated by any pressure or particle velocity increase on one or more zone boundaries in the reactive material. Such an increase creates pressure in those zones and the decomposition begins. If the pressure is relieved, the reaction rate decreases and can go to zero. This feature is important for short duration, partial decomposition reactions. If the pressure is maintained, the fraction reacted eventually reaches one and the material is completely converted to product molecules. The deflagration front rates of advance through the propellant calculated by this model for several propellants are quite close to the experimentally observed burn rate versus pressure curves.

To obtain good agreement with experimental deflagration data, the model requires an accurate description of the unreacted propellant equation of state, either an analytical fit to experimental compression data or an estimated fit based on previous experience with

similar materials. This is also true for the reaction products equation of state. The more experimental burn rate, pressure production and energy delivery data available, the better the form and constants in the reaction rate equation can be determined.

Therefore, the equations used in the burn subroutine for the pressure in the unreacted propellant

$$P_u = R1 \times e^{-R5 \cdot V_u} + R2 \times e^{-R6 \cdot V_u} + \frac{R3 \times T_u}{V_u - FRER}$$

where  $V_u$  and  $T_u$  are the relative volume and temperature respectively of the unreacted propellant. The relative density is obviously the inverse of the relative volume. The pressure  $P_p$  in the reaction products is given by:

$$P_p = A \times e^{-XP1 \times V_p} + B \times e^{-XP2 \times V_p} + \frac{G \times T_p}{V_p - CCRIT}$$

As the reaction proceeds, the unreacted and product pressures and temperatures are assumed to be equilibrated ( $T_u = T_p = T$ ,  $P = P_u = P_p$ ) and the relative volumes are additive:

$$V = (1 - F)V_u + FV_p$$

where  $V$  is the total relative volume. Other mixture assumptions can and have been used in different versions of DYNA2D/3D. The reaction rate law has the form:

$$\begin{aligned} \frac{\partial F}{\partial t} = & \text{GROW1} \times (P + \text{FREQ})^{\text{EM}} (F + \text{FMXIG})^{\text{AR1}} (1 - F + \text{FMIXG})^{\text{ES1}} \\ & + \text{GROW2} \times (P + \text{FREQ})^{\text{EN}} (F + \text{FMIXG})^{\text{AR2}} (1 - F + \text{FMIXG})^{\text{ES2}} \end{aligned}$$

If  $F$  exceeds  $\text{FMXGR}$ , the  $\text{GROW1}$  term is set equal to zero, and, if  $F$  is less than  $\text{FMNGR}$ , the  $\text{GROW2}$  term is zero. Thus, two separate (or overlapping) burn rates can be used to describe the rate at which the propellant decomposes.

This equation of state subroutine is used together with a material model to describe the propellant. In the airbag propellant case, a null material model (type #10) can be used. Material type #10 is usually used for a solid propellant or explosive when the shear modulus and yield strength are defined. The propellant material is defined by the material model and the unreacted equation of state until the reaction begins. The calculated mixture states are used until the reaction is complete and then the reaction product equation of state is used. The heat of reaction,  $\text{ENQ}$ , is assumed to be a constant and the same at all values of  $F$  but more complex energy release laws could be implemented.

History variables 4 and 7 are temperature and burn fraction, respectively. See  $\text{NEIPH}$  in  $\text{*DATABASE\_EXTENT\_BINARY}$  if these output variables are desired in the databases for post-processing.

**\*EOS\_TENSOR\_PORE\_COLLAPSE**

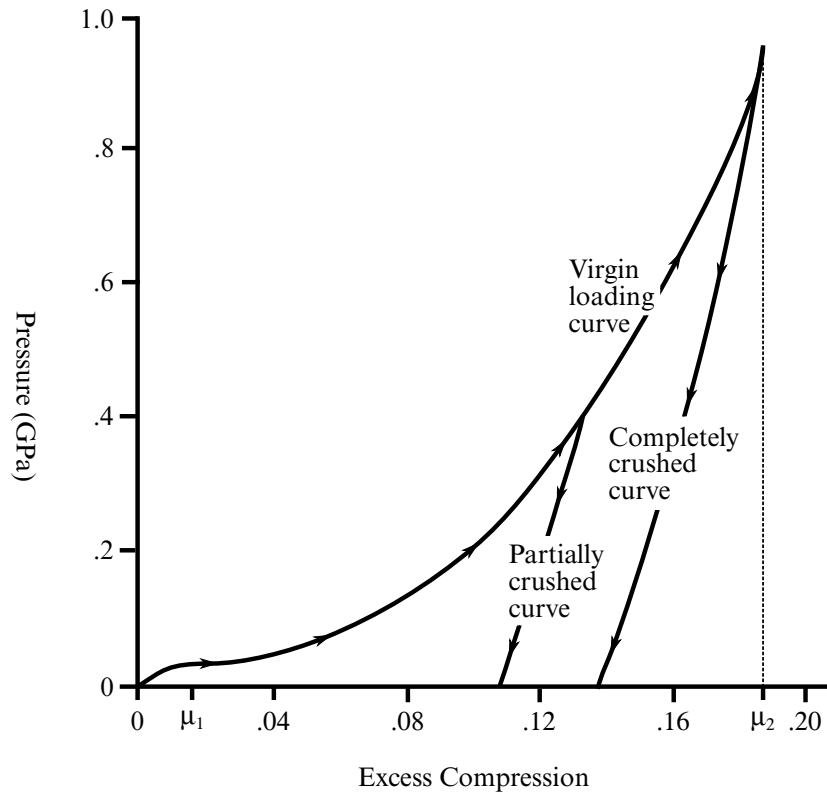
This is Equation of state Form 11.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	NLD	NCR	MU1	MU2	IE0	EC0	
Type	A8	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
NLD	Virgin loading load curve ID
NCR	Completely crushed load curve ID
MU1	Excess Compression required before any pores can collapse
MU2	Excess Compression point where the Virgin Loading Curve and the Completely Crushed Curve intersect
IE0	Initial Internal Energy
EC0	Initial Excess Compression

**Remarks:**

The pore collapse model described in the TENSOR manual [23] is no longer valid and has been replaced by a much simpler method. This is due in part to the lack of experimental data required for the more complex model. It is desired to have a close approximation of the TENSOR model in the DYNA code to enable a quality link between them. The TENSOR model defines two curves, the virgin loading curve and the completely crushed curve as shown in [Figure EOS11-1](#) also defines the excess compression point required for pore collapse to begin,  $\mu_1$ , and the excess compression point required to completely crush the material,  $\mu_2$ . From this data and the maximum excess compression the material has attained,  $u_{max}$ , the pressure for any excess compression,  $\mu$ , can be determined.



**Figure EOS11-1.** Pressure versus compaction curve

Unloading occurs along the virgin loading curve until the excess compression surpasses  $\mu_1$ . After that, the unloading follows a path between the completely crushed curve and the virgin loading curve. Reloading will follow this curve back up to the virgin loading curve. Once the excess compression exceeds  $\mu_2$ , then all unloading will follow the completely crushed curve.

For unloading between  $\mu_1$  and a partially  $\mu_2$  crushed curve is determined by the relation:

$$p_{pc}(\mu) = p_{cc} \left[ \frac{\mu_a}{1 + \mu_{max}} \frac{(1 + \mu_B)(1 + \mu)}{1 + \mu_{max}} - 1 \right]$$

where

$$\mu_B = P_{cc}^{-1}(P_{max})$$

and the subscripts “pc” and “cc” refer to the partially crushed and completely crushed states, respectively. This is more readily understood in terms of the relative volume,  $V$ .

$$V = \frac{1}{1 + \mu}$$

$$P_{pc}(V) = P_{cc} \left( \frac{V_B}{V_{min}} V \right)$$

This representation suggests that for a fixed

$$V_{\min} = \frac{1}{\mu_{\max} + 1}$$

the partially crushed curve will separate linearly from the completely crushed curve as  $V$  increases to account for pore recovery in the material.

The bulk modulus  $K$  is determined to be the slope of the current curve times one plus the excess compression

$$K = \frac{\partial P}{\partial \mu} (1 + \mu)$$

The slope  $\frac{\partial P}{\partial \mu}$  for the partially crushed curve is obtained by differentiation as:

$$\frac{\partial p_{pc}}{\partial \mu} = \frac{\partial p_{cc}}{\partial x} \bigg|_{x=\frac{(1+\mu_b)(1+\mu)}{1+\mu_{\max}}-1} \left( \frac{1 + \mu_b}{1 + \mu_{\max}} \right)$$

Simplifying,

$$K = \frac{\partial P_{cc}}{\partial \mu_a} \bigg|_{\mu_a} (1 + \mu_a)$$

where

$$\mu_a = \frac{(1 + \mu_B)(1 + \mu)}{(1 + \mu_{\max})} - 1.$$

The bulk sound speed is determined from the slope of the completely crushed curve at the current pressure to avoid instabilities in the time step.

The virgin loading and completely crushed curves are modeled with monotonic cubic-splines. An optimized vector interpolation scheme is then used to evaluate the cubic-splines. The bulk modulus and sound speed are derived from a linear interpolation on the derivatives of the cubic-splines.



**\*EOS\_IDEAL\_GAS**

Purpose: This is equation of state form 12 for modeling ideal gas. It is an alternate approach to using \*EOS\_LINEAR\_POLYNOMIAL with  $C4 = C5 = (\gamma - 1)$  to model ideal gas. This has a slightly improved energy accounting algorithm.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	CV0	CP0	CL	CQ	T0	V0	VCO
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	ADIAB							
Type	F							

**VARIABLE****DESCRIPTION**

EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
CV0	Nominal constant-volume specific heat coefficient (at STP)
CP0	Nominal constant-pressure specific heat coefficient (at STP)
CL	Linear coefficient for the variations of $C_v$ and $C_p$ versus $T$ .
CQ	Quadratic coefficient for the variations of $C_v$ and $C_p$ versus $T$ .
T0	Initial temperature
V0	Initial relative volume (see the beginning of the *EOS section)
VCO	Van der Waals covolume
ADIAB	Adiabatic flag: EQ.0.0: off EQ.1.0: on; ideal gas follows adiabatic law

**Remarks:**

1. The pressure in the ideal gas law is defined as

$$p = \rho(C_p - C_v)T$$

$$C_p = C_{p0} + C_L T + C_Q T^2$$

$$C_v = C_{v0} + C_L T + C_Q T^2$$

where  $C_p$  and  $C_v$  are the specific heat capacities at constant pressure and at constant volume, respectively.  $\rho$  is the density. The relative volume is defined as

$$v_r = \frac{V}{V_0} = \frac{(V/M)}{(V_0/M)} = \frac{v}{v_0} = \frac{\rho_0}{\rho}$$

where  $\rho_0$  is a nominal or reference density defined in the \*MAT\_NULL card. The initial pressure can then be manually computed as

$$P|_{t=0} = \rho|_{t=0}(C_P - C_V)T|_{t=0}$$

$$\rho|_{t=0} = \left\{ \frac{\rho_0}{v_r|_{t=0}} \right\}$$

$$P|_{t=0} = \left\{ \frac{\rho_0}{v_r|_{t=0}} \right\} (C_P - C_V)T|_{t=0}$$

The initial relative volume,  $v_r|_{t=0}$  (V0), initial temperature,  $T|_{t=0}$ (T0), and heat capacity information are defined in the \*EOS\_IDEAL\_GAS input. Note that the "reference" density is typically a density at a non-stressed or nominal stress state. The initial pressure should always be checked manually against simulation result.

2. With adiabatic flag on, the adiabatic state is conserved, but *exact* internal energy conservation is sacrificed.
3. The ideal gas model is good for low density gas only. Deviation from the ideal gas behavior may be indicated by the compressibility factor defined as

$$Z = \frac{Pv}{RT}$$

When  $Z$  deviates from 1, the gas behavior deviates from ideal.

4. V0 and T0 defined in this card must be the same as the time-zero ordinates for the 2 load curves defined in the \*BOUNDARY\_AMBIENT\_EOS card, if it is used. This is so that they both would consistently define the same initial state for a material.

\*EOS\_PHASE\_CHANGE

This is Equation of state Form 13. This EOS was designed for phase change from liquid to vapor, based on the homogeneous Schmidt model.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	RHOL	RHOV	CL	CV	PAMB	V0	
Type	A8	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
EOSID	Equation of state ID; a unique number or label not exceeding 8 characters must be specified.
RHOL	Density of liquid.
RHOV	Density of saturated vapor
CL	Speed of sound of liquid
CV	Speed of sound of vapor
PAMB	Ambient pressure
V0	Initial relative volume. Default = 1.

Remarks:

This model is barotropic, so the pressure is only a function of density change. Details of the model can be found in Souli et al. [2014]. Examples of applications for this model are simulations involving water hammer or fuel injection. The ambient pressure should normally be set to atmospheric pressure.

Example input for water in the MKS system (m, kg, s):

```
*EOS_PHASE_CHANGE
$ EOSID RHOL RHOV CL CV PAMB V0
1 9.9742e+2 2.095e-2 1492.00 425.00 1.e+5 0.99991
```

**\*EOS\_JWLB**

This is Equation of state Form 14. The JWLB (Jones-Wilkens-Lee-Baker) equation of state, developed by Baker [1991] and further described by Baker and Orosz [1991], describes the high pressure regime produced by overdriven detonations while retaining the low pressure expansion behavior required for standard acceleration modeling. The derived form of the equation of state is based on the JWL form due to its computational robustness and asymptotic approach to an ideal gas at high expansions. Additional exponential terms and a variable Gruneisen parameter have been added to adequately describe the high-pressure region above the Chapman-Jouguet state.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A1	A2	A3	A4	A5		
Type	A8	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	R1	R2	R3	R4	R5			
Type	F	F	F	F	F			

Card 3	1	2	3	4	5	6	7	8
Variable	AL1	AL2	AL3	AL4	AL5			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable	BL1	BL2	BL3	BL4	BL5			
Type	F	F	F	F	F			

Card 5	1	2	3	4	5	6	7	8
Variable	RL1	RL2	RL3	RL4	RL5			
Type	F	F	F	F	F			

Card 6	1	2	3	4	5	6	7	8
Variable	C	OMEGA	E	V0				
Type	I	F	F	F				

**VARIABLE****DESCRIPTION**

EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
$A_i, R_i, A_{\lambda i}, B_{\lambda i}, C, \omega$	Equation of state coefficients $A_i, R_i, A_{\lambda i}, B_{\lambda i}, R_{\lambda i}, C, \omega$ respectively. See below.
C	Equation of state coefficient, see below.
OMEGA	Equation of state coefficient, see below.
E	Energy density per unit initial volume
V0	Initial relative volume.

**Remarks:**

The JWLB equation-of-state defines the pressure as

$$p = \sum_{i=1}^5 A_i \left(1 - \frac{\lambda}{R_i V}\right) e^{-R_i V} + \frac{\lambda E}{V} + C \left(1 - \frac{\lambda}{\omega}\right) V^{-(\omega+1)}$$

$$\lambda = \sum_{i=1}^5 (A_{\lambda i} V + B_{\lambda i}) e^{-R_{\lambda i} V} + \omega$$

where V is the relative volume, E is the energy per unit initial volume, and  $A_i, R_i, A_{\lambda i}, B_{\lambda i}, R_{\lambda i}, C$ , and  $\omega$  are input constants defined above.

JWLB input constants for some common explosives as found in Baker and Stiel [1997] are given in the following table.

	TATB	LX-14	PETN	TNT	Octol 70/30
$\rho_0$ (g/cc)	1.800	1.821	1.765	1.631	1.803
E0 (Mbar)	.07040	.10205	.10910	.06656	.09590
DCJ (cm/ $\mu$ s)	.76794	.86619	.83041	.67174	.82994
PCJ (Mbar)	.23740	.31717	.29076	.18503	.29369
A1 (Mbar)	550.06	549.60	521.96	490.07	526.83
A2 (Mbar)	22.051	64.066	71.104	56.868	60.579
A3 (Mbar)	.42788	2.0972	4.4774	.82426	.91248
A4 (Mbar)	.28094	.88940	.97725	.00093	.00159
R1	16.688	34.636	44.169	40.713	52.106
R2	6.8050	8.2176	8.7877	9.6754	8.3998
R3	2.0737	20.401	25.072	2.4350	2.1339
R4	2.9754	2.0616	2.2251	.15564	.18592
C (Mbar)	.00776	.01251	.01570	.00710	.00968
$\omega$	.27952	.38375	.32357	.30270	.39023
A $\lambda_1$	1423.9	18307.	12.257	.00000	.011929
B $\lambda_1$	14387.	1390.1	52.404	1098.0	18466.
R $\lambda_1$	19.780	19.309	43.932	15.614	20.029
A $\lambda_2$	5.0364	4.4882	8.6351	11.468	5.4192
B $\lambda_2$	-2.6332	-2.6181	-4.9176	-6.5011	-3.2394
R $\lambda_2$	1.7062	1.5076	2.1303	2.1593	1.5868

\*EOS\_GASKET

This is Equation of state Form 15. This EOS works with solid elements and the thick shell using selective reduced 2 x 2 integration (ELFORM = 2 on SECTION\_TSHELL) to model the response of gaskets. For the thick shell only, it is completely decoupled from the shell material, i.e., in the local coordinate system of the shell, this model defines the normal stress,  $\sigma_{zz}$ , and does not change any of the other stress components. The model is a reduction of the \*MAT\_GENERAL\_NONLINEAR\_6DOF\_DISCRETE\_BEAM.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	LCID1	LCID2	LCID3	LCID4			
Type	A8	I	I	I	I			

Card 2	1	2	3	4	5	6	7	8
Variable	UNLOAD	K	DMPF	TFS	CFS	LOFFSET	IVS	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
LCID1	Load curve for loading.
LCID2	Load curve for unloading.
LCID3	Load curve for damping as a function of volumetric strain rate.
LCID4	Load curve for scaling the damping as a function of the volumetric strain.

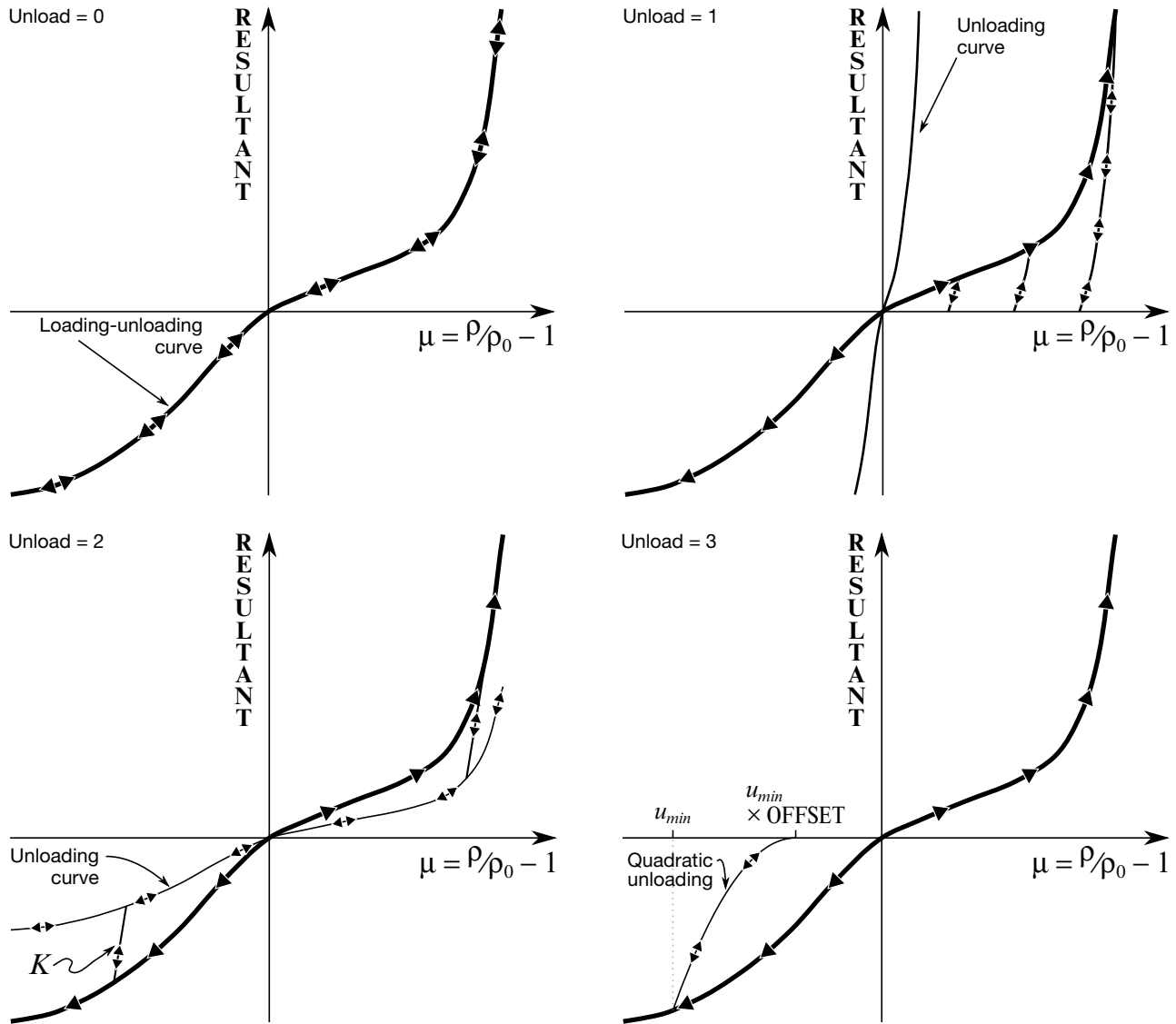


Figure EOS15-1. Load and unloading behavior.

VARIABLE	DESCRIPTION
UNLOAD	Unloading option (see <a href="#">Figure EOS15-1</a> ): EQ.0.0: Loading and unloading follow loading curve EQ.1.0: Loading follows loading curve, unloading follows unloading curve. The unloading curve ID if undefined is taken as the loading curve. EQ.2.0: Loading follows loading curve, unloading follows unloading stiffness, $K$ , to the unloading curve. The loading and unloading curves may only intersect at the origin of the axes. EQ.3.0: Quadratic unloading from peak displacement value to a



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<b>VARIABLE</b>	<b>DESCRIPTION</b>
	permanent offset.
K	Unloading stiffness, for UNLOAD = 2 only.
DMPF	Damping factor for stability. Values in the neighborhood of unity are recommended. The damping factor is properly scaled to eliminate time step size dependency.
TFS	Tensile failure strain.
CFS	Compressive failure strain.
OFFSET	Offset factor between 0 and 1.0 to determine permanent set upon unloading if the UNLOAD = 3.0. The permanent sets in compression and tension are equal to the product of this offset value and the maximum compressive and tensile displacements, respectively.
IVS	Initial volume strain.

**\*EOS\_MIE\_GRUNEISEN**

This is Equation of state Form 16, a Mie-Gruneisen form with a  $p - \alpha$  compaction model.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	GAMMA	A1	A2	A3	PEL	PCO	N
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA0	E0	V0					
Type	F	F	F					
Default	none	none	none					

**VARIABLE****DESCRIPTION**

EOSID	Equation of state identification. A unique number or label not exceeding 8 characters must be specified.
GAMMA	Gruneisen gamma.
$A_i$	Hugoniot polynomial coefficient
PEL	Crush pressure
PCO	Compaction pressure
N	Porosity exponent
ALPHA0	Initial porosity
E0	Initial internal energy
V0	Initial relative volume

**Remarks:**

The equation of state is a Mie-Gruneisen form with a polynomial Hugoniot curve and  $\alpha$  –  $\alpha$  compaction model. First, we define a history variable representing the porosity  $\alpha$  that is initialised to  $\alpha_0 > 1$ . The evolution of this variable is given as

$$\alpha(t) = \max \left\{ 1, \min \left[ \alpha_0, \min_{s \leq t} \left( 1 + (\alpha_0 - 1) \left[ \frac{p_{\text{comp}} - p(s)}{p_{\text{comp}} - p_{el}} \right]^N \right) \right] \right\}$$

where  $p(t)$  indicates the pressure at time  $t$ . For later use, we define the cap pressure as

$$p_c = p_{\text{comp}} - (p_{\text{comp}} - p_{el}) \left[ \frac{\alpha - 1}{\alpha_0 - 1} \right]^{1/N}$$

The remainder of the EOS model is given by the equations

$$p(\rho, e) = \Gamma \alpha \rho e + p_H(\eta) \left[ 1 - \frac{1}{2} \Gamma \eta \right]$$

$$p_H(\eta) = A_1 \eta + A_2 \eta^2 + A_3 \eta^3$$

together with

$$\eta(\rho) = \frac{\alpha \rho}{\alpha_0 \rho_0} - 1.$$

**\*EOS\_MURNAGHAN**

This is Equation of state Form 19. This EOS was designed to model incompressible fluid flow with SPH elements.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	GAMMA	K0	V0				
Type	A8	F	F	F				

**VARIABLE****DESCRIPTION**

EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
GAMMA, K0	Constants in the equation of state.
V0	Initial relative volume.

**Remarks:**

The Murnaghan equation of state defines pressure as

$$p = k_0 \left[ \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \right].$$

To model fluid flows accurately,  $\gamma$  is often set to 7, and  $k_0$  is chosen such that

$$c_0 = \sqrt{\frac{\gamma k_0}{\rho_0}} \geq 10v_{\max},$$

where  $v_{\max}$  is the maximum expected fluid flow velocity. This will ensure low compressibility while allowing for a relatively large time step size.

**\*EOS\_USER\_DEFINED**

These are equations of state 21-30. The user can supply his own subroutines. See also Appendix B. The keyword input has to be used for the user interface with data.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	EOST	LMC	NHV	IVECT	EO	V0	BULK
Type	A8	I	I	I	I	F	F	F

Define LMC material parameters using 8 parameters per card.

Card 2	1	2	3	4	5	6	7	8
Variable	P1	P2	P3	P4	P5	P6	P7	P8
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

EOSID	Equation of state ID, a unique number or label not exceeding 8 characters must be specified.
EOST	User equation of state type (21-30 inclusive). A number between 21 and 30 has to be chosen.
LMC	Length of material constant array which is equal to the number of material constants to be input. ( $LMC \leq 48$ )
NHV	Number of history variables to be stored, see Appendix B.
IVECT	Vectorization flag (on = 1). A vectorized user subroutine must be supplied.
EO	Initial internal energy.
V0	Initial relative volume.
BULK	Bulk modulus. This value is used in the calculation of the contact surface stiffness.
$P_i$	Material parameters $i = 1, \dots, LMC$ .



# \*MAT

LS-DYNA has historically referenced each material model by a number. As shown below, a three digit numerical designation can still be used, e.g., \*MAT\_001, and is equivalent to a corresponding descriptive designation, e.g., \*MAT\_ELASTIC. The two equivalent commands for each material model, one numerical and the other descriptive, are listed below. The numbers in square brackets (see key below) identify the element formulations for which the material model is implemented. The number in the curly brackets, { $n$ }, indicates the default number of history variables per element integration point that are stored in addition to the 7 history variables which are stored by default. Just as an example, for the type 16 fully integrated shell elements with 2 integration points through the thickness, the total number of history variables is  $8 \times (n + 7)$ . For the Belytschko-Tsay type 2 element the number is  $2 \times (n + 7)$ .

The meaning associated with particular extra history variables for a subset of material models and equations of state are tabulated at <http://www.dynasupport.com/howtos/material/history-variables>.

An additional option **TITLE** may be appended to a \*MAT keyword in which case an additional line is read in 80a format which can be used to describe the material. At present, LS-DYNA does not make use of the title. Inclusion of titles simply gives greater clarity to input decks.

### Key to numbers in square brackets

0	-	Solids (and 2D continuum elements, i.e., shell formulations 13,14,15)
1H	-	Hughes-Liu beam
1B	-	Belytschko resultant beam
1I	-	Belytschko integrated solid and tubular beams
1T	-	Truss
1D	-	Discrete beam
1SW	-	Spotweld beam
2	-	Shells
3a	-	Thick shell formulations 1,2,6
3c	-	Thick shell formulations 3,5,7
4	-	Special airbag element
5	-	SPH element
6	-	Acoustic solid
7	-	Cohesive solid
8A	-	Multi-material ALE solid (validated)

# \*MAT

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- 8B - Multi-material ALE solid (implemented but not validated<sup>1</sup>)
- 9 - Membrane element

\*MAT\_ADD\_COHESIVE<sup>2</sup> [7] {see associated material model}

\*MAT\_ADD\_EROSION<sup>2</sup>

\*MAT\_ADD\_FATIGUE

\*MAT\_ADD\_GENERALIZED\_DAMAGE<sup>2</sup> [2]

\*MAT\_ADD\_PERMEABILITY

\*MAT\_ADD\_PORE\_AIR

\*MAT\_ADD\_THERMAL\_EXPANSION<sup>2</sup>

\*MAT\_NONLOCAL<sup>2</sup>

\*MAT\_001: \*MAT\_ELASTIC [0,1H,1B,1I,1T,2,3a,3c,5,8A] {0}

\*MAT\_001\_FLUID: \*MAT\_ELASTIC\_FLUID [0,8A] {0}

\*MAT\_002: \*MAT\_{OPTION}TROPIC\_ELASTIC [0,2,3a,3c] {15}

\*MAT\_003: \*MAT\_PLASTIC\_KINEMATIC [0,1H,1I,1T,2,3a,3c,5,8A] {5}

\*MAT\_004: \*MAT\_ELASTIC\_PLASTIC\_THERMAL [0,1H,1T,2,3a,3c,5,8B] {3}

\*MAT\_005: \*MAT\_SOIL\_AND\_FOAM [0,5,3c,8A] {0}

\*MAT\_006: \*MAT\_VISCOELASTIC [0,1H,2,3a,3c,5,8B] {19}

\*MAT\_007: \*MAT\_BLATZ-KO\_RUBBER [0,2,3ac,8B] {9}

\*MAT\_008: \*MAT\_HIGH\_EXPLOSIVE\_BURN [0,5,3c,8A] {4}

\*MAT\_009: \*MAT\_NULL [0,1,2,3c,5,8A] {3}

\*MAT\_010: \*MAT\_ELASTIC\_PLASTIC\_HYDRO\_{OPTION} [0,3c,5,8B] {4}

\*MAT\_011: \*MAT\_STEINBERG [0,3c,5,8B] {5}

\*MAT\_011\_LUND: \*MAT\_STEINBERG\_LUND [0,3c,5,8B] {5}

\*MAT\_012: \*MAT\_ISOTROPIC\_ELASTIC\_PLASTIC [0,2,3a,3c,5,8B] {0}

\*MAT\_013: \*MAT\_ISOTROPIC\_ELASTIC\_FAILURE [0,3c,5,8B] {1}

\*MAT\_014: \*MAT\_SOIL\_AND\_FOAM\_FAILURE [0,3c,5,8B] {1}

\*MAT\_015: \*MAT\_JOHNSON\_COOK [0,2,3a,3c,5,8A] {6}

\*MAT\_016: \*MAT\_PSEUDO\_TENSOR [0,3c,5,8B] {6}

\*MAT\_017: \*MAT\_ORIENTED\_CRACK [0,3c] {14}

\*MAT\_018: \*MAT\_POWER\_LAW\_PLASTICITY [0,1H,2,3a,3c,5,8B] {0}

\*MAT\_019: \*MAT\_STRAIN\_RATE\_DEPENDENT\_PLASTICITY [0,2,3a,3c,5,8B] {6}

\*MAT\_020: \*MAT\_RIGID [0,1H,1B,1T,2,3a] {0}

\*MAT\_021: \*MAT\_ORTHOTROPIC\_THERMAL [0,2,3ac] {29}

\*MAT\_022: \*MAT\_COMPOSITE\_DAMAGE [0,2,3a,3c,5] {12}

\*MAT\_023: \*MAT\_TEMPERATURE\_DEPENDENT\_ORTHOTROPIC [0,2,3ac] {19}

\*MAT\_024: \*MAT\_PIECEWISE\_LINEAR\_PLASTICITY [0,1H,2,3a,3c,5,8A] {5}

\*MAT\_025: \*MAT\_GEOLOGIC\_CAP\_MODEL [0,3c,5] {12}

\*MAT\_026: \*MAT\_HONEYCOMB [0,3c] {20}

\*MAT\_027: \*MAT\_MOONEY-RIVLIN\_RUBBER [0,1T,2,3c,8B] {9}

\*MAT\_028: \*MAT\_RESULTANT\_PLASTICITY [1B,2] {5}

\*MAT\_029: \*MAT\_FORCE\_LIMITED [1B] {30}

\*MAT\_030: \*MAT\_SHAPE\_MEMORY [0,1H,2,3ac,5] {23}

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<sup>1</sup> Error associated with advection inherently leads to state variables that may be inconsistent with nonlinear constitutive routines and thus may lead to nonphysical results, nonconservation of energy, and even numerical instability in some cases. Caution is advised, particularly when using the 2<sup>nd</sup> tier of material models implemented for ALE multi-material solids (designated by [8B]) which are largely untested as ALE materials.

<sup>2</sup> These commands do not, by themselves, define a material model but rather can be used in certain cases to supplement material models



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*MAT_031:	*MAT_FRAZER_NASH_RUBBER_MODEL [0,3c,8B] {9}
*MAT_032:	*MAT_LAMINATED_GLASS [2,3a] {0}
*MAT_033:	*MAT_BARLAT_ANISOTROPIC_PLASTICITY [0,2,3a,3c] {9}
*MAT_033_96:	*MAT_BARLAT_YLD96 [2,3a] {9}
*MAT_034:	*MAT_FABRIC [4] {17}
*MAT_035:	*MAT_PLASTIC_GREEN-NAGHDI_RATE [0,3c,5,8B] {22}
*MAT_036:	*MAT_3-PARAMETER_BARLAT [2,3a] {7}
*MAT_037:	*MAT_TRANSVERSELY_ANISOTROPIC_ELASTIC_PLASTIC [2,3a] {9}
*MAT_038:	*MAT_BLATZ-KO_FOAM [0,2,3c,8B] {9}
*MAT_039:	*MAT_FLD_TRANSVERSELY_ANISOTROPIC [2,3a] {6}
*MAT_040:	*MAT_NONLINEAR_ORTHOTROPIC [0,2,3c] {17}
*MAT_041-050:	*MAT_USER_DEFINED_MATERIAL_MODELS [0,1H,1T,1D,2,3a,3c,5,8B] {0}
*MAT_051:	*MAT_BAMMAN [0,2,3a,3c,5,8B] {8}
*MAT_052:	*MAT_BAMMAN_DAMAGE [0,2,3a,3c,5,8B] {10}
*MAT_053:	*MAT_CLOSED_CELL_FOAM [0,3c,8B] {0}
*MAT_054-055:	*MAT_ENHANCED_COMPOSITE_DAMAGE [0,2,3c] {20}
*MAT_057:	*MAT_LOW_DENSITY_FOAM [0,3c,5,8B] {26}
*MAT_058:	*MAT_LAMINATED_COMPOSITE_FABRIC [2,3a] {15}
*MAT_059:	*MAT_COMPOSITE_FAILURE_{OPTION}_MODEL [0,2,3c,5] {22}
*MAT_060:	*MAT_ELASTIC_WITH_VISCOSITY [0,2,3a,3c,5,8B] {8}
*MAT_060C:	*MAT_ELASTIC_WITH_VISCOSITY_CURVE [0,2,3a,3c,5,8B] {8}
*MAT_061:	*MAT_KELVIN-MAXWELL_VISCOELASTIC [0,3c,5,8B] {14}
*MAT_062:	*MAT_VISCOUS_FOAM [0,3c,8B] {7}
*MAT_063:	*MAT_CRUSHABLE_FOAM [0,3c,5,8B] {8}
*MAT_064:	*MAT_RATE_SENSITIVE_POWERLAW_PLASTICITY [0,2,3a,3c,5,8B] {30}
*MAT_065:	*MAT_MODIFIED_ZERILLI_ARMSTRONG [0,2,3a,3c,5,8B] {6}
*MAT_066:	*MAT_LINEAR_ELASTIC_DISCRETE_BEAM [1D] {8}
*MAT_067:	*MAT_NONLINEAR_ELASTIC_DISCRETE_BEAM [1D] {14}
*MAT_068:	*MAT_NONLINEAR_PLASTIC_DISCRETE_BEAM [1D] {25}
*MAT_069:	*MAT_SID_DAMPER_DISCRETE_BEAM [1D] {13}
*MAT_070:	*MAT_HYDRAULIC_GAS_DAMPER_DISCRETE_BEAM [1D] {8}
*MAT_071:	*MAT_CABLE_DISCRETE_BEAM [1D] {8}
*MAT_072:	*MAT_CONCRETE_DAMAGE [0,3c,5,8B] {6}
*MAT_072R3:	*MAT_CONCRETE_DAMAGE_REL3 [0,3c,5] {6}
*MAT_073:	*MAT_LOW_DENSITY_VISCOUS_FOAM [0,3c,8B] {56}
*MAT_074:	*MAT_ELASTIC_SPRING_DISCRETE_BEAM [1D] {8}
*MAT_075:	*MAT_BILKHU/DUBOIS_FOAM [0,3c,5,8B] {8}
*MAT_076:	*MAT_GENERAL_VISCOELASTIC [0,2,3a,3c,5,8B] {53}
*MAT_077_H:	*MAT_HYPERELASTIC_RUBBER [0,2,3c,5,8B] {54}
*MAT_077_O:	*MAT_OGDEN_RUBBER [0,2,3c,8B] {54}
*MAT_078:	*MAT_SOIL_CONCRETE [0,3c,5,8B] {3}
*MAT_079:	*MAT_HYSTERETIC_SOIL [0,3c,5,8B] {96}
*MAT_080:	*MAT_RAMBERG-OSGOOD [0,3c,8B] {18}
*MAT_081:	*MAT_PLASTICITY_WITH_DAMAGE [0,2,3a,3c] {5}
*MAT_082(_RCDC):	*MAT_PLASTICITY_WITH_DAMAGE_ORTHO(_RCDC) [0,2,3a,3c] {22}
*MAT_083:	*MAT_FU_CHANG_FOAM [0,3c,5,8B] {54}
*MAT_084:	*MAT_WINFRITH_CONCRETE [0] {54}
*MAT_086:	*MAT_ORTHOTROPIC_VISCOELASTIC [2,3a] {17}
*MAT_087:	*MAT_CELLULAR_RUBBER [0,3c,5,8B] {19}
*MAT_088:	*MAT_MTS [0,2,3a,3c,5,8B] {5}
*MAT_089:	*MAT_PLASTICITY_POLYMER [0,2,3a,3c] {46}
*MAT_090:	*MAT_ACOUSTIC [6] {25}
*MAT_091:	*MAT_SOFT_TISSUE [0,2] {16}
*MAT_092:	*MAT_SOFT_TISSUE_VISCO [0,2] {58}

# \*MAT

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*MAT_093:	*MAT_ELASTIC_6DOF_SPRING_DISCRETE_BEAM [1D] {25}
*MAT_094:	*MAT_INELASTIC_SPRING_DISCRETE_BEAM [1D] {9}
*MAT_095:	*MAT_INELASTIC_6DOF_SPRING_DISCRETE_BEAM [1D] {25}
*MAT_096:	*MAT_BRITTLE_DAMAGE [0,8B] {51}
*MAT_097:	*MAT_GENERAL_JOINT_DISCRETE_BEAM [1D] {23}
*MAT_098:	*MAT_SIMPLIFIED_JOHNSON_COOK [0,1H,1B,1T,2,3a,3c] {6}
*MAT_099:	*MAT_SIMPLIFIED_JOHNSON_COOK_ORTHOTROPIC_DAMAGE [0,2,3a,3c] {22}
*MAT_100:	*MAT_SPOTWELD_{OPTION} [0,1SW] {6}
*MAT_100_DA:	*MAT_SPOTWELD_DAIMLERCHRYSLER [0] {6}
*MAT_101:	*MAT_GEPLASTIC_SRATE_2000a [2,3a] {15}
*MAT_102(_T):	*MAT_INV_HYPERBOLIC_SIN(_THERMAL) [0,3c,8B] {15}
*MAT_103:	*MAT_ANISOTROPIC_VISCOPLASTIC [0,2,3a,3c,5] {20}
*MAT_103_P:	*MAT_ANISOTROPIC_PLASTIC [2,3a,3c] {20}
*MAT_104:	*MAT_DAMAGE_1 [0,2,3a,3c] {11}
*MAT_105:	*MAT_DAMAGE_2 [0,2,3a,3c] {7}
*MAT_106:	*MAT_ELASTIC_VISCOPLASTIC_THERMAL [0,2,3a,3c,5] {20}
*MAT_107:	*MAT_MODIFIED_JOHNSON_COOK [0,2,3a,3c,5,8B] {15}
*MAT_108:	*MAT_ORTHO_ELASTIC_PLASTIC [2,3a] {15}
*MAT_110:	*MAT_JOHNSON_HOLMQUIST_CERAMICS [0,3c,5] {15}
*MAT_111:	*MAT_JOHNSON_HOLMQUIST_CONCRETE [0,3c,5] {25}
*MAT_112:	*MAT_FINITE_ELASTIC_STRAIN_PLASTICITY [0,3c,5] {22}
*MAT_113:	*MAT_TRIP [2,3a] {5}
*MAT_114:	*MAT_LAYERED_LINEAR_PLASTICITY [2,3a] {13}
*MAT_115:	*MAT_UNIFIED_CREEP [0,2,3a,3c,5] {1}
*MAT_115_O:	*MAT_UNIFIED_CREEP_ORTHO [0,3c,5] {1}
*MAT_116:	*MAT_COMPOSITE_LAYUP [2] {30}
*MAT_117:	*MAT_COMPOSITE_MATRIX [2] {30}
*MAT_118:	*MAT_COMPOSITE_DIRECT [2] {10}
*MAT_119:	*MAT_GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM [1D] {62}
*MAT_120:	*MAT_GURSON [0,2,3a,3c] {12}
*MAT_120_JC:	*MAT_GURSON_JC [0,2] {12}
*MAT_120_RCDC:	*MAT_GURSON_RCDC [0,2] {12}
*MAT_121:	*MAT_GENERAL_NONLINEAR_1DOF_DISCRETE_BEAM [1D] {20}
*MAT_122:	*MAT_HILL_3R [2,3a] {8}
*MAT_122_3D:	*MAT_HILL_3R_3D [0] {28}
*MAT_122_TAB:	*MAT_HILL_3R_TABULATED [2,3a] {8}
*MAT_123:	*MAT_MODIFIED_PIECEWISE_LINEAR_PLASTICITY [0,2,3a,3c,5] {11}
*MAT_124:	*MAT_PLASTICITY_COMPRESSION_TENSION [0,1H,2,3a,3c,5,8B] {7}
*MAT_125:	*MAT_KINEMATIC_HARDENING_TRANSVERSELY_ANISOTROPIC [0,2,3a,3c] {11}
*MAT_126:	*MAT_MODIFIED_HONEYCOMB [0,3c] {20}
*MAT_127:	*MAT_ARRUDA_BOYCE_RUBBER [0,3c,5] {49}
*MAT_128:	*MAT_HEART_TISSUE [0,3c] {15}
*MAT_129:	*MAT_LUNG_TISSUE [0,3c] {49}
*MAT_130:	*MAT_SPECIAL_ORTHOTROPIC [2] {35}
*MAT_131:	*MAT_ISOTROPIC_SMEARED_CRACK [0,5,8B] {15}
*MAT_132:	*MAT_ORTHOTROPIC_SMEARED_CRACK [0] {61}
*MAT_133:	*MAT_BARLAT_YLD2000 [2,3a] {9}
*MAT_134:	*MAT_VISCOELASTIC_FABRIC [9]
*MAT_135:	*MAT_WTM_STM [2,3a] {30}
*MAT_135_PLC:	*MAT_WTM_STM_PLC [2,3a] {30}
*MAT_136:	*MAT_CORUS_VEGTER [2,3a] {5}
*MAT_138:	*MAT_COHESIVE_MIXED_MODE [7] {0}
*MAT_139:	*MAT_MODIFIED_FORCE_LIMITED [1B] {35}
*MAT_140:	*MAT_VACUUM [0,8A] {0}

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*MAT_141:	*MAT_RATE_SENSITIVE_POLYMER [0,3c,8B] {6}
*MAT_142:	*MAT_TRANSVERSELY_ISOTROPIC_CRUSHABLE_FOAM [0,3c] {12}
*MAT_143:	*MAT_WOOD_{OPTION} [0,3c,5] {37}
*MAT_144:	*MAT_PITZER_CRUSHABLEFOAM [0,3c,8B] {7}
*MAT_145:	*MAT_SCHWER_MURRAY_CAP_MODEL [0,5] {50}
*MAT_146:	*MAT_1DOF_GENERALIZED_SPRING [1D] {1}
*MAT_147	*MAT_FHWA_SOIL [0,3c,5,8B] {15}
*MAT_147_N:	*MAT_FHWA_SOIL_NEBRASKA [0,3c,5,8B] {15}
*MAT_148:	*MAT_GAS_MIXTURE [0,8A] {14}
*MAT_151:	*MAT_EMMI [0,3c,5,8B] {23}
*MAT_153:	*MAT_DAMAGE_3 [0,1H,2,3a,3c]
*MAT_154:	*MAT_DESPANDE_FLECK_FOAM [0,3c,8B] {10}
*MAT_155:	*MAT_PLASTICITY_COMPRESSION_TENSION_EOS [0,3c,5,8B] {16}
*MAT_156:	*MAT_MUSCLE [1T] {0}
*MAT_157:	*MAT_ANISOTROPIC_ELASTIC_PLASTIC [0,2,3a] {5}
*MAT_158:	*MAT_RATE_SENSITIVE_COMPOSITE_FABRIC [2,3a] {54}
*MAT_159:	*MAT_CSCM_{OPTION} [0,3c,5] {22}
*MAT_160:	*MAT_ALE_INCOMPRESSIBLE
*MAT_161:	*MAT_COMPOSITE_MSC [0] {34}
*MAT_162:	*MAT_COMPOSITE_DMG_MSC [0] {40}
*MAT_163	*MAT_MODIFIED_CRUSHABLE_FOAM [0,3c,8B] {10}
*MAT_164:	*MAT_BRAIN_LINEAR_VISCOELASTIC [0] {14}
*MAT_165:	*MAT_PLASTIC_NONLINEAR_KINEMATIC [0,2,3a,3c,8B] {8}
*MAT_165B:	*MAT_PLASTIC_NONLINEAR_KINEMATIC_B [0,2]
*MAT_166:	*MAT_MOMENT_CURVATURE_BEAM [1B] {54}
*MAT_167:	*MAT_MCCORMICK [0,3c,8B] {8}
*MAT_168:	*MAT_POLYMER [0,3c,8B] {60}
*MAT_169:	*MAT_ARUP_ADHESIVE [0] {30}
*MAT_170:	*MAT_RESULTANT_ANISOTROPIC [2,3a] {67}
*MAT_171:	*MAT_STEEL_CONCENTRIC_BRACE [1B] {35}
*MAT_172:	*MAT_CONCRETE_EC2 [1H,2,3a] {64}
*MAT_173:	*MAT_MOHR_COULOMB [0,5] {52}
*MAT_174:	*MAT_RC_BEAM [1H] {22}
*MAT_175:	*MAT_VISCOELASTIC_THERMAL [0,2,3a,3c,5,8B] {86}
*MAT_176:	*MAT_QUASILINEAR_VISCOELASTIC [0,2,3a,3c,5,8B] {81}
*MAT_177:	*MAT_HILL_FOAM [0,3c] {12}
*MAT_178:	*MAT_VISCOELASTIC_HILL_FOAM [0,3c] {92}
*MAT_179:	*MAT_LOW_DENSITY_SYNTHETIC_FOAM_{OPTION} [0,3c] {77}
*MAT_181:	*MAT_SIMPLIFIED_RUBBER/FOAM_{OPTION} [0,2,3c] {39}
*MAT_183:	*MAT_SIMPLIFIED_RUBBER_WITH_DAMAGE [0,2,3c] {44}
*MAT_184:	*MAT_COHESIVE_ELASTIC [7] {0}
*MAT_185:	*MAT_COHESIVE_TH [7] {0}
*MAT_186:	*MAT_COHESIVE_GENERAL [7] {6}
*MAT_187:	*MAT_SAMP-1 [0,2,3a,3c] {38}
*MAT_188:	*MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP [0,2,3a,3c] {27}
*MAT_189:	*MAT_ANISOTROPIC_THERMOELASTIC [0,3c,8B] {21}
*MAT_190:	*MAT_FLD_3-PARAMETER_BARLAT [2,3a] {36}
*MAT_191:	*MAT_SEISMIC_BEAM [1B] {36}
*MAT_192:	*MAT_SOIL_BRICK [0,3c] {96}
*MAT_193:	*MAT_DRUCKER_PRAGER [0,3c] {24}
*MAT_194:	*MAT_RC_SHEAR_WALL [2,3a] {36}
*MAT_195:	*MAT_CONCRETE_BEAM [1H] {5}
*MAT_196:	*MAT_GENERAL_SPRING_DISCRETE_BEAM [1D] {25}
*MAT_197:	*MAT_SEISMIC_ISOLATOR [1D] {20}

# \*MAT

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\*MAT\_198: \*MAT\_JOINTED\_ROCK [0] {31}  
\*MAT\_202: \*MAT\_STEEL\_EC3 [1H] {3}  
\*MAT\_203: \*MAT\_HYSTERETIC\_REINFORCEMENT [1H,2] {64}  
\*MAT\_208: \*MAT\_BOLT\_BEAM [1D] {16}  
\*MAT\_211: \*MAT\_SPR\_JLR [1H] {60}  
\*MAT\_214: \*MAT\_DRY\_FABRIC [9]  
\*MAT\_215: \*MAT\_4A\_MICROMECH [0,2,3a,3c]  
\*MAT\_216: \*MAT\_ELASTIC\_PHASE\_CHANGE [0]  
\*MAT\_217: \*MAT\_OPTION\_TROPIC\_ELASTIC\_PHASE\_CHANGE [0]  
\*MAT\_218: \*MAT\_MOONEY-RIVLIN\_RUBBER\_PHASE\_CHANGE [0]  
\*MAT\_219: \*MAT\_CODAM2 [0,2,3a,3c]  
\*MAT\_220: \*MAT\_RIGID\_DISCRETE [0,2]  
\*MAT\_221: \*MAT\_ORTHOTROPIC\_SIMPLIFIED\_DAMAGE [0,3c,5] {17}  
\*MAT\_224: \*MAT\_TABULATED\_JOHNSON\_COOK [0,2,3a,3c,5] {17}  
\*MAT\_224\_GYS: \*MAT\_TABULATED\_JOHNSON\_COOK\_GYS [0] {17}  
\*MAT\_225: \*MAT\_VISCOPLASTIC\_MIXED\_HARDENING [0,2,3a,3c,5]  
\*MAT\_226: \*MAT\_KINEMATIC\_HARDENING\_BARLAT89 [2,3a]  
\*MAT\_230: \*MAT\_PML\_ELASTIC [0] {24}  
\*MAT\_231: \*MAT\_PML\_ACOUSTIC [6] {35}  
\*MAT\_232: \*MAT\_BIOT\_HYSTERETIC [0,2,3a] {30}  
\*MAT\_233: \*MAT\_CAZACU\_BARLAT [2,3a]  
\*MAT\_234: \*MAT\_VISCOELASTIC\_LOOSE\_FABRIC [2,3a]  
\*MAT\_235: \*MAT\_MICROMECHANICS\_DRY\_FABRIC [2,3a]  
\*MAT\_236: \*MAT\_SCC\_ON\_RCC [2,3a]  
\*MAT\_237: \*MAT\_PML\_HYSTERETIC [0] {54}  
\*MAT\_238: \*MAT\_PERT\_PIECEWISE\_LINEAR\_PLASTICITY [0,1H,2,3,5,8A]  
\*MAT\_240: \*MAT\_COHESIVE\_MIXED\_MODE\_ELASTOPLASTIC\_RATE [7] {0}  
\*MAT\_241: \*MAT\_JOHNSON\_HOLMQUIST\_JH1 [0,3c,5]  
\*MAT\_242: \*MAT\_KINEMATIC\_HARDENING\_BARLAT2000 [2,3a]  
\*MAT\_243: \*MAT\_HILL\_90 [2,3a]  
\*MAT\_244: \*MAT\_UHS\_STEEL [0,2,3a,3c,5] {35}  
\*MAT\_245: \*MAT\_PML\_{OPTION}TROPIC\_ELASTIC [0] {30}  
\*MAT\_246: \*MAT\_PML\_NULL [0] {27}  
\*MAT\_248: \*MAT\_PHS\_BMW [2] {38}  
\*MAT\_249: \*MAT\_REINFORCED\_THERMOPLASTIC [2]  
\*MAT\_249\_UDFIBER: \*MAT\_REINFORCED\_THERMOPLASTIC\_UDFIBER [2]  
\*MAT\_251: \*MAT\_TAILORED\_PROPERTIES [2] {6}  
\*MAT\_252: \*MAT\_TOUGHENED\_ADHESIVE\_POLYMER [0,7] {10}  
\*MAT\_254: \*MAT\_GENERALIZED\_PHASE\_CHANGE [0,2]  
\*MAT\_255: \*MAT\_PIECEWISE\_LINEAR\_PLASTIC\_THERMAL [0,2,3a,3c]  
\*MAT\_256: \*MAT\_AMORPHOUS\_SOLIDS\_FINITE\_STRAIN [0]  
\*MAT\_260A: \*MAT\_STOUGHTON\_NON\_ASSOCIATED\_FLOW [0,2]  
\*MAT\_260B: \*MAT\_MOHR\_NON\_ASSOCIATED\_FLOW [0,2]  
\*MAT\_261: \*MAT\_LAMINATED\_FRACTURE\_DAIMLER\_PINHO [0,2,3a,3c]  
\*MAT\_262: \*MAT\_LAMINATED\_FRACTURE\_DAIMLER\_CAMANHO [0,2,3a,3c]  
\*MAT\_264: \*MAT\_TABULATED\_JOHNSON\_COOK\_ORTHOTROPIC [0]  
\*MAT\_266: \*MAT\_TISSUE\_DISPERSED [0]  
\*MAT\_267: \*MAT\_EIGHT\_CHAIN\_RUBBER [0,5]  
\*MAT\_269: \*MAT\_BERGSTROM\_BOYCE\_RUBBER [0,5]  
\*MAT\_270: \*MAT\_CWM [0,2,5]  
\*MAT\_271: \*MAT\_POWDER [0,5]  
\*MAT\_272: \*MAT\_RHT [0,5]  
\*MAT\_273: \*MAT\_CONCRETE\_DAMAGE\_PLASTIC\_MODEL [0]  
\*MAT\_274: \*MAT\_PAPER [0,2]

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*MAT_275:	*MAT_SMOOTH_VISCOELASTIC_VISCOPLASTIC [0]
*MAT_276:	*MAT_CHRONOLOGICAL_VISCOELASTIC [2,3a,3c]
*MAT_277:	*MAT_ADHESIVE_CURING_VISCOELASTIC [0]
*MAT_278:	*MAT_CF_MICROMECHANICS [02] {3}
*MAT_279:	*MAT_COHESIVE_PAPER [7]
*MAT_280:	*MAT_GLASS [2] {32}
*MAT_293:	*MAT_COMPRF [2] {7}

For the discrete (type 6) beam elements, which are used to model complicated dampers and multi-dimensional spring-damper combinations, the following material types are available:

*MAT_066:	*MAT_LINEAR_ELASTIC_DISCRETE_BEAM [1D]
*MAT_067:	*MAT_NONLINEAR_ELASTIC_DISCRETE_BEAM [1D]
*MAT_068:	*MAT_NONLINEAR_PLASTIC_DISCRETE_BEAM [1D]
*MAT_069:	*MAT_SID_DAMPER_DISCRETE_BEAM [1D]
*MAT_070:	*MAT_HYDRAULIC_GAS_DAMPER_DISCRETE_BEAM [1D]
*MAT_071:	*MAT_CABLE_DISCRETE_BEAM [1D]
*MAT_074:	*MAT_ELASTIC_SPRING_DISCRETE_BEAM [1D]
*MAT_093:	*MAT_ELASTIC_6DOF_SPRING_DISCRETE_BEAM [1D]
*MAT_094:	*MAT_INELASTIC_SPRING_DISCRETE_BEAM [1D]
*MAT_095:	*MAT_INELASTIC_6DOF_SPRING_DISCRETE_BEAM [1D]
*MAT_119:	*MAT_GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM [1D]
*MAT_121:	*MAT_GENERAL_NONLINEAR_1DOF_DISCRETE_BEAM [1D]
*MAT_146:	*MAT_1DOF_GENERALIZED_SPRING [1D]
*MAT_196:	*MAT_GENERAL_SPRING_DISCRETE_BEAM [1D]
*MAT_197:	*MAT_SEISMIC_ISOLATOR [1D]
*MAT_208:	*MAT_BOLT_BEAM [1D]

For the discrete springs and dampers the following material types are available

*MAT_S01:	*MAT_SPRING_ELASTIC
*MAT_S02:	*MAT_DAMPER_VISCOUS
*MAT_S03:	*MAT_SPRING_ELASTOPLASTIC
*MAT_S04:	*MAT_SPRING_NONLINEAR_ELASTIC
*MAT_S05:	*MAT_DAMPER_NONLINEAR_VISCOUS
*MAT_S06:	*MAT_SPRING_GENERAL_NONLINEAR
*MAT_S07:	*MAT_SPRING_MAXWELL
*MAT_S08:	*MAT_SPRING_INELASTIC
*MAT_S13:	*MAT_SPRING_TRILINEAR_DEGRADING
*MAT_S14:	*MAT_SPRING_SQUAT_SHEARWALL
*MAT_S15:	*MAT_SPRING_MUSCLE

For ALE solids the following material types are available:

*MAT_ALE_01:	*MAT_ALE_VACUUM	(same as *MAT_140)
*MAT_ALE_02:	*MAT_ALE_GAS_MIXTURE	(same as *MAT_148)
*MAT_ALE_03:	*MAT_ALE_VISCOUS	(same as *MAT_009)
*MAT_ALE_04:	*MAT_ALE_MIXING_LENGTH	(same as *MAT_149)
*MAT_ALE_05:	*MAT_ALE_INCOMPRESSIBLE	(same as *MAT_160)
*MAT_ALE_06:	*MAT_ALE_HERSCHEL	

For SPH particles the following material type is available:



**MATERIAL MODEL REFERENCE TABLES**

The tables provided on the following pages list the material models, some of their attributes, and the general classes of physical materials to which the numerical models might be applied.

If a material model, without consideration of \*MAT\_ADD\_EROSION, \*MAT\_ADD\_THERMAL\_EXPANSION, or \*MAT\_ADD\_GENERALIZED\_DAMAGE, includes any of the following attributes, a “Y” will appear in the respective column of the table:

<b>SRATE</b>	- Strain-rate effects
<b>FAIL</b>	- Failure criteria
<b>EOS</b>	- Equation-of-State required for 3D solids and 2D continuum elements
<b>THERMAL</b>	- Thermal effects
<b>ANISO</b>	- Anisotropic/orthotropic
<b>DAM</b>	- Damage effects
<b>TENS</b>	- Tension handled differently than compression in some manner

Potential applications of the material models, in terms of classes of physical materials, are abbreviated in the table as follows:

<b>GN</b>	- General
<b>CM</b>	- Composite
<b>CR</b>	- Ceramic
<b>FL</b>	- Fluid
<b>FM</b>	- Foam
<b>GL</b>	- Glass
<b>HY</b>	- Hydrodynamic material
<b>MT</b>	- Metal
<b>PL</b>	- Plastic
<b>RB</b>	- Rubber
<b>SL</b>	- Soil, concrete, or rock
<b>AD</b>	- Adhesive or Cohesive material
<b>BIO</b>	- Biological material
<b>CIV</b>	- Civil Engineering component
<b>HT</b>	- Heat Transfer
<b>F</b>	- Fabric

Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
1	Elastic								GN, FL
2	Orthotropic Elastic (Anisotropic-solids)					Y			CM, MT
3	Plastic Kinematic/Isotropic	Y	Y						CM, MT, PL
4	Elastic Plastic Thermal				Y				MT, PL
5	Soil and Foam							Y	FM, SL
6	Linear Viscoelastic	Y							RB
7	Blatz-Ko Rubber								RB
8	High Explosive Burn			Y					HY
9	Null Material	Y	Y	Y				Y	FL, HY
10	Elastic Plastic Hydro(dynamic)		Y	Y				Y	HY, MT
11	Steinberg: Temp. Dependent Elastoplastic	Y	Y	Y	Y			Y	HY, MT
12	Isotropic Elastic Plastic								MT
13	Isotropic Elastic with Failure		Y					Y	MT
14	Soil and Foam with Failure		Y					Y	FM, SL
15	Johnson/Cook Plasticity Model	Y	Y	Y	Y		Y	Y	HY, MT
16	Pseudo Tensor Geological Model	Y	Y	Y			Y	Y	SL
17	Oriented Crack (Elastoplastic w/ Fracture)		Y	Y		Y		Y	HY, MT, PL, CR
18	Power Law Plasticity (Isotropic)	Y							MT, PL
19	Strain Rate Dependent Plasticity	Y	Y						MT, PL
20	Rigid								
21	Orthotropic Thermal (Elastic)				Y	Y			GN



**MATERIAL MODEL REFERENCE TABLES**

**\*MAT**

Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
22	Composite Damage		Y			Y		Y	CM
23	Temperature Dependent Orthotropic				Y	Y			CM
24	Piecewise Linear Plasticity (Isotropic)	Y	Y						MT, PL
25	Inviscid Two Invariant Geologic Cap		Y					Y	SL
26	Honeycomb	Y	Y			Y		Y	CM, FM, SL
27	Mooney-Rivlin Rubber							Y	RB
28	Resultant Plasticity								MT
29	Force Limited Resultant Formulation							Y	
30	Shape Memory								MT
31	Frazer-Nash Rubber							Y	RB
32	Laminated Glass (Composite)		Y						CM, GL
33	Barlat Anisotropic Plasticity (YLD96)	Y				Y			CR, MT
34	Fabric					Y		Y	F
35	Plastic-Green Naghdi Rate	Y							MT
36	Three-Parameter Barlat Plasticity	Y			Y	Y			MT
37	Transversely Anisotropic Elastic Plastic					Y			MT
38	Blatz-Ko Foam								FM, PL
39	FLD Transversely Anisotropic					Y			MT
40	Nonlinear Orthotropic		Y		Y	Y		Y	CM
41	-50 User Defined Materials	Y	Y	Y	Y	Y	Y	Y	GN
51	Bamman (Temp/Rate Dependent Plasticity)	Y			Y				GN
52	Bamman Damage	Y	Y		Y		Y		MT
53	Closed cell foam (Low density polyurethane)								FM
54	Composite Damage with Chang Failure		Y			Y	Y	Y	CM
55	Composite Damage with Tsai-Wu Failure		Y			Y	Y	Y	CM
57	Low Density Urethane Foam	Y	Y					Y	FM
58	Laminated Composite Fabric		Y			Y	Y	Y	CM, F

Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
59	Composite Failure (Plasticity Based)		Y			Y		Y	CM, CR
60	Elastic with Viscosity (Viscous Glass)	Y			Y				GL
61	Kelvin-Maxwell Viscoelastic	Y							FM
62	Viscous Foam (Crash dummy Foam)	Y							FM
63	Isotropic Crushable Foam							Y	FM
64	Rate Sensitive Powerlaw Plasticity	Y							MT
65	Zerilli-Armstrong (Rate/Temp Plasticity)	Y		Y	Y			Y	MT
66	Linear Elastic Discrete Beam	Y				Y			
67	Nonlinear Elastic Discrete Beam	Y				Y		Y	
68	Nonlinear Plastic Discrete Beam	Y	Y			Y			
69	SID Damper Discrete Beam	Y							
70	Hydraulic Gas Damper Discrete Beam	Y							
71	Cable Discrete Beam (Elastic)							Y	Cables
72	Concrete Damage (incl. Release III)	Y	Y	Y			Y	Y	SL
73	Low Density Viscous Foam	Y	Y					Y	FM
74	Elastic Spring Discrete Beam	Y	Y					Y	
75	Bilkhu/Dubois Foam							Y	FM
76	General Viscoelastic (Maxwell Model)	Y			Y			Y	RB
77	Hyperelastic and Ogden Rubber	Y						Y	RB
78	Soil Concrete		Y				Y	Y	SL
79	Hysteretic Soil (Elasto-Perfectly Plastic)		Y					Y	SL
80	Ramberg-Osgood								SL
81	Plasticity with Damage	Y	Y				Y		MT, PL
82	Plasticity with Damage Ortho	Y	Y			Y	Y		
83	Fu Chang Foam	Y	Y				Y	Y	FM
84	Winfrith Concrete	Y						Y	FM, SL
86	Orthotropic Viscoelastic	Y				Y			RB
87	Cellular Rubber	Y						Y	RB
88	MTS	Y		Y	Y				MT
89	Plasticity Polymer	Y						Y	PL

**MATERIAL MODEL REFERENCE TABLES**

**\*MAT**

Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
90	Acoustic							Y	FL
91	Soft Tissue	Y	Y			Y		Y	BIO
92	Soft Tissue (viscous)								
93	Elastic 6DOF Spring Discrete Beam	Y	Y			Y		Y	
94	Inelastic Spring Discrete Beam	Y	Y					Y	
95	Inelastic 6DOF Spring Discrete Beam	Y	Y			Y		Y	
96	Brittle Damage	Y	Y			Y	Y	Y	SL
97	General Joint Discrete Beam								
98	Simplified Johnson Cook	Y	Y						MT
99	Simpl. Johnson Cook Orthotropic Damage	Y	Y			Y	Y		MT
100	Spotweld	Y	Y				Y	Y	MT
101	GE Plastic Strain Rate	Y	Y					Y	PL
102(_T)	Inv. Hyperbolic Sin (Thermal)	Y			Y				MT, PL
103	Anisotropic Viscoplastic	Y	Y			Y			MT
103P	Anisotropic Plastic					Y			MT
104	Damage 1	Y	Y			Y	Y		MT
105	Damage 2	Y	Y				Y		MT
106	Elastic Viscoplastic Thermal	Y			Y				PL
107	Modified Johnson Cook	Y	Y		Y		Y		MT
108	Ortho Elastic Plastic					Y			
110	Johnson Holmquist Ceramics	Y	Y				Y	Y	CR, GL
111	Johnson Holmquist Concrete	Y	Y				Y	Y	SL
112	Finite Elastic Strain Plasticity	Y							PL
113	Transformation Induced Plasticity (TRIP)				Y				MT
114	Layered Linear Plasticity	Y	Y						MT, PL, CM
115	Unified Creep								GN
115_O	Unified Creep Ortho					Y			GN
116	Composite Layup					Y			CM

Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
117	Composite Matrix					Y			CM
118	Composite Direct					Y			CM
119	General Nonlinear 6DOF Discrete Beam	Y	Y			Y		Y	
120	Gurson	Y	Y				Y	Y	MT
121	General Nonlinear 1DOF Discrete Beam	Y	Y					Y	
122	Hill 3RC					Y			MT
122_3D	Hill 3R 3D					Y			MT, CM
122_TAB	Hill 3R Tabulated					Y			MT
123	Modified Piecewise Linear Plasticity	Y	Y						MT, PL
124	Plasticity Compression Tension	Y	Y					Y	MT, PL
125	Kinematic Hardening Transversely Aniso.					Y			MT
126	Modified Honeycomb	Y	Y			Y	Y	Y	CM, FM, SL
127	Arruda Boyce Rubber	Y							RB
128	Heart Tissue					Y		Y	BIO
129	Lung Tissue	Y						Y	BIO
130	Special Orthotropic					Y			
131	Isotropic Smeared Crack		Y				Y	Y	MT, CM
132	Orthotropic Smeared Crack		Y			Y	Y		MT, CM
133	Barlat YLD2000	Y			Y	Y			MT
134	Viscoelastic Fabric								
135	Weak and Strong Texture Model	Y	Y			Y			MT
136	Corus Vegter					Y			MT
138	Cohesive Mixed Mode		Y			Y	Y	Y	AD
139	Modified Force Limited						Y	Y	
140	Vacuum								
141	Rate Sensitive Polymer	Y							PL
142	Transversely Isotropic Crushable Foam							Y	FM

**MATERIAL MODEL REFERENCE TABLES**

**\*MAT**

Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
143	Wood	Y	Y			Y	Y	Y	Wood
144	Pitzer Crushable Foam	Y						Y	FM
145	Schwer Murray Cap Model	Y	Y				Y	Y	SL
146	1DOF Generalized Spring	Y							
147	FWHA Soil	Y					Y	Y	SL
147N	FHWA Soil Nebraska	Y					Y	Y	SL
148	Gas Mixture				Y				FL
151	Evolving Microstructural Model of Inelast.	Y	Y		Y	Y	Y		MT
153	Damage 3	Y	Y				Y		MT, PL
154	Deshpande Fleck Foam		Y						FM
155	Plasticity Compression Tension EOS	Y	Y	Y				Y	Ice
156	Muscle	Y						Y	BIO
157	Anisotropic Elastic Plastic					Y			MT, CM
158	Rate-Sensitive Composite Fabric	Y	Y			Y	Y	Y	CM
159	CSCM	Y	Y				Y	Y	SL
160	ALE incompressible								
161,162	Composite MSC (Dmg)	Y	Y			Y	Y	Y	CM
163	Modified Crushable Foam	Y						Y	FM
164	Brain Linear Viscoelastic	Y							BIO
165	Plastic Nonlinear Kinematic		Y						MT
165B	Plastic Nonlinear Kinematic_B								MT
166	Moment Curvature Beam	Y	Y					Y	CIV
167	McCormick	Y							MT
168	Polymer				Y			Y	PL
169	Arup Adhesive	Y	Y			Y		Y	AD
170	Resultant Anisotropic					Y			PL
171	Steel Concentric Brace						Y	Y	CIV
172	Concrete EC2		Y		Y			Y	SL, MT
173	Mohr Coulomb					Y		Y	SL

Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
174	RC Beam						Y	Y	SL
175	Viscoelastic Thermal	Y			Y			Y	RB
176	Quasilinear Viscoelastic	Y	Y				Y	Y	BIO
177	Hill Foam							Y	FM
178	Viscoelastic Hill Foam (Ortho)	Y						Y	FM
179	Low Density Synthetic Foam	Y	Y			Y	Y	Y	FM
181	Simplified Rubber/Foam	Y	Y				Y	Y	RB, FM
183	Simplified Rubber with Damage	Y					Y	Y	RB
184	Cohesive Elastic		Y					Y	AD
185	Cohesive TH		Y			Y	Y	Y	AD
186	Cohesive General		Y			Y	Y	Y	AD
187	Semi-Analytical Model for Polymers – 1	Y	Y				Y	Y	PL
188	Thermo Elasto Viscoelastic Creep	Y			Y				MT
189	Anisotropic Thermoelastic				Y	Y			
190	Flow limit diagram 3-Parameter Barlat		Y			Y		Y	MT
191	Seismic Beam							Y	CIV
192	Soil Brick					Y			SL
193	Drucker Prager							Y	SL
194	RC Shear Wall		Y				Y	Y	CIV
195	Concrete Beam	Y	Y				Y	Y	CIV
196	General Spring Discrete Beam	Y						Y	
197	Seismic Isolator	Y	Y			Y		Y	CIV
198	Jointed Rock		Y			Y		Y	SL
202	Steel EC3								CIV
203	Hysteretic Reinforcement		Y				Y	Y	CV
208	Bolt Beam		Y				Y	Y	MT
211	SPR JLR	Y	Y						MT
214	Dry Fabric	Y	Y			Y	Y	Y	
215	4A Micromec	Y	Y			Y	Y		CM,PL
216	Elastic Phase Change								GN
217	Orthotropic Elastic Phase Change					Y			GN

**MATERIAL MODEL REFERENCE TABLES**

**\*MAT**

Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
218	Mooney Rivlin Rubber Phase Change							Y	RB
219	CODAM2		Y			Y	Y	Y	CM
220	Rigid Discrete								
221	Orthotropic Simplified Damage		Y			Y	Y	Y	CM
224	Tabulated Johnson Cook	Y	Y	Y	Y		Y	Y	HY, MT, PL
224_GYS	Tabulated Johnson Cook GYS	Y	Y	Y	Y		Y	Y	HY, MT, PL
225	Viscoplastic Mixed Hardening	Y	Y						MT, PL
226	Kinematic hardening Barlat 89					Y			MT
230	Elastic Perfectly Matched Layer (PML)	Y							SL
231	Acoustic PML								FL
232	Biot Linear Hysteretic Material	Y							SL
233	Cazacu Barlat					Y		Y	MT
234	Viscoelastic Loose Fabric	Y	Y			Y		Y	Fabric
235	Micromechanic Dry Fabric					Y		Y	Fabric
236	SCC_on_RCC		Y			Y		Y	CM, CR
237	Biot Hysteretic PML	Y							SL
238	Piecewise linear plasticity (PERT)	Y	Y						MT, PL
240	Cohesive mixed mode	Y	Y			Y	Y	Y	AD
241	Johnson Holmquist JH1	Y	Y				Y	Y	CR, GL
242	Kinematic hardening Barlat 2000					Y			MT
243	Hill 90	Y			Y	Y			MT
244	UHS Steel	Y			Y				MT
245	Orthotropic/anisotropic PML	Y							SL
246	Null material PML			Y					FL
248	PHS BMW	Y			Y	Y			MT

Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
249	Reinforced Thermoplastic				Y	Y		Y	CM, F
249_	Reinforced Thermoplastic UDFiber				Y	Y		Y	CM, F
251	Tailored Properties	Y	Y						MT, PL
252	Toughened Adhesive Polymer	Y	Y		Y	Y	Y	Y	AD
254	Generalized Phase Change	Y			Y				MT
255	Piecewise linear plastic thermal	Y	Y		Y			Y	MT
256	Amorphous solid (finite strain)	Y						Y	GL
260A	Stoughton non-associated flow	Y				Y			MT
260B	Mohr non-associated flow	Y	Y		Y	Y	Y		MT
261	Laminated Fracture Daimler Pinho		Y			Y	Y	Y	CM
262	Laminated Fracture Daimler Camanho		Y			Y	Y	Y	CM
264	Tabulated Johnson Cook Orthotropic Plasticity	Y	Y	Y	Y	Y	Y	Y	HY, MT, PL
266	Dispersed tissue					Y			BIO
267	Eight chain rubber	Y				Y			RB, PL
269	Bergström Boyce rubber	Y							RB
270	Welding material				Y				MT,PL
271	Powder compaction							Y	CR,SL
272	RHT concrete model	Y	Y				Y	Y	SL,CIV
273	Concrete damage plastic	Y	Y				Y	Y	SL
274	Paper					Y		Y	CM,PL
275	Smooth viscoelastic viscoplastic	Y							MT,PL
276	Chronological viscoelastic	Y			Y				RB
277	Adhesive curing viscoelastic	Y			Y				PL,RB
278	CF Micromechanics	Y	Y		Y	Y			CM
279	Cohesive Paper		Y					Y	AD
280	Glass					Y	Y	Y	GL
293	COMPRF	Y				Y		Y	CM
A01	ALE Vacuum								FL
A02	ALE Gas Mixture				Y				FL



Material Number And Description		SRATE	FAIL	EOS	THERMAL	ANISO	DAM	TENS	APPS
A03	ALE Viscous			Y				Y	FL
A04	ALE Mixing Length								FL
A05	ALE Incompressible								FL
A06	ALE Herschel			Y				Y	FL
SPH01	SPH Viscous			Y				Y	FL
S1	Spring Elastic (Linear)								
S2	Damper Viscous (Linear)	Y							
S3	Spring Elastoplastic (Isotropic)								
S4	Spring Nonlinear Elastic	Y						Y	
S5	Damper Nonlinear Viscous	Y						Y	
S6	Spring General Nonlinear							Y	
S7	Spring Maxwell (3-Parameter Viscoelastic)	Y							
S8	Spring Inelastic (Tension or Compression)							Y	
S13	Spring Trilinear Degrading		Y				Y		CIV
S14	Spring Squat Shearwall						Y		CIV
S15	Spring Muscle	Y						Y	BIO
B1	Seatbelt							Y	
T01	Thermal Isotropic				Y				HT
T02	Thermal Orthotropic				Y	Y			HT
T03	Thermal Isotropic (Temp Dependent)				Y				HT
T04	Thermal Orthotropic (Temp Dependent)				Y	Y			HT
T05	Thermal Discrete Beam				Y				HT
T07	Thermal CWM (Welding)				Y				HT
T08	Thermal Orthotropic(Temp dep-load curve)				Y	Y			HT
T09	Thermal Isotropic (Phase Change)				Y				HT
T10	Thermal Isotropic (Temp dep-load curve)				Y				HT
T11	Thermal User Defined				Y				HT

# ALPHABETIZED MATERIALS LIST

## Alphabetized Materials List

Material Keyword	Number
*EOS_GASKET	*EOS_015
*EOS_GRUNEISEN	*EOS_004
*EOS_IDEAL_GAS	*EOS_012
*EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE	*EOS_007
*EOS_JWL	*EOS_002
*EOS_JWLB	*EOS_014
*EOS_LINEAR_POLYNOMIAL	*EOS_001
*EOS_LINEAR_POLYNOMIAL_WITH_ENERGY_LEAK	*EOS_006
*EOS_MIE_GRUNEISEN	*EOS_016
*EOS_MURNAGHAN	*EOS_019
*EOS_PHASE_CHANGE	*EOS_013
*EOS_PROPELLENT_DEFLAGRATION	*EOS_010
*EOS_RATIO_OF_POLYNOMIALS	*EOS_005
*EOS_SACK_TUESDAY	*EOS_003
*EOS_TABULATED	*EOS_009
*EOS_TABULATED_COMPACTION	*EOS_008
*EOS_TENSOR_PORE_COLLAPSE	*EOS_011
*EOS_USER_DEFINED	*EOS_021-*EOS_030
*MAT_{OPTION}TROPIC_ELASTIC	*MAT_002
*MAT_1DOF_GENERALIZED_SPRING	*MAT_146
*MAT_3-PARAMETER_BARLAT	*MAT_036
*MAT_4A_MICROMECH	*MAT_215
*MAT_ACOUSTIC	*MAT_090
*MAT_ADD_AIRBAG_PEROSITY_LEAKAGE	
*MAT_ADD_COHESIVE	
*MAT_ADD_EROSION	
*MAT_ADD_FATIGUE	

# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_ADD_GENERALIZED_DAMAGE	
*MAT_ADD_PERMEABILITY	
*MAT_ADD_PORE_AIR	
*MAT_ADD_THERMAL_EXPANSION	
*MAT_ADHESIVE_CURING_VISCOELASTIC	*MAT_277
*MAT_ALE_GAS_MIXTURE	*MAT_ALE_02
*MAT_ALE_HERSHEL	*MAT_ALE_06
*MAT_ALE_INCOMPRESSIBLE	*MAT_160
*MAT_ALE_MIXING_LENGTH	*MAT_ALE_04
*MAT_ALE_VACUUM	*MAT_ALE_01
*MAT_ALE_VISCOIS	*MAT_ALE_03
*MAT_AMORPHOUS_SOLIDS_FINITE_STRAIN	*MAT_256
*MAT_ANISOTROPIC_ELASTIC	*MAT_002_ANISO
*MAT_ANISOTROPIC_ELASTIC_PLASTIC	*MAT_157
*MAT_ANISOTROPIC_PLASTIC	*MAT_103_P
*MAT_ANISOTROPIC_THERMOELASTIC	*MAT_189
*MAT_ANISOTROPIC_VISCOPLASTIC	*MAT_103
*MAT_ARRUDA_BOYCE_RUBBER	*MAT_127
*MAT_ARUP_ADHESIVE	*MAT_169
*MAT_BAMMAN	*MAT_051
*MAT_BAMMAN_DAMAGE	*MAT_052
*MAT_BARLAT_ANISOTROPIC_PLASTICITY	*MAT_033
*MAT_BARLAT_YLD2000	*MAT_133
*MAT_BARLAT_YLD96	*MAT_033_96
*MAT_BERGSTROM_BOYCE_RUBBER	*MAT_269
*MAT_BILKHU/DUBOIS_FOAM	*MAT_075
*MAT_BIOT_HYSTERETIC	*MAT_232
*MAT_BLATZ-KO_FOAM	*MAT_038
*MAT_BLATZ-KO_RUBBER	*MAT_007
*MAT_BOLT_BEAM	*MAT_208

# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_BRAIN_LINEAR_VISCOELASTIC	*MAT_164
*MAT_BRITTLE_DAMAGE	*MAT_096
*MAT_CABLE_DISCRETE_BEAM	*MAT_071
*MAT_CAZACU_BARLAT	*MAT_233
*MAT_CELLULAR_RUBBER	*MAT_087
*MAT_CF_MICROMECHANICS	*MAT_278
*MAT_CHRONOLOGICAL_VISCOELASTIC	*MAT_276
*MAT_CLOSED_CELL_FOAM	*MAT_053
*MAT_CODAM2	*MAT_219
*MAT_COHESIVE_ELASTIC	*MAT_184
*MAT_COHESIVE_GENERAL	*MAT_186
*MAT_COHESIVE_MIXED_MODE	*MAT_138
*MAT_COHESIVE_MIXED_MODE_ELASTOPLASTIC_RATE	*MAT_240
*MAT_COHESIVE_PAPER	*MAT_279
*MAT_COHESIVE_TH	*MAT_185
*MAT_COMPOSITE_DAMAGE	*MAT_022
*MAT_COMPOSITE_DIRECT	*MAT_118
*MAT_COMPOSITE_DMG_MSC	*MAT_162
*MAT_COMPOSITE_FAILURE_{OPTION}_MODEL	*MAT_059
*MAT_COMPOSITE_LAYUP	*MAT_116
*MAT_COMPOSITE_MATRIX	*MAT_117
*MAT_COMPOSITE_MSC	*MAT_161
*MAT_COMPRF	*MAT_293
*MAT_CONCRETE_BEAM	*MAT_195
*MAT_CONCRETE_DAMAGE	*MAT_072
*MAT_CONCRETE_DAMAGE_PLASTIC_MODEL	*MAT_273
*MAT_CONCRETE_DAMAGE_REL3	*MAT_072R3
*MAT_CONCRETE_EC2	*MAT_172
*MAT_CORUS_VEGTER	*MAT_136
*MAT_CRUSHABLE_FOAM	*MAT_063

# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_CSCM_{OPTION}	*MAT_159
*MAT_CWM	*MAT_270
*MAT_DAMAGE_1	*MAT_104
*MAT_DAMAGE_2	*MAT_105
*MAT_DAMAGE_3	*MAT_153
*MAT_DAMPER_NONLINEAR_VISCOUS	*MAT_S05
*MAT_DAMPER_VISCOUS	*MAT_S02
*MAT_DESHPANDE_FLECK_FOAM	*MAT_154
*MAT_DRUCKER_PRAGER	*MAT_193
*MAT_DRY_FABRIC	*MAT_214
*MAT_EIGHT_CHAIN_RUBBER	*MAT_267
*MAT_ELASTIC	*MAT_001
*MAT_ELASTIC_6DOF_SPRING_DISCRETE_BEAM	*MAT_093
*MAT_ELASTIC_FLUID	*MAT_001_FLUID
*MAT_ELASTIC_PHASE_CHANGE	*MAT_216
*MAT_ELASTIC_PLASTIC_HYDRO_{OPTION}	*MAT_010
*MAT_ELASTIC_PLASTIC_THERMAL	*MAT_004
*MAT_ELASTIC_SPRING_DISCRETE_BEAM	*MAT_074
*MAT_ELASTIC_VISCOPLASTIC_THERMAL	*MAT_106
*MAT_ELASTIC_WITH_VISCOSITY	*MAT_060
*MAT_ELASTIC_WITH_VISCOSITY_CURVE	*MAT_060C
*MAT_EMMI	*MAT_151
*MAT_ENHANCED_COMPOSITE_DAMAGE	*MAT_054-055
*MAT_EXTENDED_3-PARAMETER_BARLAT	*MAT_036E
*MAT_FABRIC	*MAT_034
*MAT_FABRIC_MAP	*MAT_034M
*MAT_FHWA_SOIL	*MAT_147
*MAT_FHWA_SOIL_NEBRASKA	*MAT_147_N
*MAT_FINITE_ELASTIC_STRAIN_PLASTICITY	*MAT_112
*MAT_FLD_3-PARAMETER_BARLAT	*MAT_190

# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_FLD_TRANSVERSELY_ANISOTROPIC	*MAT_039
*MAT_FORCE_LIMITED	*MAT_029
*MAT_FRAZER_NASH_RUBBER_MODEL	*MAT_031
*MAT_FU_CHANG_FOAM	*MAT_083
*MAT_GAS_MIXTURE	*MAT_148
*MAT_GENERAL_JOINT_DISCRETE_BEAM	*MAT_097
*MAT_GENERAL_NONLINEAR_1DOF_DISCRETE_BEAM	*MAT_121
*MAT_GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM	*MAT_119
*MAT_GENERAL_SPRING_DISCRETE_BEAM	*MAT_196
*MAT_GENERAL_VISCOELASTIC	*MAT_076
*MAT_GENERALIZED_PHASE_CHANGE	*MAT_254
*MAT_GEOLOGIC_CAP_MODEL	*MAT_025
*MAT_GEPLASTIC_SRATE_2000a	*MAT_101
*MAT_GLASS	*MAT_280
*MAT_GURSON	*MAT_120
*MAT_GURSON_JC	*MAT_120_JC
*MAT_GURSON_RCDC	*MAT_120_RCDC
*MAT_HEART_TISSUE	*MAT_128
*MAT_HIGH_EXPLOSIVE_BURN	*MAT_008
*MAT_HILL_3R	*MAT_122
*MAT_HILL_3R_3D	*MAT_122_3D
*MAT_HILL_3R_TABULATED	*MAT_122_TAB
*MAT_HILL_90	*MAT_243
*MAT_HILL_FOAM	*MAT_177
*MAT_HONEYCOMB	*MAT_026
*MAT_HYDRAULIC_GAS_DAMPER_DISCRETE_BEAM	*MAT_070
*MAT_HYPERELASTIC_RUBBER	*MAT_077_H
*MAT_HYSTERETIC_REINFORCEMENT	*MAT_203
*MAT_HYSTERETIC_SOIL	*MAT_079
*MAT_INELASTC_6DOF_SPRING_DISCRETE_BEAM	*MAT_095

# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_INELASTIC_6DOF_SPRING_DISCRETE_BEAM	*MAT_095
*MAT_INELASTIC_SPRING_DISCRETE_BEAM	*MAT_094
*MAT_INV_HYPERBOLIC_SIN(_THERMAL)	*MAT_102(_T)
*MAT_ISOTROPIC_ELASTIC_FAILURE	*MAT_013
*MAT_ISOTROPIC_ELASTIC_PLASTIC	*MAT_012
*MAT_ISOTROPIC_SMEARED_CRACK	*MAT_131
*MAT_JOHNSON_COOK	*MAT_015
*MAT_JOHNSON_HOLMQUIST_CERAMICS	*MAT_110
*MAT_JOHNSON_HOLMQUIST_CONCRETE	*MAT_111
*MAT_JOHNSON_HOLMQUIST_JH1	*MAT_241
*MAT_JOINTED_ROCK	*MAT_198
*MAT_KELVIN-MAXWELL_VISCOELASTIC	*MAT_061
*MAT_KINEMATIC_HARDENING_BARLAT2000	*MAT_242
*MAT_KINEMATIC_HARDENING_BARLAT89	*MAT_226
*MAT_KINEMATIC_HARDENING_TRANSVERSELY_ANISOTROPIC	*MAT_125
*MAT_LAMINATED_COMPOSITE_FABRIC	*MAT_058
*MAT_LAMINATED_FRACTURE_DAIMLER_CAMANHO	*MAT_262
*MAT_LAMINATED_FRACTURE_DAIMLER_PINHO	*MAT_261
*MAT_LAMINATED_GLASS	*MAT_032
*MAT_LAYERED_LINEAR_PLASTICITY	*MAT_114
*MAT_LINEAR_ELASTIC_DISCRETE_BEAM	*MAT_066
*MAT_LOW_DENSITY_FOAM	*MAT_057
*MAT_LOW_DENSITY_SYNTHETIC_FOAM_{OPTION}	*MAT_179
*MAT_LOW_DENSITY_VISCOUS_FOAM	*MAT_073
*MAT_LUNG_TISSUE	*MAT_129
*MAT_MCCORMICK	*MAT_167
*MAT_MICROMECHANICS_DRY_FABRIC	*MAT_235
*MAT_MODIFIED_CRUSHABLE_FOAM	*MAT_163
*MAT_MODIFIED_FORCE_LIMITED	*MAT_139
*MAT_MODIFIED_HONEYCOMB	*MAT_126

# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_MODIFIED_JOHNSON_COOK	*MAT_107
*MAT_MODIFIED_PIECEWISE_LINEAR_PLASTICITY	*MAT_123
*MAT_MODIFIED_ZERILLI_ARMSTRONG	*MAT_065
*MAT_MOHR_COULOMB	*MAT_173
*MAT_MOHR_NON_ASSOCIATED_FLOW	*MAT_260B
*MAT_MOMENT_CURVATURE_BEAM	*MAT_166
*MAT_MOONEY-RIVLIN_RUBBER	*MAT_027
*MAT_MOONEY-RIVLIN_RUBBER_PHASE_CHANGE	*MAT_218
*MAT_MTS	*MAT_088
*MAT_MUSCLE	*MAT_156
*MAT_NONLINEAR_ELASTIC_DISCRETE_BEAM	*MAT_067
*MAT_NONLINEAR_ORTHOTROPIC	*MAT_040
*MAT_NONLINEAR_PLASTIC_DISCRETE_BEAM	*MAT_068
*MAT_NONLOCAL	
*MAT_NULL	*MAT_009
*MAT_OGDEN_RUBBER	*MAT_077_O
*MAT_OPTION_TROPIC_ELASTIC	*MAT_002
*MAT_OPTION_TROPIC_ELASTIC_PHASE_CHANGE	*MAT_217
*MAT_ORIENTED_CRACK	*MAT_017
*MAT_ORTHO_ELASTIC_PLASTIC	*MAT_108
*MAT_ORTHOTROPIC_SIMPLIFIED_DAMAGE	*MAT_221
*MAT_ORTHOTROPIC_SMEARED_CRACK	*MAT_132
*MAT_ORTHOTROPIC_THERMAL	*MAT_021
*MAT_ORTHOTROPIC_VISCOELASTIC	*MAT_086
*MAT_PAPER	*MAT_274
*MAT_PERT_PIECEWISE_LINEAR_PLASTICITY	*MAT_238
*MAT_PHS_BMW	*MAT_248
*MAT_PIECEWISE_LINEAR_PLASTIC_THERMAL	*MAT_255
*MAT_PIECEWISE_LINEAR_PLASTICITY	*MAT_024
*MAT_PITZER_CRUSHABLEFOAM	*MAT_144



# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_PLASTIC_GREEN-NAGHDI_RATE	*MAT_035
*MAT_PLASTIC_KINEMATIC	*MAT_003
*MAT_PLASTIC_NONLINEAR_KINEMATIC	*MAT_165
*MAT_PLASTIC_NONLINEAR_KINEMATIC_B	*MAT_165B
*MAT_PLASTICITY_COMPRESSION_TENSION	*MAT_124
*MAT_PLASTICITY_COMPRESSION_TENSION_EOS	*MAT_155
*MAT_PLASTICITY_POLYMER	*MAT_089
*MAT_PLASTICITY_WITH_DAMAGE	*MAT_081
*MAT_PLASTICITY_WITH_DAMAGE_ORTHO(_RCDC)	*MAT_082(_RCDC)
*MAT_PML_{OPTION}TROPIC_ELASTIC	*MAT_245
*MAT_PML_ACOUSTIC	*MAT_231
*MAT_PML_ELASTIC	*MAT_230
*MAT_PML_ELASTIC_FLUID	*MAT_230
*MAT_PML_HYSTERETIC	*MAT_237
*MAT_PML_NULL	*MAT_246
*MAT_POLYMER	*MAT_168
*MAT_POWDER	*MAT_271
*MAT_POWER_LAW_PLASTICITY	*MAT_018
*MAT_PSEUDO_TENSOR	*MAT_016
*MAT_QUASILINEAR_VISCOELASTIC	*MAT_176
*MAT_RAMBERG-OSGOOD	*MAT_080
*MAT_RATE_SENSITIVE_COMPOSITE_FABRIC	*MAT_158
*MAT_RATE_SENSITIVE_POLYMER	*MAT_141
*MAT_RATE_SENSITIVE_POWERLAW_PLASTICITY	*MAT_064
*MAT_RC_Beam	*MAT_174
*MAT_RC_SHEAR_WALL	*MAT_194
*MAT_REINFORCED_THERMOPLASTIC	*MAT_249
*MAT_REINFORCED_THERMOPLASTIC_UDFIBER	*MAT_249_ UDFIBER
*MAT_RESULTANT_ANISOTROPIC	*MAT_170

# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_RESULTANT_PLASTICITY	*MAT_028
*MAT_RHT	*MAT_272
*MAT_RIGID	*MAT_020
*MAT_RIGID_DISCRETE	*MAT_220
*MAT_SAMP-1	*MAT_187
*MAT_SCC_ON_RCC	*MAT_236
*MAT_SCHWER_MURRAY_CAP_MODEL	*MAT_145
*MAT_SEATBELT	*MAT_B01
*MAT_SEISMIC_BEAM	*MAT_191
*MAT_SEISMIC_ISOLATOR	*MAT_197
*MAT_SHAPE_MEMORY	*MAT_030
*MAT_SID_DAMPER_DISCRETE_BEAM	*MAT_069
*MAT_SIMPLIFIED_JOHNSON_COOK	*MAT_098
*MAT_SIMPLIFIED_JOHNSON_COOK_ORTHOTROPIC_DAMAGE	*MAT_099
*MAT_SIMPLIFIED_RUBBER/FOAM_{OPTION}	*MAT_181
*MAT_SIMPLIFIED_RUBBER_WITH_DAMAGE	*MAT_183
*MAT_SMOOTH_VISCOELASTIC_VISCOPLASTIC	*MAT_275
*MAT_SOFT_TISSUE	*MAT_091
*MAT_SOFT_TISSUE_VISCO	*MAT_092
*MAT_SOIL_AND_FOAM	*MAT_005
*MAT_SOIL_AND_FOAM_FAILURE	*MAT_014
*MAT_SOIL_BRICK	*MAT_192
*MAT_SOIL_CONCRETE	*MAT_078
*MAT_SPECIAL_ORTHOTROPIC	*MAT_130
*MAT_SPH_VISCOUS	*MAT_SPH_01
*MAT_SPOTWELD_{OPTION}	*MAT_100
*MAT_SPOTWELD_DAIMLERCHRYSLER	*MAT_100_DA
*MAT_SPR_JLR	*MAT_211
*MAT_SPRING_ELASTIC	*MAT_S01
*MAT_SPRING_ELASTOPLASTIC	*MAT_S03

# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_SPRING_GENERAL_NONLINEAR	*MAT_S06
*MAT_SPRING_INELASTIC	*MAT_S08
*MAT_SPRING_MAXWELL	*MAT_S07
*MAT_SPRING_MUSCLE	*MAT_S15
*MAT_SPRING_NONLINEAR_PLASTIC	*MAT_S04
*MAT_SPRING_SQUAT_SHEARWALL	*MAT_S14
*MAT_SPRING_TRILINEAR_DEGRADING	*MAT_S13
*MAT_STEEL_CONCENTRIC_BRACE	*MAT_171
*MAT_STEEL_EC3	*MAT_202
*MAT_STEINBERG	*MAT_011
*MAT_STEINBERG_LUND	*MAT_011_LUND
*MAT_STOUGHTON_NON_ASSOCIATED_FLOW	*MAT_260A
*MAT_STRAIN_RATE_DEPENDENT_PLASTICITY	*MAT_019
*MAT_TABULATED_JOHNSON_COOK	*MAT_224
*MAT_TABULATED_JOHNSON_COOK_GYS	*MAT_224_GYS
*MAT_TABULATED_JOHNSON_COOK_ORTHO_PLASTICITY	*MAT_264
*MAT_TAILORED_PROPERTIES	*MAT_251
*MAT_TEMPERATURE_DEPENDENT_ORTHOTROPIC	*MAT_023
*MAT_THERMAL_CHEMICAL_REACTION	*MAT_T06
*MAT_THERMAL_CWM	*MAT_T07
*MAT_THERMAL_DISCRETE_BEAM	*MAT_T05
*MAT_THERMAL_ISOTROPIC	*MAT_TO1
*MAT_THERMAL_ISOTROPIC_PHASE_CHANGE	*MAT_T09
*MAT_THERMAL_ISOTROPIC_TD	*MAT_T03
*MAT_THERMAL_ISOTROPIC_TD_LC	*MAT_T10
*MAT_THERMAL_OPTION	*MAT_T00
*MAT_THERMAL_ORTHOTROPIC	*MAT_T02
*MAT_THERMAL_ORTHOTROPIC_TD	*MAT_T04
*MAT_THERMAL_ORTHOTROPIC_TD_LC	*MAT_T08
*MAT_THERMAL_USER_DEFINED	*MAT_T11

# ALPHABETIZED MATERIALS LIST

Material Keyword	Number
*MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP	*MAT_188
*MAT_TISSUE_DISPERSED	*MAT_266
*MAT_TOUGHENED_ADHESIVE_POLYMER	*MAT_252
*MAT_TRANSVERSELY_ANISOTROPIC_ELASTIC_PLASTIC	*MAT_037
*MAT_TRANSVERSELY_ISOTROPIC_CRUSHABLE_FOAM	*MAT_142
*MAT_TRIP	*MAT_113
*MAT_UHS_STEEL	*MAT_244
*MAT_UNIFIED_CREEP	*MAT_115
*MAT_UNIFIED_CREEP_ORTHO	*MAT_115_O
*MAT_USER_DEFINED_MATERIAL_MODELS	*MAT_041-050
*MAT_VACUUM	*MAT_140
*MAT_VISCOELASTIC	*MAT_006
*MAT_VISCOELASTIC_FABRIC	*MAT_134
*MAT_VISCOELASTIC_HILL_FOA	*MAT_178
*MAT_VISCOELASTIC_LOOSE_FABRIC	*MAT_234
*MAT_VISCOELASTIC_THERMAL	*MAT_175
*MAT_VISCOPLASTIC_MIXED_HARDENING	*MAT_225
*MAT_VISCOUS_FOAM	*MAT_062
*MAT_WINFIRTH_CONCRETE_REINFORCEMENT	*MAT_084_REINF
*MAT_WINFRITH_CONCRETE	*MAT_084
*MAT_WOOD_{OPTION}	*MAT_143
*MAT_WTM_STM	*MAT_135
*MAT_WTM_STM_PLC	*MAT_135_PLC

\*MAT\_ADD\_AIRBAG\_POROSITY\_LEAKAGE

This command allows users to model porosity leakage through non-fabric material when such material is used as part of control volume, airbag. It applies to both \*AIRBAG\_HYBRID and \*AIRBAG\_WANG\_NEFSKE.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	FLC/X2	FAC/X3	ELA	FVOPT	X0	X1	
Type	I	F	F	F	F	F	F	
Default	none	none	1.0	none	none	none	none	

**VARIABLE**

**DESCRIPTION**

MID Material ID for which the porosity leakage property applies

FLC/X2 If  $X0 \neq 0$  and  $X0 \neq 1$   
 X2 is one of the coefficients of the porosity in the equation of Anagonye and Wang [1999]. (Defined below in description for X0/X1)

If  $X0 = 0$   
 GE.0.0: X2, in this context named FLC, is an optional fabric porous leakage flow coefficient.

LT.0.0: |FLC| is the load curve ID of the curve defining FLC versus time.

If  $X0 = 1$   
 GE.0.0: See  $X0 = 0$  above.  
 LT.0.0: |FLC| is the load curve ID defining FLC versus the stretching ratio defined as  $r_s = A/A_0$ . See notes below.

FAC/X3 If  $X0 \neq 0$  and  $X0 \neq 1$   
 X3 is one of the coefficients of the porosity in the equation of Anagonye and Wang [1999]. (Defined below in description for X0/X1)

If  $X0 = 0$  and  $FVOPT < 7$   
 GE.0.0: X3, in this context named FAC, is an optional fabric characteristic parameter.

LT.0.0: |FAC| is the load curve ID of the curve defining FAC versus absolute pressure.

If X0 = 1 and FVOPT < 7

GE.0.0: See X0 = 0 and FVOPT < 7 above.

LT.0.0: |FAC| is the load curve ID defining FAC versus the pressure ratio defined as  $r_p = P_{\text{air}}/P_{\text{bag}}$ . See remark 3 of \*MAT\_FABRIC.

If (X0 = 0 or X0 = 1) and (FVOPT = 7 or FVOPT = 8)

GE.0.0: See X0 = 0 and FVOPT < 7 above.

LT.0.0: FAC defines leakage volume flux rate versus absolute pressure. The volume flux (per area) rate (per time) has the unit of velocity and it is equivalent to relative porous gas speed.

$$\left[ \frac{d(\text{Vol}_{\text{flux}})}{dt} \right] = \frac{[\text{volume}]}{[\text{area}]} \frac{1}{[\text{time}]} = \frac{[\text{length}]}{[\text{time}]} = [\text{velocity}],$$

ELA

Effective leakage area for blocked fabric, ELA.

LT.0.0: |ELA| is the load curve ID of the curve defining ELA versus time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas.

FVOPT

Fabric venting option.

EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered.

EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered.

EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered.

EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered.

EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered.

EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered.

EQ.7: Leakage is based on gas volume outflow versus pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.

EQ.8: Leakage is based on gas volume outflow versus pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.

X0, X1

Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area:

$$A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$$

**\*MAT\_ADD\_COHESIVE**

The ADD\_COHESIVE option offers the possibility to use a selection of three dimensional material models in LS-DYNA in conjunction with cohesive elements.

Usually the cohesive elements (ELFORM = 19 and 20 of \*SECTION\_SOLID) can only be used with a small number of material models (41-50, 138, 184, 185, 186, 240). But with this additional keyword, a larger number of standard three dimensional material models can be used that would only be available for solid elements in general. Currently the following material models are supported: 1, 3, 4, 6, 15, 24, 41-50, 81, 82, 89, 96, 98, 103, 104, 105, 106, 107, 115, 120, 123, 124, 141, 168, 173, 187, 188, 193, 224, 225, 252, and 255.

Card 1	1	2	3	4	5	6	7	8
Variable	PID	ROFLG	INTFAIL	THICK				
Type	I	F	F	F				
Default	none	0.0	0.0	0.0				

**VARIABLE****DESCRIPTION**

PID

Part ID for which the cohesive property applies.

ROFLG

Flag for whether density is specified per unit area or volume.

EQ.0.0: Density specified per unit volume (default).

EQ.1.0: Density specified per unit area for controlling the mass of cohesive elements with an initial volume of zero.

INTFAIL

The number of integration points required for the cohesive element to be deleted. If it is zero, the element will *not* be deleted even if it satisfies the failure criterion. The value of INTFAIL may range from 1 to 4 with 1 the recommended value.

THICK

Thickness of the adhesive layer.

EQ.0.0: The actual thickness of the cohesive element is used.

GT.0.0: User specified thickness.



**Remarks:**

Cohesive elements possess 3 kinematic variables, namely, two relative displacements  $\delta_1, \delta_2$  in tangential directions and one relative displacement  $\delta_3$  in normal direction. In a corresponding constitutive model, they are used to compute 3 associated traction stresses  $t_1, t_2$ , and  $t_3$ . For example, in the elastic case (\*MAT\_COHESIVE\_ELASTIC):

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} E_T & 0 & 0 \\ 0 & E_T & 0 \\ 0 & 0 & E_N \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}.$$

Hypoelastic three dimensional material models for standard solid elements, however, are formulated with respect to 6 independent strain rates and 6 associated stress rates. For isotropic elasticity (\*MAT\_ELASTIC):

$$\begin{bmatrix} \dot{\sigma}_{xx} \\ \dot{\sigma}_{yy} \\ \dot{\sigma}_{zz} \\ \dot{\sigma}_{xy} \\ \dot{\sigma}_{yz} \\ \dot{\sigma}_{zx} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{bmatrix} \begin{bmatrix} \dot{\epsilon}_{xx} \\ \dot{\epsilon}_{yy} \\ \dot{\epsilon}_{zz} \\ \dot{\epsilon}_{xy} \\ \dot{\epsilon}_{yz} \\ \dot{\epsilon}_{zx} \end{bmatrix}.$$

To be able to use such three dimensional material models in a cohesive element environment, an assumption is necessary to transform 3 relative displacements to 6 strain rates. Therefore, it is assumed that neither lateral expansion nor in-plane shearing is possible for the cohesive element. Therefore,

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} \rightarrow \begin{bmatrix} \dot{\epsilon}_{xx} \\ \dot{\epsilon}_{yy} \\ \dot{\epsilon}_{zz} \\ \dot{\epsilon}_{xy} \\ \dot{\epsilon}_{yz} \\ \dot{\epsilon}_{zx} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\delta}_3/(t + \delta_3) \\ 0 \\ \dot{\delta}_2/(t + \delta_3) \\ \dot{\delta}_1/(t + \delta_3) \end{bmatrix},$$

where  $t$  is the initial thickness of the adhesive layer; see parameter THICK. These strain rates are then used in a three dimensional constitutive model to obtain new Cauchy stresses, where 3 components can finally be used for the cohesive element:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} \rightarrow \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} \sigma_{zx} \\ \sigma_{yz} \\ \sigma_{zz} \end{bmatrix}.$$

If this keyword is used in combination with a three dimensional material model, the output to d3plot or elout is organized as with other material models for cohesive elements; see for example \*MAT\_184. Instead of the usual six stress components, three traction stresses are written into those databases. The in-plane shear traction along the 1-2 edge replaces the  $x$

component of stress, the orthogonal in-plane shear traction replaces the  $y$  component of stress, and the traction in the normal direction replaces the  $z$  component of stress.

**\*MAT\_ADD\_EROSION**

Many of the constitutive models in LS-DYNA do not allow failure and erosion. The ADD\_EROSION option provides a way of including failure in these models. This option can also be applied to constitutive models that already include other failure/erosion criterion.

For the non-damage options, each of the failure criteria defined here are applied independently, and once a sufficient number of those criteria are satisfied according to NCS, the element is deleted from the calculation.

In addition to erosion, the “generalized incremental stress-state dependent damage model” (GISSMO) or alternative “damage initiation and evolution models” (DIEM) are available as described in the remarks. See variable IDAM. For DIEM, NCS has a special meaning, see description below for details.

This option applies to nonlinear element formulations including the 2D continuum elements, 3D solid elements, 2D and 3D SPH particles, 3D shell elements, and thick shell elements. Beam formulations 1 and 11 currently support the erosion but not the damage and evolution models.

**NOTE:** that all \*MAT\_ADD\_EROSION commands in a model can be disabled by using \*CONTROL\_MAT.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	EXCL	MXPRES	MNEPS	EFFEPS	VOLEPS	NUMFIP	NCS
Type	A8	F	F	F	F	F	F	F
Default	none	none	0.0	0.0	0.0	0.0	1.0	1.0/0.0

Card 2	1	2	3	4	5	6	7	8
Variable	MNPRES	SIGP1	SIGVM	MXEPS	EPSSH	SIGTH	IMPULSE	FAILTM
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

**\*MAT****\*MAT\_ADD\_EROSION**

The following card is optional:

Card 3	1	2	3	4	5	6	7	8
Variable	IDAM	DMGTYP	LCSDG	ECRIT	DMGEXP	DCRIT	FADEXP	LCREGD
Type	A8	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0

Additional card for IDAM > 0.

Card 4	1	2	3	4	5	6	7	8
Variable	SIZFLG	REFSZ	NAHSV	LCSRS	SHRF	BIAXF		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

**Damage Initiation and Evolution Card Pairs.** For IDAM < 0 include | IDAM | pairs of Cards 5 and 6.

Card 5	1	2	3	4	5	6	7	8
Variable	DITYP	P1	P2	P3				
Type	F	F	F	F				
Default	0.0	0.0	0.0	0.0				

Card 6	1	2	3	4	5	6	7	8
Variable	DEtyp	DCTYP	Q1	Q2				
Type	F	F	F	F				
Default	0.0	0.0	0.0	0.0				

**Optional Cards with additional failure criteria.**

Card 7	1	2	3	4	5	6	7	8
Variable	LCFLD	NSFF	EPSTHIN	ENGCRT	RADCRT	LCEPS12	LCEPS13	LCEPSMX
Type	F	F	F	F	F	I	I	I
Default	0.0	0.0	0.0	0.0	0.0	0	0	0

Card 8	1	2	3	4	5	6	7	8
Variable			MXTMP					
Type			F					
Default			none					

**VARIABLE**

**DESCRIPTION**

MID

Material identification for which this erosion definition applies. A unique number or label not exceeding 8 characters must be specified.

VARIABLE	DESCRIPTION
EXCL	<p>The exclusion number, which applies to the failure values defined on Cards 1, 2, and 7. When any of the failure values on these cards are set to the exclusion number, the associated failure criterion is not invoked. Or in other words, only the failure values not set to the exclusion number are invoked. The default value of EXCL is 0.0, which eliminates all failure criteria from consideration that have their constants left blank or set to 0.0.</p> <p>As an example, to prevent a material from developing tensile pressure, the user could specify an unusual value for the exclusion number, e.g., 1234, set MNPRES to 0.0, and set all the remaining failure values to 1234. However, use of an exclusion number may be considered nonessential since the same effect could be achieved without use of the exclusion number by setting MNPRES to a very small negative value.</p>
MXPRES	<p>Maximum pressure at failure, <math>P_{\max}</math>. If the value is exactly zero, it is automatically excluded to maintain compatibility with old input files.</p>
MNEPS	<p>Minimum principal strain at failure, <math>\epsilon_{\min}</math>. If the value is exactly zero, it is automatically excluded to maintain compatibility with old input files.</p>
EFFEPS	<p>Maximum effective strain at failure:</p> $\epsilon_{\text{eff}} = \sum_{ij} \sqrt{\frac{2}{3}} \epsilon_{ij}^{\text{dev}} \epsilon_{ij}^{\text{dev}}.$ <p>If the value is exactly zero, it is automatically excluded to maintain compatibility with old input files. If the value is negative, then <math> \text{EFFEPS} </math> is the effective plastic strain to failure. In combination with cohesive elements, EFFEPS is the maximum effective in-plane strain.</p>
VOLEPS	<p>Volumetric strain at failure,</p> $\epsilon_{\text{vol}} = \epsilon_{11} + \epsilon_{22} + \epsilon_{33},$ <p>or</p> $\ln(\text{relative volume}).$ <p>VOLEPS can be a positive or negative number depending on whether the failure is in tension or compression, respectively. If the value is exactly zero, it is automatically excluded to maintain compatibility with old input files.</p>

VARIABLE	DESCRIPTION
NUMFIP	<p>Number of failed integration points prior to element deletion. The default is unity. See Remark 11.</p> <p>LT.0.0 (IDAM = 0): Only for shells.  NUMFIP  is the percentage of integration points which must exceed the failure criterion before element fails. If NUMFIP &lt; -100, then  NUMFIP  -100 is the number of failed integration points prior to element deletion.</p> <p>LT.0.0 (IDAM ≠ 0): Only for shells.  NUMFIP  is the percentage of layers which must fail before element fails. For shell formulations with 4 integration points per layer, the layer is considered failed if any of the integration points in the layer fails.</p>
NCS	<p>This meaning of this input depends on whether the damage option DIEM is used or not.</p> <p>IDAM.GE.0: Number of failure conditions to satisfy before failure occurs. For example, if SIGP1 and SIGVM are defined and if NCS = 2, both failure criteria must be met before element deletion can occur. The default is set to unity.</p> <p>IDAM.LT.0: Plastic strain increment between evaluation of damage instability and evolution criteria. See DIEM description, the default is zero.</p>
MNPRES	Minimum pressure at failure, $P_{\min}$ .
SIGP1	Principal stress at failure, $\sigma_{\max}$ .
SIGVM	Equivalent stress at failure, $\bar{\sigma}_{\max}$ . The equivalent stress at failure is made a function of the effective strain rate by setting SIGVM to the negative of the appropriate load curve ID.
MXEPS	Maximum principal strain at failure, $\varepsilon_{\max}$ . The maximum principal strain at failure is made a function of the effective strain rate by setting MXEPS to the negative of the appropriate load curve ID.
EPSSH	Tensorial shear strain at failure, $\gamma_{\max}/2$ .
SIGTH	Threshold stress, $\sigma_0$ .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
IMPULSE	Stress impulse for failure, $K_f$ .
FAILTM	Failure time. When the problem time exceeds the failure time, the material is removed.
IDAM	Flag for damage model. EQ.0: no damage model is used. EQ.1: GISSMO damage model. LT.0: -IDAM represents the number of damage initiation and evolution model (DIEM) criteria to be applied
DMGTYP	<p><u>For GISSMO damage type the following applies.</u> DMGTYP is interpreted digit-wise as follows:</p> $DMGTYP = [NM] = M + 10 \times N$ <p>M.EQ.0: Damage is accumulated, no coupling to flow stress, no failure. M.EQ.1: Damage is accumulated, element failure occurs for <math>D = 1</math>. Coupling of damage to flow stress depending on parameters, see remarks below. N.EQ.0: Equivalent plastic strain is the driving quantity for the damage. (To be more precise, it's the history variable that LS-PrePost blindly labels as "plastic strain". What this history variable actually represents depends on the material model.) N.GT.0: The Nth additional history variable is the driving quantity for damage. These additional history variables are the same ones flagged by the *DATABASE_EXTENT_BINARY keyword's NEIPS and NEIPH fields. For example, for solid elements with *MAT_187 setting <math>N = 6</math> chooses volumetric plastic strain as the driving quantity for the GISSMO damage.</p> <p><u>For IDAM.LT.0 the following applies.</u></p> <p>EQ.0: No action is taken EQ.1: Damage history is initiated based on values of initial plastic strains and initial strain tensor, this is to be used in multistage analyses</p>



VARIABLE	DESCRIPTION
LCSDG	<p>Failure strain curve/table or function. Only for GISSMO (IDAM = 1).</p> <p>GT.0.0: Load curve ID or Table ID. Load curve defines equivalent plastic strain to failure vs. triaxiality. Table defines for each Lode parameter value (between -1 and 1) a load curve ID giving the equivalent plastic strain to failure vs. triaxiality for that Lode parameter value.</p> <p>LT.0.0:  LCSDG  is the ID of a function (*DEFINE_FUNCTION) with the arguments triaxiality and Lode parameter:  <math>f(\eta, L)</math></p>
ECRIT	<p>Critical plastic strain (material instability), see below.</p> <p>LT.0.0:  ECRIT  is either a load curve ID defining critical equivalent plastic strain versus triaxiality or a table ID defining critical equivalent plastic strain as a function of triaxiality and Lode parameter (as in LCSDG).</p> <p>EQ.0.0: Fixed value DCRIT defining critical damage is read (see below)</p> <p>GT.0.0: Fixed value for stress-state independent critical equivalent plastic strain.</p>
DMGEXP	<p>Exponent for nonlinear damage accumulation, see remarks.</p>
DCRIT	<p>Damage threshold value (critical damage). If a Load curve of critical plastic strain or fixed value is given by ECRIT, input is ignored.</p>
FADEXP	<p>Exponent for damage-related stress fadeout.</p> <p>LT.0.0:  FADEXP  is load curve ID defining element-size dependent fading exponent.</p> <p>GT.0.0: Constant fading exponent.</p>

VARIABLE	DESCRIPTION
LCREGD	<p>Load curve ID or Table ID defining element size dependent regularization factors for equivalent plastic strain to failure in the GISSMO damage model. This feature can also be used with the standard (non-GISSMO) failure criteria of Cards 1 (MXPRES, MNEPS, EFFEPS, VOLEPS), 2 (MNPRES, SIGP1, SIGVM, MXEPS, EPSSH, IMPULSE) and 4 (LCFLD, EPSTHIN), i.e. when IDAM = 0. Tables are only supported in GISSMO.</p> <p>GT.0.0: Load curve ID (reg. factor vs. element size) or Table ID (reg. factor vs. element size curves vs. effective rate)</p> <p>LT.0.0:  LCREGD  is Table ID (reg. factor vs. element size curves vs. triaxiality). This table provides an alternative to the use of SHRF and BIAXF for defining the effect of triaxiality on element size regularization of equivalent plastic strain to failure.</p>
SIZFLG	<p>Flag for method of element size determination.</p> <p>EQ.0: (default) Element size is determined in undeformed configuration as square root of element area (shells), or cubic root of element volume (solids), respectively.</p> <p>EQ.1: Element size is updated every time step, and determined as mean edge length (this option was added to ensure compatibility with *MAT_120, and is not recommended for general purpose).</p>
REFSZ	<p>Reference element size, for which an additional output of damage will be generated. This is necessary to ensure the applicability of resulting damage quantities when transferred to different mesh sizes.</p>
NAHSV	<p>Number of history variables from damage model which should be stored in standard material history array for Postprocessing. See remarks.</p>
LCSRS	<p>Load curve ID defining failure strain scaling factor for LCSDG vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate. <i>The curve should not extrapolate to zero or failure may occur at low strain.</i></p> <p>GT.0: scale ECRIT, too</p> <p>LT.0: do not scale ECRIT.</p>

VARIABLE	DESCRIPTION
SHRF	Reduction factor for regularization for shear stress states. This parameter can be defined between -1.0 and +1.0. See remarks below.
BIAXF	Reduction factor for regularization for biaxial stress states. This parameter can be defined between -1.0 and +1.0. See remarks below.
DITYP	Damage initiation type EQ.0.0: Ductile based on stress triaxiality EQ.1.0: Shear EQ.2.0: MSFLD EQ.3.0: FLD EQ.4.0: Ductile based on normalized principal stress
P1	Damage initiation parameter DITYP.EQ.0.0: Load curve/table ID representing plastic strain at onset of damage as function of stress triaxiality $\eta$ and optionally plastic strain rate. DITYP.EQ.1.0: Load curve/table ID representing plastic strain at onset of damage as function of shear influence $\theta$ and optionally plastic strain rate. DITYP.EQ.2.0: Load curve/table ID representing plastic strain at onset of damage as function of ratio of principal plastic strain rates $\alpha$ and optionally plastic strain rate. DITYP.EQ.3.0: Load curve/table ID representing plastic strain at onset of damage as function of ratio of principal plastic strain rates $\alpha$ and optionally plastic strain rate. DITYP.EQ.4.0: Load curve/table ID representing plastic strain at onset of damage as function of stress state parameter $\beta$ and optionally plastic strain rate.
P2	Damage initiation parameter DITYP.EQ.0.0: Not used DITYP.EQ.1.0: Pressure influence parameter $k_s$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	DITYP.EQ.2.0: Layer specification EQ.0: Mid layer EQ.1: Outer layer
	DITYP.EQ.3.0: Layer specification EQ.0: Mid layer EQ.1: Outer layer
	DITYP.EQ.4.0: Triaxiality influence parameter $k_d$
P3	Damage initiation parameter DITYP.EQ.0.0: Not used DITYP.EQ.1.0: Not used DITYP.EQ.2.0: Initiation formulation EQ.0: Direct EQ.1: Incremental DITYP.EQ.3.0: Initiation formulation EQ.0: Direct EQ.1: Incremental DITYP.EQ.4.0: Not used
DETYP	Damage evolution type EQ.0.0: Linear softening, evolution of damage is a function of the plastic displacement after the initiation of damage. EQ.1.0: Linear softening, evolution of damage is a function of the fracture energy after the initiation of damage.
DCTYP	Damage composition option for multiple criteria EQ.-1.0: Damage not coupled to stress EQ.0.0: Maximum EQ.1.0: Multiplicative
Q1	Damage evolution parameter DETYP.EQ.0.0: Plastic displacement at failure, $u_f^p$ , a negative value corresponds to a table ID for $u_f^p$ as a function of triaxiality and damage. DETYP.EQ.1.0: Fracture energy at failure, $G_f$ .

VARIABLE	DESCRIPTION
Q2	Set to 1.0 to output information to log files (messag and d3hsp) when an integration point fails.
LCFLD	Load curve ID or Table ID. Load curve defines the Forming Limit Diagram, where minor engineering strains in percent are defined as abscissa values and major engineering strains in percent are defined as ordinate values. Table defines for each strain rate an associated FLD curve. The forming limit diagram is shown in <a href="#">Figure M39-1</a> . In defining the curve, list pairs of minor and major strains starting with the left most point and ending with the right most point. This criterion is only available for shell elements.
NSFF	Number of explicit time step cycles for stress fade-out used in the LCFLD criterion. Default is 10.
EPSTHIN	Thinning strain at failure for thin and thick shells. GT.0.0: individual thinning for each integration point from z-strain LT.0.0: averaged thinning strain from element thickness change
ENGCRT	Critical energy for nonlocal failure criterion, see <a href="#">Item 9</a> below.
RADCRT	Critical radius for nonlocal failure criterion, see <a href="#">Item 9</a> below.
LCEPS12	Load curve ID defining in-plane shear strain limit $\gamma_{12}^c$ vs. element size. See <a href="#">Remark 10</a> .
LCEPS13	Load curve ID defining through-thickness shear strain limit $\gamma_{13}^c$ vs. element size. See <a href="#">Remark 10</a> .
LCEPSMX	Load curve ID defining in-plane major strain limit $\varepsilon_1^c$ vs. element size. See <a href="#">Remark 10</a> .
MXTMP	Maximum temperature at failure.

**In addition to failure time, supported criteria for failure are:**

1.  $P \geq P_{\max}$ , where  $P$  is the pressure (positive in compression), and  $P_{\max}$  is the maximum pressure at failure.
2.  $\varepsilon_3 \leq \varepsilon_{\min}$ , where  $\varepsilon_3$  is the minimum principal strain, and  $\varepsilon_{\min}$  is the minimum principal strain at failure.

3.  $P \leq P_{\min}$ , where  $P$  is the pressure (positive in compression), and  $P_{\min}$  is the minimum pressure at failure.
4.  $\sigma_1 \geq \sigma_{\max}$ , where  $\sigma_1$  is the maximum principal stress, and  $\sigma_{\max}$  is the maximum principal stress at failure.
5.  $\sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}} \geq \bar{\sigma}_{\max}$ , where  $\sigma'_{ij}$  are the deviatoric stress components, and  $\bar{\sigma}_{\max}$  is the equivalent stress at failure.
6.  $\varepsilon_1 \geq \varepsilon_{\max}$ , where  $\varepsilon_1$  is the maximum principal strain, and  $\varepsilon_{\max}$  is the maximum principal strain at failure.
7.  $\gamma_1 \geq \gamma_{\max}/2$ , where  $\gamma_1$  is the maximum tensorial shear strain =  $(\varepsilon_1 - \varepsilon_3)/2$ , and  $\gamma_{\max}$  is the engineering shear strain at failure.
8. The Tuler-Butcher criterion,

$$\int_0^t [\max(0, \sigma_1 - \sigma_0)]^2 dt \geq K_f,$$

where  $\sigma_1$  is the maximum principal stress,  $\sigma_0$  is a specified threshold stress,  $\sigma_1 \geq \sigma_0 \geq 0$ , and  $K_f$  is the stress impulse for failure. Stress values below the threshold value are too low to cause fracture even for very long duration loadings.

9. A nonlocal failure criterion which is mainly intended for windshield impact can be defined via ENGCRT, RADCRT, and one additional "main" failure criterion (only SIGP1 is available at the moment). All three parameters should be defined for one part, namely the windshield glass and the glass should be discretized with shell elements. The course of events of this nonlocal failure model is as follows: If the main failure criterion SIGP1 is fulfilled, the corresponding element is flagged as center of impact, but no element erosion takes place yet. Then, the internal energy of shells inside a circle, defined by RADCRT, around the center of impact is tested against the product of the given critical energy ENGCRT and the "area factor". The area factor is defined as,

$$\text{Area Factor} = \frac{\text{total area of shell elements found inside the circle}}{2\pi \times \text{RADCRT}^2}$$

The reason for having two times the circle area in the denominator is that we expect two layers of shell elements, as would typically be the case for laminated windshield glass.. If this energy criterion is exceeded, all elements of the part are now allowed to be eroded by the main failure criterion.

10. An element size dependent mixed-mode fracture criterion (MMFC) can be defined for shell elements using load curves LCEPS12, LCEPS23, and LCEPSMX. Failure happens if NCS (see above) of these three criteria are met

$$\begin{aligned}
\text{LCEPS12: } \gamma_{12} &= \frac{1}{2}(\varepsilon_1 - \varepsilon_2) \geq \gamma_{12}^c(l_e) & \text{if } -2.0 \leq \varepsilon_2/\varepsilon_1 \leq -0.5 \\
\text{LCEPS13: } \gamma_{13} &= \frac{1}{2}(\varepsilon_1 - \varepsilon_3) \geq \gamma_{13}^c(l_e) & \text{if } -0.5 \leq \varepsilon_2/\varepsilon_1 \leq 1.0 \\
\text{LCEPSMX: } \varepsilon_1 &\geq \varepsilon_1^c(l_e) & \text{if } -0.5 \leq \varepsilon_2/\varepsilon_1 \leq 1.0
\end{aligned}$$

where  $\gamma_{12}$  and  $\gamma_{13}$  are in-plane and through-thickness shear strains,  $\varepsilon_1$  and  $\varepsilon_2$  are in-plane major and minor strains, and  $\varepsilon_3$  is the through-thickness strain. The characteristic element size is  $l_e$  and it is computed as the square root of the shell element area. More details can be found in Zhu & Zhu (2011).

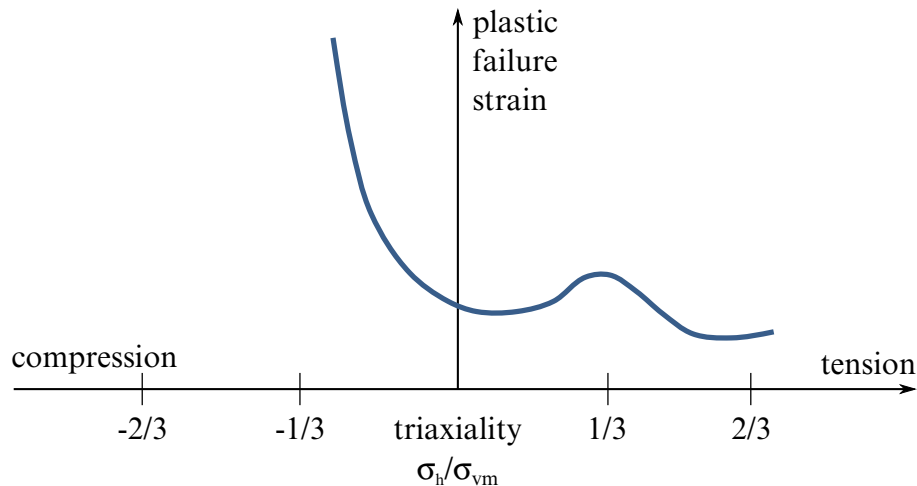
11. When  $\text{IDAM} = 0$ , there are 3 ways to specify how shell elements are eroded and removed from the calculation.
  - a) When  $\text{NUMFIP} > 0$ , elements erode when  $\text{NUMFIP}$  points fail.
  - b) When  $-100 \leq \text{NUMFIP} < 0$ , elements erode when  $|\text{NUMFIP}|$  percent of the integration points fail.
  - c) When  $\text{NUMFIP} < -100$ , elements erode when  $|\text{NUMFIP}| - 100$  integration points fail.

For  $\text{NUMFIP} > 0$  and  $-100 \leq \text{NUMFIP} < 0$ , layers retain full strength until the element is eroded. For  $\text{NUMFIP} < -100$ , the stress at an integration point immediately drops to zero when failure is detected at that integration point.

When  $\text{IDAM} \neq 0$ , there are 2 ways to specify how shell elements are eroded and removed from the calculation.

- a) When  $\text{NUMFIP} > 0$ , elements erode when  $\text{NUMFIP}$  points fail.
- b) When  $\text{NUMFIP} < 0$ , elements erode when  $|\text{NUMFIP}|$  percent of the layers fail.

A layer fails if any integration point within that layer fails. When  $\text{IDAM} = 0$ , erosion is in terms of failed points, not layers.



**Figure 2-1.** Typical failure curve for metal sheet, modeled with shell elements.

## DAMAGE MODELS

### GISSMO:

The GISSMO damage model is a phenomenological formulation that allows for an incremental description of damage accumulation, including softening and failure. It is intended to provide a maximum in variability for the description of damage for a variety of metallic materials (e.g. \*MAT\_024, \*MAT\_036, \*MAT\_103, ...). The input of parameters is based on tabulated data, allowing the user to directly convert test data to numerical input.

The model is based on an incremental formulation of damage accumulation:

$$\Delta D = \frac{\text{DMGEXP} \times D^{(1 - \frac{1}{\text{DMGEXP}})}}{\varepsilon_f} \Delta \varepsilon_p$$

where,

$D$  Damage value ( $0 \leq D \leq 1$ ). For numerical reasons,  $D$  is initialized to a value of 1.E-20 for all damage types in the first time step

$\varepsilon_f$  Equivalent plastic strain to failure, determined from LCSDG as a function of the current triaxiality value  $\eta$  (and Lode parameter  $L$  as an option).

A typical failure curve LCSDG for metal sheet, modelled with shell elements is shown in [Figure 2-1](#) Triaxiality should be monotonically increasing in this curve. A reasonable range for triaxiality is -2/3 to 2/3 if shell elements are used (plane stress).

For 3-dimensional stress states (solid elements), the possible range of triaxiality goes from  $-\infty$  to  $+\infty$ , but to get a good resolution in the internal load



curve discretization (depending on parameter LCINT of \*CONTROL\_SOLUTION) one should define lower limits, e.g. -1 to 1 if LCINT = 100 (default).

$\Delta\varepsilon_p$  Equivalent plastic strain increment

For constant values of failure strain, this damage rate can be integrated to get a relation of damage and actual equivalent plastic strain:

$$D = \left( \frac{\varepsilon_p}{\varepsilon_f} \right)^{\text{DMGEXP}}, \quad \text{for } \varepsilon_f = \text{constant}$$

Triaxiality  $\eta$  as a measure of the current stress state is defined as

$$\eta = \frac{\sigma_H}{\sigma_M}, \quad \text{with hydrostatic stress } \sigma_H \text{ and equivalent von Mises stress } \sigma_M.$$

Lode parameter  $L$  as an additional measure of the current stress state is defined as

$$L = \frac{27}{2} \frac{J_3}{\sigma_M^3}, \quad \text{with third invariant of the stress deviator } J_3.$$

For DMGTYP.EQ.0, damage is accumulated according to the description above, yet no softening and failure is taken into account. Thus, parameters ECRIT, DCRIT and FADEXP will not have any influence. This option can be used to calculate pre-damage in multi-stage deformations without influencing the simulation results.

For DMGTYP.EQ.1, elements will be deleted if  $D \geq 1$ .

Depending on the set of parameters given by ECRIT (or DCRIT) and FADEXP, a Lemaitre-type coupling of damage and stress (*effective stress concept*) can be used.

Three principal ways of damage definition can be used:

1. Input of a fixed value of critical plastic strain (ECRIT.GT.0.)

As soon as the magnitude of plastic strain reaches this value, the current damage parameter  $D$  is stored as critical damage DCRIT and the damage coupling flag is set to unity, in order to facilitate an identification of critical elements in postprocessing. From this point on, damage is coupled to the stress tensor using the following relation:

$$\sigma = \tilde{\sigma} \left[ 1 - \left( \frac{D - \text{DCRIT}}{1 - \text{DCRIT}} \right)^{\text{FADEXP}} \right]$$

This leads to a continuous reduction of stress, up to the load-bearing capacity completely vanishing as  $D$  reaches unity. The fading exponent FADEXP can be defined element size dependent, to allow for the consideration of an element-size dependent amount of energy to be dissipated during element fade-out.

2. Input of a load curve defining critical plastic strain vs. triaxiality (ECRIT < 0.), pointing to load curve ID |ECRIT|. This allows for a definition of triaxiality-dependent material instability, which takes account of that instability and localization will occur depending on the actual load case. This offers the possibility to use a transformed Forming Limit Diagram as an input for the expected onset of softening and localization. Using this load curve, the instability measure  $F$  is accumulated using the following relation, which is similar to the accumulation of damage  $D$  except for the instability curve is used as an input:

$$\Delta F = \frac{\text{DMGEXP}}{\varepsilon_{p,loc}} F^{\left(1 - \frac{1}{\text{DMGEXP}}\right)} \Delta \varepsilon_p$$

with,

$F$  Instability measure ( $0 \leq F \leq 1$ ).

$\varepsilon_{p,loc}$  Equivalent plastic strain to instability, determined from ECRIT

$\Delta \varepsilon_p$  Equivalent plastic strain increment

As soon as the instability measure  $F$  reaches unity, the current value of damage  $D$  in the respective element is stored. Damage will from this point on be coupled to the flow stress using the relation described above.

3. If no input for ECRIT is made, parameter DCRIT will be considered.

Coupling of Damage to the stress tensor starts if this value (*damage threshold*) is exceeded ( $0 \leq \text{DCRIT} \leq 1$ ). Coupling of damage to stress is done using the relation described above.

This input allows for the use of extreme values also – for example, DCRIT = 1.0 would lead to no coupling at all, and element deletion under full load (brittle fracture).

The values of SHRF and BIAXF generally lie between 0.0 and 1.0 where 0.0 means full regularization and 1.0 means no regularization under shear (triaxiality = 0.0 for SHRF = 1.0) or biaxial tension (triaxiality = 2/3 for BIAXF = 1.0). Any other intermediate triaxiality follows a linear interpolation between triaxiality 0.0 and 1/3 and also between triaxiality 1/3 and 2/3. Notice that a full regularization is always undertaken for a one-dimensional tensile stress state (triaxiality = 1/3) according to the factors defined under LCREGD. For the sake of generalization, both SHRF and BIAXF can also assume negative values (e.g., SHRF = -1.0 and BIAXF = -1.0). In this case, regularization is affected not at triaxialities 0.0 and 2/3 but rather at the triaxialities where the failure curve (LCSDG) crosses the instability curve (-ECRIT). The use of a triaxiality-dependent regularization approach may be necessary because simple regularization only depending on the element size can be unrealistic for certain stress states.

**History Variables:**

History variables of the GISSMO damage model are written to the post-processing database only if NAHSV > 0. As well, NEIPH and NEIPS must be set in \*DATABASE\_EXTENT\_BINARY. The damage history variables start at position ND, which is displayed in d3hsp file, e.g. "first damage history variable = 6" means that ND = 6. For example, if you wish to view the damage parameter (first GISSMO history variable) for a \*MAT\_024 shell element, you must set NEIPS = 6 and NAHSV = 1. In LS-PrePost, access the damage parameter as history variable #6.

<u>Variable</u>	<u>Description</u>
ND	Damage parameter $D$ , ( $10^{-20} < D \leq 1$ )
ND+1	Damage threshold DCRIT
ND+2	Domain flag for damage coupling (0: no coupling, 1: coupling)
ND+3	Triaxiality variable $\sigma_H/\sigma_M$
ND+4	Equivalent plastic strain
ND+5	Regularization factor for failure strain (determined from LCREGD)
ND+6	Exponent for stress fading FADEXP
ND+7	Calculated element size
ND+8	Instability measure F
ND+9	Resultant damage parameter $D$ for element size REFSZ
ND+10	Resultant damage threshold DCRIT for element size REFSZ
ND+11	Averaged triaxiality
ND+12	Lode parameter value (only calculated if LCSDG refers to a table)
ND+13	Alternative damage value: $D^{1/DMGEXP}$
ND+14	Averaged Lode parameter

**DIEM (Damage Initiation and Evolution Models):**

As an alternative to GISSMO, the user may invoke up to 5 damage initiation and evolution criteria. For the sake of efficiency, the parameter NCS can be used to only check these criteria in quantified increments of plastic strain. In other words, the criteria are only checked when the effective plastic strain goes beyond NCS,  $2 \times$  NCS,  $3 \times$  NCS, etc. For NCS = 0 the checks are performed in each step there is plastic flow, a reasonable value of NCS could for instance be NCS = 0.0001. The following theory applies to the DIEM option.

Assuming that  $n$  initiation/evolution types have been specified in the input deck ( $n = -IDAM$ ) there is defined at each integration point a damage initiation variable,  $\omega_D^i$ , and an evolution history variable  $D^i$ , such that,

$$\omega_D^i \in [0, \infty)$$

and

$$D^i \in [0,1], \quad i = 1, \dots, n.$$

These are initially set to zero and evolve with the deformation of the elements according to rules associated with the specific damage initiation and evolution type chosen, see below for details.

These quantities can be post-processed as ordinary material history variables and their positions in the history variables array is given in `d3hsp`, search for the string *Damage history listing*. The damage initiation variables do not influence the results but serve to indicate the onset of damage. As an alternative, the keyword `*DEFINE_MATERIAL_HISTORIES` can be used to output the instability and damage, following

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>		
<b>Label</b>	<b>Attributes</b>	<b>Description</b>
Instability	- - - -	Maximum initiation variable, $\max_{i=1, \dots, n} \omega_D^i$
Damage	- - - -	Effective damage $D$ , see below

The damage evolution variables govern the damage in the material and are used to form the global damage  $D \in [0,1]$ . Each criterion is of either of DCTYP set to maximum (DCTYP = 0) or multiplicative (DCTYP = 1), or one could choose to not couple damage to the stress by setting DCTYP = -1. This means that the damage value is calculated and stored, but it is not affecting the stress as for the other options, so if all DCTYP are set to -1 there will be no damage or failure. Letting  $I_{\max}$  denote the set of evolution types with DCTYP set to maximum and  $I_{\text{mult}}$  denote the set of evolution types with DCTYP set to multiplicative the global damage,  $D$ , is defined as

$$D = \max(D_{\max}, D_{\text{mult}}),$$

where

$$D_{\max} = \max_{i \in I_{\max}} D^i$$

and,

$$D_{\text{mult}} = 1 - \prod_{i \in I_{\text{mult}}} (1 - D^i).$$

The damage variable relates the macroscopic (damaged) to microscopic (true) stress by

$$\sigma = (1 - D)\tilde{\sigma}.$$

Once the damage has reached the level of  $D_{\text{erode}}$  (=0.99 by default) the stress is set to zero and the integration point is assumed failed and not processed thereafter. For NUMFIP > 0, a shell element is eroded and removed from the finite element model when NUMFIP integration points have failed. For NUMFIP < 0, a shell element is eroded and removed from the finite element model when -NUMFIP percent of the layers have failed.

### DAMAGE INITIATION, $\omega_D$

For each evolution type  $i$ ,  $\omega_D^i$  governs the onset of damage. For  $i \neq j$  the evolution of  $\omega_D^i$  is independent from the evolution of  $\omega_D^j$ . The following list enumerates the algorithms for modelling damage initiation.

In this subsection we suppress the superscripted  $i$  indexing the evolution type.

#### Ductility Based on Stress Triaxiality (DITYP.EQ.0):

For the ductile initiation option a function  $\varepsilon_D^p = \varepsilon_D^p(\eta, \dot{\varepsilon}^p)$  represents the plastic strain at onset of damage (P1). This is a function of stress triaxiality defined as

$$\eta = -p/q$$

with  $p$  being the pressure and  $q$  the von Mises equivalent stress. Optionally this can be defined as a table with the second dependency being on the effective plastic strain rate  $\dot{\varepsilon}^p$ . The damage initiation history variable evolves according to

$$\omega_D = \int_0^{\varepsilon^p} \frac{d\varepsilon^p}{\varepsilon_D^p}.$$

#### Shear (DITYP.EQ.1):

For the shear initiation option a function  $\varepsilon_D^p = \varepsilon_D^p(\theta, \dot{\varepsilon}^p)$  represents the plastic strain at onset of damage (P1). This is a function of a shear stress function defined as

$$\theta = (q + k_s p) / \tau$$

with  $p$  being the pressure,  $q$  the von Mises equivalent stress and  $\tau$  the maximum shear stress defined as a function of the principal stress values

$$\tau = (\sigma_{\text{major}} - \sigma_{\text{minor}}) / 2.$$

Introduced here is also the pressure influence parameter  $k_s$  (P2). Optionally this can be defined as a table with the second dependency being on the effective plastic strain rate  $\dot{\varepsilon}^p$ . The damage initiation history variable evolves according to

$$\omega_D = \int_0^{\varepsilon^p} \frac{d\varepsilon^p}{\varepsilon_D^p}$$

**MSFLD (DITYP.EQ.2):**

For the MSFLD initiation option a function  $\varepsilon_D^p = \varepsilon_D^p(\alpha, \dot{\varepsilon}^p)$  represents the plastic strain at onset of damage (P1). This is a function of the ratio of principal plastic strain rates defined as

$$\alpha = \frac{\dot{\varepsilon}_{\text{minor}}^p}{\dot{\varepsilon}_{\text{major}}^p}$$

The MSFLD criterion is only relevant for shells and the principal strains should be interpreted as the in-plane principal strains. For simplicity the plastic strain evolution in this formula is assumed to stem from an associated von Mises flow rule and whence

$$\alpha = \frac{s_{\text{minor}}}{s_{\text{major}}}$$

with  $s$  being the deviatoric stress. This insures that the calculation of  $\alpha$ , is in a sense, robust at the expense of being slightly inaccurate for materials with anisotropic yield functions and/or non-associated flow rules. Optionally this can be defined as a table with the second dependency being on the effective plastic strain rate  $\dot{\varepsilon}^p$ . For  $\dot{\varepsilon}^p = 0$  the value of  $\varepsilon_D^p$  is set to a large number to prevent onset of damage for no plastic evolution. Furthermore, the plastic strain used in this failure criteria is a modified effective plastic strain that only evolves when the pressure is negative, i.e., the material is not affected in compression.

This modified plastic strain can be monitored as the second history variable of the initiation history variables in the binary output database. For  $P3 = 0$ , the damage initiation history variable is calculated directly from the ratio of (modified) plastic strain and the critical plastic strain

$$\omega_D = \max_{t \leq T} \frac{\varepsilon^p}{\varepsilon_D^p}$$

This should be interpreted as the maximum value up to this point in time. If  $P3 = 1$  the damage initiation history variable is instead incrementally updated from the ratio of (modified) plastic strain and the critical plastic strain

$$\omega_D = \int_0^{\varepsilon^p} \frac{d\varepsilon^p}{\varepsilon_D^p}$$

For this initiation option P2 is used to determine the layer in the shell where the criterion is evaluated, if  $P2 = 0$  the criterion is evaluated in the mid-layer only whereas if  $P2 = 1$  it is evaluated in the outer layers only (bottom and top). This can be used to distinguish between a membrane instability typically used for FLD evaluations ( $P2 = 0$ ), and a bending instability ( $P2 = 1$ ). As soon as  $\omega_D$  reaches 1 in any of the integration points of interest, *all*

integration points in the shell goes over in damage mode, i.e., subsequent damage is applied to the entire element.

### FLD (DITYP.EQ.3):

The FLD initiation criterion is identical to MSFLD with one subtle difference: the plastic strain used for evaluating the criteria is not accounting for the sign of the hydrostatic stress, but is identical to the effective plastic strain directly from the underlying material model. In other words, it is not the modified plastic strain used in the MSFLD criterion, but apart from that it is an identical criterion.

### Ductile based on normalized principal stress (DITYP.EQ.4):

For the ductile initiation option the plastic strain at the onset of damage (P1) is taken as a function of  $\beta$  and  $\dot{\varepsilon}^p$ , that is  $\varepsilon_D^p = \varepsilon_D^p(\beta, \dot{\varepsilon}^p)$ , where  $\beta$  is the normalized principal stress

$$\beta = (q + k_d p) / \sigma_{\text{major}}$$

where  $p$  is the pressure,  $q$  is the von Mises equivalent stress,  $\sigma_{\text{major}}$  is the major principal stress, and where  $k_d$  is the pressure influence parameter specified in the P2 field. Optionally, this can be defined as a table with the second dependency being on the effective plastic strain rate  $\dot{\varepsilon}^p$ . The damage initiation history variable evolves according to

$$\omega_D = \int_0^{\varepsilon^p} \frac{d\varepsilon^p}{\varepsilon_D^p}$$

## DAMAGE EVOLUTION, $D$

For the evolution of the associated damage variable  $D$  we introduce the plastic displacement  $u^P$  which evolves according to

$$\dot{u}^p = \begin{cases} 0 & \omega_D < 1 \\ l\dot{\varepsilon}^p & \omega_D \geq 1 \end{cases}$$

with  $l$  being a characteristic length of the element. Fracture energy is related to plastic displacement as follows

$$G_f = \int_0^{u_f^p} \sigma_y du^p$$

where  $\sigma_y$  is the yield stress. The following list enumerates the algorithms available for modelling damage.

### Linear (DETYP.EQ.0):

With this option the damage variable evolves linearly with the plastic displacement

$$\dot{D} = \frac{\dot{u}^p}{u_f^p}$$

with  $u_f^p$  being the plastic displacement at failure (Q1). If Q1 is negative, then -Q1 refers to a table that defines  $u_f^p$  as a function of triaxiality and damage, i.e.,  $u_f^p = u_f^p(\eta, D)$ , and importantly the damage evolution law is changed generalized to

$$\dot{D} = \frac{\dot{u}^p}{\frac{\partial u_f^p}{\partial D}}$$

**Linear (DETYP.EQ.1):**

With this option the damage variable evolves linearly as follows

$$\dot{D} = \frac{\dot{u}^p}{u_f^p}$$

where  $u_f^p = 2G_f / \sigma_{y0}$  and  $\sigma_{y0}$  is the yield stress when failure criterion is reached.



**\*MAT\_ADD\_FATIGUE**

The ADD\_FATIGUE option defines the S-N fatigue property of a material model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	LCID	LTYPE	A	B	STHRES	SNLIMT	SNTYPE
Type	I	I	I	F	F	F	I	I
Default	none	-1	0	0.0	0.0	none	0	0

**S-N Curve Segment Cards.** Include one card for each additional S-N curve segment. Between zero and seven of these cards may be included in the deck. This input ends at the next keyword ("\*") card.

Card 2	1	2	3	4	5	6	7	8
Variable				Ai	Bi	STHRESi		
Type				F	F	F		
Default				0.0	0.0	none		

**VARIABLE****DESCRIPTION**

MID	Material identification for which the fatigue property applies.
LCID	S-N fatigue curve ID. GT.0: S-N fatigue curve ID EQ.-1: S-N fatigue curve uses equation $NS^b = a$ EQ.-2: S-N fatigue curve uses equation $\log(S) = a - b \log(N)$ EQ.-3: S-N fatigue curve uses equation $S = a N^b$
LTYPE	Type of S-N curve. EQ.0: Semi-log interpolation (default) EQ.1: Log-Log interpolation EQ.2: Linear-Linear interpolation

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A	Material parameter $a$ in S-N fatigue equation.
B	Material parameter $b$ in S-N fatigue equation.
STHRES	Fatigue threshold stress if the S-N curve is defined by equation (LCID < 0).
SNLIMT	<p><u>If LCID &gt; 0</u></p> <p>SNLIMT determines the algorithm used when stress is lower than the lowest stress on S-N curve.</p> <p>EQ.0: use the life at the last point on S-N curve</p> <p>EQ.1: extrapolation from the last two points on S-N curve</p> <p>EQ.2: infinity.</p> <p><u>If LCID &lt; 0</u></p> <p>SNLIMT determines the algorithm used when stress is lower than STHRES.</p> <p>EQ.0: use the life at STHRES</p> <p>EQ.1: <i>Ignored.</i> only applicable for LCID &gt; 0</p> <p>EQ.2: infinity.</p>
SNTYPE	<p>Stress type of S-N curve.</p> <p>EQ.0: stress range (default)</p> <p>EQ.1: stress amplitude.</p>
$A_i$	Material parameter $a$ in S-N fatigue equation for the $i$ -th segment.
$B_i$	Material parameter $b$ in S-N fatigue equation for the $i$ -th segment.
STHRES $_i$	Fatigue threshold stress for the $i$ -th segment.

**Remarks:**

1. S-N curves can be defined by \*DEFINE\_CURVE, or for LCID < 0 by

$$NS^b = a$$

when LCID = -1 or for LCID = -2

$$\log(S) = a - b \log(N)$$

or for LCID = -3

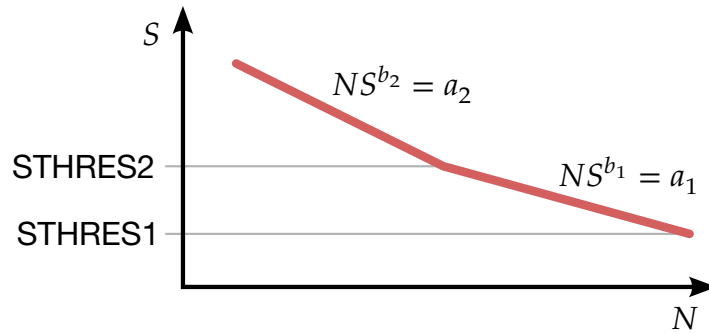


Figure 2-2. S-N Curve having multiple slopes

$$S = a N^b$$

where  $N$  is the number of cycles for fatigue failure and  $S$  is the stress amplitude. Note that the two equations can be converted to each other, with some minor algebraic manipulation on the constants  $a$  and  $b$ .

To define S-N curve with multiple slopes, the S-N curve can be split into multiple segments and each segment is defined by a set of parameters  $A_i$ ,  $B_i$  and  $STHRES_i$ . Up to 8 sets of the parameters ( $A_i$ ,  $B_i$  and  $STHRES_i$ ) can be defined. The lower limit of the  $i$ -th segment is represented by the threshold stress  $STHRES_i$ , as shown in Figure 2-2. This only applies to the case where the S-N curve is defined by equations (LCID = -1 or LCID = -2)

2. This model is applicable to frequency domain fatigue analysis, defined by the keywords: \*FREQUENCY\_DOMAIN\_RANDOM\_VIBRATION\_FATIGUE, and \*FREQUENCY\_DOMAIN\_SSD\_FATIGUE (see these keywords for further details).

**\*MAT\_ADD\_GENERALIZED\_DAMAGE**

This option provides a way of including generalized (tensor type) damage and failure in standard LS-DYNA material models. The basic idea is to apply a general damage model (e.g. GISSMO) using several history variables as damage driving quantities at the same time. With that feature it may be possible to obtain e.g. anisotropic damage behavior or separate stress degradation for volumetric and deviatoric deformations. A maximum of three simultaneous damage evolutions (i.e. definition of 3 history variables) is possible. A detailed description of this model can be found in Erhart et al. [2017].

This option currently applies to shell element types 1, 2, 3, 4, 16, and 17 and solid element types -2, -1, 1, 2, 3, 4, 10, 13, 15, 16, and 17.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	IDAM	DTYP	REFSZ	NUMFIP		PDDT	NHIS
Type	I	I	I	F	F		I	I
Default	none	0	0	0.0	1.0		0	1

Card 2	1	2	3	4	5	6	7	8
Variable	HIS1	HIS2	HIS3	IFLG1	IFLG2	IFLG3		
Type	I	I	I	I	I	I		
Default	0	none	none	0	0	0		

Card 3	1	2	3	4	5	6	7	8
Variable	D11	D22	D33	D44	D55	D66		
Type	I	I	I	I	I	I		
Default	none	none	none	none	none	none		

Card 4 (shells)	1	2	3	4	5	6	7	8
Variable	D12	D21	D24	D42	D14	D41		
Type	I	I	I	I	I	I		
Default	none	none	none	none	none	none		

Card 4 (solids)	1	2	3	4	5	6	7	8
Variable	D12	D21	D23	D32	D13	D31		
Type	I	I	I	I	I	I		
Default	none	none	none	none	none	none		

**Damage definition cards for IDAM = 1 (GISSMO).**

2 x NHIS cards have to be defined, i.e. two cards for each history variable.

First Card for history variable HISn:

Card 5...	1	2	3	4	5	6	7	8
Variable	LCSDG	ECRIT	DMGEXP	DCRIT	FADEXP	LCREG		
Type	I	F	F	F	F	I		
Default	0	0.0	1.0	0.0	1.0	0		

Second Card for history variable HISn:

Card 6...	1	2	3	4	5	6	7	8
Variable	LCSRS	SHRF	BIAXF	LCDLIM				
Type	I	F	F	I				
Default	0	0.0	0.0	0				

**VARIABLE****DESCRIPTION**

MID	Material ID for which this generalized damage definition applies.
IDAM	Flag for damage model. EQ.0: no damage model is used. EQ.1: GISSMO damage model.
DTYP	Flag for damage behavior. EQ.0: Damage is accumulated, no coupling to flow stress, no failure. EQ.1: Damage is accumulated, element failure occurs for $D = 1$ .
REFSZ	Reference element size, for which an additional output of damage will be generated. This is necessary to ensure the applicability of resulting damage quantities when transferred to different mesh sizes.
NUMFIP	Number of failed integration points prior to element deletion. The default is unity. LT.0: $ \text{NUMFIP} $ is the percentage of layers which must fail before element fails.

VARIABLE	DESCRIPTION
PDDT	<p>Pre-defined damage tensors. If non-zero, damage tensor coefficients D11 to D66 on cards 3 and 4 will be ignored. See remarks for details.</p> <p>EQ.0: No pre-defined damage tensor is used.</p> <p>EQ.1: Isotropic damage tensor.</p> <p>EQ.2: 2-parameter isotropic damage tensor for volumetric-deviatoric split.</p> <p>EQ.3: Anisotropic damage tensor as in MAT_104 (FLAG = -1).</p> <p>EQ.4: 3-parameter damage tensor associated with IFLG1 = 2.</p>
NHIS	<p>Number of history variables as driving quantities (min = 1, max = 3).</p>
HISn	<p>Choice of variable as driving quantity for damage, called "history value" in the following.</p> <p>EQ.0: Equivalent plastic strain rate is the driving quantity for the damage if IFLG1 = 0. Alternatively if IFLG1 = 1, components of the plastic strain rate tensor are driving quantities for damage (see remarks).</p> <p>GT.0: The rate of the additional history variable HISn is the driving quantity for damage. IFLG1 should be set to 0.</p> <p>LT.0: *DEFINE_FUNCTION IDs defining the damage driving quantities as a function of the components of the plastic strain rate tensor, IFLG1 should be set to 1.</p>
IFLG1	<p>Damage driving quantities</p> <p>EQ.0: Rates of history variables HISn.</p> <p>EQ.1: Specific components of the plastic strain rate tensor, see remarks for details.</p> <p>EQ.2: Predefined functions of plastic strain rate components for orthotropic damage model, HISn inputs will be ignored, IFLG2 should be set to 1. This option is for shell elements only.</p>

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
IFLG2	Damage strain coordinate system EQ.0: Local element system (shells) or global system (solids). EQ.1: Material system, only applicable for non-isotropic material models. Supported models for shells: all materials with AOPT feature. Supported models for solids: 22, 33, 41-50, 103, 122, 157, 233. EQ.2: Principal strain system (rotating). EQ.3: Principal strain system (fixed when instability/coupling starts).
IFLG3	Erosion criteria and damage coupling system EQ.0: Erosion occurs when one of the damage parameters computed reaches unity, the damage tensor components are based on the individual damage parameters d1 to d3. EQ.1: Erosion occurs when a single damage parameter D reaches unity, the damage tensor components are based on this single damage parameter.
D11...D31	DEFINE_FUNCTION IDs for damage tensor coefficients, see remarks.
LCSDG	Load curve ID defining corresponding history value to failure vs. triaxiality.
ECRIT	Critical history value (material instability), see below. LT.0.0:   ECRIT   is load curve ID defining critical history value vs. triaxiality. EQ.0.0: Fixed value DCRIT defining critical damage is read. GT.0.0: Fixed value for stress-state independent critical history value.
DMGEXP	Exponent for nonlinear damage accumulation.
DCRIT	Damage threshold value (critical damage). If a Load curve of critical history value or fixed value is given by ECRIT, input is ignored.



VARIABLE	DESCRIPTION
FADEXP	Exponent for damage-related stress fadeout. LT.0.0:  FADEXP  is load curve ID defining element-size dependent fading exponent. GT.0.0: Constant fading exponent.
LCREG	Load curve ID defining element size dependent regularization factors for history value to failure.
LCSRS	Load curve ID defining failure history value scaling factor for LCSDG vs. history value rate. If the first rate value in the curve is negative, it is assumed that all rate values are given as natural logarithm of the history rate. GT.0: scale ECRIT, too LT.0: do not scale ECRIT.
SHRF	Reduction factors for regularization at triaxiality = 0 (shear)
BIAXF	Reduction factors for regularization at triaxiality = 2/3 (biaxial)
LCDLIM	Load curve ID defining damage limit values as a function of triaxiality. Damage can be restricted to values less than 1.0 to prevent further stress reduction and failure for certain triaxialities.

**Remarks:**

The GISSMO damage model is described in detail in the remarks of \*MAT\_ADD\_EROSION. If NHIS = 1 and HIS1 = 0 is used, this new feature ("MAGD") behaves just the same as before ("GISSMO"). The main difference with this new keyword is that up to 3 independent but simultaneous damage evolutions are possible. Therefore, parameters LCSDG, ECRIT, DMGEXP, DCRIT, FADEXP, LCREGD, LCSRS, SHRF, BIAXF, and LCDLIM can be defined separately for each history variable.

The relation between nominal (damaged) stresses  $\sigma_{ij}$  and effective (undamaged) stresses  $\tilde{\sigma}_{ij}$  is now expressed as

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{21} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{31} & D_{32} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \tilde{\sigma}_{11} \\ \tilde{\sigma}_{22} \\ \tilde{\sigma}_{33} \\ \tilde{\sigma}_{12} \\ \tilde{\sigma}_{23} \\ \tilde{\sigma}_{31} \end{bmatrix}$$

with damage tensor  $D$ . Each damage tensor coefficient  $D_{ij}$  can be defined via \*DEFINE\_FUNCTION as a function of damage parameters  $d_1$  to  $d_3$ . For simple isotropic damage driven by plastic strain (NHIS = 1, HIS1 = 0, IFLG1 = IFLG2 = IFLG3 = 0) that would be

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix} = (1 - d_1) \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{\sigma}_{11} \\ \tilde{\sigma}_{22} \\ \tilde{\sigma}_{33} \\ \tilde{\sigma}_{12} \\ \tilde{\sigma}_{23} \\ \tilde{\sigma}_{31} \end{bmatrix}$$

That means the following function should be defined for D11 to D66 (Card 3):

```
*DEFINE_FUNCTION
1,D11toD66
func1(d1,d2,d3)=(1.0-d1)
```

and all entries in Card 4 can be left empty or equal zero in that case.

If GISSMO (IDAM = 1) is used, the damage parameters used in those functions are internally replaced by

$$d_i \rightarrow \left( \frac{d_i - DCRIT_i}{1 - DCRIT_i} \right)^{FADEXP_i}$$

In the case of plane stress (shell) elements, coupling between normal stresses and shear stresses is implemented and the damage tensor is defined as below :

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ 0 \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 & D_{14} & 0 & 0 \\ D_{21} & D_{22} & 0 & D_{24} & 0 & 0 \\ 0 & 0 & D_{33} & 0 & 0 & 0 \\ D_{41} & D_{42} & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \tilde{\sigma}_{11} \\ \tilde{\sigma}_{22} \\ 0 \\ \tilde{\sigma}_{12} \\ \tilde{\sigma}_{23} \\ \tilde{\sigma}_{31} \end{bmatrix}$$

Since the evaluation of \*DEFINE\_FUNCTION for variables D11 to D66 is relatively time consuming, pre-defined damage tensors (PDDT) can be used. Currently the following options are available for shell elements:

PDDT = 1	$(1 - D_1) \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
----------	--

PDDT = 2	$\begin{bmatrix} 1 - \frac{2}{3}D_1 - \frac{1}{3}D_2 & \frac{1}{3}D_1 - \frac{1}{3}D_2 & 0 & 0 & 0 & 0 \\ \frac{1}{3}D_1 - \frac{1}{3}D_2 & 1 - \frac{2}{3}D_1 - \frac{1}{3}D_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
PDDT = 3	$\begin{bmatrix} 1 - D_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 - D_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - \frac{1}{2}(D_1 + D_2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - \frac{1}{2}D_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - \frac{1}{2}D_1 \end{bmatrix}$
PDDT = 4	$\begin{bmatrix} 1 - D_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 - D_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 - D_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

and the following ones for solid elements:

PDDT = 1	$(1 - D_1) \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
PDDT = 2	$\begin{bmatrix} 1 - \frac{2}{3}D_1 - \frac{1}{3}D_2 & \frac{1}{3}D_1 - \frac{1}{3}D_2 & \frac{1}{3}D_1 - \frac{1}{3}D_2 & 0 & 0 & 0 \\ \frac{1}{3}D_1 - \frac{1}{3}D_2 & 1 - \frac{2}{3}D_1 - \frac{1}{3}D_2 & \frac{1}{3}D_1 - \frac{1}{3}D_2 & 0 & 0 & 0 \\ \frac{1}{3}D_1 - \frac{1}{3}D_2 & \frac{1}{3}D_1 - \frac{1}{3}D_2 & 1 - \frac{2}{3}D_1 - \frac{1}{3}D_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - D_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - D_1 \end{bmatrix}$

PDDT = 3	$1 - D_1$	0	0	0	0	0
	0	$1 - D_2$	0	0	0	0
	0	0	$1 - D_3$	0	0	0
	0	0	0	$1 - \frac{1}{2}(D_1 + D_2)$	0	0
	0	0	0	0	$1 - \frac{1}{2}(D_2 + D_3)$	0
	0	0	0	0	0	$1 - \frac{1}{2}(D_3 + D_1)$

**History Variables:**

The increment of the damage parameter is computed in GISSMO based on a driving quantity that has the dimension of a strain rate :

$$\dot{d} = nd^{1-1/n} \frac{\dot{HIS}_i}{epf}$$

The history variables defined by the user through HISi should thus have the dimension of a strain as the rate is computed internally by MAT\_ADD\_GENERALIZED\_DAMAGE:

$$\dot{HIS}_i = \frac{HIS_i(t^{n+1}) - HIS_i(t^n)}{t^{n+1} - t^n}$$

History variables can either come directly from associated material models (IFLG1 = 0 and HISi > 0), or they can be equivalent to plastic strain rate tensor components (IFLG1 = 1 and HISi = 0):

$$\dot{HIS}_1 = \dot{\epsilon}_{xx}^p, \dot{HIS}_2 = \dot{\epsilon}_{xx}^p, \dot{HIS}_3 = \dot{\epsilon}_{xy}^p \quad (\text{IFLG2} = 0)$$

$$\dot{HIS}_1 = \dot{\epsilon}_{aa}^p, \dot{HIS}_2 = \dot{\epsilon}_{bb}^p, \dot{HIS}_3 = \dot{\epsilon}_{ab}^p \quad (\text{IFLG2} = 1)$$

$$\dot{HIS}_1 = \dot{\epsilon}_1^p, \dot{HIS}_2 = \dot{\epsilon}_2^p, \dot{HIS}_3 = 0 \quad (\text{IFLG2} = 2)$$

or they can be provided via \*DEFINE\_FUNCTIONS by the user (IFLG1 = 1 and HISi < 0):

$$\dot{HIS}_i = f_i(\dot{\epsilon}_{xx}^p, \dot{\epsilon}_{yy}^p, \dot{\epsilon}_{zz}^p, \dot{\epsilon}_{xy}^p, \dot{\epsilon}_{yz}^p, \dot{\epsilon}_{zx}^p) \quad (\text{IFLG2} = 0)$$

$$\dot{HIS}_i = f_i(\dot{\epsilon}_{aa}^p, \dot{\epsilon}_{bb}^p, \dot{\epsilon}_{zz}^p, \dot{\epsilon}_{ab}^p, \dot{\epsilon}_{bz}^p, \dot{\epsilon}_{za}^p) \quad (\text{IFLG2} = 1)$$

$$\dot{HIS}_i = f_i(\dot{\epsilon}_1^p, \dot{\epsilon}_2^p) \quad (\text{IFLG2} = 2)$$

e.g. the following example defines a history variable (HISi = -1234) as function of the transverse shear strains in material coordinate system a-b-z for shells:

\*DEFINE\_FUNCTION

1234

fhis1(eaa, ebb, ezz, eab, ebz, eza) = 1.1547\*sqrt(ebz\*\*2+eza\*\*2)

The plastic strain rate tensor is not always available in the material law and is estimated as:

$$\dot{\epsilon}^p = \frac{\dot{\epsilon}_{eff}^p}{\dot{\epsilon}_{eff}} \left[ \dot{\epsilon} - \frac{\dot{\epsilon}_{vol}}{3} \delta \right]$$

This is a good approximation for isochoric materials with small elastic strains (e.g. metals) and correct for J2 plasticity.

The following table gives an overview of the driving quantities used for incrementing the damage in function of the input parameters (strain superscript “p” for “plastic” is omitted for convenience):

IFLG1	IFLG2	HISi > 0	HISi = 0	HISi < 0
0	0	HIS <sub>i</sub>	$\dot{\epsilon}$	N/A
0	1	HIS <sub>i</sub>	N/A	N/A
0	2	HIS <sub>i</sub>	N/A	N/A
1	0	N/A	$\dot{\epsilon}_{ij}$	$f(\dot{\epsilon}_{ij})$
1	1	N/A	$\dot{\epsilon}_{ij}^{mat}$	$f(\dot{\epsilon}_{ij}^{mat})$
1	2	N/A	$\dot{\epsilon}_i$	$f(\dot{\epsilon}_i)$
2	0	N/A		
2	1	Preprogrammed functions of plastic strain rate		
2	2	N/A		

**Postprocessing History Variables:**

History variables of the GENERALIZED\_DAMAGE model are written to the post-processing database behind those already occupied by the material model which is used in combination:

*Variable*    *Description*

ND    Triaxiality variable  $\sigma_H/\sigma_M$

ND+1    Lode parameter value

ND+2    Single damage parameter  $D$ , ( $10^{-20} < D \leq 1$ ), only for IFLG3 = 1

ND+3	Damage parameter $d_1$
ND+4	Damage parameter $d_2$
ND+5	Damage parameter $d_3$
ND+6	Damage threshold DCRIT <sub>1</sub>
ND+7	Damage threshold DCRIT <sub>2</sub>
ND+8	Damage threshold DCRIT <sub>3</sub>
ND+12	History variable HIS <sub>1</sub>
ND+13	History variable HIS <sub>2</sub>
ND+14	History variable HIS <sub>3</sub>
ND+15	Angle between principal and material axes
ND+21	Characteristic element size (used in LCREG)

For instance, ND = 6 for \*MAT\_024, ND = 9 for \*MAT\_036, or ND = 23 for \*MAT\_187. Exact information of the variable locations can be found in the d3hsp section "MAGD damage history listing".

\*MAT\_ADD\_PERMEABILITY

For consolidation calculations.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	PERM	(blank)	(blank)	THEXP	LCKZ		
Type	I	F			F	I		
Default	none	none			0.0	none		

VARIABLE	DESCRIPTION
MID	Material identification – must be same as the structural material.
PERM	Permeability
THEXP	Undrained volumetric thermal expansion coefficient (Units: 1/temperature). If negative, then –THEXP is the ID of a load curve giving thermal expansion coefficient (y-axis) versus temperature (x-axis).
LCKZ	Load curve giving factor on PERM versus z-coordinate. (X-axis – z-coordinate, yaxis – non dimensional factor)

**Remarks:**

The units of PERM are length/time (volume flow rate of water per unit area per gradient of pore pressure head).

THEXP represents the thermal expansion of the material caused by the pore fluid. It should be set equal to  $n\alpha_w$ , where n is the porosity of the soil and  $\alpha_w$  is the volumetric thermal expansion coefficient of the pore fluid. If the pore fluid is water, the thermal expansion coefficient varies strongly with temperature; a curve of coefficient versus temperature may be input instead of a constant value.

See notes under \*CONTROL\_PORE\_FLUID

**\*MAT\_ADD\_PORE\_AIR**

For pore air pressure calculations.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	PA_RHO	PA_PRE	PORE				
Type	I	I	F	F				
Default	none	AIR_RO	AIR_RO	1.				
Remarks	1			1, 2				

Card 2	1	2	3	4	5	6	7	8
Variable	PERM1	PERM2	PERM3	CDARCY	CDF	LCPGD1	LCPGD2	LCPGD3
Type	F	F	F	F	F	I	I	I
Default	0.	PERM1	PERM1	1.	0.	none	LCPGD1	LCPGD1
Remarks	2, 3, 4, 5	2, 3, 4, 5	2, 3, 4, 5	1	1, 5	6	6	6

**VARIABLE****DESCRIPTION**

MID	Material identification – must be same as the structural material.
PA_RHO	Initial density of pore air, default to atmospheric air density, AIR_RO, defined in *CONTROL_PORE_AIR
PA_PRE	Initial pressure of pore air, default to atmospheric air pressure, AIR_P, defined in *CONTROL_PORE_AIR
PORE	Porosity, ratio of pores to total volume, default to 1.
PERM[1-3]	Permeability of pore air along $x$ , $y$ and $z$ -direction. If less than 0 – PERM[1-3] is taken to be the curve ID defining the permeability coefficient as a function of volume ratio of current volume to volume in the stress free state.



VARIABLE	DESCRIPTION
CDARCY	Coefficient of Darcy's law
CDF	Coefficient of Dupuit-Forchheimer law
LCPGD1~3	Curves defining non-linear Darcy's laws along x, y and z-directions, see Remarks 6.

### Remarks:

- Card 1.** This card must be defined for all materials requiring consideration of pore air pressure. The pressure contribution of pore air is  $(\rho - \rho_{\text{atm}})RT \times \text{PORE}$ , where  $\rho$  and  $\rho_{\text{atm}}$  are the current and atmospheric air density,  $R$  is air's gas constant,  $T$  is atmospheric air temperature and PORE is the porosity. The values for  $R$ ,  $T$  and PORE are assumed to be constant during simulation.
- Permeability Model.** The unit of PERMi is  $[\text{Length}]^3[\text{time}]/[\text{mass}]$ , (air flow velocity per gradient of excess pore pressure), i.e.

$$(\text{CDARCY} + \text{CDF} \times |v_i|) \times \text{PORE} \times v_i = \text{PERMi} \times \frac{\partial P_a}{\partial x_i}, \quad i = 1,2,3$$

where  $v_i$  is the pore air flow velocity along the  $i^{\text{th}}$  direction,  $\partial P_a / \partial x_i$  is the pore air pressure gradient along the  $i^{\text{th}}$  direction, and  $x_1 = x$ ,  $x_2 = y$ ,  $x_3 = z$ .

- Default Values for PERM2 and PERM3.** PERM2 and PERM3 are assumed to be equal to PERM1 when they are not defined. A definition of "0" means no permeability.
- Local Coordinate Systems.** (x,y,z), or (1,2,3), refers to the local material coordinate system (a,b,c) when MID is an orthotropic material, such as \*MAT\_002 or \*MAT\_142; otherwise it refers to the global coordinate system.
- CDF for Viscosity.** CDF can be used to consider the viscosity effect for high speed air flow
- Nonlinearity.** LCPGD $i$  can be used to define a non-linear Darcy's law as follows:

$$(\text{CDARCY} + \text{CDF} \times |v_i|) \times \text{PORE} \times v_i = \text{PERMi} \times f_i \frac{\partial P_a}{\partial x_i}, \quad i = 1,2,3$$

where  $f_i$  is value of the function defined by the LCPGD $i$  field. The linear version of Darcy's law of [Remark 2](#), can be recovered when the LCPGD $i$  curves are defined as straight lines of slope of 1.

**\*MAT\_ADD\_THERMAL\_EXPANSION**

The ADD\_THERMAL\_EXPANSION option is used to occupy an arbitrary material model in LS-DYNA with a thermal expansion property. This option applies to all nonlinear solid, shell, thick shell and beam elements and all material models except those models which use resultant formulations such as \*MAT\_RESULTANT\_PLASTICITY and \*MAT\_SPECIAL\_ORTHOTROPIC. Orthotropic expansion effects are supported for anisotropic materials.

Card 1	1	2	3	4	5	6	7	8
Variable	PID	LCID	MULT	LCIDY	MULTY	LCIDZ	MULTZ	
Type	I	I	F	I	F	I	F	
Default	none	none	1.0	LCID	MULT	LCID	MULT	

**VARIABLE****DESCRIPTION**

PID	Part ID for which the thermal expansion property applies
LCID	For isotropic material models, LCIDY, MULTY, LCIDZ, and MULTZ are ignored, and LCID is the load curve ID defining the thermal expansion coefficient as a function of temperature. If zero, the thermal expansion coefficient is constant and equal to MULT. For anisotropic material models, LCID and MULT define the thermal expansion coefficient in the local material a-direction.
MULT	Scale factor scaling load curve given by LCID.
LCIDY	Load curve ID defining the thermal expansion coefficient in local material b-direction as a function of temperature. If zero, the thermal expansion coefficient in the local material b-direction is constant and equal to MULTY. If MULTY = 0 as well, LCID and MULT define the thermal expansion coefficient in the local material b-direction.
MULTY	Scale factor scaling load curve given by LCIDY.

VARIABLE	DESCRIPTION
LCIDZ	Load curve ID defining the thermal expansion coefficient in local material c-direction as a function of temperature. If zero, the thermal expansion coefficient in the local material c-direction is constant and equal to MULTZ. If MULTZ = 0 as well, LCID and MULT define the thermal expansion coefficient in the local material c-direction.
MULTZ	Scale factor scaling load curve given by LCIDZ.

**Remarks:**

When invoking the isotropic thermal expansion property (no use of the local y and z parameters) for a material, the stress update is based on the elastic strain rates given by

$$\dot{\epsilon}_{ij}^e = \dot{\epsilon}_{ij} - \alpha(T)\dot{T}\delta_{ij}$$

rather than on the total strain rates  $\dot{\epsilon}_{ij}$ . For a material with the stress based on the deformation gradient  $F_{ij}$ , the elastic part of the deformation gradient is used for the stress computations

$$F_{ij}^e = J_T^{-1/3}F_{ij}$$

where  $J_T$  is the thermal Jacobian. The thermal Jacobian is updated using the rate given by

$$\dot{J}_T = 3\alpha(T)\dot{T}J_T.$$

For orthotropic properties, which apply only to materials with anisotropy, these equations are generalized to

$$\dot{\epsilon}_{ij}^e = \dot{\epsilon}_{ij} - \alpha_k(T)\dot{T}q_{ik}q_{jk}$$

and

$$F_{ij}^e = F_{ik}\beta_l^{-1}Q_{kl}Q_{jl}$$

where the  $\beta_i$  are updated as

$$\dot{\beta}_i = \alpha_i(T)\dot{T}\beta_i.$$

Here  $q_{ij}$  represents the matrix with material directions with respect to the current configuration whereas  $Q_{ij}$  are the corresponding directions with respect to the initial configuration. For (shell) materials with multiple layers of different anisotropy directions, the mid surface layer determines the orthotropy for the thermal expansion.

**\*MAT\_NONLOCAL**

In nonlocal failure theories, the failure criterion depends on the state of the material within a radius of influence which surrounds the integration point. An advantage of nonlocal failure is that mesh size sensitivity on failure is greatly reduced leading to results which converge to a unique solution as the mesh is refined.

Without a nonlocal criterion, strains will tend to localize randomly with mesh refinement leading to results which can change significantly from mesh to mesh. The nonlocal failure treatment can be a great help in predicting the onset and the evolution of material failure. This option can be used with two and three-dimensional solid elements, and three-dimensional shell elements and thick shell elements. This option applies to a subset of elastoplastic materials that include a damage-based failure criterion.

Card 1	1	2	3	4	5	6	7	8
Variable	IDNL	PID	P	Q	L	NFREQ	NHV	
Type	I	I	F	F	F	I	I	
Default	none	none	none	none	none	none	none	

**History Cards.** Include as many cards as needed to set NHV variables. *One card 2 will be read even if NHV = 0.*

Card 2	1	2	3	4	5	6	7	8
Variable	NL1	NL2	NL3	NL4	NL5	NL6	NL7	NL8
Type	I	I	I	I	I	I	I	I
Default	none	none	none	none	none	none	none	none

**Symmetry Plane Cards.** Define one card for each symmetry plane. Up to six symmetry planes can be defined. The next “\*” card terminates this input.

Cards 3	1	2	3	4	5	6	7	8
Variable	XC1	YC1	ZC1	XC2	YC2	ZC2		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

VARIABLE	DESCRIPTION
IDNL	Nonlocal material input ID.
PID	Part ID for nonlocal material.
P	Exponent of weighting function. A typical value might be 8 depending somewhat on the choice of L. See equations below.
Q	Exponent of weighting function. A typical value might be 2. See equations below.
L	Characteristic length. This length should span a few elements. See equations below.
NFREQ	Number of time steps between searching for integration points that lie in the neighborhood. Nonlocal smoothing will be done each cycle using these neighbors until the next search is done. The neighbor search can add significant computational time so it is suggested that NFREQ be set to value of 10 to 100 depending on the problem. This parameter may be somewhat problem dependent. If NFREQ = 0, a single search will be done at the start of the calculation.
NHV	Define the number of history variables for nonlocal treatment.
NL1, ..., NL8	Identifies the history variable(s) for nonlocal treatment. Define NHV values (maximum of 8 values per line).
XC1, YC1, ZC1	Coordinate of point on symmetry plane.
XC2, YC2, ZC2	Coordinate of a point along the normal vector.

**Remarks:**

For elastoplastic material models in LS-DYNA which use the plastic strain as a failure criterion, setting the variable NL1 to 1 would tag plastic strain for nonlocal treatment. A sampling of other history variables that can be tagged for nonlocal treatment are listed in the table below. The value in the third column in the table below corresponds to the history variable number as tabulated at <http://www.dynasupport.com/howtos/material/history-variables>. Note that the  $NL_n$  value is the history variable number plus 1.

Material Model Name		*MAT_NONLOCAL NL $n$ Value	History Variable Number
JOHNSON_COOK	15	5 (shells); 7 (solids)	4 (shells); 6 (solids)
PLASTICITY_WITH_DAMAGE	81	2	1
DAMAGE_1	104	4	3
DAMAGE_2	105	2	1
JOHNSON_HOLMQUIST_CONCRETE	111	2	1
GURSON	120	2	1

In applying the nonlocal equations to shell and thick shell elements, integration points lying in the same plane within the radius determined by the characteristic length are considered. Therefore, it is important to define the connectivity of the shell elements consistently within the part ID, e.g., so that the outer integration points lie on the same surface.

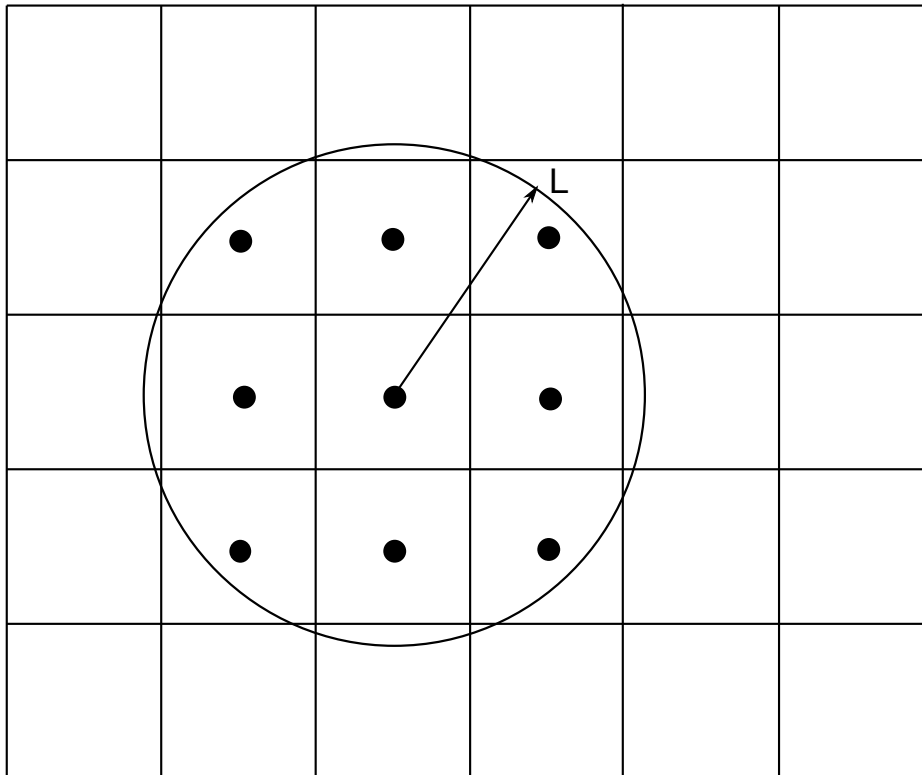
The equations and our implementation are based on the implementation by Worswick and Lalbin [1999] of the nonlocal theory to Pijaudier-Cabot and Bazant [1987]. Let  $\Omega_r$  be the neighborhood of radius,  $L_r$ , of element  $e_r$  and  $\{e_i\}_{i=1,\dots,N_r}$  the list of elements included in  $\Omega_r$ , then

$$\dot{f}_r = \dot{f}(x_r) = \frac{1}{W_r} \int_{\Omega_r} \dot{f}_{\text{local}} w(x_r - y) dy \approx \frac{1}{W_r} \sum_{i=1}^{N_r} \dot{f}_{\text{local}}^i w_{ri} V_i$$

where

$$W_r = W(x_r) = \int w(x_r - y) dy \approx \sum_{i=1}^{N_r} w_{ri} V_i$$

$$w_{ri} = w(x_r - y_i) = \frac{1}{\left[1 + \left(\frac{\|x_r - y_i\|}{L}\right)^p\right]^q}$$



**Figure 2-3.** Here  $\dot{f}_r$  and  $x_r$  are respectively the nonlocal rate of increase of damage and the center of the element  $e_r$ , and  $\dot{f}_{local}^i$ ,  $V_i$  and  $y_i$  are respectively the local rate of increase of damage, the volume and the center of element  $e_i$ .

**\*MAT\_ELASTIC\_{OPTION}**

This is Material Type 1. This is an isotropic hypoelastic material and is available for beam, shell, and solid elements in LS-DYNA. A specialization of this material allows the modeling of fluids.

Available options include:

<BLANK>

FLUID

such that the keyword cards appear:

\*MAT\_ELASTIC or MAT\_001

\*MAT\_ELASTIC\_FLUID or MAT\_001\_FLUID

The fluid option is valid for solid elements only.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	DA	DB	K	
Type	A8	F	F	F	F	F	F	
Default	none	none	none	0.0	0.0	0.0	0.0	

Additional card for FLUID keyword option.

Card 2	1	2	3	4	5	6	7	8
Variable	VC	CP						
Type	F	F						
Default	none	1.0E+20						

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label not exceeding 8 characters must be specified.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
DA	Axial damping factor (used for Belytschko-Schwer beam, type 2, only).
DB	Bending damping factor (used for Belytschko-Schwer beam, type 2, only).
K	Bulk Modulus (define for fluid option only).
VC	Tensor viscosity coefficient, values between .1 and .5 should be okay.
CP	Cavitation pressure (default = 1.0e+20).

**Remarks:**

This hypoelastic material model may not be stable for finite (large) strains. If large strains are expected, a hyperelastic material model, e.g., \*MAT\_002, would be more appropriate.

The axial and bending damping factors are used to damp down numerical noise. The update of the force resultants,  $F_i$ , and moment resultants,  $M_i$ , includes the damping factors:

$$F_i^{n+1} = F_i^n + \left(1 + \frac{DA}{\Delta t}\right) \Delta F_i^{n+\frac{1}{2}}$$

$$M_i^{n+1} = M_i^n + \left(1 + \frac{DB}{\Delta t}\right) \Delta M_i^{n+\frac{1}{2}}$$

The history variable labeled as "plastic strain" by LS-PrePost is actually volumetric strain in the case of \*MAT\_ELASTIC.

Truss elements include a damping stress given by

$$\sigma = 0.05\rho cL/\Delta t$$

where  $\rho$  is the mass density,  $c$  is the material wave speed,  $L$  is the element length, and  $\Delta t$  is the computation time step.

For the fluid option, the bulk modulus field,  $K$ , must be defined, and both the Young's modulus and Poisson's ratio fields are ignored. With the fluid option, fluid-like behavior is obtained where the bulk modulus,  $K$ , and pressure rate,  $p$ , are given by:

$$K = \frac{E}{3(1 - 2\nu)}$$

$$\dot{p} = -K\dot{\epsilon}_{ii}$$

and the shear modulus is set to zero. A tensor viscosity is used which acts only the deviatoric stresses,  $S_{ij}^{n+1}$ , given in terms of the damping coefficient as:

$$S_{ij}^{n+1} = VC \times \Delta L \times a \times \rho \dot{\epsilon}'_{ij}$$

where  $\Delta L$  is a characteristic element length,  $a$  is the fluid bulk sound speed,  $\rho$  is the fluid density, and  $\dot{\epsilon}'_{ij}$  is the deviatoric strain rate.

\*MAT\_OPTIONTROPIC\_ELASTIC

This is Material Type 2. This material is valid for modeling the elastic-orthotropic behavior of solids, shells, and thick shells. An anisotropic option is available for solid elements. For orthotropic solids an isotropic frictional damping is available.

In the case of solids, stresses are calculated not from incremental strains but rather from the deformation gradient. Also for solids, the elastic constants are formulated in terms of second Piola-Kirchhoff stress and Green’s strain, however, Cauchy stress is output.

In the case of shells, the stress update is incremental and the elastic constants are formulated in terms of Cauchy stress and true strain.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

Available options include:

ORTHO

ANISO

such that the keyword cards appear:

\*MAT\_ORTHOTROPIC\_ELASTIC or MAT\_002 (4 cards follow)

\*MAT\_ANISOTROPIC\_ELASTIC or MAT\_002\_ANIS (5 cards follow)

**Orthotropic Card 1.** Card 1 for ORTHO keyword option.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
Type	A8	F	F	F	F	F	F	F

**Orthotropic Card 2.** Card 2 for ORTHO keyword option.

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT	G	SIGF		
Type	F	F	F	F	F	F		

**Anisotropic Card 1.** Card 1 for ANISO keyword option.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	C11	C12	C22	C13	C23	C33
Type	A8	F	F	F	F	F	F	F

**Anisotropic Card 2.** Card 2 for ANISO keyword option.

Card 2	1	2	3	4	5	6	7	8
Variable	C14	C24	C34	C44	C15	C25	C35	C45
Type	F	F	F	F	F	F	F	F

**Anisotropic Card 3.** Card 3 for ANISO keyword option.

Card 3	1	2	3	4	5	6	7	8
Variable	C55	C16	C26	C36	C46	C56	C66	AOPT
Type	F	F	F	F	F	F	F	F

**Local Coordinate System Card 1.** Required for all keyword options

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	IHIS
Type	F	F	F	F	F	F	I	F

**Local Coordinate System Card 2.** Required for all keyword options

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	REF
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID Material identification. A unique number or label not exceeding 8 characters must be specified.

RO Mass density.

**Define for the ORTHO option only:**

EA  $E_a$ , Young's modulus in  $a$ -direction.

EB  $E_b$ , Young's modulus in  $b$ -direction.

EC  $E_c$ , Young's modulus in  $c$ -direction (nonzero value required but not used for shells).

PRBA  $\nu_{ba}$ , Poisson's ratio in the  $ba$  direction.

PRCA  $\nu_{ca}$ , Poisson's ratio in the  $ca$  direction.

PRCB  $\nu_{cb}$ , Poisson's ratio in the  $cb$  direction.

GAB  $G_{ab}$ , shear modulus in the  $ab$  direction.

GBC  $G_{bc}$ , shear modulus in the  $bc$  direction.

GCA  $G_{ca}$ , shear modulus in the  $ca$  direction.

**Due to symmetry define the upper triangular  $C_{ij}$ 's for the ANISO option only:**

C11 The 1, 1 term in the  $6 \times 6$  anisotropic constitutive matrix. Note that 1 corresponds to the  $a$  material direction

C12 The 1, 2 term in the  $6 \times 6$  anisotropic constitutive matrix. Note that 2 corresponds to the  $b$  material direction

⋮

⋮

C66 The 6, 6 term in the  $6 \times 6$  anisotropic constitutive matrix.

**Define AOPT for both options:**

- AOPT            Material axes option, see [Figure M2-1](#).
- EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in part (a) of [Figure M2-1](#). The **a**-direction is from node 1 to node 2 of the element. The **b**-direction is orthogonal to the a-direction and is in the plane formed by nodes 1, 2, and 4. When this option is used in two-dimensional planar and axisymmetric analysis, it is critical that the nodes in the element definition be numbered counterclockwise for this option to work correctly.
- EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only.
- EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with \*DEFINE\_COORDINATE\_VECTOR.
- EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.
- EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, **P**, which define the centerline axis. This option is for solid elements only.
- LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on \*DEFINE\_COORDINATE\_NODES, \*DEFINE\_COORDINATE\_SYSTEM or \*DEFINE\_COORDINATE\_VECTOR). Available in R3 version of 971 and later.
- G                Shear modulus for frequency independent damping. Frequency independent damping is based of a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250-1000 times greater than SIGF. This option applies only to solid elements.

SIGF	Limit stress for frequency independent, frictional, damping.
XP, YP, ZP	Define coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	Define components of vector $\mathbf{a}$ for AOPT = 2.
MACF	Material axes change flag for brick elements: EQ.1: No change, default, EQ.2: switch material axes $a$ and $b$ , EQ.3: switch material axes $a$ and $c$ , EQ.4: switch material axes $b$ and $c$ .
IHIS	Flag for anisotropic stiffness terms initialization (for solid elements only). EQ.0: C11, C12, ... from Cards 1, 2, and 3 are used. EQ.1: C11, C12, ... are initialized by *INITIAL_STRESS_SOLID's history data.
V1, V2, V3	Define components of vector $\mathbf{v}$ for AOPT = 3 and 4.
D1, D2, D3	Define components of vector $\mathbf{d}$ for AOPT = 2.
BETA	Material angle in degrees for AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.

**Remarks:**

The material law that relates stresses to strains is defined as:

$$\mathbf{C} = \mathbf{T}^T \mathbf{C}_L \mathbf{T}$$

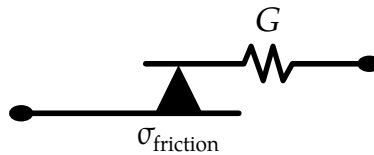
where  $\mathbf{T}$  is a transformation matrix, and  $\mathbf{C}_L$  is the constitutive matrix defined in terms of the material constants of the orthogonal material axes,  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ . The inverse of  $\mathbf{C}_L$  for the orthotropic case is defined as:

$$\mathbf{C}_L^{-1} = \begin{bmatrix} \frac{1}{E_a} & -\frac{\nu_{ba}}{E_b} & -\frac{\nu_{ca}}{E_c} & 0 & 0 & 0 \\ -\frac{\nu_{ab}}{E_a} & \frac{1}{E_b} & -\frac{\nu_{cb}}{E_c} & 0 & 0 & 0 \\ -\frac{\nu_{ac}}{E_a} & -\frac{\nu_{bc}}{E_b} & \frac{1}{E_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{ab}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{bc}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{ca}} \end{bmatrix}$$

where,

$$\frac{\nu_{ab}}{E_a} = \frac{\nu_{ba}}{E_b}, \quad \frac{\nu_{ca}}{E_c} = \frac{\nu_{ac}}{E_a}, \quad \frac{\nu_{cb}}{E_c} = \frac{\nu_{bc}}{E_b}.$$

The frequency independent damping is obtained by having a spring and slider in series as shown in the following sketch:



This option applies only to orthotropic solid elements and affects only the deviatoric stresses.

The procedure for describing the principle material directions is now explained for solid and shell elements for this material model and other anisotropic materials. We will call the material coordinate system the {**a**, **b**, **c**} coordinate system. The AOPT options illustrated in [Figure M2-1](#) define the preliminary {**a**, **b**, **c**} system for all elements of the parts that use the material, but this is not the final material direction. The {**a**, **b**, **c**} system defined by the AOPT options may be offset by a final rotation about the **c**-axis. The offset angle we call BETA.

For solid elements, the BETA angle is specified in one of two ways. When using AOPT = 3, the BETA parameter defines the offset angle for all elements that use the material. The BETA parameter has no meaning for the other AOPT options. Alternatively, a BETA angle can be defined for individual solid elements as described in remark 5 for \*ELEMENT\_SOLID\_ORTHO. The beta angle by the ORTHO option is available for all values of AOPT, and it overrides the BETA angle on the \*MAT card for AOPT = 3.

The directions determined by the material AOPT options may be overridden for individual elements as described in remark 3 for \*ELEMENT\_SOLID\_ORTHO. However, be aware that for materials with AOPT = 3, the final {**a**, **b**, **c**} system will be the system defined on the element card rotated about **c**-axis by the BETA angle specified on the \*MAT card.



There are two fundamental differences between shell and solid element orthotropic materials. First, the **c**-direction is always normal to a shell element such that the **a**-direction and **b**-directions are within the plane of the element. Second, for some anisotropic materials, shell elements may have unique fiber directions within each layer through the thickness of the element so that a layered composite can be modeled with a single element.

When AOPT = 0 is used in two-dimensional planar and axisymmetric analysis, it is critical that the nodes in the element definition be numbered counterclockwise for this option to work correctly.

Because shell elements have their **c**-axes defined by the element normal, AOPT = 1 and AOPT = 4 are not available for shells. Also, AOPT = 2 requires only the vector **a** be defined since **d** is not used. The shell procedure projects the inputted **a**-direction onto each element surface.

Similar to solid elements, the {**a, b, c**} coordinate system determined by AOPT is then modified by a rotation about the **c**-axis which we will call  $\phi$ . For those materials that allow a unique rotation angle for each integration point through the element thickness, the rotation angle is calculated by

$$\phi_i = \beta + \beta_i$$

where  $\beta$  is a rotation for the element, and  $\beta_i$  is the rotation for the *i*'th layer of the element. The  $\beta$  angle can be input using the BETA parameter on the \*MAT data, or will be overridden for individual elements if the BETA keyword option for \*ELEMENT\_SHELL is used. The  $\beta_i$  angles are input using the ICOMP = 1 option of \*SECTION\_SHELL or with \*PART\_COMPOSITE. If  $\beta$  or  $\beta_i$  is omitted, they are assumed to be zero.

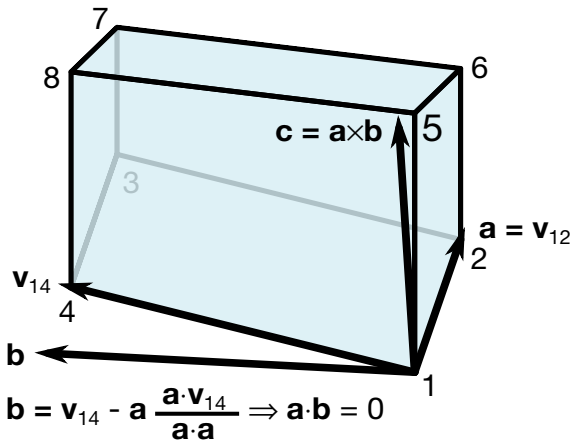
All anisotropic shell materials have the BETA parameter on the \*MAT card available for both AOPT = 0 and AOPT = 3, except for materials 91 and 92 which have it available (but called FANG instead of BETA) for AOPT = 0, 2, and 3.

All anisotropic shell materials allow an angle for each integration point through the thickness,  $\beta_i$ , except for materials 2, 86, 91, 92, 117, 130, 170, 172, and 194.

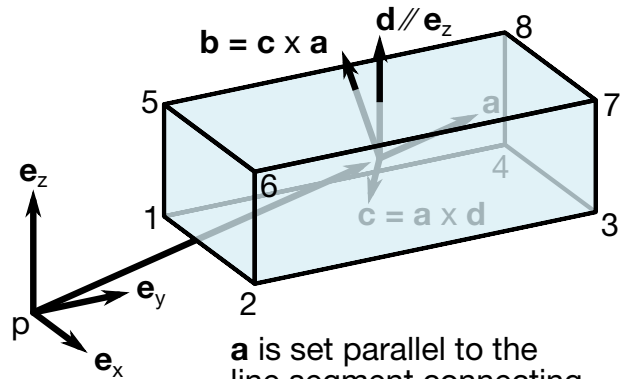
This discussion of material direction angles in shell elements also applies to thick shell elements which allow modeling of layered composites using \*INTEGRATION\_SHELL or \*PART\_COMPOSITE\_TSHELL.

Illustration of AOPT: Figure M2-1

**AOPT = 0.0**



**AOPT = 1.0 (solid only)**

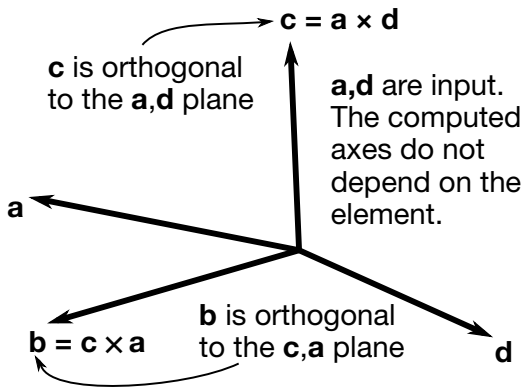


$\mathbf{a}$  is set parallel to the line segment connecting  $p$  to the element center.

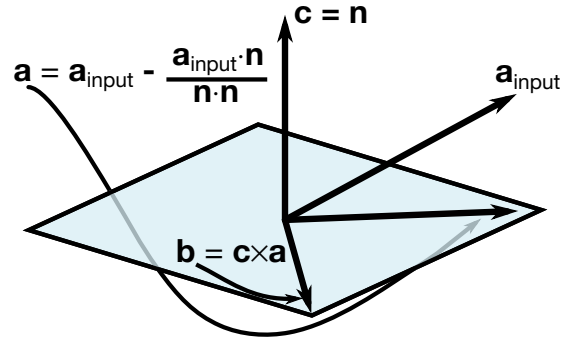
$\mathbf{d}$  is set parallel to  $\mathbf{e}_z$ .

input( $p$ )  $\rightarrow$   $\{\mathbf{a}\} \rightarrow \{\mathbf{c}\} \rightarrow \{\mathbf{b}\}$

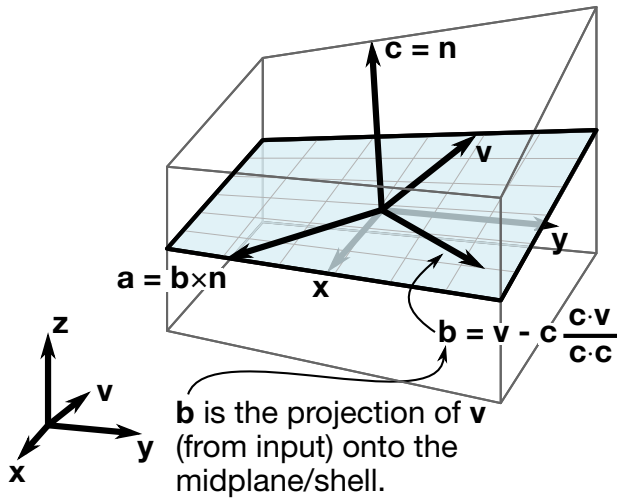
**AOPT = 2.0 (solid)**



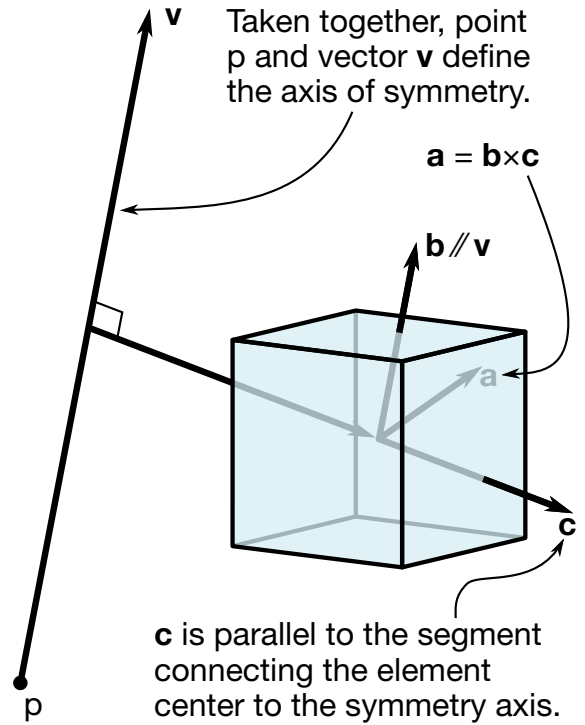
**AOPT = 2.0 (shell)**

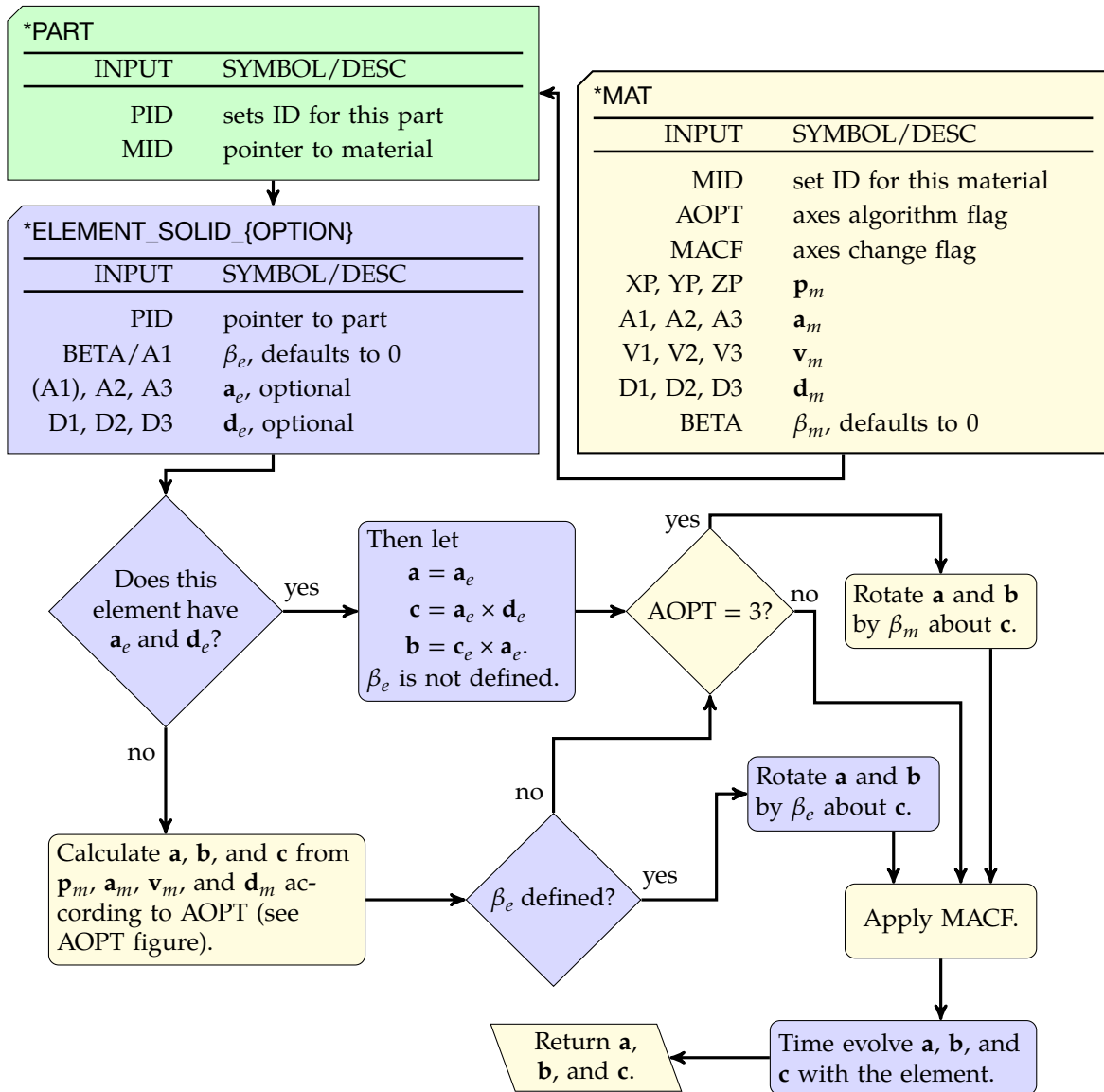


**AOPT = 3.0**

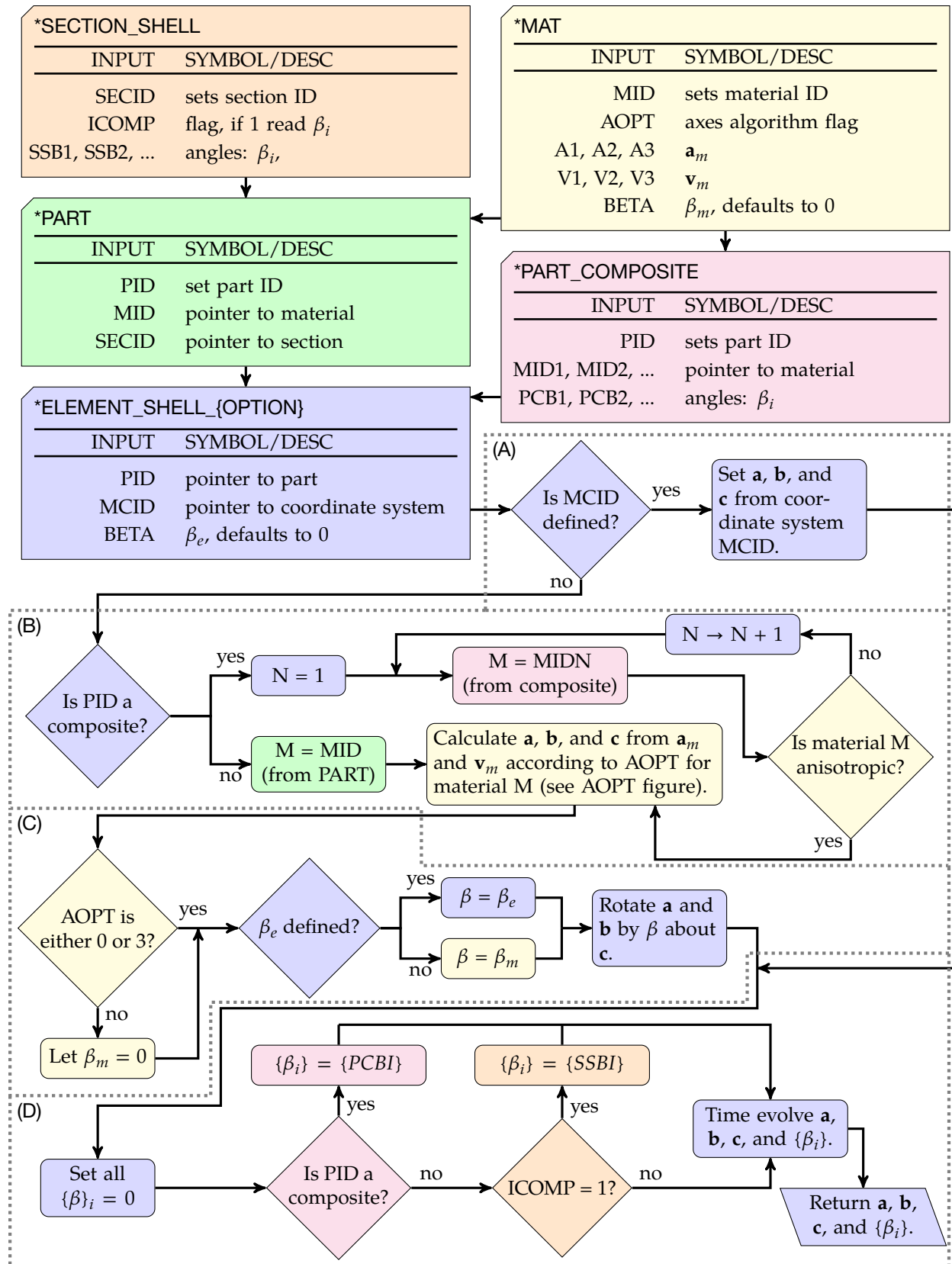


**AOPT = 4.0 (solid only)**





**Figure M2-2.** Flow chart showing how for each solid element LS-DYNA determines the vectors  $\{a, b, c\}$  from the input.



**Figure M2-3.** Flowchart for shells: (a) check for coordinate system ID; (b) process AOPT; (c) determine  $\beta$ ; and (d) for each layer determine  $\beta_i$ .

**\*MAT\_PLASTIC\_KINEMATIC**

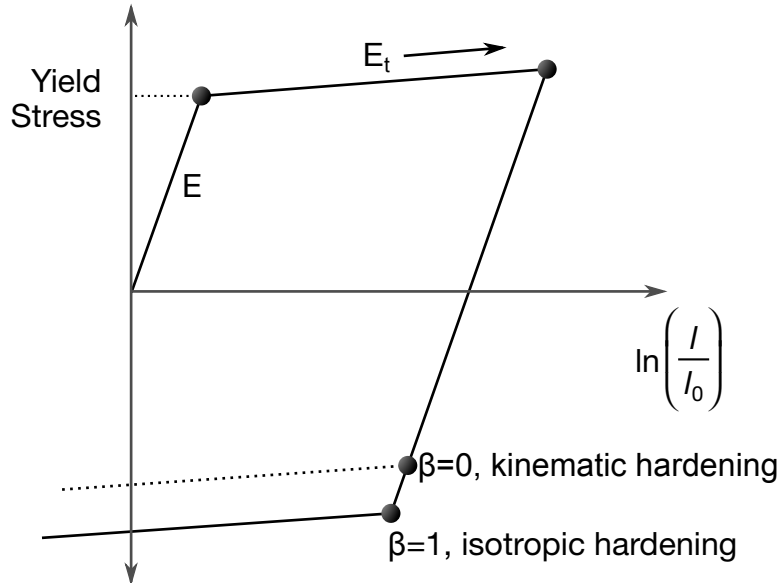
This is Material Type 3. This model is suited to model isotropic and kinematic hardening plasticity with the option of including rate effects. It is a very cost effective model and is available for beam (Hughes-Liu and Truss), shell, and solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ETAN	BETA	
Type	A8	F	F	F	F	F	F	
Default	none	none	none	none	none	0.0	0.0	

Card 2	1	2	3	4	5	6	7	8
Variable	SRC	SRP	FS	VP				
Type	F	F	F	F				
Default	0.0	0.0	1.E+20	0.0				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
ETAN	Tangent modulus, see <a href="#">Figure M3-1</a>
BETA	Hardening parameter, $0 < \beta' < 1$ . See comments below.
SRC	Strain rate parameter, $C$ , for Cowper Symonds strain rate model, see below. If zero, rate effects are not considered..



**Figure M3-1.** Elastic-plastic behavior with kinematic and isotropic hardening where  $l_0$  and  $l$  are undeformed and deformed lengths of uniaxial tension specimen.  $E_t$  is the slope of the bilinear stress strain curve.

VARIABLE	DESCRIPTION
SRP	Strain rate parameter, $P$ , for Cowper Symonds strain rate model, see below. If zero, rate effects are not considered.
FS	Effective plastic strain for eroding elements.
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default), EQ.1.0: Viscoplastic formulation (recommended)

### Remarks:

Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate. A fully viscoplastic formulation is optional which incorporates the Cowper and Symonds formulation within the yield surface. To ignore strain rate effects set both SRC and SRP to zero.

Kinematic, isotropic, or a combination of kinematic and isotropic hardening may be specified by varying  $\beta'$  between 0 and 1. For  $\beta'$  equal to 0 and 1, respectively, kinematic

and isotropic hardening are obtained as shown in [Figure M3-1](#). For isotropic hardening,  $\beta' = 1$ , Material Model 12, \*MAT\_ISOTROPIC\_ELASTIC\_PLASTIC, requires less storage and is more efficient. Whenever possible, Material 12 is recommended for solid elements, but for shell elements it is less accurate and thus Material 12 is not recommended in this case.



**\*MAT\_ELASTIC\_PLASTIC\_THERMAL**

This is Material Type 4. Temperature dependent material coefficients can be defined. A maximum of eight temperatures with the corresponding data can be defined. A minimum of two points is needed. When this material type is used it is necessary to define nodal temperatures by activating a coupled analysis or by using another option to define the temperatures such as \*LOAD\_THERMAL\_LOAD\_CURVE, or \*LOAD\_THERMAL\_VARIABLE.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0						
Type	A8	F						

Card 2	1	2	3	4	5	6	7	8
Variable	T1	T2	T3	T4	T5	T6	T7	T8
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	E1	E2	E3	E4	E5	E6	E7	E8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	PR1	PR2	PR3	PR4	PR5	PR6	PR7	PR8
Type	F	F	F	F	F	F	F	F

No defaults are assumed.

Card 5	1	2	3	4	5	6	7	8
Variable	ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8
Type	F	F	F	F	F	F	F	F

Card 6	1	2	3	4	5	6	7	8
Variable	SIGY1	SIGY2	SIGY3	SIGY4	SIGY5	SIGY6	SIGY7	SIGY8
Type	F	F	F	F	F	F	F	F

Card 7	1	2	3	4	5	6	7	8
Variable	ETAN1	ETAN2	ETAN3	ETAN4	ETAN5	ETAN6	ETAN7	ETAN8
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
TI	Temperatures. The minimum is 2, the maximum is 8.
EI	Corresponding Young's moduli at temperature TI.
PRI	Corresponding Poisson's ratios.
ALPHA1	Corresponding coefficients of thermal expansion.
SIGY1	Corresponding yield stresses.
ETAN1	Corresponding plastic hardening moduli.

**Remarks:**

The stress update for this material follows the standard approach to hypo-elastoplasticity, using Jaumann rate for objectivity. The rate of Cauchy stress  $\sigma$  can in principal be expressed as

$$\dot{\sigma} = C(\dot{\epsilon} - \dot{\epsilon}_T - \dot{\epsilon}_p) + \dot{C}C^{-1}\sigma$$

where  $C$  is the temperature dependent isotropic elasticity tensor,  $\dot{\epsilon}$  is the rate-of-deformation,  $\dot{\epsilon}_T$  is the thermal strain rate and  $\dot{\epsilon}_p$  is the plastic strain rate. The coefficient of thermal expansion is defined as the instantaneous value. Thus, the thermal strain rate becomes

$$\dot{\epsilon}_T = \alpha \dot{T} I$$

where  $\alpha$  is the temperature dependent thermal expansion coefficient,  $\dot{T}$  is the rate of temperature and  $I$  is the identity tensor. Associated von Mises plasticity is adopted, resulting in

$$\dot{\epsilon}_p = \dot{\epsilon}_p \frac{3s}{2\bar{\sigma}}$$

where  $\dot{\epsilon}_p$  is the effective plastic strain rate,  $s$  is the deviatoric stress tensor and  $\bar{\sigma}$  is the von Mises effective stress. The last term accounts for stress changes due to change in stiffness with respect to temperature, using the total elastic strain defined as  $\epsilon_e = C^{-1}\sigma$ . A way to intuitively understand this contribution, for small displacement elasticity if neglecting everything but the temperature dependent elasticity parameters, we have

$$\dot{\sigma} = \frac{d}{dt}(C\epsilon)$$

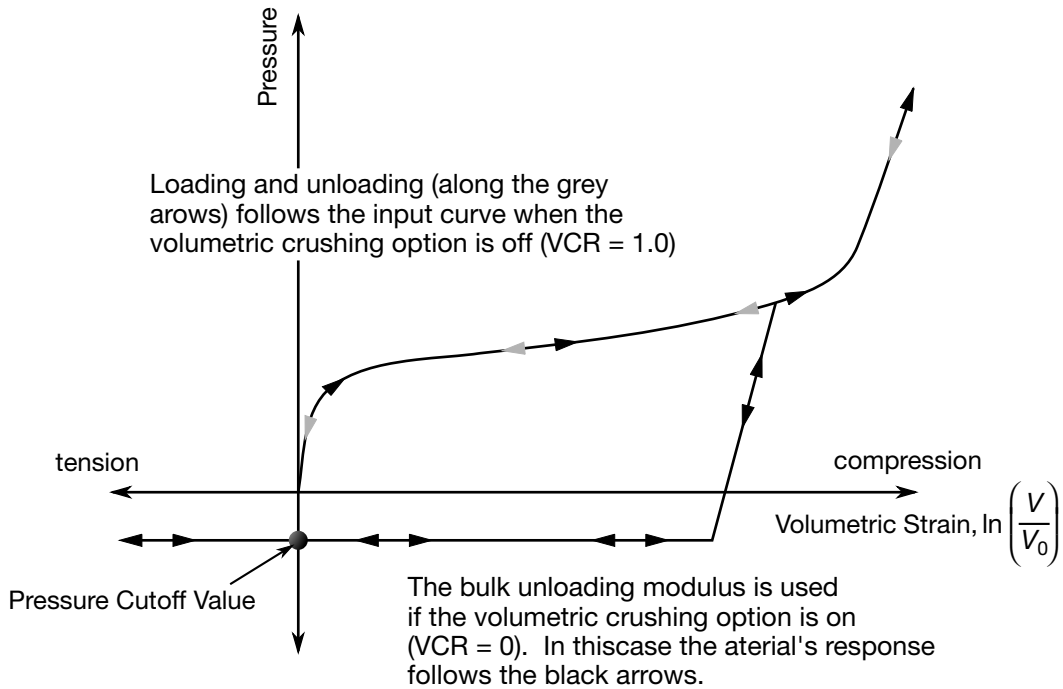
as a special case, showing that the stress may change without any change in strain.

At least two temperatures and their corresponding material properties must be defined. The analysis will be terminated if a material temperature falls outside the range defined in the input. If a thermo-elastic material is considered, do not define SIGY and ETAN.



Card 6	1	2	3	4	5	6	7	8
Variable	P9	P10						
Type	F	F						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G	Shear modulus.
KUN	Bulk modulus for unloading used for VCR = 0.0.
A0	Yield function constant for plastic yield function below.
A1	Yield function constant for plastic yield function below.
A2	Yield function constant for plastic yield function below.
PC	Pressure cutoff for tensile fracture (< 0).
VCR	Volumetric crushing option: EQ.0.0: on, EQ.1.0: loading and unloading paths are the same.
REF	Use reference geometry to initialize the pressure. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY. This option does not initialize the deviatoric stress state. EQ.0.0: off, EQ.1.0: on.
LCID	Load curve ID for compressive pressure (ordinate) as a function of volumetric strain (abscissa). If LCID is defined, then the curve is used instead of the input for EPS1..., and P1.... It makes no difference whether the values of volumetric strain in the curve are input as positive or negative since internally, a negative sign is applied to the absolute value of each abscissa entry. The response is extended to being temperature dependent if LCID refers to a table.



**Figure M5-1.** Pressure versus volumetric strain curve for soil and crushable foam model. The volumetric strain is given by the natural logarithm of the relative volume,  $V$ .

VARIABLE	DESCRIPTION
EPS1, ...	Volumetric strain values in pressure vs. volumetric strain curve (see Remarks below). A maximum of 10 values including 0.0 are allowed and a minimum of 2 values are necessary. If EPS1 is not 0.0 then a point (0.0,0.0) will be automatically generated and a maximum of nine values may be input.
P1, P2, ..., PN	Pressures corresponding to volumetric strain values given on Cards 3 and 4.

**Remarks:**

Pressure is positive in compression. Volumetric strain is given by the natural log of the relative volume and is negative in compression. Relative volume is a ratio of the current volume to the initial volume at the start of the calculation. The tabulated data should be given in order of increasing compression. If the pressure drops below the cutoff value specified, it is reset to that value. For a detailed description we refer to Kreig [1972].

The deviatoric perfectly plastic yield function,  $\phi$ , is described in terms of the second invariant  $J_2$ ,

$$J_2 = \frac{1}{2} s_{ij}s_{ij},$$

pressure,  $p$ , and constants  $a_0$ ,  $a_1$ , and  $a_2$  as:

$$\phi = J_2 - [a_0 + a_1p + a_2p^2].$$

On the yield surface  $J_2 = \frac{1}{3}\sigma_y^2$  where  $\sigma_y$  is the uniaxial yield stress, i.e.,

$$\sigma_y = [3(a_0 + a_1p + a_2p^2)]^{1/2}$$

there is no strain hardening on this surface.

To eliminate the pressure dependence of the yield strength, set:

$$a_1 = a_2 = 0 \quad \text{and} \quad a_0 = \frac{1}{3}\sigma_y^2.$$

This approach is useful when a von Mises type elastic-plastic model is desired for use with the tabulated volumetric data.

The history variable labeled as “plastic strain” by LS-PrePost is actually plastic volumetric strain. Note that when  $VCR = 1.0$ , plastic volumetric strain is zero.

**\*MAT\_VISCOELASTIC**

This is Material Type 6. This model allows the modeling of viscoelastic behavior for beams (Hughes-Liu), shells, and solids. Also see \*MAT\_GENERAL\_VISCOELASTIC for a more general formulation.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	G0	GI	BETA		
Type	A8	F	F	F	F	F		

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
BULK	Elastic bulk modulus. LT.0.0:  BULK  is load curve of bulk modulus as a function of temperature.
G0	Short-time shear modulus, see equations below. LT.0.0:  G0  is load curve of short-time shear modulus as a function of temperature.
GI	Long-time (infinite) shear modulus, $G_\infty$ . LT.0.0:  GI  is load curve of long-time shear modulus as a function of temperature.
BETA	Decay constant. LT.0.0:  BETA  is load curve of decay constant as a function of temperature.

**Remarks:**

The shear relaxation behavior is described by [Hermann and Peterson, 1968]:

$$G(t) = G_\infty + (G_0 - G_\infty)\exp(-\beta t)$$

A Jaumann rate formulation is used



$$\overset{\nabla}{\sigma}'_{ij} = 2 \int_0^t G(t - \tau) D'_{ij}(\tau) d\tau$$

where the prime denotes the deviatoric part of the stress rate,  $\overset{\nabla}{\sigma}'_{ij}$ , and the strain rate,  $D_{ij}$ .

**\*MAT\_BLATZ-KO\_RUBBER**

This is Material Type 7. This one parameter material allows the modeling of nearly incompressible continuum rubber. The Poisson's ratio is fixed to 0.463.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G	REF				
Type	A8	F	F	F				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G	Shear modulus.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.

**Remarks:**

The strain energy density potential for the Blatz-Ko rubber is

$$W(\mathbf{C}) = \frac{G}{2} \left[ I_1 - 3 + \frac{1}{\beta} (I_3^{-\beta} - 1) \right]$$

where  $G$  is the shear modulus,  $I_1$  and  $I_3$  are the first and third invariants of the right Cauchy-Green tensor  $\mathbf{C} = \mathbf{F}^T \mathbf{F}$  and

$$\beta = \frac{\nu}{1 - 2\nu}.$$

The second Piola-Kirchhoff stress is computed as

$$\mathbf{S} = 2 \frac{\partial W}{\partial \mathbf{C}} = G [\mathbf{I} - I_3^{-\beta} \mathbf{C}^{-1}]$$

from which the Cauchy stress is obtained by a push-forward from the reference to current configuration divided by the relative volume  $J = \det(\mathbf{F})$ ,

$$\boldsymbol{\sigma} = \frac{1}{J} \mathbf{F} \mathbf{S} \mathbf{F}^T = \frac{G}{J} [\mathbf{B} - I_3^{-\beta} \mathbf{I}].$$

Here we used  $\mathbf{B} = \mathbf{F} \mathbf{F}^T$  to denote the left Cauchy-Green tensor, and the Poisson ratio  $\nu$  above is set internally to  $\nu = 0.463$ , also see Blatz and Ko [1962].

**\*MAT\_HIGH\_EXPLOSIVE\_BURN**

This is Material Type 8. It allows the modeling of the detonation of a high explosive. In addition an equation of state must be defined. See Wilkins [1969] and Giroux [1973].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	D	PCJ	BETA	K	G	SIGY
Type	A8	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
D	Detonation velocity.
PCJ	Chapman-Jouget pressure.
BETA	Beta burn flag, BETA (see comments below): EQ.0.0: beta and programmed burn, EQ.1.0: beta burn only, EQ.2.0: programmed burn only.
K	Bulk modulus (BETA = 2.0 only).
G	Shear modulus (BETA = 2.0 only).
SIGY	$\sigma_y$ , yield stress (BETA = 2.0 only).

**Remarks:**

Burn fractions,  $F$ , which multiply the equations of states for high explosives, control the release of chemical energy for simulating detonations. At any time, the pressure in a high explosive element is given by:

$$p = Fp_{\text{eos}}(V, E)$$

where  $p_{\text{eos}}$  is the pressure from the equation of state (either types 2, 3, or 14),  $V$  is the relative volume, and  $E$  is the internal energy density per unit initial volume.

In the initialization phase, a lighting time  $t_l$  is computed for each element by dividing the distance from the detonation point to the center of the element by the detonation velocity  $D$ . If multiple detonation points are defined, the closest detonation point determines  $t_l$ . The burn fraction  $F$  is taken as the maximum

$$F = \max(F_1, F_2)$$

Where

$$F_1 = \begin{cases} \frac{2 (t - t_l) D A_{e_{\max}}}{3v_e} & \text{if } t > t_l \\ 0 & \text{if } t \leq t_l \end{cases}$$

$$F_2 = \beta = \frac{1 - V}{1 - V_{CJ}}$$

where  $V_{CJ}$  is the Chapman-Jouguet relative volume and  $t$  is current time. If  $F$  exceeds 1, it is reset to 1. This calculation of the burn fraction usually requires several time steps for  $F$  to reach unity, thereby spreading the burn front over several elements. After reaching unity,  $F$  is held constant. This burn fraction calculation is based on work by Wilkins [1964] and is also discussed by Giroux [1973].

If the beta burn option is used,  $BETA = 1.0$ , any volumetric compression will cause detonation and

$$F = F_2$$

and  $F_1$  is not computed.

If programmed burn is used,  $BETA = 2.0$ , the undetonated high explosive material will behave as an elastic perfectly plastic material if the bulk modulus, shear modulus, and yield stress are defined. Therefore, with this option the explosive material can compress without causing detonation. The location and time of detonation is controlled by \*INITIAL\_DETONATION.

As an option, the high explosive material can behave as an elastic perfectly-plastic solid prior to detonation. In this case we update the stress tensor, to an elastic trial stress,  $*s_{ij}^{n+1}$ ,

$$*s_{ij}^{n+1} = s_{ij}^n + s_{ip}\Omega_{pj} + s_{jp}\Omega_{pi} + 2G\dot{\epsilon}'_{ij}dt$$

where  $G$  is the shear modulus, and  $\dot{\epsilon}'_{ij}$  is the deviatoric strain rate. The von Mises yield condition is given by:

$$\phi = J_2 - \frac{\sigma_y^2}{3}$$

where the second stress invariant,  $J_2$ , is defined in terms of the deviatoric stress components as

$$J_2 = \frac{1}{2}s_{ij}s_{ij}$$

and the yield stress is  $\sigma_y$ . If yielding has occurred, i.e.,  $\varphi > 0$ , the deviatoric trial stress is scaled to obtain the final deviatoric stress at time  $n+1$ :

$$s_{ij}^{n+1} = \frac{\sigma_y}{\sqrt{3}J_2} * s_{ij}^{n+1}$$

If  $\varphi \leq 0$ , then

$$s_{ij}^{n+1} = * s_{ij}^{n+1}$$

Before detonation pressure is given by the expression

$$p^{n+1} = K \left( \frac{1}{V^{n+1}} - 1 \right)$$

where  $K$  is the bulk modulus. Once the explosive material detonates:

$$s_{ij}^{n+1} = 0$$

and the material behaves like a gas.

\*MAT\_NULL

This is Material Type 9.

In the case of solids and thick shells, this material allows equations of state to be considered without computing deviatoric stresses. Optionally, a viscosity can be defined. Also, erosion in tension and compression is possible.

Beams and shells that use this material type are completely bypassed in the element processing; however, the mass of the null beam or shell elements is computed and added to the nodal points which define the connectivity. The mass of null beams is ignored if the value of the density is less than 1.e-11. The Young’s modulus and Poisson’s ratio are used only for setting the contact stiffness, and it is recommended that reasonable values be input. The variables PC, MU, TEROD, and EDROD do not apply to beams and shells. Historically, null beams and/or null shells have been used as an aid in modeling of contact but this practice is now seldom needed.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PC	MU	TEROD	CEROD	YM	PR
Type	A8	F	F	F	F	F	F	F
Defaults	none	none	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
PC	Pressure cutoff ( $\leq 0.0$ ). See Remark 4.
MU	Dynamic viscosity $\mu$ (optional). See Remark 1.
TEROD	Relative volume, $\frac{V}{V_0}$ , for erosion in tension. Typically, use values greater than unity. If zero, erosion in tension is inactive.
CEROD	Relative volume, $\frac{V}{V_0}$ , for erosion in compression. Typically, use values less than unity. If zero, erosion in compression is inactive.
YM	Young’s modulus (used for null beams and shells only)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PR	Poisson's ratio (used for null beams and shells only)

**Remarks:**

These remarks apply to solids and thick shells only.

1. When used with solids or thick shells, this material must be used with an equation-of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\epsilon}'_{ij}$$

$$\left[\frac{N}{m^2}\right] \sim \left[\frac{N}{m^2}s\right] \left[\frac{1}{s}\right]$$

is computed for nonzero  $\mu$  where  $\dot{\epsilon}'_{ij}$  is the deviatoric strain rate.  $\mu$  is the dynamic viscosity. For example, in SI unit system,  $\mu$  may have a unit of [Pa\*s].

2. Null material has no shear stiffness (except from viscosity) and hourglass control must be used with great care. In some applications, the default hourglass coefficient may lead to significant energy losses. In general for fluid, the hourglass coefficient QM should be small (in the range 1.0E-6 to 1.0E-4) and the hourglass type IHQ should be set to 1 (default).
3. The Null material has no yield strength and behaves in a fluid-like manner.
4. The cut-off pressure, PC, must be defined to allow for a material to “numerically” cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.





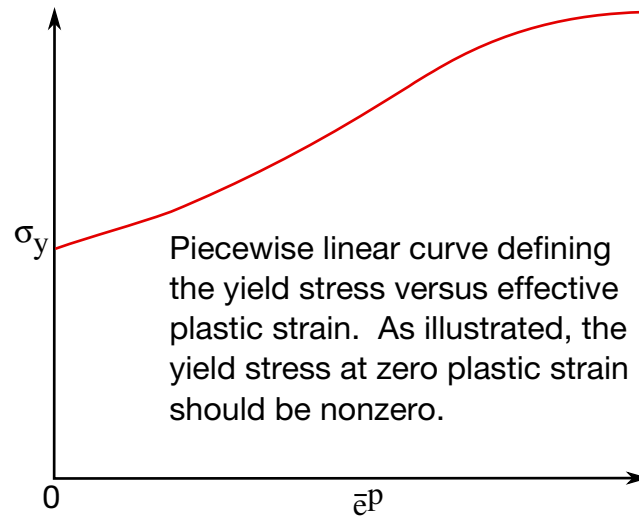
Card 3	1	2	3	4	5	6	7	8
Variable	EPS9	EPS10	EPS11	EPS12	EPS13	EPS14	EPS15	EPS16
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	ES9	ES10	ES11	ES12	ES13	ES14	ES15	ES16
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G	Shear modulus.
SIG0	Yield stress, see comment below.
EH	Plastic hardening modulus, see definition below.
PC	Pressure cutoff ( $\leq 0.0$ ). If zero, a cutoff of $-\infty$ is assumed.
FS	Effective plastic strain at which erosion occurs.



**Figure M10-1.** Effective stress versus effective plastic strain curve. See EPS and ES input.

VARIABLE	DESCRIPTION
CHARL	Characteristic element thickness for deletion. This applies to 2D solid elements that lie on a boundary of a part. If the boundary element thins down due to stretching or compression, and if it thins to a value less than CHARL, the element will be deleted. The primary application of this option is to predict the break-up of axisymmetric shaped charge jets.
A1	Linear pressure hardening coefficient.
A2	Quadratic pressure hardening coefficient.
SPALL	Spall type: EQ.0.0: default set to "1.0", EQ.1.0: tensile pressure is limited by PC, i.e., $p$ is always $\geq PC$ , EQ.2.0: if $\sigma_{\max} \geq -PC$ element spalls and tension, $p < 0$ , is never allowed, EQ.3.0: $p < PC$ element spalls and tension, $p < 0$ , is never allowed.
EPS	Effective plastic strain (True). Define up to 16 values. Care must be taken that the full range of strains expected in the analysis is covered. Linear extrapolation is used if the strain values exceed the maximum input value.
ES	Effective stress. Define up to 16 values.

**Remarks:**

If ES and EPS are undefined, the yield stress and plastic hardening modulus are taken from SIG0 and EH. In this case, the bilinear stress-strain curve shown in M10-1 is obtained with hardening parameter,  $\beta = 1$ . The yield strength is calculated as

$$\sigma_y = \sigma_0 + E_h \bar{\epsilon}^p + (a_1 + p a_2) \max[p, 0]$$

The quantity  $E_h$  is the plastic hardening modulus defined in terms of Young's modulus,  $E$ , and the tangent modulus,  $E_t$ , as follows

$$E_h = \frac{E_t E}{E - E_t}$$

and  $p$  is the pressure taken as positive in compression.

If ES and EPS are specified, a curve like that shown in M10-1 may be defined. Effective stress is defined in terms of the deviatoric stress tensor,  $s_{ij}$ , as:

$$\bar{\sigma} = \left( \frac{3}{2} s_{ij} s_{ij} \right)^{1/2}$$

**and effective plastic strain by:**

$$\bar{\epsilon}^p = \int_0^t \left( \frac{2}{3} D_{ij}^p D_{ij}^p \right)^{1/2} dt,$$

where  $t$  denotes time and  $D_{ij}^p$  is the plastic component of the rate of deformation tensor. In this case the plastic hardening modulus on Card 1 is ignored and the yield stress is given as

$$\sigma_y = f(\bar{\epsilon}^p),$$

where the value for  $f(\bar{\epsilon}^p)$  is found by interpolation from the data curve.

A choice of three spall models is offered to represent material splitting, cracking, and failure under tensile loads. The pressure limit model, SPALL = 1, limits the hydrostatic tension to the specified value,  $p_{\text{cut}}$ . If pressures more tensile than this limit are calculated, the pressure is reset to  $p_{\text{cut}}$ . This option is not strictly a spall model, since the deviatoric stresses are unaffected by the pressure reaching the tensile cutoff, and the pressure cutoff value,  $p_{\text{cut}}$ , remains unchanged throughout the analysis.

The maximum principal stress spall model, SPALL = 2, detects spall if the maximum principal stress,  $\sigma_{\text{max}}$ , exceeds the limiting value  $-p_{\text{cut}}$ . Note that the negative sign is required because  $p_{\text{cut}}$  is measured positive in compression, while  $\sigma_{\text{max}}$  is positive in tension. Once spall is detected with this model, the deviatoric stresses are reset to zero, and no hydrostatic tension ( $p < 0$ ) is permitted. If tensile pressures are calculated, they are reset to

0 in the spalled material. Thus, the spalled material behaves as a rubble or incohesive material.

The hydrostatic tension spall model,  $SPALL = 3$ , detects spall if the pressure becomes more tensile than the specified limit,  $p_{cut}$ . Once spall is detected the deviatoric stresses are reset to zero, and nonzero values of pressure are required to be compressive (positive). If hydrostatic tension ( $p < 0$ ) is subsequently calculated, the pressure is reset to 0 for that element.

This model is applicable to a wide range of materials, including those with pressure-dependent yield behavior. The use of 16 points in the yield stress versus effective plastic strain curve allows complex post-yield hardening behavior to be accurately represented. In addition, the incorporation of an equation of state permits accurate modeling of a variety of different materials. The spall model options permit incorporation of material failure, fracture, and disintegration effects under tensile loads.

The **STOCHASTIC** option allows spatially varying yield and failure behavior. See **\*DEFINE\_STOCHASTIC\_VARIATION** for additional information.

**\*MAT\_STEINBERG**

This is Material Type 11. This material is available for modeling materials deforming at very high strain rates ( $> 10^5$ ) and can be used with solid elements. The yield strength is a function of temperature and pressure. An equation of state determines the pressure.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G0	SIG0	BETA	N	GAMA	SIGM
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	B	BP	H	F	A	TMO	GAMO	SA
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	PC	SPALL	RP	FLAG	MMN	MMX	ECO	EC1
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	EC2	EC3	EC4	EC5	EC6	EC7	EC8	EC9
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G0	Basic shear modulus.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SIGO	$\sigma_o$ , see defining equations.
BETA	$\beta$ , see defining equations.
N	n, see defining equations.
GAMA	$\gamma_i$ , initial plastic strain, see defining equations.
SIGM	$\sigma_m$ , see defining equations.
B	b, see defining equations.
BP	b', see defining equations.
H	h, see defining equations.
F	f, see defining equations.
A	Atomic weight (if = 0.0, R' must be defined).
TMO	$T_{m0}$ , see defining equations.
GAMO	$\gamma_o$ , see defining equations.
SA	a, see defining equations.
PC	Pressure cutoff (default = -1.e+30)
SPALL	Spall type: EQ.0.0: default set to "2.0", EQ.1.0: $p \geq PC$ , EQ.2.0: if $\sigma_{max} \geq -PC$ element spalls and tension, $p < 0$ , is never allowed, EQ.3.0: $p < PC$ element spalls and tension, $p < 0$ , is never allowed.
RP	R'. If $R' \neq 0.0$ , A is not defined.
FLAG	Set to 1.0 for $\mu$ coefficients for the cold compression energy fit. Default is $\eta$ .
MMN	$\mu_{min}$ or $\eta_{min}$ . Optional $\mu$ or $\eta$ minimum value.
MMX	$\mu_{max}$ or $\eta_{max}$ . Optional $\mu$ or $\eta$ maximum value.

VARIABLE	DESCRIPTION
EC0, ..., EC9	Cold compression energy coefficients (optional).

**Remarks:**

Users who have an interest in this model are encouraged to study the paper by Steinberg and Guinan which provides the theoretical basis. Another useful reference is the KOVEC user's manual.

In terms of the foregoing input parameters, we define the shear modulus,  $G$ , before the material melts as:

$$G = G_0 \left[ 1 + bpV^{1/3} - h \left( \frac{E_i - E_c}{3R'} - 300 \right) \right] e^{\frac{-fE_i}{E_m - E_i}}$$

where  $p$  is the pressure,  $V$  is the relative volume,  $E_c$  is the cold compression energy:

$$E_c(x) = \int_0^x p dx - \frac{900R' \exp(ax)}{(1-x)^{2(\gamma_0 - a - 1/2)'}}$$

$$x = 1 - V,$$

and  $E_m$  is the melting energy:

$$E_m(x) = E_c(x) + 3R'T_m(x)$$

which is in terms of the melting temperature  $T_m(x)$ :

$$T_m(x) = \frac{T_{mo} \exp(2ax)}{V^{2(\gamma_0 - a - 1/3)}}$$

and the melting temperature at  $\rho = \rho_0$ ,  $T_{mo}$ .

In the above equation  $R'$  is defined by

$$R' = \frac{R\rho}{A}$$

where  $R$  is the gas constant and  $A$  is the atomic weight. If  $R'$  is not defined, LS-DYNA computes it with  $R$  in the cm-gram-microsecond system of units.

The yield strength  $\sigma_y$  is given by:

$$\sigma_y = \sigma'_0 \left[ 1 + b'pV^{1/3} - h \left( \frac{E_i - E_c}{3R'} - 300 \right) \right] e^{\frac{-fE_i}{E_m - E_i}}$$

if  $E_m$  exceeds  $E_i$ . Here,  $\sigma'_0$  is given by:

$$\sigma'_0 = \sigma_0 [1 + \beta(\gamma_i + \bar{\epsilon}^p)]^n$$



where  $\sigma_0$  is the initial yield stress and  $\gamma_i$  is the initial plastic strain. If the work-hardened yield stress  $\sigma'_0$  exceeds  $\sigma_m$ ,  $\sigma'_0$  is set equal to  $\sigma_m$ . After the materials melt,  $\sigma_y$  and  $G$  are set to one half their initial value.

If the coefficients EC0, ..., EC9 are not defined above, LS-DYNA will fit the cold compression energy to a ten term polynomial expansion either as a function of  $\mu$  or  $\eta$  depending on the input variable, FLAG, as:

$$E_c(\eta^i) = \sum_{i=0}^9 EC_i \eta^i$$

$$E_c(\mu^i) = \sum_{i=0}^9 EC_i \mu^i$$

where  $EC_i$  is the  $i$ th coefficient and:

$$\eta = \frac{\rho}{\rho_o}$$

$$\mu = \frac{\rho}{\rho_o} - 1$$

A linear least squares method is used to perform the fit.

A choice of three spall models is offered to represent material splitting, cracking, and failure under tensile loads. The pressure limit model, SPALL = 1, limits the hydrostatic tension to the specified value,  $p_{cut}$ . If pressures more tensile than this limit are calculated, the pressure is reset to  $p_{cut}$ . This option is not strictly a spall model, since the deviatoric stresses are unaffected by the pressure reaching the tensile cutoff, and the pressure cutoff value,  $p_{cut}$ , remains unchanged throughout the analysis. The maximum principal stress spall model, SPALL = 2, detects spall if the maximum principal stress,  $\sigma_{max}$ , exceeds the limiting value  $-p_{cut}$ . Note that the negative sign is required because  $p_{cut}$  is measured positive in compression, while  $\sigma_{max}$  is positive in tension. Once spall is detected with this model, the deviatoric stresses are reset to zero, and no hydrostatic tension ( $p < 0$ ) is permitted. If tensile pressures are calculated, they are reset to 0 in the spalled material. Thus, the spalled material behaves as a rubble or incohesive material. The hydrostatic tension spall model, SPALL = 3, detects spall if the pressure becomes more tensile than the specified limit,  $p_{cut}$ . Once spall is detected the deviatoric stresses are reset to zero, and nonzero values of pressure are required to be compressive (positive). If hydrostatic tension ( $p < 0$ ) is subsequently calculated, the pressure is reset to 0 for that element.

This model is applicable to a wide range of materials, including those with pressure-dependent yield behavior. In addition, the incorporation of an equation of state permits accurate modeling of a variety of different materials. The spall model options permit incorporation of material failure, fracture, and disintegration effects under tensile loads.



Card 5	1	2	3	4	5	6	7	8
Variable	UK	C1	C2	YP	YA	YM		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G0	Basic shear modulus.
SIGO	$\sigma_0$ , see defining equations.
BETA	$\beta$ , see defining equations.
N	$n$ , see defining equations.
GAMA	$\gamma_i$ , initial plastic strain, see defining equations.
SIGM	$\sigma_m$ , see defining equations.
B	$b$ , see defining equations.
BP	$b'$ , see defining equations.
H	$h$ , see defining equations.
F	$f$ , see defining equations.
A	Atomic weight (if = 0.0, $R'$ must be defined).
TMO	$T_{m0}$ , see defining equations.
GAMO	$\gamma_0$ , see defining equations.
SA	$a$ , see defining equations.
PC	$p_{cut}$ or $-\sigma_f$ (default = $-1.e+30$ )

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SPALL	Spall type: EQ.0.0: default set to "2.0", EQ.1.0: $p \geq p_{\min}$ , EQ.2.0: if $\sigma_{\max} \geq -p_{\min}$ element spalls and tension, $p < 0$ , is never allowed, EQ.3.0: $p < -p_{\min}$ element spalls and tension, $p < 0$ , is never allowed.
RP	R'. If R'≠0.0, A is not defined.
FLAG	Set to 1.0 for $\mu$ coefficients for the cold compression energy fit. Default is $\eta$ .
MMN	$\mu_{\min}$ or $\eta_{\min}$ . Optional $\mu$ or $\eta$ minimum value.
MMX	$\mu_{\max}$ or $\eta_{\max}$ . Optional $\mu$ or $\eta$ maximum value.
EC0, ..., EC9	Cold compression energy coefficients (optional).
UK	Activation energy for rate dependent model.
C1	Exponent prefactor in rate dependent model.
C2	Coefficient of drag term in rate dependent model.
YP	Peierls stress for rate dependent model.
YA	A thermal yield stress for rate dependent model.
YMAX	Work hardening maximum for rate model.

**Remarks:**

This model is similar in theory to the \*MAT\_STEINBERG above but with the addition of rate effects. When rate effects are included, the yield stress is given by:

$$\sigma_y = \{Y_T(\dot{\epsilon}_p, T) + Y_{Af}(\epsilon_p)\} \frac{G(p, T)}{G_0}$$

There are two imposed limits on the yield stress. The first is on the thermal yield stress:

$$Y_{Af}(\epsilon_p) = Y_A [1 + \beta(\gamma_i + \epsilon^p)]^n \leq Y_{\max}$$

and the second is on the thermal part:

$$Y_T \leq Y_P$$

R' is the heat capacity per unit volume. Most handbooks give the heat capacity per unit mass or per mole. To obtain R', multiply the heat capacity per unit mass by the initial density, and to obtain R' from the heat capacity per mole, divide it by the mass per mole and then multiply the result by the initial density. The mass per mole in grams equals the atomic weight.

For example, the heat capacity per mole for aluminum is 24.2 J/mole/K, the density is 2.70 g/cc, and the atomic weight is 13. The heat capacity per cubic centimeter is therefore  $(24.2 \text{ J/mole/K}) / (13\text{g/mole}) \times (2.70\text{g/cc}) = 5.026 \text{ J/cc/K}$ . To convert it to J/m<sup>3</sup>/K, multiply the result by 10<sup>6</sup> cc/m<sup>3</sup> to obtain a final heat capacity of 5.026e6 J/m<sup>3</sup>/K.

**\*MAT\_ISOTROPIC\_ELASTIC\_PLASTIC**

This is Material Type 12. This is a very low cost isotropic plasticity model for three-dimensional solids. In the plane stress implementation for shell elements, a one-step radial return approach is used to scale the Cauchy stress tensor to if the state of stress exceeds the yield surface. This approach to plasticity leads to inaccurate shell thickness updates and stresses after yielding. This is the only model in LS-DYNA for plane stress that does not default to an iterative approach.

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	G	SIGY	ETAN	BULK		
Type	A8	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G	Shear modulus.
SIGY	Yield stress.
ETAN	Plastic hardening modulus.
BULK	Bulk modulus, K.

**Remarks:**

Here the pressure is integrated in time

$$\dot{p} = -K\dot{\epsilon}_{ii}$$

where  $\dot{\epsilon}_{ii}$  is the volumetric strain rate.

**\*MAT\_ISOTROPIC\_ELASTIC\_FAILURE**

This is Material Type 13. This is a non-iterative plasticity with simple plastic strain failure model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G	SIGY	ETAN	BULK		
Type	A8	F	F	F	F	F		
Default	none	none	none	none	0.0	none		

Card 2	1	2	3	4	5	6	7	8
Variable	EPF	PRF	REM	TREM				
Type	F	F	F	F				
Default	none	0.0	0.0	0.0				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G	Shear modulus.
SIGY	Yield stress.
ETAN	Plastic hardening modulus.
BULK	Bulk modulus.
EPF	Plastic failure strain.
PRF	Failure pressure ( $\leq 0.0$ ).

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
REM	Element erosion option: EQ.0.0: failed element eroded after failure, NE.0.0: element is kept, no removal except by $\Delta t$ below.
TREM	$\Delta t$ for element removal: EQ.0.0: $\Delta t$ is not considered (default), GT.0.0: element eroded if element time step size falls below $\Delta t$ .

**Remarks:**

When the effective plastic strain reaches the failure strain or when the pressure reaches the failure pressure, the element loses its ability to carry tension and the deviatoric stresses are set to zero, i.e., the material behaves like a fluid. If  $\Delta t$  for element removal is defined the element removal option is ignored.

The element erosion option based on  $\Delta t$  must be used cautiously with the contact options. Nodes to surface contact is recommended with all nodes of the eroded brick elements included in the node list. As the elements are eroded the mass remains and continues to interact with the master surface.



**\*MAT\_SOIL\_AND\_FOAM\_FAILURE**

This is Material Type 14. The input for this model is the same as for \*MATERIAL\_SOIL\_-AND\_FOAM (Type 5); however, when the pressure reaches the tensile failure pressure, the element loses its ability to carry tension. It should be used only in situations when soils and foams are confined within a structure or are otherwise confined by nodal boundary conditions.

**\*MAT\_JOHNSON\_COOK\_{OPTION}**

Available options include:

<BLANK>

STOCHASTIC

This is Material Type 15. The Johnson/Cook strain and temperature sensitive plasticity is sometimes used for problems where the strain rates vary over a large range and adiabatic temperature increases due to plastic heating cause material softening. When used with solid elements this model requires an equation-of-state. If thermal effects and damage are unimportant, the much less expensive \*MAT\_SIMPLIFIED\_JOHNSON\_COOK model is recommended. The simplified model can be used with beam elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	G	E	PR	DTF	VP	RATEOP
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	A	B	N	C	M	TM	TR	EPS0
Type	F	F	F	F	F	F	F	F
Default	none	0.0	0.0	0.0	none	none	none	none

Card 3	1	2	3	4	5	6	7	8
Variable	CP	PC	SPALL	IT	D1	D2	D3	D4
Type	F	F	F	F	F	F	F	F
Default	none	0.0	2.0	0.0	0.0	0.0	0.0	0.0

Card 4	1	2	3	4	5	6	7	8
Variable	D5	C2/P/XNP	EROD	EFMIN	NUMINT			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	10 <sup>-6</sup>	0.			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
G	Shear modulus. G and an equation-of-state are required for element types that use a 3D stress update, e.g., solids, 2D shell forms 13-15, tshell forms 3, 5, and 7. For other element types, G is ignored, and E and PR must be provided.
E	Young's Modulus (see note above pertaining to G).
PR	Poisson's ratio (see note above pertaining to G).
DTF	Minimum time step size for automatic element deletion (shell elements). The element will be deleted when the solution time step size drops below DTF × TSSFAC where TSSFAC is the time step scale factor defined by *CONTROL_TIMESTEP.
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default), EQ.1.0: Viscoplastic formulation.
RATEOP	Form of strain-rate term. RATEOP is ignored if VP = 0. EQ.0.0: Log-Linear Johnson-Cook (default), EQ.1.0: Log-Quadratic Huh-Kang (2 parameters), EQ.2.0: Exponential Allen-Rule-Jones, EQ.3.0: Exponential Cowper-Symonds (2 parameters). EQ.4.0: Nonlinear rate coefficient (2 parameters).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A	See equations below
B	See equations below
N	See equations below
C	See equations below
M	See equations below
TM	Melt temperature
TR	Room temperature
EPSO	Quasi-static threshold strain rate. Ideally, this value represents the highest strain rate for which no rate adjustment to the flow stress is needed, and is input in units of $[\text{time}]^{-1}$ . For example, if strain rate effects on the flow stress first become apparent at strain rates greater than $10^{-2}\text{s}^{-1}$ and the system of units for the model input is kg, mm, ms, then EPSO should be set to $10^{-5}$ , which is $10^{-2}\text{s}^{-1}$ in units of ms.
CP	Specific heat (superseded by heat capacity in *MAT_THERMAL_OPTION if a coupled thermal/structural analysis)
PC	Tensile failure stress or tensile pressure cutoff ( $PC < 0.0$ )
SPALL	<p>Spall type:</p> <p>EQ.0.0: default set to "2.0".</p> <p>EQ.1.0: Tensile pressure is limited by PC, i.e., <math>p</math> is always <math>\geq PC</math>.</p> <p><i>Shell Element Specific Behavior:</i></p> <p>EQ.2.0: Shell elements are deleted when <math>\sigma_{\max} \geq -PC</math>.</p> <p>EQ.3.0: Shell elements are deleted when <math>p &lt; PC</math>.</p> <p><i>Solid Element Specific Behavior</i></p> <p>EQ.2.0: For solid elements <math>\sigma_{\max} \geq -PC</math> resets tensile stresses to zero. Compressive stress are still allowed.</p> <p>EQ.3.0: For solid elements <math>p &lt; PC</math> resets the pressure to zero thereby disallowing tensile pressure.</p>

VARIABLE	DESCRIPTION												
IT	<p>Plastic strain iteration option. This input applies to solid elements only since it is always necessary to iterate for the shell element plane stress condition.</p> <p>EQ.0.0: no iterations (default),</p> <p>EQ.1.0: accurate iterative solution for plastic strain. Much more expensive than default.</p>												
D1 - D5	<p>Failure parameters, see equations below. A negative input of D3 will be converted to its absolute value.</p>												
C2/P/XNP	<p>Optional strain-rate parameter.</p> <table border="1"> <thead> <tr> <th>Field</th> <th>Var</th> <th>Model</th> </tr> </thead> <tbody> <tr> <td>C2</td> <td><math>C_2</math></td> <td>Huh-Kang</td> </tr> <tr> <td>P</td> <td><math>P</math></td> <td>Cowper-Symonds</td> </tr> <tr> <td>XNP</td> <td><math>n'</math></td> <td>Nonlinear Rate Coefficient</td> </tr> </tbody> </table> <p>These models are documented in the remarks.</p>	Field	Var	Model	C2	$C_2$	Huh-Kang	P	$P$	Cowper-Symonds	XNP	$n'$	Nonlinear Rate Coefficient
Field	Var	Model											
C2	$C_2$	Huh-Kang											
P	$P$	Cowper-Symonds											
XNP	$n'$	Nonlinear Rate Coefficient											
EROD	<p>Erosion Flag:</p> <p>EQ.0.0: default, element erosion allowed.</p> <p>NE.0.0: element does not erode; deviatoric stresses set to zero when element fails.</p>												
EFMIN	<p>The lower bound for calculated strain at fracture (see equation).</p>												
NUMINT	<p>Number of through thickness integration points which must fail before the shell element is deleted. (If zero, all points must fail.)</p> <p>Since nodal fiber rotations limit strains at active integration points, the default, which is to require that <i>all</i> integration points fail, is not recommended, because elements undergoing large strain are often not deleted using this criterion. Better results may be obtained when NUMINT is set to 1 or a number less than one half of the number of through thickness points.</p> <p>For example, if four through thickness points are used, NUMINT should not exceed 2, even for fully integrated shells which have 16 integration points.</p>												

**Remarks:**

Johnson and Cook express the flow stress as

$$\sigma_y = (A + B\bar{\epsilon}^n)(1 + c \ln \dot{\epsilon}^*)(1 - T^{*m})$$

Where,

$A, B, c, n,$  and  $m =$  input constants

$\bar{\epsilon}^p =$  effective plastic strain

$$\dot{\epsilon}^* = \begin{cases} \frac{\dot{\epsilon}}{\text{EPS0}} & \text{for VP.EQ.0 (normalized effective total strain-rate)} \\ \frac{\dot{\epsilon}^p}{\text{EPS0}} & \text{for VP.EQ.1 (normalized effective plastic strain rate)} \end{cases}$$

$$T^* = \text{homologous temperature} = \frac{T - T_{\text{room}}}{T_{\text{melt}} - T_{\text{room}}}$$

The quantity  $T - T_{\text{room}}$  is stored as extra history variable 5. In the case of a mechanical-only analysis, this is the adiabatic temperature increase calculated as

$$T - T_{\text{room}} = (\text{internal energy}) / (C_p \times \rho \times V_0)$$

Where,

$C_p$  and  $\rho =$  input constants

$V_0 =$  initial volume

In a coupled thermal/mechanical analysis,  $T - T_{\text{room}}$  includes heating/cooling from all sources, not just adiabatic heating from the internal energy.

Constants for a variety of materials are provided in Johnson and Cook [1983]. A fully viscoplastic formulation is optional (VP) which incorporates the rate equations within the yield surface. An additional cost is incurred but the improvement is that results can be dramatic.

Due to nonlinearity in the dependence of flow stress on plastic strain, an accurate value of the flow stress requires iteration for the increment in plastic strain. However, by using a Taylor series expansion with linearization about the current time, we can solve for  $\sigma_y$  with sufficient accuracy to avoid iteration.

The strain at fracture is given by

$$\epsilon^f = \max([D_1 + D_2 \exp D_3 \sigma^*][1 + D_4 \ln \dot{\epsilon}^*][1 + D_5 T^*], \text{EFMIN})$$

where  $\sigma^*$  is the ratio of pressure divided by effective stress

$$\sigma^* = \frac{p}{\sigma_{\text{eff}}}$$

Fracture occurs when the damage parameter

$$D = \sum \frac{\Delta \bar{\epsilon}^p}{\epsilon^f}$$

reaches the value of 1.  $D$  is stored as extra history variable 4 in shell elements and extra history variable 6 in solid elements.

A choice of three spall models is offered to represent material splitting, cracking, and failure under tensile loads. The pressure limit model limits the minimum hydrostatic pressure to the specified value,  $p \geq p_{\min}$ . If pressures more tensile than this limit are calculated, the pressure is reset to  $p_{\min}$ . This option is not strictly a spall model since the deviatoric stresses are unaffected by the pressure reaching the tensile cutoff and the pressure cutoff value  $p_{\min}$  remains unchanged throughout the analysis. The maximum principal stress spall model detects spall if the maximum principal stress,  $\sigma_{\max}$ , exceeds the limiting value  $\sigma_p$ . Once spall in solids is detected with this model, the deviatoric stresses are reset to zero and no hydrostatic tension is permitted. If tensile pressures are calculated, they are reset to 0 in the spalled material. Thus, the spalled material behaves as rubble. The hydrostatic tension spall model detects spall if the pressure becomes more tensile than the specified limit,  $p_{\min}$ . Once spall in solids is detected with this model, the deviatoric stresses are set to zero and the pressure is required to be compressive. If hydrostatic tension is calculated then the pressure is reset to 0 for that element.

In addition to the above failure criterion, this material model also supports a shell element deletion criterion based on the maximum stable time step size for the element,  $\Delta t_{\max}$ . Generally,  $\Delta t_{\max}$  goes down as the element becomes more distorted. To assure stability of time integration, the global LS-DYNA time step is the minimum of the  $\Delta t_{\max}$  values calculated for all elements in the model. Using this option allows the selective deletion of elements whose time step  $\Delta t_{\max}$  has fallen below the specified minimum time step,  $\Delta t_{\text{crit}}$ . Elements which are severely distorted often indicate that material has failed and supports little load, but these same elements may have very small time steps and therefore control the cost of the analysis. This option allows these highly distorted elements to be deleted from the calculation, and, therefore, the analysis can proceed at a larger time step, and, thus, at a reduced cost. Deleted elements do not carry any load, and are deleted from all applicable slide surface definitions. Clearly, this option must be judiciously used to obtain accurate results at a minimum cost.

Material type 15 is applicable to the high rate deformation of many materials including most metals. Unlike the Steinberg-Guinan model, the Johnson-Cook model remains valid down to lower strain rates and even into the quasistatic regime. Typical applications include explosive metal forming, ballistic penetration, and impact.

### **Optional Strain Rate Forms:**

The standard Johnson-Cook strain rate term is linear in the logarithm of the strain rate:

$$1 + C \ln \dot{\epsilon}^*$$

Some additional data fitting capability can be obtained by using the quadratic form proposed by Huh & Kang [2002]:

$$1 + C \ln \dot{\epsilon}^* + C_2 (\ln \dot{\epsilon}^*)^2$$

Three additional exponential forms are available, one due to Allen, Rule & Jones [1997],

$$(\dot{\epsilon}^*)^c$$

the Cowper-Symonds-like [1958] form

$$1 + \left( \frac{\dot{\epsilon}_{\text{eff}}^p}{C} \right)^{\frac{1}{P}}$$

and the nonlinear rate coefficient,

$$1 + C (\epsilon_{\text{eff}}^p)^{n'} \ln \dot{\epsilon}^*.$$

The four additional rate forms (RATEOP = 1, 2, 3 or 4) are currently available for solid & shell elements but only when the viscoplastic rate option is active (VP = 1). If VP is set to zero, RATEOP is ignored. See Huh and Kang [2002], Allen, Rule, and Jones [1997], and Cowper and Symonds [1958].

The STOCHASTIC option allows spatially varying yield and failure behavior. See \*DEFINE\_STOCHASTIC\_VARIATION for additional information.



**\*MAT\_PSEUDO\_TENSOR**

This is Material Type 16. This model has been used to analyze buried steel reinforced concrete structures subjected to impulsive loadings.

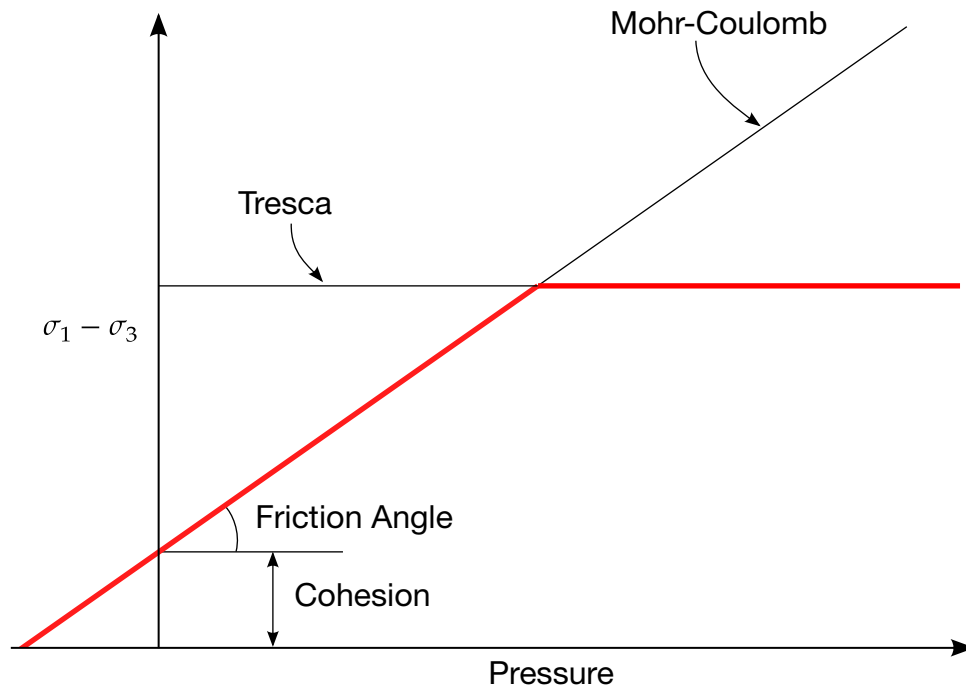
Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	G	PR				
Type	A8	F	F	F				
Default	none	none	none	none				

Card 2	1	2	3	4	5	6	7	8
Variable	SIGF	A0	A1	A2	A0F	A1F	B1	PER
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Card 3	1	2	3	4	5	6	7	8
Variable	ER	PRR	SIGY	ETAN	LCP	LCR		
Type	F	F	F	F	F	F		
Default	0.0	0.0	none	0.0	none	none		



<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G	Shear modulus.
PR	Poisson's ratio.
SIGF	Tensile cutoff (maximum principal stress for failure).
A0	Cohesion.
A1	Pressure hardening coefficient.
A2	Pressure hardening coefficient.
A0F	Cohesion for failed material.
A1F	Pressure hardening coefficient for failed material.
B1	Damage scaling factor (or exponent in Mode II.C).
PER	Percent reinforcement.
ER	Elastic modulus for reinforcement.
PRR	Poisson's ratio for reinforcement.
SIGY	Initial yield stress.
ETAN	Tangent modulus/plastic hardening modulus.
LCP	Load curve ID giving rate sensitivity for principal material, see *DEFINE_CURVE.
LCR	Load curve ID giving rate sensitivity for reinforcement, see *DEFINE_CURVE.
$X_n$	Effective plastic strain, damage, or pressure. See discussion below.
$YS_n$	Yield stress (Mode I) or scale factor (Mode II.B or II.C).



**Figure M16-1.** Mohr-Coulomb surface with a Tresca Limit.

### Remarks:

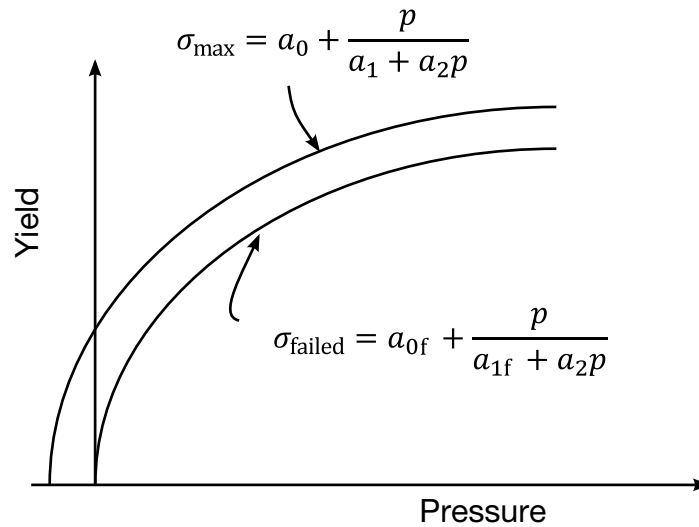
This model can be used in two major modes - a simple tabular pressure-dependent yield surface, and a potentially complex model featuring two yield versus pressure functions with the means of migrating from one curve to the other. For both modes, load curve LCP is taken to be a strain rate multiplier for the yield strength. Note that this model must be used with equation-of-state type 8, 9 or 11.

### Response Mode I. Tabulated Yield Stress Versus Pressure

This model is well suited for implementing standard geologic models like the Mohr-Coulomb yield surface with a Tresca limit, as shown in [Figure M16-1](#). Examples of converting conventional triaxial compression data to this type of model are found in (Desai and Siriwardane, 1984). Note that under conventional triaxial compression conditions, the LS-DYNA input corresponds to an ordinate of  $\sigma_1 - \sigma_3$  rather than the more widely used  $\frac{\sigma_1 - \sigma_3}{2}$ , where  $\sigma_1$  is the maximum principal stress and  $\sigma_3$  is the minimum principal stress.

This material combined with equation-of-state type 9 (saturated) has been used very successfully to model ground shocks and soil-structure interactions at pressures up to 100kbars (approximately  $1.5 \times 10^6$  psi).

To invoke Mode I of this model, set  $a_0, a_1, a_2, b_1, a_{0f}$ , and  $a_{1f}$  to zero. The tabulated values of pressure should then be specified on cards 4 and 5, and the corresponding values of yield



**Figure M16-2.** Two-curve concrete model with damage and failure

stress should be specified on cards 6 and 7. The parameters relating to reinforcement properties, initial yield stress, and tangent modulus are not used in this response mode, and should be set to zero.

### Simple tensile failure

Note that  $a_{1f}$  is reset internally to  $1/3$  even though it is input as zero; this defines a failed material curve of slope  $3p$ , where  $p$  denotes pressure (positive in compression). In this case the yield strength is taken from the tabulated yield vs. pressure curve until the maximum principal stress ( $\sigma_1$ ) in the element exceeds the tensile cutoff  $\sigma_{cut}$  (input as variable SIGF). When  $\sigma_1 > \sigma_{cut}$  is detected, the yield strength is scaled back by a fraction of the distance between the two curves in each of the next 20 time steps so that after those 20 time steps, the yield strength is defined by the failure curve. The only way to inhibit this feature is to set  $\sigma_{cut}$  (SIGF) arbitrarily large.

### Response Mode II. Two Curve Model with Damage and Failure

This approach uses two yield versus pressure curves of the form

$$\sigma_y = a_0 + \frac{p}{a_1 + a_2 p}$$

The upper curve is best described as the maximum yield strength curve and the lower curve is the failed material curve. There are a variety of ways of moving between the two curves and each is discussed below.

#### MODE II. A: Simple tensile failure

Define  $a_0$ ,  $a_1$ ,  $a_2$ ,  $a_{0f}$  and  $a_{1f}$ , set  $b_1$  to zero, and leave cards 4 through 7 blank. In this case the yield strength is taken from the maximum yield curve until the maximum principal stress ( $\sigma_1$ ) in the element exceeds the tensile cutoff ( $\sigma_{cut}$ ). When  $\sigma_1 > \sigma_{cut}$  is detected, the yield strength is scaled back by a fraction of the distance between the two curves in each of

the next 20 time steps so that after those 20 time steps, the yield strength is defined by the failure curve.

### Mode II.B: Tensile failure plus plastic strain scaling

Define  $a_0, a_1, a_2, a_{0f}$  and  $a_{1f}$ , set  $b_1$  to zero, and user cards 4 through 7 to define a scale factor,  $\eta$ , versus effective plastic strain. LS-DYNA evaluates  $\eta$  at the current effective plastic strain and then calculated the yield stress as

$$\sigma_{\text{yield}} = \sigma_{\text{failed}} + \eta(\sigma_{\text{max}} - \sigma_{\text{failed}})$$

where  $\sigma_{\text{max}}$  and  $\sigma_{\text{failed}}$  are found as shown in [Figure M16-2](#). This yield strength is then subject to scaling for tensile failure as described above. This type of model allows the description of a strain hardening or softening material such as concrete.

### Mode II.C: Tensile failure plus damage scaling

The change in yield stress as a function of plastic strain arises from the physical mechanisms such as internal cracking, and the extent of this cracking is affected by the hydrostatic pressure when the cracking occurs. This mechanism gives rise to the "confinement" effect on concrete behavior. To account for this phenomenon, a "damage" function was defined and incorporated. This damage function is given the form:

$$\lambda = \int_0^{\varepsilon^p} \left(1 + \frac{p}{\sigma_{\text{cut}}}\right)^{-b_1} d\varepsilon^p$$

Define  $a_0, a_1, a_2, a_{0f}$  and  $a_{1f}$ , and  $b_1$ . Cards 4 though 7 now give  $\eta$  as a function of  $\lambda$  and scale the yield stress as

$$\sigma_{\text{yield}} = \sigma_{\text{failed}} + \eta(\sigma_{\text{max}} - \sigma_{\text{failed}})$$

and then apply any tensile failure criteria.

### Mode II Concrete Model Options

Material Type 16 Mode II provides for the automatic internal generation of a simple "generic" model from concrete if A0 is negative then SIGF is assumed to be the unconfined concrete compressive strength,  $f'_c$  and  $-A0$  is assumed to be a conversion factor from LS-DYNA pressure units to psi. (For example, if the model stress units are MPa, A0 should be set to  $-145$ .) In this case the parameter values generated internally are

$$\begin{aligned}
 f'_c &= \text{SIGF} & a_1 &= \frac{1}{3} & a_{0f} &= 0 \\
 \sigma_{cut} &= 1.7 \left( \frac{f_c'^2}{-A0} \right)^{\frac{1}{3}} & a_2 &= \frac{1}{3f'_c} & a_{1f} &= 0.385 \\
 a_0 &= \frac{f'_c}{4}
 \end{aligned}$$

Note that these  $a_{0f}$  and  $a_{1f}$  defaults will be overridden by non zero entries on Card 3. If plastic strain or damage scaling is desired, Cards 5 through 8 and  $b_1$  should be specified in the input. When  $a_0$  is input as a negative quantity, the equation-of-state can be given as 0 and a trilinear EOS Type 8 model will be automatically generated from the unconfined compressive strength and Poisson's ratio. The EOS 8 model is a simple pressure versus volumetric strain model with no internal energy terms, and should give reasonable results for pressures up to 5kbar (approximately 75,000 psi).

### Mixture model

A reinforcement fraction,  $f_r$ , can be defined (indirectly as PER/100) along with properties of the reinforcement material. The bulk modulus, shear modulus, and yield strength are then calculated from a simple mixture rule, i.e., for the bulk modulus the rule gives:

$$K = (1 - f_r)K_m + f_r K_r$$

where  $K_m$  and  $K_r$  are the bulk moduli for the geologic material and the reinforcement material, respectively. This feature should be used with caution. It gives an isotropic effect in the material instead of the true anisotropic material behavior. A reasonable approach would be to use the mixture elements only where the reinforcing exists and plain elements elsewhere. When the mixture model is being used, the strain rate multiplier for the principal material is taken from load curve N1 and the multiplier for the reinforcement is taken from load curve N2.

### A Suggestion

The LLNL DYNA3D manual from 1991 [Whirley and Hallquist] suggests using the damage function (Mode II.C.) in Material Type 16 with the following set of parameters:

$$\begin{aligned}
 a_0 &= \frac{f'_c}{4} & a_2 &= \frac{1}{3f'_c} & a_{1f} &= 1.5 \\
 a_1 &= \frac{1}{3} & a_{0f} &= \frac{f'_c}{10} & b_1 &= 1.25
 \end{aligned}$$

**\*MAT\_016****\*MAT\_PSEUDO\_TENSOR**

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and a damage table of:

Card 4:	0.0 5.17E-04	8.62E-06 6.38E-04	2.15E-05 7.98E-04	3.14E-05	3.95E-04
Card 5:	9.67E-04 4.00E-03	1.41E-03 4.79E-03	1.97E-03 0.909	2.59E-03	3.27E-03
Card 6:	0.309 0.790	0.543 0.630	0.840 0.469	0.975	1.000
Card 7:	0.383 0.086	0.247 0.056	0.173 0.0	0.136	0.114

This set of parameters should give results consistent with Dilger, Koch, and Kowalczyk, [1984] for plane concrete. It has been successfully used for reinforced structures where the reinforcing bars were modeled explicitly with embedded beam and shell elements. The model does not incorporate the major failure mechanism - separation of the concrete and reinforcement leading to catastrophic loss of confinement pressure. However, experience indicates that this physical behavior will occur when this model shows about 4% strain.



**\*MAT\_ORIENTED\_CRACK**

This is Material Type 17. This material may be used to model brittle materials which fail due to large tensile stresses.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ETAN	FS	PRF
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	none	0.0

Optional card for crack propagation to adjacent elements (see remarks):

Card 2	1	2	3	4	5	6	7	8
Variable	SOFT	CVELO						
Type	F	F						
Default	1.0	0.0						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
ETAN	Plastic hardening modulus.
FS	Fracture stress.
PRF	Failure or cutoff pressure ( $\leq 0.0$ ).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SOFT	Factor by which the fracture stress is reduced when a crack is coming from failed neighboring element. See remarks.
CVELO	Crack propagation velocity. See remarks.

**Remarks:**

This is an isotropic elastic-plastic material which includes a failure model with an oriented crack. The von Mises yield condition is given by:

$$\phi = J_2 - \frac{\sigma_y^2}{3}$$

where the second stress invariant,  $J_2$ , is defined in terms of the deviatoric stress components as

$$J_2 = \frac{1}{2} s_{ij} s_{ij}$$

and the yield stress,  $\sigma_y$ , is a function of the effective plastic strain,  $\epsilon_{\text{eff}}^p$ , and the plastic hardening modulus,  $E_p$ :

$$\sigma_y = \sigma_0 + E_p \epsilon_{\text{eff}}^p$$

The effective plastic strain is defined as:

$$\epsilon_{\text{eff}}^p = \int_0^t d\epsilon_{\text{eff}}^p$$

where

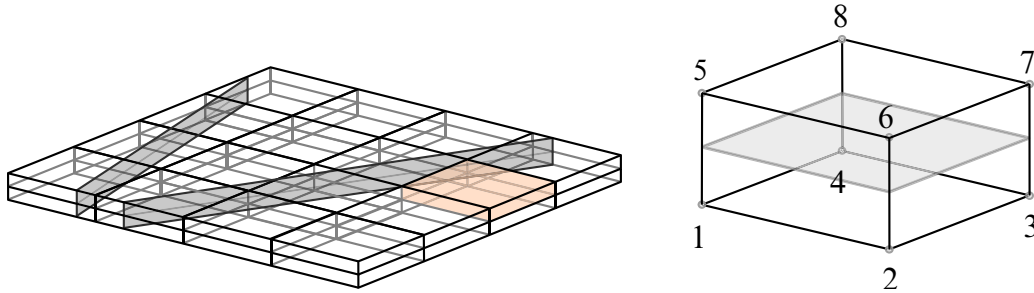
$$d\epsilon_{\text{eff}}^p = \sqrt{\frac{2}{3} d\epsilon_{ij}^p d\epsilon_{ij}^p}$$

and the plastic tangent modulus is defined in terms of the input tangent modulus,  $E_t$ , as

$$E_p = \frac{EE_t}{E - E_t}$$

Pressure in this model is found from evaluating an equation of state. A pressure cutoff can be defined such that the pressure is not allowed to fall below the cutoff value.

The oriented crack fracture model is based on a maximum principal stress criterion. When the maximum principal stress exceeds the fracture stress,  $\sigma_f$ , the element fails on a plane perpendicular to the direction of the maximum principal stress. The normal stress and the two shear stresses on that plane are then reduced to zero. This stress reduction is done according to a delay function that reduces the stresses gradually to zero over a small number of time steps. This delay function procedure is used to reduce the ringing that may



**Figure M17-1.** Thin structure (2 elements over thickness) with cracks and necessary element numbering.

otherwise be introduced into the system by the sudden fracture. The number of steps for stress reduction is 20 by default (CVELO = 0.0) or it is internally computed if CVELO > 0.0 is given:

$$n_{\text{steps}} = \text{int} \left[ \frac{L_e}{\text{CVELO} \times \Delta t} \right]$$

where  $L_e$  is characteristic element length and  $\Delta t$  is time step size.

After a tensile fracture, the element will not support tensile stress on the fracture plane, but in compression will support both normal and shear stresses. The orientation of this fracture surface is tracked throughout the deformation, and is updated to properly model finite deformation effects. If the maximum principal stress subsequently exceeds the fracture stress in another direction, the element fails isotropically. In this case the element completely loses its ability to support any shear stress or hydrostatic tension, and only compressive hydrostatic stress states are possible. Thus, once isotropic failure has occurred, the material behaves like a fluid.

This model is applicable to elastic or elastoplastic materials under significant tensile or shear loading when fracture is expected. Potential applications include brittle materials such as ceramics as well as porous materials such as concrete in cases where pressure hardening effects are not significant.

Crack propagation behavior to adjacent elements can be controlled via parameter SOFT for thin, shell-like structures (e.g. only 2 or 3 solids over thickness). Additionally, LS-DYNA has to know where the plane or solid element midplane is at each integration point for projection of crack plane on this element midplane. Therefore, element numbering has to be as shown in [Figure M17-1](#). Only solid element type 1 is supported with that option at the moment.

**\*MAT\_POWER\_LAW\_PLASTICITY**

This is Material Type 18. This is an isotropic plasticity model with rate effects which uses a power law hardening rule.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	K	N	SRC	SRP
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	SIGY	VP	EPSF					
Type	F	F	F					
Default	0.0	0.0	0.0					

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
K	Strength coefficient.
N	Hardening exponent.
SRC	Strain rate parameter, $C$ , if zero, rate effects are ignored.
SRP	Strain rate parameter, $P$ , if zero, rate effects are ignored.

VARIABLE	DESCRIPTION
SIGY	Optional input parameter for defining the initial yield stress, $\sigma_y$ . Generally, this parameter is not necessary and the strain to yield is calculated as described below. LT.0.02: $\varepsilon_{yp} = \text{SIGY}$ GT.0.02: See below.
EPSF	Plastic failure strain for element deletion.
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default), EQ.1.0: Viscoplastic formulation.

**Remarks:**

Elastoplastic behavior with isotropic hardening is provided by this model. The yield stress,  $\sigma_y$ , is a function of plastic strain and obeys the equation:

$$\sigma_y = k\varepsilon^n = k(\varepsilon_{yp} + \bar{\varepsilon}^p)^n$$

where  $\varepsilon_{yp}$  is the elastic strain to yield and  $\bar{\varepsilon}^p$  is the effective plastic strain (logarithmic). If SIGY is set to zero, the strain to yield is found by solving for the intersection of the linearly elastic loading equation with the strain hardening equation:

$$\begin{aligned}\sigma &= E\varepsilon \\ \sigma &= k \varepsilon^n\end{aligned}$$

which gives the elastic strain at yield as:

$$\varepsilon_{yp} = \left(\frac{E}{k}\right)^{\left[\frac{1}{n-1}\right]}$$

If SIGY is nonzero and greater than 0.02 then:

$$\varepsilon_{yp} = \left(\frac{\sigma_y}{k}\right)^{\left[\frac{1}{n}\right]}$$

Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate. A fully viscoplastic formulation is optional which incorporates the Cowper and Symonds formulation within the yield surface. An additional cost is incurred but the improvement in results can be dramatic.

**\*MAT\_STRAIN\_RATE\_DEPENDENT\_PLASTICITY**

This is Material Type 19. A strain rate dependent material can be defined. For an alternative, see Material Type 24. Required is a curve for the yield stress versus the effective strain rate. Optionally, Young's modulus and the tangent modulus can also be defined versus the effective strain rate. Also, optional failure of the material can be defined either by defining a von Mises stress at failure as a function of the effective strain rate (valid for solids/shells/thick shells) or by defining a minimum time step size (only for shells).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	VP			
Type	A8	F	F	F	F			
Default	none	none	none	none	0.0			

Card 2	1	2	3	4	5	6	7	8
Variable	LC1	ETAN	LC2	LC3	LC4	TDEL	RDEF	
Type	F	F	F	F	F	F	F	
Default	none	0.0	0.0	0.0	0.0	0.0	0.0	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default), EQ.1.0: Viscoplastic formulation

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LC1	Load curve ID defining the yield stress $\sigma_0$ as a function of the effective strain rate.
ETAN	Tangent modulus, $E_t$
LC2	Load curve ID defining Young's modulus as a function of the effective strain rate (available only when VP = 0; not recommended).
LC3	Load curve ID defining tangent modulus as a function of the effective strain rate (optional).
LC4	Load curve ID defining von Mises stress at failure as a function of the effective strain rate (optional).
TDEL	Minimum time step size for automatic element deletion. Use for shells only.
RDEF	Redefinition of failure curve: EQ.1.0: Effective plastic strain, EQ.2.0: Maximum principal stress and absolute value of minimum principal stress, EQ.3.0: Maximum principal stress (release 5 of v.971)

**Remarks:**

In this model, a load curve is used to describe the yield strength  $\sigma_0$  as a function of effective strain rate  $\dot{\epsilon}$  where

$$\dot{\epsilon} = \left( \frac{2}{3} \dot{\epsilon}'_{ij} \dot{\epsilon}'_{ij} \right)^{1/2}$$

and the prime denotes the deviatoric component. The strain rate is available for post-processing as the first stored history variable. If the viscoplastic option is active, the plastic strain rate is output; otherwise, the effective strain rate defined above is output.

The yield stress is defined as

$$\sigma_y = \sigma_0(\dot{\epsilon}) + E_p \bar{\epsilon}^p$$

where  $\bar{\epsilon}^p$  is the effective plastic strain and  $E_p$  is given in terms of Young's modulus and the tangent modulus by

$$E_p = \frac{EE_t}{E - E_t}$$



Both Young's modulus and the tangent modulus may optionally be made functions of strain rate by specifying a load curve ID giving their values as a function of strain rate. If these load curve ID's are input as 0, then the constant values specified in the input are used.

Note that all load curves used to define quantities as a function of strain rate must have the same number of points at the same strain rate values. This requirement is used to allow vectorized interpolation to enhance the execution speed of this constitutive model.

This model also contains a simple mechanism for modeling material failure. This option is activated by specifying a load curve ID defining the effective stress at failure as a function of strain rate. For solid elements, once the effective stress exceeds the failure stress the element is deemed to have failed and is removed from the solution. For shell elements the entire shell element is deemed to have failed if all integration points through the thickness have an effective stress that exceeds the failure stress. After failure the shell element is removed from the solution.

In addition to the above failure criterion, this material model also supports a shell element deletion criterion based on the maximum stable time step size for the element,  $\Delta t_{\max}$ . Generally,  $\Delta t_{\max}$  goes down as the element becomes more distorted. To assure stability of time integration, the global LS-DYNA time step is the minimum of the  $\Delta t_{\max}$  values calculated for all elements in the model. Using this option allows the selective deletion of elements whose time step  $\Delta t_{\max}$  has fallen below the specified minimum time step,  $\Delta t_{\text{crit}}$ . Elements which are severely distorted often indicate that material has failed and supports little load, but these same elements may have very small time steps and therefore control the cost of the analysis. This option allows these highly distorted elements to be deleted from the calculation, and, therefore, the analysis can proceed at a larger time step, and, thus, at a reduced cost. Deleted elements do not carry any load, and are deleted from all applicable slide surface definitions. Clearly, this option must be judiciously used to obtain accurate results at a minimum cost.

A fully viscoplastic formulation is optional which incorporates the rate formulation within the yield surface. An additional cost is incurred but the improvement in results can be dramatic.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus. Reasonable values have to be chosen for contact analysis (choice of penalty), see Remarks below.
PR	Poisson's ratio. Reasonable values have to be chosen for contact analysis (choice of penalty), see Remarks below.
N	MADYMO3D 5.4 coupling flag, n: EQ.0: use normal LS-DYNA rigid body updates, GT.0: the rigid body is coupled to MADYMO 5.4 ellipsoid number n LT.0: the rigid body is coupled to MADYMO 5.4 plane number  n .
COUPLE	Coupling option if applicable: EQ.-1: attach VDA surface in ALIAS (defined in the eighth field) and automatically generate a mesh for viewing the surface in LS-PREPOST. MADYMO 5.4 / CAL3D coupling option: EQ.0: the undeformed geometry input to LS-DYNA corresponds to the local system for MADYMO 5.4 / CAL3D. The finite element mesh is input, EQ.1: the undeformed geometry input to LS-DYNA corresponds to the global system for MADYMO 5.4 / CAL3D, EQ.2: generate a mesh for the ellipsoids and planes internally in LS-DYNA.
M	MADYMO3D 5.4 coupling flag, m: EQ.0: use normal LS-DYNA rigid body updates, EQ.m: this rigid body corresponds to MADYMO rigid body number m. Rigid body updates are performed by MADYMO.
ALIAS	VDA surface alias name, see Appendix L.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RE	MADYMO 6.0.1 External Reference Number
CMO	Center of mass constraint option, CMO: EQ.+1.0: constraints applied in global directions, EQ.0.0: no constraints, EQ.-1.0: constraints applied in local directions (SPC constraint).
CON1	First constraint parameter: If CMO = +1.0, then specify global translational constraint: EQ.0: no constraints, EQ.1: constrained x displacement, EQ.2: constrained y displacement, EQ.3: constrained z displacement, EQ.4: constrained x and y displacements, EQ.5: constrained y and z displacements, EQ.6: constrained z and x displacements, EQ.7: constrained x, y, and z displacements.  If CMO = -1.0, then specify local coordinate system ID. See *DEFINE_COORDINATE_OPTION: This coordinate system is fixed in time.
CON2	Second constraint parameter: If CMO = +1.0, then specify global rotational constraint: EQ.0: no constraints, EQ.1: constrained x rotation, EQ.2: constrained y rotation, EQ.3: constrained z rotation, EQ.4: constrained x and y rotations, EQ.5: constrained y and z rotations, EQ.6: constrained z and x rotations, EQ.7: constrained x, y, and z rotations.

VARIABLE	DESCRIPTION
	<p>If CM0 = -1.0, then specify local (SPC) constraint:</p> <p>EQ.000000: no constraint,            EQ.100000: constrained x translation,            EQ.010000: constrained y translation,            EQ.001000: constrained z translation,            EQ.000100: constrained x rotation,            EQ.000010: constrained y rotation,            EQ.000001: constrained z rotation.</p> <p>Any combination of local constraints can be achieved by adding the number 1 into the corresponding column.</p>
LCO	Local coordinate system number for output.
A1 - V3	<p>Alternative method for specifying local system below:</p> <p>Define two vectors <b>a</b> and <b>v</b>, fixed in the rigid body which are used for output and the user defined airbag sensor subroutines. The output parameters are in the directions <b>a</b>, <b>b</b>, and <b>c</b> where the latter are given by the cross products <math>\mathbf{c} = \mathbf{a} \times \mathbf{v}</math> and <math>\mathbf{b} = \mathbf{c} \times \mathbf{a}</math>. This input is optional.</p>

### Remarks:

The rigid material type 20 provides a convenient way of turning one or more parts comprised of beams, shells, or solid elements into a rigid body. Approximating a deformable body as rigid is a preferred modeling technique in many real world applications. For example, in sheet metal forming problems the tooling can properly and accurately be treated as rigid. In the design of restraint systems the occupant can, for the purposes of early design studies, also be treated as rigid. Elements which are rigid are bypassed in the element processing and no storage is allocated for storing history variables; consequently, the rigid material type is very cost efficient.

Two unique rigid part ID's may not share common nodes unless they are merged together using the rigid body merge option. A rigid body may be made up of disjoint finite element meshes, however. LS-DYNA assumes this is the case since this is a common practice in setting up tooling meshes in forming problems.

All elements which reference a given part ID corresponding to the rigid material should be contiguous, but this is not a requirement. If two disjoint groups of elements on opposite sides of a model are modeled as rigid, separate part ID's should be created for each of the

contiguous element groups if each group is to move independently. This requirement arises from the fact that LS-DYNA internally computes the six rigid body degrees-of-freedom for each rigid body (rigid material or set of merged materials), and if disjoint groups of rigid elements use the same part ID, the disjoint groups will move together as one rigid body.

Inertial properties for rigid materials may be defined in either of two ways. By default, the inertial properties are calculated from the geometry of the constituent elements of the rigid material and the density specified for the part ID. Alternatively, the inertial properties and initial velocities for a rigid body may be directly defined, and this overrides data calculated from the material property definition and nodal initial velocity definitions.

Young's modulus,  $E$ , and Poisson's ratio,  $\nu$  are used for determining sliding interface parameters if the rigid body interacts in a contact definition. Realistic values for these constants should be defined since unrealistic values may contribute to numerical problem in contact.

Constraint directions for rigid materials (CMO equal to +1 or -1) are fixed, that is, not updated, with time. To impose a constraint on a rigid body such that the constraint direction is updated as the rigid body rotates, use `*BOUNDARY_PRESCRIBED_MOTION_RIGID_LOCAL`.

It is strongly advised that nodal constraints, e.g., by `*BOUNDARY_SPC_OPTION`, not be applied to nodes of a rigid body as doing so may compromise the intended constraints in the case of an explicit simulation. Such SPCs will be skipped in an implicit simulation and a warning issued.

If the intended constraints are not with respect to the calculated center-of-mass of the rigid body, `*CONSTRAINED_JOINT_OPTION` may often be used to obtain the desired effect. This approach typically entails defining a second rigid body which is fully constrained and then defining a joint between the two rigid bodies. Another alternative for defining rigid body constraints that are not with respect to the calculated center-of-mass of the rigid body is to manually specify the initial center-of-mass location using `*PART_INERTIA`. When using `*PART_INERTIA`, a full set of mass properties must be specified and the user must understand that the dynamic behavior of the rigid body is affected by its mass properties.

**For coupling with MADYMO 5.4.1, only basic coupling is available.**

The coupling flags (N and M) must match with SYSTEM and ELLIPSOID/PLANE in the MADYMO input file and the coupling option (COUPLE) must be defined.

**For coupling with MADYMO 6.0.1, both basic and extended coupling are available:**

1. **Basic Coupling:** The external reference number (RE) must match with the external reference number in the MADYMO XML input file. The coupling option (COUPLE) must be defined.

2. Extended Coupling: Under this option MADYMO will handle the contact between the MADYMO and LS-DYNA models. The external reference number (RE) and the coupling option (COUPLE) are not needed. All coupling surfaces that interface with the MADYMO models need to be defined in \*CONTACT\_COUPLING.





**Required for failure.**

Card 5	1	2	3	4	5	6	7	8
Variable	A1	A11	A2	A5	A55	A4	NIP	
Type	F	F	F	F	F	F	F	

**Additional card 5 required for curing.**

Card 5	1	2	3	4	5	6	7	8
Variable	K1	K2	C1	C2	M	N	R	
Type	F	F	F	F	F	F	F	

**Additional card 6 required for curing.**

Card 6	1	2	3	4	5	6	7	8
Variable	LCCHA	LCCHB	LCCHC	LCAA	LCAB	LCAC		
Type	I	I	I	I	I	I		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
EA	$E_a$ , Young's modulus in $a$ -direction.
EB	$E_b$ , Young's modulus in $b$ -direction.
EC	$E_c$ , Young's modulus in $c$ -direction.
PRBA	$\nu_{ba}$ , Poisson's ratio, $ba$ .
PRCA	$\nu_{ca}$ , Poisson's ratio, $ca$ .
PRCB	$\nu_{cb}$ , Poisson's ratio, $cb$ .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GAB	$G_{ab}$ , Shear modulus, $ab$ .
GBC	$G_{bc}$ , Shear modulus, $bc$ .
GCA	$G_{ca}$ , Shear modulus, $ca$ .
AA	$\alpha_a$ , coefficient of thermal expansion in the $a$ -direction.
AB	$\alpha_b$ , coefficient of thermal expansion in the $b$ -direction.
AC	$\alpha_c$ , coefficient of thermal expansion in the $c$ -direction.
AOPT	<p>Material axes option (see MAT_OPTION TROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>\mathbf{p}</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MACF	Material axes change flag for brick elements: EQ.1: No change, default, EQ.2: switch material axes a and b, EQ.3: switch material axes a and c, EQ.4: switch material axes b and c.
XP, YP, ZP	Coordinates of point <b>p</b> for AOPT = 1 and 4.
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3 and 4.
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 1 (shells only) and AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_-BETA or *ELEMENT_SOLID_ORTHO.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.
A1, A11, A2	Coefficients for the matrix dominated failure criterion.
A5, A55, A4	Coefficients for the fiber dominated failure criterion.
K1	Parameter $k_1$ for Kamal model. For details see remark below.
K2	Parameter $k_2$ for Kamal model.
C1	Parameter $c_1$ for Kamal model.
C2	Parameter $c_2$ for Kamal model.
M	Exponent $m$ for Kamal model.
N	Exponent $n$ for Kamal model.
R	Gas constant for Kamal model.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCCHA	Load curve for $\gamma_a$ , coefficient of chemical shrinkage in the $a$ -direction. Input $\gamma_a$ as function of state of cure $\beta$ .
LCCHB	Load curve for $\gamma_b$ , coefficient of chemical shrinkage in the $b$ -direction. Input $\gamma_b$ as function of state of cure $\beta$ .
LCCHC	Load curve for $\gamma_c$ , coefficient of chemical shrinkage in the $c$ -direction. Input $\gamma_c$ as function of state of cure $\beta$ .
LCAA	Load curve or table ID for $\alpha_a$ . If defined parameter AA is ignored. <u>IF LCID</u> Input $\alpha_a$ versus temperature. <u>IF TABID:</u> Input $\alpha_a$ as functions of state of cure (table values) and temperatures (see *DEFINE_TABLE)
LCAB	Load curve ID for $\alpha_b$ . If defined parameter AB is ignored. See LCAA for further details.
LCAC	Load curve ID for $\alpha_c$ . If defined parameter AC is ignored. See LCAA for further details.

**Remarks:**

In the implementation for three-dimensional continua a total Lagrangian formulation is used. In this approach the material law that relates second Piola-Kirchhoff stress  $\mathbf{S}$  to the Green-St. Venant strain  $\mathbf{E}$  is

$$\mathbf{S} = \mathbf{C} \cdot \mathbf{E} = \mathbf{T}^T \mathbf{C}_I \mathbf{T} \cdot \mathbf{E}$$

where  $\mathbf{T}$  is the transformation matrix [Cook 1974].

$$\mathbf{T} = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & l_1 m_1 & m_1 n_1 & n_1 l_1 \\ l_2^2 & m_2^2 & n_2^2 & l_2 m_2 & m_2 n_2 & n_2 l_2 \\ l_3^2 & m_3^2 & n_3^2 & l_3 m_3 & m_3 n_3 & n_3 l_3 \\ 2l_1 l_2 & 2m_1 m_2 & 2n_1 n_2 & (l_1 m_2 + l_2 m_1) & (m_1 n_2 + m_2 n_1) & (n_1 l_2 + n_2 l_1) \\ 2l_2 l_3 & 2m_2 m_3 & 2n_2 n_3 & (l_2 m_3 + l_3 m_2) & (m_2 n_3 + m_3 n_2) & (n_2 l_3 + n_3 l_2) \\ 2l_3 l_1 & 2m_3 m_1 & 2n_3 n_1 & (l_3 m_1 + l_1 m_3) & (m_3 n_1 + m_1 n_3) & (n_3 l_1 + n_1 l_3) \end{bmatrix}$$

$l_i, m_i, n_i$  are the direction cosines

$$x'_i = l_i x_1 + m_i x_2 + n_i x_3 \text{ for } i = 1, 2, 3$$

and  $x'_i$  denotes the material axes. The constitutive matrix  $\mathbf{C}_l$  is defined in terms of the material axes as

$$\mathbf{C}_l^{-1} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{21}}{E_{22}} & -\frac{\nu_{31}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & -\frac{\nu_{32}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_{11}} & -\frac{\nu_{23}}{E_{22}} & \frac{1}{E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{31}} \end{bmatrix}$$

where the subscripts denote the material axes, i.e.,

$$\nu_{ij} = \nu_{x'_i x'_j} \quad \text{and} \quad E_{ii} = E_{x'_i}$$

Since  $\mathbf{C}_l$  is symmetric

$$\frac{\nu_{12}}{E_{11}} = \frac{\nu_{21}}{E_{22}}, \dots$$

The vector of Green-St. Venant strain components is

$$\mathbf{E}^T = [E_{11}, E_{22}, E_{33}, E_{12}, E_{23}, E_{31}]$$

which include the local thermal strains which are integrated in time:

$$\varepsilon_{aa}^{n+1} = \varepsilon_{aa}^n + \alpha_a (T^{n+1} - T^n)$$

$$\varepsilon_{bb}^{n+1} = \varepsilon_{bb}^n + \alpha_b (T^{n+1} - T^n)$$

$$\varepsilon_{cc}^{n+1} = \varepsilon_{cc}^n + \alpha_c (T^{n+1} - T^n)$$

where  $T$  is temperature. After computing  $S_{ij}$  we then obtain the Cauchy stress:

$$\sigma_{ij} = \frac{\rho}{\rho_0} \frac{\partial x_i}{\partial X_k} \frac{\partial x_j}{\partial X_l} S_{kl}$$

This model will predict realistic behavior for finite displacement and rotations as long as the strains are small.

In the implementation for shell elements, the stresses are integrated in time and are updated in the corotational coordinate system. In this procedure the local material axes are assumed to remain orthogonal in the deformed configuration. This assumption is valid if the strains remain small.

The failure models were derived by William Feng. The first one defines the matrix dominated failure mode,

$$F_m = A_1(I_1 - 3) + A_{11}(I_1 - 3)^2 + A_2(I_2 - 3) - 1$$

and the second defines the fiber dominated failure mode,

$$F_f = A_5(I_5 - 1) + A_{55}(I_5 - 1)^2 + A_4(I_4 - 1) - 1.$$

When either is greater than zero, the integration point fails, and the element is deleted after NIP integration points fail.

The coefficients  $A_i$  are defined in the input and the invariants  $I_i$  are the strain invariants

$$I_1 = \sum_{\alpha=1,3} C_{\alpha\alpha}$$

$$I_2 = \frac{1}{2} [I_1^2 - \sum_{\alpha,\beta=1,3} C_{\alpha\beta}^2]$$

$$I_3 = \det(\mathbf{C})$$

$$I_4 = \sum_{\alpha,\beta,\gamma=1,3} V_\alpha C_{\alpha\gamma} C_{\gamma\beta} V_\beta$$

$$I_5 = \sum_{\alpha,\beta=1,3} V_\alpha C_{\alpha\beta} V_\beta$$

and  $\mathbf{C}$  is the Cauchy strain tensor and  $\mathbf{V}$  is the fiber direction in the undeformed state. By convention in this material model, the fiber direction is aligned with the  $a$  direction of the local orthotropic coordinate system.

The curing option implies that orthotropic chemical shrinkage is to be considered, resulting from a curing process in the material. The state of cure  $\beta$  is an internal material variable that follows the Kamal model

$$\frac{d\beta}{dt} = (K_1 + K_2\beta^m)(1 - \beta)^n \quad \text{with} \quad K_1 = k_1 e^{-\frac{c_1}{RT}}, \quad K_2 = k_2 e^{-\frac{c_2}{RT}}$$

and chemical strains are introduced:

$$\varepsilon_{aa}^{n+1} = \varepsilon_{aa}^n + \gamma_a(\beta^{n+1} - \beta^n)$$

$$\varepsilon_{bb}^{n+1} = \varepsilon_{bb}^n + \gamma_b(\beta^{n+1} - \beta^n)$$

$$\varepsilon_{cc}^{n+1} = \varepsilon_{cc}^n + \gamma_c(\beta^{n+1} - \beta^n)$$

The coefficients  $\gamma_a, \gamma_b, \gamma_c$  can be defined as functions of the state of cure  $\beta$ . Furthermore, the coefficients of thermal expansion  $\alpha_a, \alpha_b, \alpha_c$  can also be defined as functions of the state of cure  $\beta$  and the temperature  $T$ , if the curing option is used.

**\*MAT\_COMPOSITE\_DAMAGE**

This is Material Type 22. An orthotropic material with optional brittle failure for composites can be defined following the suggestion of [Chang and Chang 1987a, 1987b]. Three failure criteria are possible, see the LS-DYNA Theory Manual. By using the user defined integration rule, see \*INTEGRATION\_SHELL, the constitutive constants can vary through the shell thickness.

For all shells, except the DKT formulation, laminated shell theory can be activated to properly model the transverse shear deformation. Lamination theory is applied to correct for the assumption of a uniform constant shear strain through the thickness of the shell. For sandwich shells where the outer layers are much stiffer than the inner layers, the response will tend to be too stiff unless lamination theory is used. To turn on lamination theory see \*CONTROL\_SHELL.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	EA	EB	EC	PRBA	PRCA	PRCB
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	KFAIL	AOPT	MACF	ATRACK	
Type	F	F	F	F	F	I	I	
Default	none	none	none	0.0	0.0	0	0	

**\*MAT\_022****\*MAT\_COMPOSITE\_DAMAGE**

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

Card 5	1	2	3	4	5	6	7	8
Variable	SC	XT	YT	YC	ALPH	SN	SYZ	SZX
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
EA	$E_a$ , Young's modulus in $a$ -direction.
EB	$E_b$ , Young's modulus in $b$ -direction.
EC	$E_c$ , Young's modulus in $c$ -direction.
PRBA	$\nu_{ba}$ , Poisson ratio, $ba$ .



<b>VARIABLE</b>	<b>DESCRIPTION</b>
PRCA	$\nu_{ca}$ , Poisson ratio, <i>ca</i> .
PRCB	$\nu_{cb}$ , Poisson ratio, <i>cb</i> .
GAB	$G_{ab}$ , Shear modulus, <i>ab</i> .
GBC	$G_{bc}$ , Shear modulus, <i>bc</i> .
GCA	$G_{ca}$ , Shear modulus, <i>ca</i> .
KFAIL	Bulk modulus of failed material. Necessary for compressive failure.
AOPT	Material axes option (see MAT_OPTION TROPIC_ELASTIC for a more complete description): <ul style="list-style-type: none"> <li>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA.</li> <li>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <i>a</i>-direction. This option is for solid elements only.</li> <li>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</li> <li>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <i>v</i> with the element normal.</li> <li>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <b>v</b>, and an originating point, <b>p</b>, which define the centerline axis. This option is for solid elements only.</li> <li>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</li> </ul>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MACF	Material axes change flag for brick elements: EQ.1: No change, default, EQ.2: switch material axes $a$ and $b$ , EQ.3: switch material axes $a$ and $c$ , EQ.4: switch material axes $b$ and $c$ .
ATRACK	Material a-axis tracking flag (shell elements only) EQ.0: a-axis rotates with element (default) EQ.1: a-axis also tracks deformation
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3 and 4.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_-BETA or *ELEMENT_SOLID_ORTHO.
SC	Shear strength, $ab$ plane, see the LS-DYNA Theory Manual.
XT	Longitudinal tensile strength, $a$ -axis, see the LS-DYNA Theory Manual.
YT	Transverse tensile strength, $b$ -axis.
YC	Transverse compressive strength, $b$ -axis (positive value).
ALPH	Shear stress parameter for the nonlinear term, see the LS-DYNA Theory Manual. Suggested range 0 – 0.5.
SN	Normal tensile strength ( <i>solid elements only</i> )
SYZ	Transverse shear strength ( <i>solid elements only</i> )
SZX	Transverse shear strength ( <i>solid elements only</i> )

**Remarks:**

1. **History Data.** The number of additional integration point variables for shells written to the d3plot database is specified using the \*DATABASE\_EXTENT\_BINARY

RY keyword on the NEIPS field. These additional history variables are enumerated below:

History Variable <sup>3</sup>	Description	Value	LS-PrePost history variable
ef( <i>i</i> )	tensile fiber mode	1 - elastic 0 - failed	See table below
cm( <i>i</i> )	tensile matrix mode		1
ed( <i>i</i> )	compressive matrix mode		2

The following components are stored as element component 7 instead of the effective plastic strain. Note that ef(*i*) for *i* = 1,2,3 is not retrievable.

Description	Integration point
$\frac{1}{nip} \sum_{i=1}^{nip} ef(i)$	1
$\frac{1}{nip} \sum_{i=1}^{nip} cm(i)$	2
$\frac{1}{nip} \sum_{i=1}^{nip} ed(i)$	3
ef( <i>i</i> ) for <i>i</i> > 3	<i>i</i>

- The ATRACK Field.** The initial material directions are set using AOPT and the related data. By default, the material directions in shell elements are updated each cycle based on the rotation of the 1-2 edge, or else the rotation of all edges if the invariant node numbering option is set on \*CONTROL\_ACCURACY. When ATRACK=1, an optional scheme is used in which the *a*-direction of the material tracks element deformation as well as rotation.

At the start of the calculation, a line is passed through each element center in the direction of the material *a*-axis. This line will intersect the edges of the element at two points. The referential coordinates of these two points are stored, and then used throughout the calculation to locate these points in the deformed geometry. The material *a*-axis is assumed to be in the direction of the line that passes through both points. If ATRACK = 0, the layers of a layered composite will always rotate together. However, if ATRACK = 1, the layers can rotate independently which

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<sup>3</sup> *i* ranges over the shell integration points.

may be more accurate, particularly for shear deformation. This option is available only for shell elements.



**Second Temperature Card**

Card 5	1	2	3	4	5	6	7	8
Variable	AAi	ABi	ACi	GABi	GBCi	GCAi	Ti	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
AOPT	<p>Material axes option (see MAT_{OPTION}TROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <i>a</i>-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>\mathbf{p}</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and</p>

VARIABLE	DESCRIPTION
	later.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see for more details). EQ.0.0: off, EQ.1.0: on.
MACF	Material axes change flag for brick elements: EQ.1: No change, default, EQ.2: switch material axes $a$ and $b$ , EQ.3: switch material axes $a$ and $c$ , EQ.4: switch material axes $b$ and $c$ .
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3 and 4.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO.
EAI	$E_a$ , Young's modulus in $a$ -direction at temperature $T_i$ .
EBI	$E_b$ , Young's modulus in $b$ -direction at temperature $T_i$ .
ECI	$E_c$ , Young's modulus in $c$ -direction at temperature $T_i$ .
PRBAI	$\nu_{ba}$ , Poisson's ratio $ba$ at temperature $T_i$ .
PRCAI	$\nu_{ca}$ , Poisson's ratio $ca$ at temperature $T_i$ .
PRCBI	$\nu_{cb}$ , Poisson's ratio $cb$ at temperature $T_i$ .
AAI	$\alpha_a$ , coefficient of thermal expansion in $a$ -direction at temperature $T_i$ .
ABI	$\alpha_b$ , coefficient of thermal expansion in $b$ -direction at temperature $T_i$ .
ACI	$\alpha_c$ , coefficient of thermal expansion in $c$ -direction at temperature $T_i$ .

VARIABLE	DESCRIPTION
GAB <i>i</i>	$G_{ab}$ , Shear modulus $ab$ at temperature $T_i$ .
GBC <i>i</i>	$G_{bc}$ , Shear modulus $bc$ at temperature $T_i$ .
GCA <i>i</i>	$G_{ca}$ , Shear modulus $ca$ at temperature $T_i$ .
$T_i$	$i^{\text{th}}$ temperature

**Remarks:**

In the implementation for three-dimensional continua a total Lagrangian formulation is used. In this approach the material law that relates second Piola-Kirchhoff stress  $\mathbf{S}$  to the Green-St. Venant strain  $\mathbf{E}$  is

$$\mathbf{S} = \mathbf{C} \cdot \mathbf{E} = \mathbf{T}^T \mathbf{C}_I \mathbf{T} \cdot \mathbf{E}$$

where  $\mathbf{T}$  is the transformation matrix [Cook 1974].

$$\mathbf{T} = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & l_1 m_1 & m_1 n_1 & n_1 l_1 \\ l_2^2 & m_2^2 & n_2^2 & l_2 m_2 & m_2 n_2 & n_2 l_2 \\ l_3^2 & m_3^2 & n_3^2 & l_3 m_3 & m_3 n_3 & n_3 l_3 \\ 2l_1 l_2 & 2m_1 m_2 & 2n_1 n_2 & (l_1 m_2 + l_2 m_1) & (m_1 n_2 + m_2 n_1) & (n_1 l_2 + n_2 l_1) \\ 2l_2 l_3 & 2m_2 m_3 & 2n_2 n_3 & (l_2 m_3 + l_3 m_2) & (m_2 n_3 + m_3 n_2) & (n_2 l_3 + n_3 l_2) \\ 2l_3 l_1 & 2m_3 m_1 & 2n_3 n_1 & (l_3 m_1 + l_1 m_3) & (m_3 n_1 + m_1 n_3) & (n_3 l_1 + n_1 l_3) \end{bmatrix}$$

$l_i, m_i, n_i$  are the direction cosines

$$x'_i = l_i x_1 + m_i x_2 + n_i x_3 \text{ for } i = 1, 2, 3$$

and  $x'_i$  denotes the material axes. The temperature dependent constitutive matrix  $\mathbf{C}_I$  is defined in terms of the material axes as

$$\mathbf{C}_I^{-1} = \begin{bmatrix} \frac{1}{E_{11}(T)} & -\frac{\nu_{21}(T)}{E_{22}(T)} & -\frac{\nu_{31}(T)}{E_{33}(T)} & 0 & 0 & 0 \\ -\frac{\nu_{12}(T)}{E_{11}(T)} & \frac{1}{E_{22}(T)} & -\frac{\nu_{32}(T)}{E_{33}(T)} & 0 & 0 & 0 \\ -\frac{\nu_{13}(T)}{E_{11}(T)} & -\frac{\nu_{23}(T)}{E_{22}(T)} & \frac{1}{E_{33}(T)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}(T)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}(T)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{31}(T)} \end{bmatrix}$$

where the subscripts denote the material axes, i.e.,



$$v_{ij} = v_{x'_i x'_j} \quad \text{and} \quad E_{ii} = E_{x'_i}$$

Since  $\mathbf{C}_l$  is symmetric

$$\frac{v_{12}}{E_{11}} = \frac{v_{21}}{E_{22}}, \dots$$

The vector of Green-St. Venant strain components is

$$\mathbf{E}^T = [E_{11}, E_{22}, E_{33}, E_{12}, E_{23}, E_{31}]$$

which include the local thermal strains which are integrated in time:

$$\varepsilon_{aa}^{n+1} = \varepsilon_{aa}^n + \alpha_a \left( T^{n+\frac{1}{2}} \right) [T^{n+1} - T^n]$$

$$\varepsilon_{bb}^{n+1} = \varepsilon_{bb}^n + \alpha_b \left( T^{n+\frac{1}{2}} \right) [T^{n+1} - T^n]$$

$$\varepsilon_{cc}^{n+1} = \varepsilon_{cc}^n + \alpha_c \left( T^{n+\frac{1}{2}} \right) [T^{n+1} - T^n]$$

where  $T$  is temperature. After computing  $S_{ij}$  we then obtain the Cauchy stress:

$$\sigma_{ij} = \frac{\rho}{\rho_0} \frac{\partial x_i}{\partial X_k} \frac{\partial x_j}{\partial X_l} S_{kl}$$

This model will predict realistic behavior for finite displacement and rotations as long as the strains are small.

For shell elements, the stresses are integrated in time and are updated in the corotational coordinate system. In this procedure the local material axes are assumed to remain orthogonal in the deformed configuration. This assumption is valid if the strains remain small.

# \*MAT\_024

# \*MAT\_PIECEWISE\_LINEAR\_PLASTICITY

## \*MAT\_PIECEWISE\_LINEAR\_PLASTICITY\_{OPTION}

Available options include:

<BLANK>

LOG\_INTERPOLATION

STOCHASTIC

MIDFAIL

This is Material Type 24, which is an elasto-plastic material with an arbitrary stress versus strain curve and arbitrary strain rate dependency can be defined. See also Remark below. Also, failure based on a plastic strain or a minimum time step size can be defined. For another model with a more comprehensive failure criteria see MAT\_MODIFIED\_PIECEWISE\_LINEAR\_PLASTICITY. If considering laminated or sandwich shells with non-uniform material properties (this is defined through the user specified integration rule), the model, MAT\_LAYERED\_LINEAR\_PLASTICITY, is recommended. If solid elements are used and if the elastic strains before yielding are finite, the model, MAT\_FINITE\_ELASTIC\_STRAIN\_PLASTICITY, treats the elastic strains using a hyperelastic formulation.

The LOG\_INTERPOLATION option interpolates the strain rate effect in table LCSS with logarithmic interpolation.

The STOCHASTIC option allows spatially varying yield and failure behavior. See \*DEFINE\_STOCHASTIC\_VARIATION for additional information.

The MIDFAIL option is available only for shell elements. When included on the keyword line, this option causes failure to be checked only at the mid-plane of the element. If an element has an even number of layers, failure is checked in the two layers closest to the mid-plane.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ETAN	FAIL	TDEL
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	10 <sup>21</sup>	0

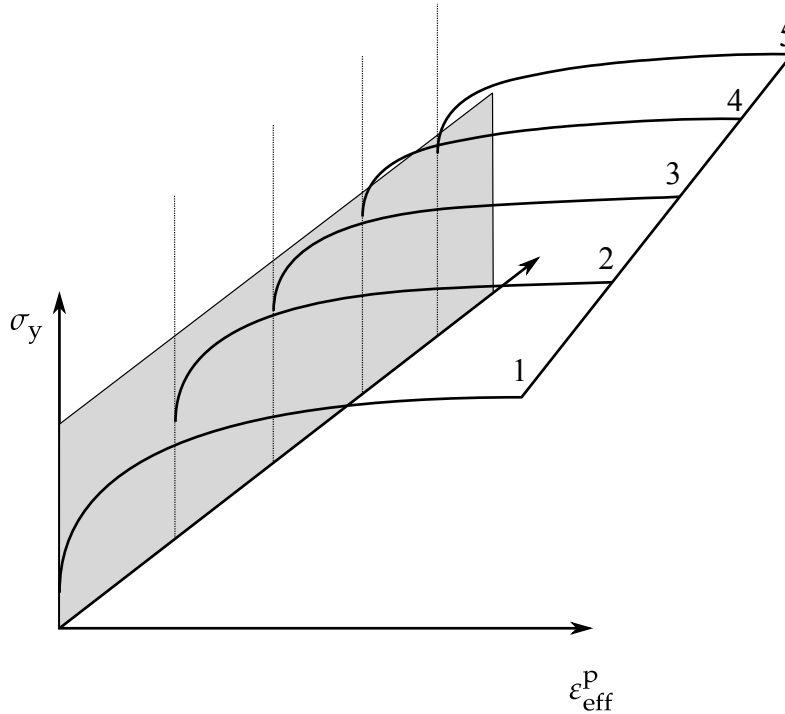
Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR	VP			
Type	F	F	F	F	F			
Default	0	0	0	0	0			

Card 3	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.



**Figure M24-1.** Rate effects may be accounted for by defining a table of curves. If a table ID is specified a curve ID is given for each strain rate, see \*DEFINE\_TABLE. Intermediate values are found by interpolating between curves. Effective plastic strain versus yield stress is expected. If the strain rate values fall out of range, extrapolation is not used; rather, either the first or last curve determines the yield stress depending on whether the rate is low or high, respectively.

VARIABLE	DESCRIPTION
ETAN	Tangent modulus, ignored if (LCSS.GT.0) is defined.
FAIL	<p>Failure flag.</p> <p>LT.0.0: User defined failure subroutine, <code>matusr_24</code> in <code>dyn21.F</code>, is called to determine failure</p> <p>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.</p> <p>GT.0.0: Effective plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.</p>
TDEL	Minimum time step size for automatic element deletion.
C	Strain rate parameter, $C$ , see formula below.
P	Strain rate parameter, $P$ , see formula below.

VARIABLE	DESCRIPTION
LCSS	<p>Load curve ID or Table ID.</p> <p><b>Load Curve.</b> When LCSS is a Load curve ID, it is taken as defining effective stress versus effective plastic strain. If defined EPS1 - EPS8 and ES1 - ES8 are ignored.</p> <p><b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See <a href="#">Figure M24-1</a>. When the strain rate falls below the minimum value, the stress versus effective plastic strain curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value the stress versus effective plastic strain curve for the highest value of strain rate is used. The strain rate parameters: C and P, the curve ID, LCSR, EPS1 - EPS8, and ES1 - ES8 are ignored if a Table ID is defined. Linear interpolation between the discrete strain rates is used by default; logarithmic interpolation is used when the LOG_INTERPOLATION option is invoked.</p> <p><b>Logarithmically Defined Tables.</b> An alternative way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the <i>first</i> value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. There is some additional computational cost associated invoking logarithmic interpolation.</p>
LCSR	<p>Load curve ID defining strain rate scaling effect on yield stress. If LCSR is negative, the load curve is evaluated using a binary search for the correct interval for the strain rate. The binary search is slower than the default incremental search, but in cases where large changes in the strain rate may occur over a single time step, it is more robust. This option is not necessary for the viscoplastic formulation.</p>
VP	<p>Formulation for rate effects:</p> <p>EQ.-1.0: Cowper-Symonds with deviatoric strain rate rather than total,</p> <p>EQ.0.0: Scale yield stress (default),</p> <p>EQ.1.0: Viscoplastic formulation.</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EPS1 - EPS8	Effective plastic strain values (optional; supersedes SIGY, ETAN). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. <b>WARNING:</b> If the first point is nonzero the yield stress is extrapolated to determine the initial yield. If this option is used SIGY and ETAN are ignored and may be input as zero.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8.

**Remarks:**

The stress strain behavior may be treated by a bilinear stress strain curve by defining the tangent modulus, ETAN. Alternately, a curve of effective stress vs. effective plastic strain similar to that shown in [Figure M10-1](#) may be defined by (EPS1, ES1) - (EPS8, ES8); however, a curve ID (LCSS) may be referenced instead if eight points are insufficient. The cost is roughly the same for either approach. Note that in the special case of uniaxial stress, true stress vs. true plastic strain is equivalent to effective stress vs. effective plastic strain. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate.  $\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$ . If VP = -1. The deviatoric strain rates are used instead.

If the viscoplastic option is active, VP = 1.0, and if SIGY is > 0 then the dynamic yield stress is computed from the sum of the static stress,  $\sigma_y^s(\epsilon_{\text{eff}}^p)$ , which is typically given by a load curve ID, and the initial yield stress, SIGY, multiplied by the Cowper-Symonds rate term as follows:

$$\sigma_y(\epsilon_{\text{eff}}^p, \dot{\epsilon}_{\text{eff}}^p) = \sigma_y^s(\epsilon_{\text{eff}}^p) + \text{SIGY} \times \left(\frac{\dot{\epsilon}_{\text{eff}}^p}{C}\right)^{1/p}$$

where the plastic strain rate is used. With this latter approach similar results can be obtained between this model and material model: \*MAT\_ANISOTROPIC\_VISCOPLASTIC. If SIGY = 0, the following equation is used instead where the static stress,  $\sigma_y^s(\epsilon_{\text{eff}}^p)$ , must be defined by a load curve:

$$\sigma_y(\epsilon_{eff}^p, \dot{\epsilon}_{eff}^p) = \sigma_y^s(\epsilon_{eff}^p) \left[ 1 + \left( \frac{\dot{\epsilon}_{eff}^p}{C} \right)^{1/p} \right]$$

This latter equation is always used if the viscoplastic option is off.

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor versus strain rate is defined.
3. If different stress versus strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in \*DEFINE\_TABLE has to be used, see [Figure M24-1](#).

A fully viscoplastic formulation is optional (variable VP) which incorporates the different options above within the yield surface. An additional cost is incurred over the simple scaling but the improvement in results can be dramatic.

For implicit calculations on this material involving severe nonlinear hardening the radial return method may result in inaccurate stress-strain response. By setting IACC = 1 on \*CONTROL\_ACCURACY activates a fully iterative plasticity algorithm, which will remedy this. This is not to be confused with the MITER flag on \*CONTROL\_SHELL, which governs the treatment of the plane stress assumption for shell elements. If failure is applied with this option, incident failure will initiate damage, and the stress will continuously degrade to zero before erosion for a deformation of 1% plastic strain. So for instance, if the failure strain is FAIL = 0.05, then the element is eroded when  $\bar{\epsilon}^p = 0.06$  and the material goes from intact to completely damaged between  $\bar{\epsilon}^p = 0.05$  and  $\bar{\epsilon}^p = 0.06$ . The reason is to enhance implicit performance by maintaining continuity in the internal forces.

For a nonzero failure strain, \*DEFINE\_MATERIAL\_HISTORIES can be used to output the failure indicator.

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>					
Label	Attributes				Description
Instability	-	-	-	-	Failure indicator $\epsilon_{eff}^p / \epsilon_{fail}^p$ , see FAIL
Plastic Strain Rate	-	-	-	-	Effective plastic strain rate $\dot{\epsilon}_{eff}^p$

**\*MAT\_GEOLOGIC\_CAP\_MODEL**

This is Material Type 25. This is an inviscid two invariant geologic cap model. This material model can be used for geomechanical problems or for materials as concrete, see references cited below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	G	ALPHA	THETA	GAMMA	BETA
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	R	D	W	X0	C	N		
Type	F	F	F	F	F	F		

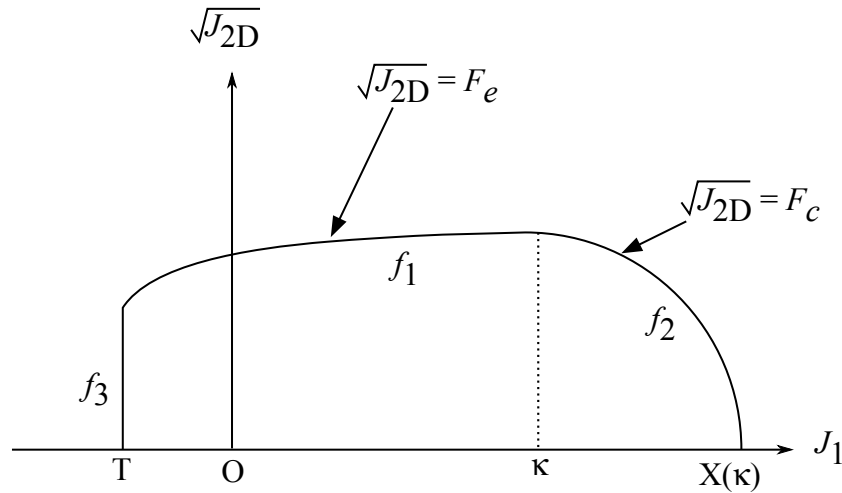
Card 3	1	2	3	4	5	6	7	8
Variable	PLOT	FTYPE	VEC	TOFF				
Type	F	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
BULK	Initial bulk modulus, K.
G	Initial Shear modulus.
ALPHA	Failure envelope parameter, $\alpha$ .
THETA	Failure envelope linear coefficient, $\theta$ .
GAMMA	Failure envelope exponential coefficient, $\gamma$ .



VARIABLE	DESCRIPTION
BETA	Failure envelope exponent, $\beta$ .
R	Cap, surface axis ratio.
D	Hardening law exponent.
W	Hardening law coefficient.
X0	Hardening law exponent, $X_0$ .
C	Kinematic hardening coefficient, $\bar{c}$ .
N	Kinematic hardening parameter.
PLOT	<p>Save the following variable for plotting in LS-PrePost, to be labeled there as "effective plastic strain:"</p> <p>EQ.1: hardening parameter, <math>\kappa</math></p> <p>EQ.2: cap -J1 axis intercept, <math>X(\kappa)</math></p> <p>EQ.3: volumetric plastic strain <math>\varepsilon_v^p</math></p> <p>EQ.4: first stress invariant, <math>J_1</math></p> <p>EQ.5: second stress invariant, <math>\sqrt{J_2}</math></p> <p>EQ.6: not used</p> <p>EQ.7: not used</p> <p>EQ.8: response mode number</p> <p>EQ.9: number of iterations</p>
FTYPE	<p>Formulation flag:</p> <p>EQ.1: soils (Cap surface may contract)</p> <p>EQ.2: concrete and rock (Cap doesn't contract)</p>
VEC	<p>Vectorization flag:</p> <p>EQ.0: vectorized (fixed number of iterations)</p> <p>EQ.1: fully iterative</p> <p>If the vectorized solution is chosen, the stresses might be slightly off the yield surface; however, on vector computers a much more efficient solution is achieved.</p>
TOFF	Tension Cut Off, $TOFF < 0$ (positive in compression).



**Figure M25-1.** The yield surface of the two-invariant cap model in pressure  $\sqrt{J_{2D}} - J_1$  space. Surface  $f_1$  is the failure envelope,  $f_2$  is the cap surface, and  $f_3$  is the tension cutoff.

**Remarks:**

The implementation of an extended two invariant cap model, suggested by Stojko [1990], is based on the formulations of Simo, et al. [1988, 1990] and Sandler and Rubin [1979]. In this model, the two invariant cap theory is extended to include nonlinear kinematic hardening as suggested by Isenberg, Vaughan, and Sandler [1978]. A brief discussion of the extended cap model and its parameters is given below.

The cap model is formulated in terms of the invariants of the stress tensor. The square root of the second invariant of the deviatoric stress tensor,  $\sqrt{J_{2D}}$  is found from the deviatoric stresses  $\mathbf{s}$  as

$$\sqrt{J_{2D}} \equiv \sqrt{\frac{1}{2} S_{ij} S_{ij}}$$

and is the objective scalar measure of the distortional or shearing stress. The first invariant of the stress,  $J_1$ , is the trace of the stress tensor.

The cap model consists of three surfaces in  $\sqrt{J_{2D}} - J_1$  space, as shown in [Figure M25-1](#). First, there is a failure envelope surface, denoted  $f_1$  in the figure. The functional form of  $f_1$  is

$$f_1 = \sqrt{J_{2D}} - \min[F_e(J_1), T_{mises}],$$

where  $F_e$  is given by

$$F_e(J_1) \equiv \alpha - \gamma \exp(-\beta J_1) + \theta J_1$$

and  $T_{mises} \equiv |X(\kappa_n) - L(\kappa_n)|$ . This failure envelop surface is fixed in  $\sqrt{J_{2D}} - J_1$  space, and therefore does not harden unless kinematic hardening is present. Next, there is a cap surface, denoted  $f_2$  in the figure, with  $f_2$  given by

$$f_2 = \sqrt{J_{2D}} - F_c(J_1, K)$$

where  $F_c$  is defined by

$$F_c(J_1, \kappa) \equiv \frac{1}{R} \sqrt{[X(\kappa) - L(\kappa)]^2 - [J_1 - L(\kappa)]^2},$$

$X(\kappa)$  is the intersection of the cap surface with the  $J_1$  axis

$$X(\kappa) = \kappa + RF_e(\kappa),$$

and  $L(\kappa)$  is defined by

$$L(\kappa) \equiv \begin{cases} \kappa & \text{if } \kappa > 0 \\ 0 & \text{if } \kappa \leq 0 \end{cases}$$

The hardening parameter  $\kappa$  is related to the plastic volume change  $\varepsilon_v^p$  through the hardening law

$$\varepsilon_v^p = W\{1 - \exp[-D(X(\kappa) - X_0)]\}$$

Geometrically,  $\kappa$  is seen in the figure as the  $J_1$  coordinate of the intersection of the cap surface and the failure surface. Finally, there is the tension cutoff surface, denoted  $f_3$  in the figure. The function  $f_3$  is given by

$$f_3 \equiv T - J_1$$

where  $T$  is the input material parameter which specifies the maximum hydrostatic tension sustainable by the material. The elastic domain in  $\sqrt{J_{2D}} - J_1$  space is then bounded by the failure envelope surface above, the tension cutoff surface on the left, and the cap surface on the right.

An additive decomposition of the strain into elastic and plastic parts is assumed:

$$\varepsilon = \varepsilon^e + \varepsilon^p,$$

where  $\varepsilon^e$  is the elastic strain and  $\varepsilon^p$  is the plastic strain. Stress is found from the elastic strain using Hooke's law,

$$\sigma = \mathbf{C}(\varepsilon - \varepsilon^p),$$

where  $\sigma$  is the stress and  $\mathbf{C}$  is the elastic constitutive tensor.

The yield condition may be written

$$f_1(s) \leq 0$$

$$f_2(s, \kappa) \leq 0$$

$$f_3(s) \leq 0$$

and the plastic consistency condition requires that

$$\dot{\lambda}_k f_k = 0$$

$$k = 1,2,3$$

$$\dot{\lambda}_k \geq 0$$

where  $\lambda_k$  is the plastic consistency parameter for surface k. If  $f_k < 0$  then,  $\dot{\lambda}_k = 0$  and the response is elastic. If  $f_k > 0$  then surface k is active and  $\dot{\lambda}_k$  is found from the requirement that  $\dot{f}_k = 0$ .

Associated plastic flow is assumed, so using Koiter's flow rule the plastic strain rate is given as the sum of contribution from all of the active surfaces,

$$\dot{\epsilon}^p = \sum_{k=1}^3 \dot{\lambda}_k \frac{\partial f_k}{\partial s}$$

One of the major advantages of the cap model over other classical pressure-dependent plasticity models is the ability to control the amount of dilatancy produced under shear loading. Dilatancy is produced under shear loading as a result of the yield surface having a positive slope in  $\sqrt{J_{2D}} - J$  space, so the assumption of plastic flow in the direction normal to the yield surface produces a plastic strain rate vector that has a component in the volumetric (hydrostatic) direction (see [Figure M25-1](#)). In models such as the Drucker-Prager and Mohr-Coulomb, this dilatancy continues as long as shear loads are applied, and in many cases produces far more dilatancy than is experimentally observed in material tests. In the cap model, when the failure surface is active, dilatancy is produced just as with the Drucker-Prager and Mohr-Coulomb models. However, the hardening law permits the cap surface to contract until the cap intersects the failure envelope at the stress point, and the cap remains at that point. The local normal to the yield surface is now vertical, and therefore the normality rule assures that no further plastic volumetric strain (dilatancy) is created. Adjustment of the parameters that control the rate of cap contractions permits experimentally observed amounts of dilatancy to be incorporated into the cap model, thus producing a constitutive law which better represents the physics to be modeled.

Another advantage of the cap model over other models such as the Drucker-Prager and Mohr-Coulomb is the ability to model plastic compaction. In these models all purely volumetric response is elastic. In the cap model, volumetric response is elastic until the stress point hits the cap surface. Therefore, plastic volumetric strain (compaction) is generated at a rate controlled by the hardening law. Thus, in addition to controlling the amount of dilatancy, the introduction of the cap surface adds another experimentally observed response characteristic of geological material into the model.

The inclusion of kinematic hardening results in hysteretic energy dissipation under cyclic loading conditions. Following the approach of Isenberg, et al. [1978] a nonlinear kinematic hardening law is used for the failure envelope surface when nonzero values of  $\alpha$  and  $N$  are specified. In this case, the failure envelope surface is replaced by a family of yield surfaces bounded by an initial yield surface and a limiting failure envelope surface. Thus, the shape

of the yield surfaces described above remains unchanged, but they may translate in a plane orthogonal to the J axis,

Translation of the yield surfaces is permitted through the introduction of a “back stress” tensor,  $\alpha$ . The formulation including kinematic hardening is obtained by replacing the stress  $\sigma$  with the translated stress tensor  $\eta \equiv \sigma - \alpha$  in all of the above equation. The history tensor  $\alpha$  is assumed deviatoric, and therefore has only 5 unique components. The evolution of the back stress tensor is governed by the nonlinear hardening law

$$\dot{\alpha} = \bar{c}\bar{F}(\sigma, \alpha)\dot{e}^p$$

where  $\bar{c}$  is a constant,  $\bar{F}$  is a scalar function of  $\sigma$  and  $\alpha$  and  $\dot{e}^p$  is the rate of deviatoric plastic strain. The constant may be estimated from the slope of the shear stress - plastic shear strain curve at low levels of shear stress.

The function  $\bar{F}$  is defined as

$$\bar{F} \equiv \max \left[ 0, 1 - \frac{(\sigma - \alpha)\alpha}{2NF_e(J_1)} \right]$$

where N is a constant defining the size of the yield surface. The value of N may be interpreted as the radial distance between the outside of the initial yield surface and the inside of the limit surface. In order for the limit surface of the kinematic hardening cap model to correspond with the failure envelope surface of the standard cap model, the scalar parameter  $\alpha$  must be replaced  $\alpha - N$  in the definition  $F_e$ .

The cap model contains a number of parameters which must be chosen to represent a particular material, and are generally based on experimental data. The parameters  $\alpha$ ,  $\beta$ ,  $\theta$ , and  $\gamma$  are usually evaluated by fitting a curve through failure data taken from a set of triaxial compression tests. The parameters W, D, and  $X_0$  define the cap hardening law. The value W represents the void fraction of the uncompressed sample and D governs the slope of the initial loading curve in hydrostatic compression. The value of R is the ration of major to minor axes of the quarter ellipse defining the cap surface. Additional details and guidelines for fitting the cap model to experimental data are found in Chen and Baladi [1985].



Card 5	1	2	3	4	5	6	7	8
Variable	D1	D2	D3	TSEF	SSEF	V1	V2	V3
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus for compacted honeycomb material.
PR	Poisson's ratio for compacted honeycomb material.
SIGY	Yield stress for fully compacted honeycomb.
VF	Relative volume at which the honeycomb is fully compacted.
MU	$\mu$ , material viscosity coefficient. (default=.05) Recommended.
BULK	Bulk viscosity flag: EQ.0.0: bulk viscosity is not used. This is recommended. EQ.1.0: bulk viscosity is active and $\mu = 0$ . This will give results identical to previous versions of LS-DYNA.
LCA	Load curve ID, see *DEFINE_CURVE, for sigma-aa versus either relative volume or volumetric strain. See notes below.
LCB	Load curve ID, see *DEFINE_CURVE, for sigma-bb versus either relative volume or volumetric strain. Default LCB = LCA. See notes below.
LCC	Load curve ID, see *DEFINE_CURVE, for sigma-cc versus either relative volume or volumetric strain. Default LCC = LCA. See notes below.
LCS	Load curve ID, see *DEFINE_CURVE, for shear stress versus either relative volume or volumetric strain. Default LCS = LCA. Each component of shear stress may have its own load curve. See notes below.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCAB	Load curve ID, see *DEFINE_CURVE, for sigma-ab versus either relative volume or volumetric strain. Default LCAB = LCS. See notes below.
LCBC	Load curve ID, see *DEFINE_CURVE, for sigma-bc versus either relative volume or volumetric strain. Default LCBC = LCS. See notes below.
LCCA	Load curve ID, see *DEFINE_CURVE, or sigma-ca versus either relative volume or volumetric strain. Default LCCA = LCS. See notes below.
LCSR	Load curve ID, see *DEFINE_CURVE, for strain-rate effects defining the scale factor versus strain rate. This is optional. The curves defined above are scaled using this curve.
EAAU	Elastic modulus $E_{aaU}$ in uncompressed configuration.
EBBU	Elastic modulus $E_{bbU}$ in uncompressed configuration.
ECCU	Elastic modulus $E_{ccU}$ in uncompressed configuration.
GABU	Shear modulus $G_{abU}$ in uncompressed configuration.
GBCU	Shear modulus $G_{bcU}$ in uncompressed configuration.
GCAU	Shear modulus $G_{caU}$ in uncompressed configuration.
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description): <ul style="list-style-type: none"> <li>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</li> <li>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction.</li> <li>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR</li> <li>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element nor-</li> </ul>



VARIABLE	DESCRIPTION
	<p>mal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>p</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later..</p>
MACF	<p>Material axes change flag:</p> <p>EQ.1: No change, default,</p> <p>EQ.2: switch material axes a and b,</p> <p>EQ.3: switch material axes a and c,</p> <p>EQ.4: switch material axes b and c.</p>
XP YP ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1 A2 A3	Components of vector $\mathbf{a}$ for AOPT = 2.
D1 D2 D3	Components of vector $\mathbf{d}$ for AOPT = 2.
V1 V2 V3	Define components of vector $\mathbf{v}$ for AOPT = 3 and 4.
TSEF	Tensile strain at element failure (element will erode).
SSEF	Shear strain at element failure (element will erode).

**Remarks:**

For efficiency it is strongly recommended that the load curve ID's: LCA, LCB, LCC, LCS, LCAB, LCBC, and LCCA, contain exactly the same number of points with corresponding strain values on the abscissa. If this recommendation is followed the cost of the table lookup is insignificant. Conversely, the cost increases significantly if the abscissa strain values are not consistent between load curves.

The behavior before compaction is orthotropic where the components of the stress tensor are uncoupled, i.e., an  $a$  component of strain will generate resistance in the local  $a$ -direction with no coupling to the local  $b$  and  $c$  directions. The elastic moduli vary, from their initial values to the fully compacted values at  $V_f$ , linearly with the relative volume  $V$ :

$$E_{aa} = E_{aa0} + \beta(E - E_{aa0})$$

$$E_{bb} = E_{bb0} + \beta(E - E_{bb0})$$

$$E_{cc} = E_{cc0} + \beta(E - E_{cc0})$$

$$G_{ab} = G_{ab0} + \beta(G - G_{ab0})$$

$$G_{bc} = G_{bc0} + \beta(G - G_{bc0})$$

$$G_{ca} = G_{ca0} + \beta(G - G_{ca0})$$

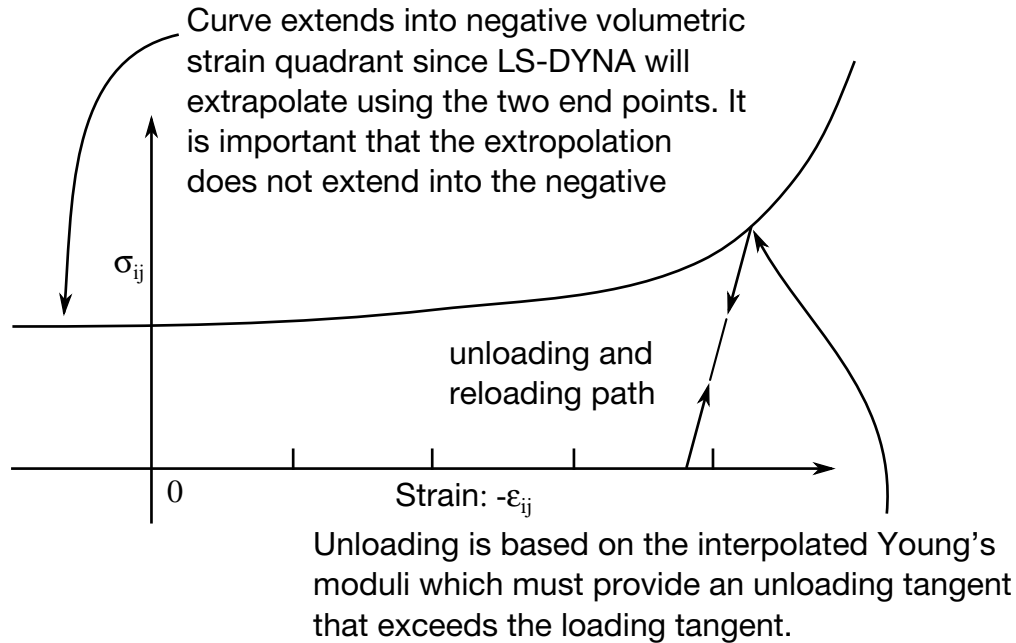
where

$$\beta = \max \left[ \min \left( \frac{1 - V}{1 - V_f}, 1 \right), 0 \right]$$

and  $G$  is the elastic shear modulus for the fully compacted honeycomb material

$$G = \frac{E}{2(1 + \nu)}$$

The relative volume,  $V$ , is defined as the ratio of the current volume to the initial volume. Typically,  $V = 1$  at the beginning of a calculation. The viscosity coefficient  $\mu$  (MU) should be set to a small number (usually .02 - .10 is okay). Alternatively, the two bulk viscosity coefficients on the control cards should be set to very small numbers to prevent the development of spurious pressures that may lead to undesirable and confusing results. The latter is not recommended since spurious numerical noise may develop.



**Figure M26-1.** Stress quantity versus volumetric strain. Note that the “yield stress” at a volumetric strain of zero is non-zero. In the load curve definition, see \*DEFINE\_CURVE, the “time” value is the volumetric strain and the “function” value is the yield stress.

The load curves define the magnitude of the average stress as the material changes density (relative volume), see [Figure M26-1](#). Each curve related to this model must have the same number of points and the same abscissa values. There are two ways to define these curves, **a**) as a function of relative volume ( $V$ ) or **b**) as a function of volumetric strain defined as:

$$\varepsilon_V = 1 - V$$

In the former, the first value in the curve should correspond to a value of relative volume slightly less than the fully compacted value. In the latter, the first value in the curve should be less than or equal to zero, corresponding to tension, and increase to full compaction. **Care should be taken when defining the curves so that extrapolated values do not lead to negative yield stresses.**

At the beginning of the stress update each element's stresses and strain rates are transformed into the local element coordinate system. For the uncompacted material, the trial stress components are updated using the elastic interpolated moduli according to:

$$\sigma_{aa}^{n+1\text{trial}} = \sigma_{aa}^n + E_{aa}\Delta\varepsilon_{aa}$$

$$\sigma_{bb}^{n+1\text{trial}} = \sigma_{bb}^n + E_{bb}\Delta\varepsilon_{bb}$$

$$\sigma_{cc}^{n+1\text{trial}} = \sigma_{cc}^n + E_{cc}\Delta\varepsilon_{cc}$$

$$\sigma_{ab}^{n+1\text{trial}} = \sigma_{ab}^n + 2G_{ab}\Delta\varepsilon_{ab}$$

$$\sigma_{bc}^{n+1\text{trial}} = \sigma_{bc}^n + 2G_{bc}\Delta\varepsilon_{bc}$$

$$\sigma_{ca}^{n+1\text{trial}} = \sigma_{ca}^n + 2G_{ca}\Delta\varepsilon_{ca}$$

Each component of the updated stresses is then independently checked to ensure that they do not exceed the permissible values determined from the load curves; e.g., if

$$\left| \sigma_{ij}^{n+1\text{trial}} \right| > \lambda \sigma_{ij}(V)$$

then

$$\sigma_{ij}^{n+1} = \sigma_{ij}(V) \frac{\lambda \sigma_{ij}^{n+1\text{trial}}}{\left| \lambda \sigma_{ij}^{n+1\text{trial}} \right|}$$

On Card 2  $\sigma_{ij}(V)$  is defined by LCA for the aa stress component, LCB for the bb component, LCC for the cc component, and LCS for the ab, bc, ca shear stress components. The parameter  $\lambda$  is either unity or a value taken from the load curve number, LCSR, that defines  $\lambda$  as a function of strain-rate. Strain-rate is defined here as the Euclidean norm of the deviatoric strain-rate tensor.

For fully compacted material it is assumed that the material behavior is elastic-perfectly plastic and the stress components updated according to:

$$s_{ij}^{\text{trial}} = s_{ij}^n + 2G\Delta\varepsilon_{ij}^{\text{dev}}{}^{n+1/2}$$

where the deviatoric strain increment is defined as

$$\Delta\varepsilon_{ij}^{\text{dev}} = \Delta\varepsilon_{ij} - \frac{1}{3}\Delta\varepsilon_{kk}\delta_{ij}$$

Now a check is made to see if the yield stress for the fully compacted material is exceeded by comparing

$$s_{\text{eff}}^{\text{trial}} = \left( \frac{3}{2} s_{ij}^{\text{trial}} s_{ij}^{\text{trial}} \right)^{1/2}$$

the effective trial stress to the defined yield stress, SIGY. If the effective trial stress exceeds the yield stress the stress components are simply scaled back to the yield surface

$$s_{ij}^{n+1} = \frac{\sigma_y}{s_{\text{eff}}^{\text{trial}}} s_{ij}^{\text{trial}}.$$

Now the pressure is updated using the elastic bulk modulus, K

$$p^{n+1} = p^n - K\Delta\varepsilon_{kk}{}^{n+1/2}$$

where

$$K = \frac{E}{3(1-2\nu)}$$

to obtain the final value for the Cauchy stress

$$\sigma_{ij}^{n+1} = s_{ij}^{n+1} - p^{n+1}\delta_{ij}$$

After completing the stress update transform the stresses back to the global configuration.

For `*CONSTRAINED_TIED_NODES_WITH_FAILURE`, the failure is based on the volume strain instead to the plastic strain.

**\*MAT\_MOONEY-RIVLIN\_RUBBER**

This is Material Type 27. A two-parametric material model for rubber can be defined.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PR	A	B	REF		
Type	A8	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LCID				
Type	F	F	F	F				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
PR	Poisson's ratio (value between 0.49 and 0.5 is recommended, smaller values may not work).
A	Constant, see literature and equations defined below.
B	Constant, see literature and equations defined below.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.

If the values on Card 2 are nonzero, then a least squares fit is computed from the uniaxial data provided by the curve LCID superceding the A and B values on Card 1. If the A and B fields are left blank on Card 1 then the variables on Card 2 *must* be nonzero.

SGL Specimen gauge length  $l_0$ , see [Figure M27-1](#).

VARIABLE	DESCRIPTION
SW	Specimen width, see <a href="#">Figure M27-1</a> .
ST	Specimen thickness, see <a href="#">Figure M27-1</a> .
LCID	Curve ID, see *DEFINE_CURVE, giving the force versus actual change $\Delta L$ in the gauge length. See also <a href="#">Figure M27-2</a> for an alternative definition.

**Remarks:**

The strain energy density function is defined as:

$$W = A(I - 3) + B(II - 3) + C(III^{-2} - 1) + D(III - 1)^2$$

where

$$C = 0.5A + B$$

$$D = \frac{A(5\nu - 2) + B(11\nu - 5)}{2(1 - 2\nu)}$$

$$\nu = \text{Poisson's ratio}$$

$$2(A + B) = \text{shear modulus of linear elasticity}$$

$$I, II, III = \text{invariants of right Cauchy-Green Tensor } C.$$

The load curve definition that provides the uniaxial data should give the change in gauge length,  $\Delta L$ , versus the corresponding force. In compression both the force and the change in gauge length must be specified as negative values. In tension the force and change in gauge length should be input as positive values. The principal stretch ratio in the uniaxial direction,  $\lambda_1$ , is then given by

$$\lambda_1 = \frac{L_0 + \Delta L}{L_0}$$

with  $L_0$  being the initial length and  $L$  being the actual length.

Alternatively, the stress versus strain curve can also be input by setting the gauge length, thickness, and width to unity (1.0) and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force, see [Figure M27-1](#).

The least square fit to the experimental data is performed during the initialization phase and is a comparison between the fit and the actual input is provided in the d3hsp file. It is a good idea to visually check to make sure it is acceptable. The coefficients  $A$  and  $B$  are also printed in the output file. It is also advised to use the material driver (see Appendix K) for checking out the material model.

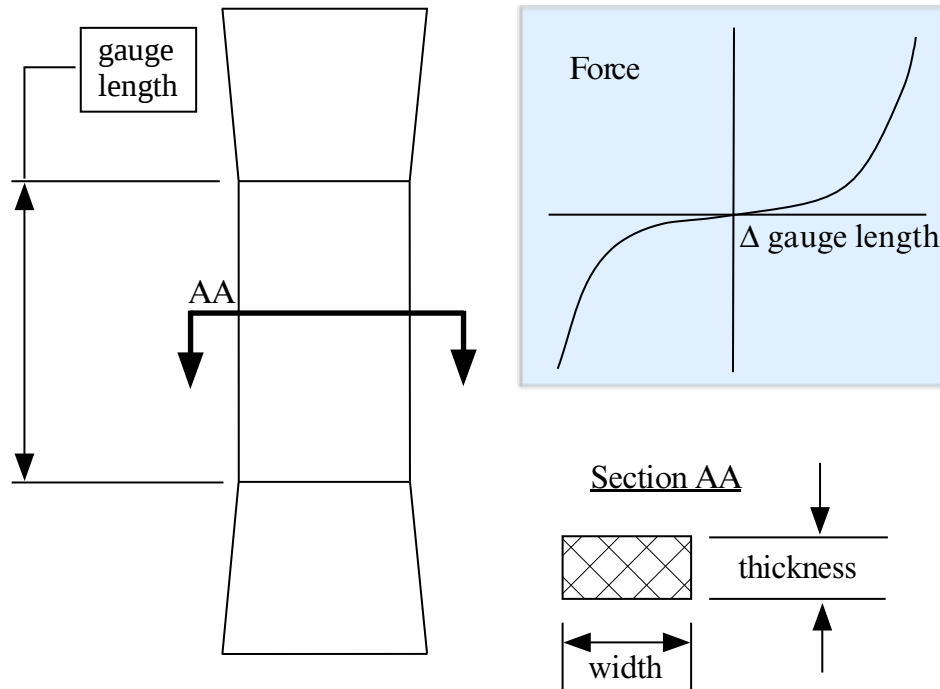


Figure M27-1. Uniaxial specimen for experimental data

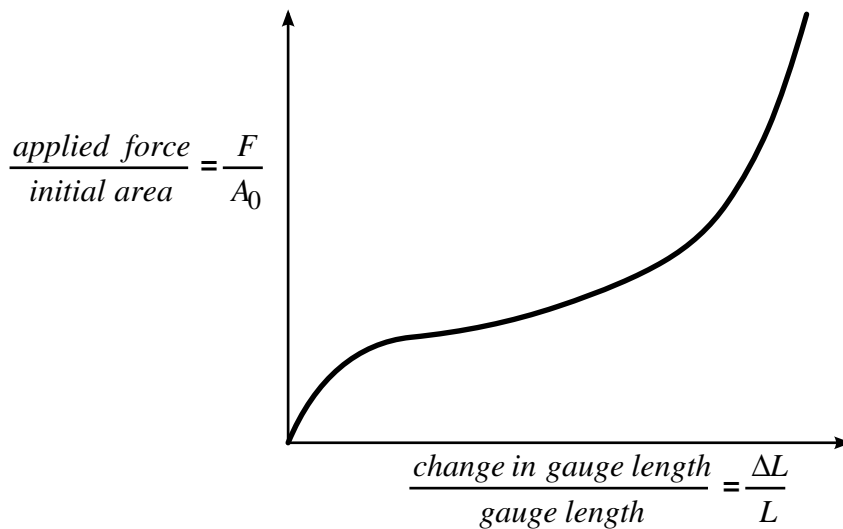


Figure M27-2 The stress versus strain curve can be used instead of the force versus the change in the gauge length by setting the gauge length, thickness, and width to unity (1.0) and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force. \*MAT\_077\_O is a better alternative for fitting data resembling the curve above. \*MAT\_027 will provide a poor fit to a curve that exhibits an strong upturn in slope as strains become large.



**\*MAT\_RESULTANT\_PLASTICITY**

This is Material Type 28. A resultant formulation for beam and shell elements including elasto-plastic behavior can be defined. This model is available for the Belytschko-Schwer beam, the C<sup>0</sup> triangular shell, the Belytschko-Tsay shell, and the fully integrated type 16 shell. For beams, the treatment is elastic-perfectly plastic, but for shell elements isotropic hardening is approximately modeled. For a detailed description we refer to the LS-DYNA Theory Manual. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ETAN		
Type	A8	F	F	F	F	F		
Default	none	none	none	none	none	0.0		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
ETAN	Plastic hardening modulus (for shells only)

**\*MAT\_FORCE\_LIMITED**

This is Material Type 29. With this material model, for the Belytschko-Schwer beam only, plastic hinge forming at the ends of a beam can be modeled using curve definitions. Optionally, collapse can also be modeled. See also \*MAT\_139.

Description: FORCE LIMITED Resultant Formulation

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	DF	AOPT	YTFLAG	ASOFT
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	0.0	0.0	0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	M1	M2	M3	M4	M5	M6	M7	M8
Type	F	F	F	F	F	F	F	F
Default	none	0	0	0	0	0	0	0

Card 3	1	2	3	4	5	6	7	8
Variable	LC1	LC2	LC3	LC4	LC5	LC6	LC7	LC8
Type	F	F	F	F	F	F	F	F
Default	none	0	0	0	0	0	0	0

Card 4	1	2	3	4	5	6	7	8
Variable	LPS1	SFS1	LPS2	SFS2	YMS1	YMS2		
Type	F	F	F	F	F	F		
Default	0	1.0	LPS1	1.0	1.0E+20	YMS1		

Card 5	1	2	3	4	5	6	7	8
Variable	LPT1	SFT1	LPT2	SFT2	YMT1	YMT2		
Type	F	F	F	F	F	F		
Default	0	1.0	LPT1	1.0	1.0E+20	YMT1		

Card 6	1	2	3	4	5	6	7	8
Variable	LPR	SFR	YMR					
Type	F	F	F					
Default	0	1.0	1.0E+20					

**VARIABLE**

**DESCRIPTION**

- MID Material identification. A unique number or label not exceeding 8 characters must be specified.
- RO Mass density
- E Young's modulus
- PR Poisson's ratio
- DF Damping factor, see definition in notes below. A proper control for the timestep has to be maintained by the user!

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	Axial load curve option: EQ.0.0: axial load curves are force versus strain, EQ.1.0: axial load curves are force versus change in length. LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
YTFLAG	Flag to allow beam to yield in tension: EQ.0.0: beam does not yield in tension, EQ.1.0: beam can yield in tension.
ASOFT	Axial elastic softening factor applied once hinge has formed. When a hinge has formed the stiffness is reduced by this factor. If zero, this factor is ignored.
M1, M2, ..., M8	Applied end moment for force versus (strain/change in length) curve. At least one must be defined. A maximum of 8 moments can be defined. The values should be in ascending order.
LC1, LC2, ..., LC8	Load curve ID (see *DEFINE_CURVE) defining axial force (collapse load) versus strain/change in length (see AOPT) for the corresponding applied end moment. Define the same number as end moments. Each curve must contain the same number of points.
LPS1	Load curve ID for plastic moment versus rotation about s-axis at node 1. If zero, this load curve is ignored.
SFS1	Scale factor for plastic moment versus rotation curve about s-axis at node 1. Default = 1.0.
LPS2	Load curve ID for plastic moment versus rotation about s-axis at node 2. Default: is same as at node 1.
SFS2	Scale factor for plastic moment versus rotation curve about s-axis at node 2. Default: is same as at node 1.
YMS1	Yield moment about s-axis at node 1 for interaction calculations (default set to 1.0E+20 to prevent interaction).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
YMS2	Yield moment about s-axis at node 2 for interaction calculations (default set to YMS1).
LPT1	Load curve ID for plastic moment versus rotation about t-axis at node 1. If zero, this load curve is ignored.
SFT1	Scale factor for plastic moment versus rotation curve about t-axis at node 1. Default = 1.0.
LPT2	Load curve ID for plastic moment versus rotation about t-axis at node 2. Default: is the same as at node 1.
SFT2	Scale factor for plastic moment versus rotation curve about t-axis at node 2. Default: is the same as at node 1.
YMT1	Yield moment about t-axis at node 1 for interaction calculations (default set to 1.0E+20 to prevent interactions)
YMT2	Yield moment about t-axis at node 2 for interaction calculations (default set to YMT1)
LPR	Load curve ID for plastic torsional moment versus rotation. If zero, this load curve is ignored.
SFR	Scale factor for plastic torsional moment versus rotation (default = 1.0).
YMR	Torsional yield moment for interaction calculations (default set to 1.0E+20 to prevent interaction)

**Remarks:**

This material model is available for the Belytschko resultant beam element only. Plastic hinges form at the ends of the beam when the moment reaches the plastic moment. The moment versus rotation relationship is specified by the user in the form of a load curve and scale factor. The points of the load curve are (plastic rotation in radians, plastic moment). Both quantities should be positive for all points, with the first point being (zero, initial plastic moment). Within this constraint any form of characteristic may be used, including flat or falling curves. Different load curves and scale factors may be specified at each node and about each of the local s and t axes.

Axial collapse occurs when the compressive axial load reaches the collapse load. Collapse load versus collapse deflection is specified in the form of a load curve. The points of the load curve are either (true strain, collapse force) or (change in length, collapse force). Both

quantities should be entered as positive for all points, and will be interpreted as compressive. The first point should be (zero, initial collapse load).

The collapse load may vary with end moment as well as with deflections. In this case several load-deflection curves are defined, each corresponding to a different end moment. Each load curve should have the same number of points and the same deflection values. The end moment is defined as the average of the absolute moments at each end of the beam and is always positive.

Stiffness-proportional damping may be added using the damping factor  $\lambda$ . This is defined as follows:

$$\lambda = \frac{2 \times \zeta}{\omega}$$

where  $\zeta$  is the damping factor at the reference frequency  $\omega$  (in radians per second). For example if 1% damping at 2Hz is required

$$\lambda = \frac{2 \times 0.01}{2\pi \times 2} = 0.001592$$

If damping is used, a small timestep may be required. LS-DYNA does not check this so to avoid instability it may be necessary to control the timestep via a load curve. As a guide, the timestep required for any given element is multiplied by  $0.3L/c\lambda$  when damping is present (L = element length, c = sound speed).

### **Moment Interaction:**

Plastic hinges can form due to the combined action of moments about the three axes. This facility is activated only when yield moments are defined in the material input. A hinge forms when the following condition is first satisfied.

$$\left( \frac{M_r}{M_{r\text{yield}}} \right)^2 + \left( \frac{M_s}{M_{s\text{yield}}} \right)^2 + \left( \frac{M_t}{M_{t\text{yield}}} \right)^2 \geq 1$$

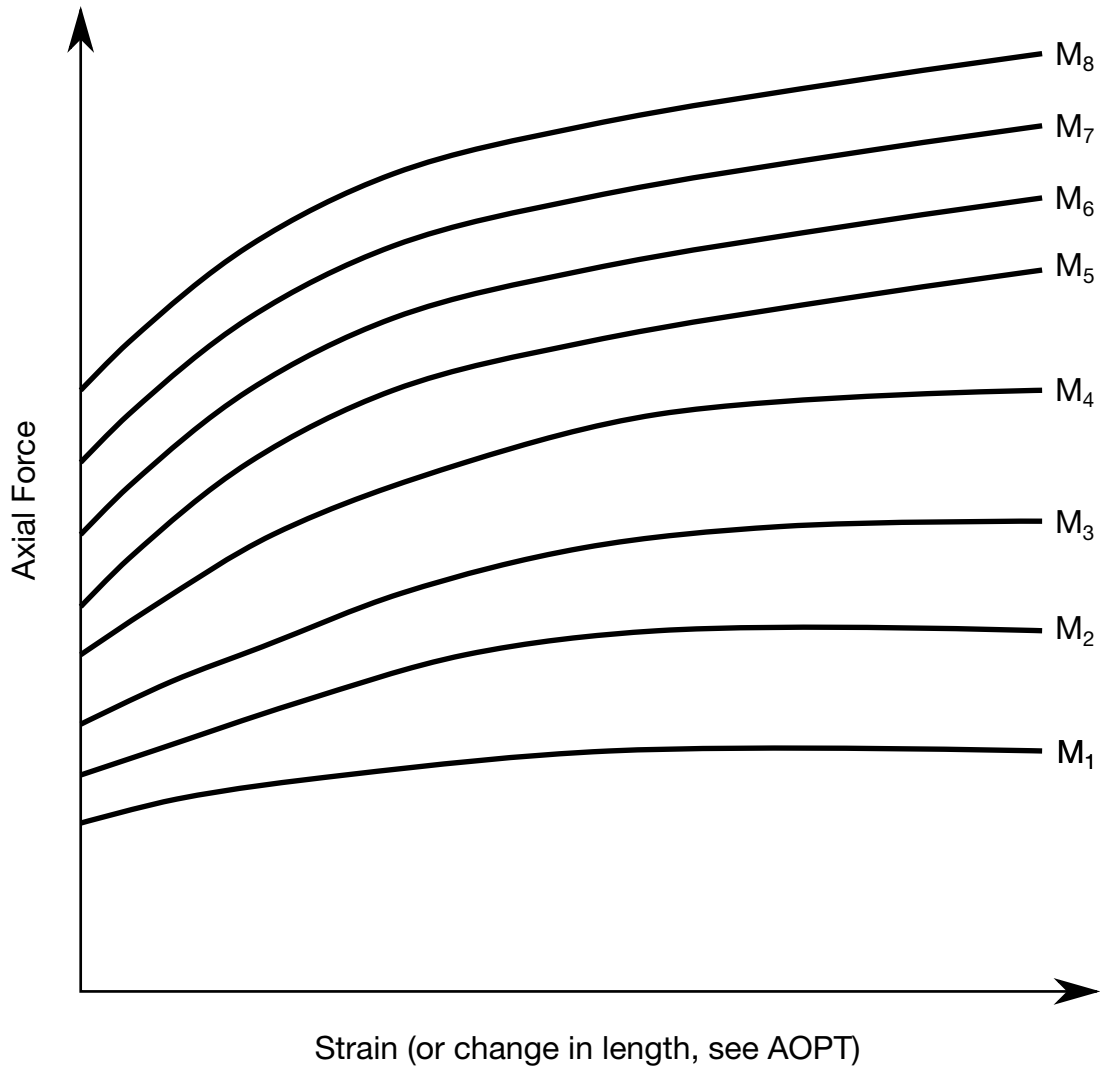
where,

$M_r, M_s, M_t$ , = current moment

$M_{r\text{yield}}, M_{s\text{yield}}, M_{t\text{yield}}$  = yield moment

Note that scale factors for hinge behavior defined in the input will also be applied to the yield moments: for example,  $M_{s\text{yield}}$  in the above formula is given by the input yield moment about the local axis times the input scale factor for the local s axis. For strain-softening characteristics, the yield moment should generally be set equal to the initial peak of the moment-rotation load curve.

On forming a hinge, upper limit moments are set. These are given by



**Figure M29-1.** The force magnitude is limited by the applied end moment. For an intermediate value of the end moment LS-DYNA interpolates between the curves to determine the allowable force value.

$$M_{r\text{upper}} = \max\left(M_r, \frac{M_{r\text{yield}}}{2}\right)$$

and similar conditions hold for  $M_{s\text{upper}}$  and  $M_{t\text{upper}}$ .

Thereafter, the plastic moments will be given by

$$M_{rp} = \min(M_{r\text{upper}}, M_{r\text{curve}})$$

where,

$M_{rp}$  = current plastic moment

$M_{r\text{curve}}$  = moment from load curve at the current rotation scaled by the scale factor.

$M_{s_p}$  and  $M_{t_p}$  satisfy similar conditions.

The effect of this is to provide an upper limit to the moment that can be generated; it represents the softening effect of local buckling at a hinge site. Thus if a member is bent about its local s-axis it will then be weaker in torsion and about its local t-axis. For moment-softening curves, the effect is to trim off the initial peak (although if the curves subsequently harden, the final hardening will also be trimmed off).

It is not possible to make the plastic moment vary with axial load.



\*MAT\_SHAPE\_MEMORY

This is material type 30. This material model describes the superelastic response present in shape-memory alloys (SMA), that is the peculiar material ability to undergo large deformations with a full recovery in loading-unloading cycles (See Figure M30-1). The material response is always characterized by a hysteresis loop. See the references by Auricchio, Taylor and Lubliner [1997] and Auricchio and Taylor [1997]. This model is available for shells, solids, and Hughes-Liu beam elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A8	F	F	F				
Default	none	none	none	none				

Card 2	1	2	3	4	5	6	7	8
Variable	SIG_ASS	SIG_ASF	SIG_SAS	SIG_SAF	EPSL	ALPHA	YMRT	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	0.0	0.0	

**Optional Load Curve Card (starting with R7.1).** Load curves for mechanically induced phase transitions.

Card 3	1	2	3	4	5	6	7	8
Variable	LCID_AS	LCID_SA						
Type	I	I						
Default	none	none						

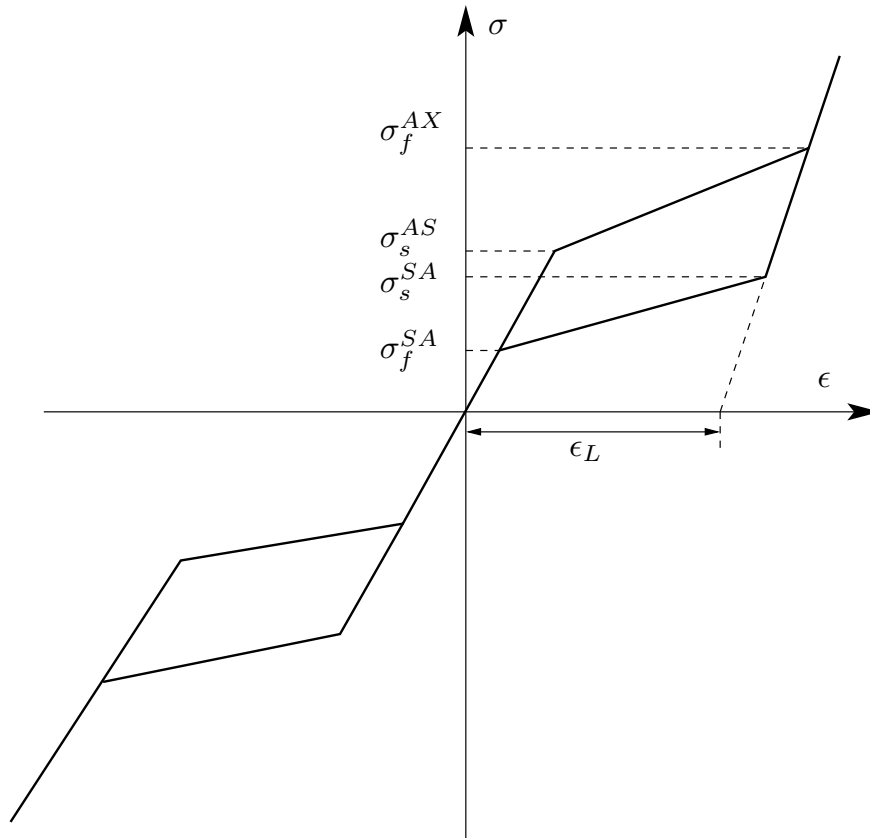
<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Density
E	Young's modulus
PR	Poisson's ratio
SIG_ASS	Starting value for the forward phase transformation (conversion of austenite into martensite) in the case of a uniaxial tensile state of stress. A load curve for SIG_ASS as a function of temperature is specified by using the negative of the load curve ID number.
SIG_ASF	Final value for the forward phase transformation (conversion of austenite into martensite) in the case of a uniaxial tensile state of stress. SIG_ASF as a function of temperature is specified by using the negative of the load curve ID number.
SIG_SAS	Starting value for the reverse phase transformation (conversion of martensite into austenite) in the case of a uniaxial tensile state of stress. SIG_SAS as a function of temperature is specified by using the negative of the load curve ID number.
SIG_SAF	Final value for the reverse phase transformation (conversion of martensite into austenite) in the case of a uniaxial tensile state of stress. SIG_SAF as a function of temperature is specified by using the negative of the load curve ID number.
EPSL	Recoverable strain or maximum residual strain. It is a measure of the maximum deformation obtainable all the martensite in one direction.
ALPHA	Parameter measuring the difference between material responses in tension and compression (set alpha = 0 for no difference). Also, see the following Remark.
YMRT	Young's modulus for the martensite if it is different from the modulus for the austenite. Defaults to the austenite modulus if it is set to zero.
LCID_AS	Load curve ID or Table ID for the <i>forward</i> phase change (conversion of austenite into martensite).

1. When *LCID\_AS* is a load curve ID the curve is taken to be

VARIABLE	DESCRIPTION
LCID_SA	<p data-bbox="589 254 1409 323">effective stress versus martensite fraction (ranging from 0 to 1).</p> <ol data-bbox="524 365 1409 877" style="list-style-type: none"> <li data-bbox="524 365 1409 512">2. When <i>LCID_AS</i> is a table ID the table defines for each phase transition rate (derivative of martensite fraction) a load curve ID specifying the stress versus martensite fraction for that phase transition rate.  The stress versus martensite fraction curve for the lowest value of the phase transition rate is used, if the phase transition rate falls below the minimum value. Likewise, the stress versus martensite fraction curve for the highest value of phase transition rate is used if phase transition rate exceeds the maximum value.</li> <li data-bbox="524 806 1409 877">3. The values of SIG_ASS and SIG_ASF are overwritten when this option is used.</li> </ol> <p data-bbox="475 905 1409 974">Load curve ID or Table ID for <i>reverse</i> phase change (conversion of martensite into austenite).</p> <ol data-bbox="524 1016 1409 1680" style="list-style-type: none"> <li data-bbox="524 1016 1409 1129">1. When <i>LCID_SA</i> is a load curve ID the curve is taken to be effective stress versus martensite fraction (ranging from 0 to 1).</li> <li data-bbox="524 1163 1409 1570">2. When <i>LCID_SA</i> is a table ID the table defines for each phase transition rate (derivative of martensite fraction) a load curve ID specifying the stress versus martensite fraction for that phase transition rate.  The stress versus martensite fraction curve for the lowest value of the phase transition rate is used, if the phase transition rate falls below the minimum value. Likewise, the stress versus martensite fraction curve for the highest value of phase transition rate is used if phase transition rate exceeds the maximum value.</li> <li data-bbox="524 1604 1409 1680">3. The values of SIG_ASS and SIG_ASF are overwritten when this option is used.</li> </ol>

**Remarks:**

The material parameter alpha,  $\alpha$ , measures the difference between material responses in tension and compression. In particular, it is possible to relate the parameter  $\alpha$  to the initial



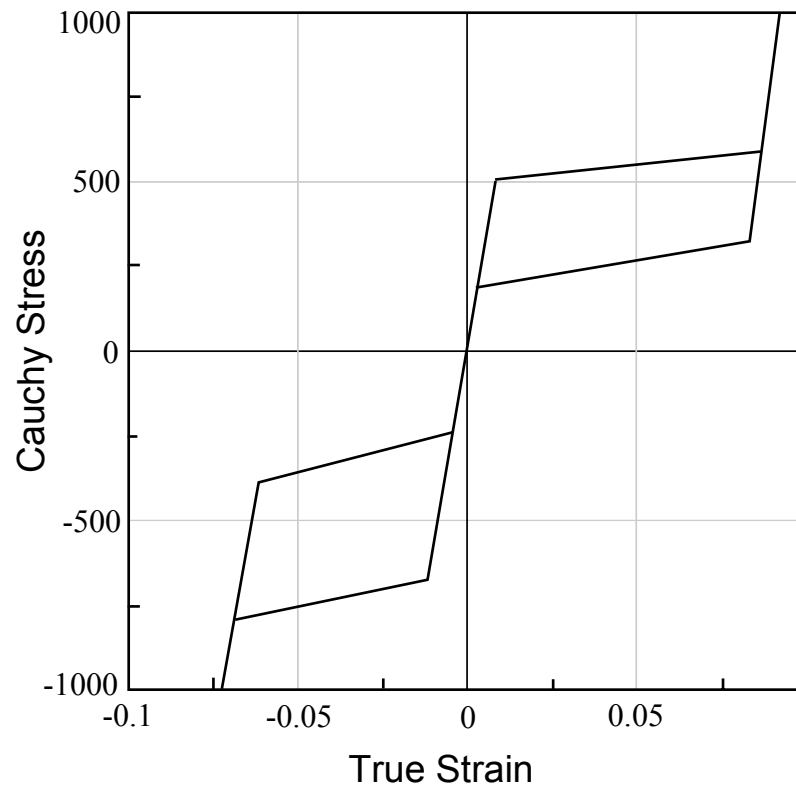
**Figure M30-1.** Superelastic Behavior for a Shape Memory Material

stress value of the austenite into martensite conversion, indicated respectively as  $\sigma_s^{AS,+}$  and  $\sigma_s^{AS,-}$ , according to the following expression:

$$\alpha = \frac{\sigma_s^{AS,-} - \sigma_s^{AS,+}}{\sigma_s^{AS,-} + \sigma_s^{AS,+}}$$

In the following, the results obtained from a simple test problem is reported. The material properties are set as:

E	60000 MPa
PR	0.3
SIG_ASS	520 MPa
SIG_ASF	600 MPa
SIG_SAS	300 MPa
SIG_SAF	200 MPa



**Figure M30-2.** Complete loading-unloading test in tension and compression.

EPSL	0.07
ALPHA	0.12
YMRT	50000 MPa

The investigated problem is the complete loading-unloading test in tension and compression. The uniaxial Cauchy stress versus the logarithmic strain is plotted in [Figure M30-2](#).

**\*MAT\_FRAZER\_NASH\_RUBBER\_MODEL**

This is Material Type 31. This model defines rubber from uniaxial test data. It is a modified form of the hyperelastic constitutive law first described in Kenchington [1988]. See also the notes below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PR	C100	C200	C300	C400	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	C110	C210	C010	C020	EXIT	EMAX	EMIN	REF
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LCID				
Type	F	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
PR	Poisson's ratio. Values between .49 and .50 are suggested.
C100	C100 (EQ.1.0 if term is in the least squares fit.)
C200	C200 (EQ.1.0 if term is in the least squares fit.)
C300	C300 (EQ.1.0 if term is in the least squares fit.)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
C400	C400 (EQ.1.0 if term is in the least squares fit.)
C110	C110 (EQ.1.0 if term is in the least squares fit.)
C210	C210 (EQ.1.0 if term is in the least squares fit.)
C010	C010 (EQ.1.0 if term is in the least squares fit.)
C020	C020 (EQ.1.0 if term is in the least squares fit.)
EXIT	Exit option: EQ.0.0: stop if strain limits are exceeded (recommended), NE.0.0: continue if strain limits are exceeded. The curve is then extrapolated.
EMAX	Maximum strain limit, (Green-St, Venant Strain).
EMIN	Minimum strain limit, (Green-St, Venant Strain).
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.
SGL	Specimen gauge length, see <a href="#">Figure M27-1</a> .
SW	Specimen width, see <a href="#">Figure M27-1</a> .
ST	Specimen thickness, see <a href="#">Figure M27-1</a> .
LCID	Load curve ID, see DEFINE_CURVE, giving the force versus actual change in gauge length. See also <a href="#">Figure M27-2</a> for an alternative definition.

**Remarks:**

The constants can be defined directly or a least squares fit can be performed if the uniaxial data (SGL, SW, ST and LCID) is available. If a least squares fit is chosen, then the terms to be included in the energy functional are flagged by setting their corresponding coefficients

to unity. If all coefficients are zero the default is to use only the terms involving  $I_1$  and  $I_2$ .  $C_{100}$  defaults to unity if the least square fit is used.

The strain energy functional,  $U$ , is defined in terms of the input constants as:

$$U = C_{100}I_1 + C_{200}I_1^2 + C_{300}I_1^3 + C_{400}I_1^4 + C_{110}I_1I_2 + C_{210}I_1^2I_2 + C_{010}I_2 + C_{020}I_2^2 + f(J)$$

where the invariants can be expressed in terms of the deformation gradient matrix,  $F_{ij}$ , and the Green-St. Venant strain tensor,  $E_{ij}$ :

$$J = |F_{ij}|$$

$$I_1 = E_{ii}$$

$$I_2 = \frac{1}{2!} \delta_{pq}^{ij} E_{pi} E_{qj}$$

The derivative of  $U$  with respect to a component of strain gives the corresponding component of stress

$$S_{ij} = \frac{\partial U}{\partial E_{ij}}$$

here,  $S_{ij}$ , is the second Piola-Kirchhoff stress tensor.

The load curve definition that provides the uniaxial data should give the change in gauge length,  $\Delta L$ , and the corresponding force. In compression both the force and the change in gauge length must be specified as negative values. In tension the force and change in gauge length should be input as positive values. The principal stretch ratio in the uniaxial direction,  $\lambda_1$ , is then given by

$$\lambda = \frac{L_o + \Delta L}{L_o}$$

Alternatively, the stress versus strain curve can also be input by setting the gauge length, thickness, and width to unity and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force, see [Figure M27-2](#). The least square fit to the experimental data is performed during the initialization phase and a comparison between the fit and the actual input is provided in the printed file. It is a good idea to visually check the fit to make sure it is acceptable. The coefficients  $C_{100}$  -  $C_{020}$  are also printed in the output file.



\*MAT\_LAMINATED\_GLASS

This is Material Type 32. With this material model, a layered glass including polymeric layers can be modeled. Failure of the glass part is possible. See notes below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EG	PRG	SYG	ETG	EFG	EP
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	PRP	SYP	ETP					
Type	F	F	F					

**Integration Point Cards.** Define 1-4 cards specifying up to 32 values. If less than 4 cards are input, reading is stopped by a "\*" control card.

Card 3	1	2	3	4	5	6	7	8
Variable	F1	F2	F3	F4	F5	F6	F7	F8
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
EG	Young's modulus for glass
PRG	Poisson's ratio for glass
SYG	Yield stress for glass
ETG	Plastic hardening modulus for glass

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EFG	Plastic strain at failure for glass
EP	Young's modulus for polymer
PRP	Poisson's ratio for polymer
SYP	Yield stress for polymer
ETP	Plastic hardening modulus for polymer
F1, ..., FN	Integration point material: $f_n = 0.0$ : glass, $f_n = 1.0$ : polymer. A user-defined integration rule must be specified, see *INTEGRATION_SHELL. See remarks below.

**Remarks:**

Isotropic hardening for both materials is assumed. The material to which the glass is bonded is assumed to stretch plastically without failure. A user defined integration rule specifies the thickness of the layers making up the glass.  $F_i$  defines whether the integration point is glass (0.0) or polymer (1.0). The material definition,  $F_i$ , has to be given for the same number of integration points (NIPTS) as specified in the rule. A maximum of 32 layers is allowed.

If the recommended user defined rule is not defined, the default integration rules are used. The location of the integration points in the default rules are defined in the \*SECTION\_SHELL keyword description.

**\*MAT\_BARLAT\_ANISOTROPIC\_PLASTICITY**

This is Material Type 33. This model was developed by Barlat, Lege, and Brem [1991] for modeling anisotropic material behavior in forming processes. The finite element implementation of this model is described in detail by Chung and Shah [1992] and is used here. It is based on a six parameter model, which is ideally suited for 3D continuum problems, see notes below. For sheet forming problems, material 36 based on a 3-parameter model is recommended.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	K	E0	N	M
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	A	B	C	F	G	H	LCID	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	BETA						
Type	F	F						

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus, $E$ .
PR	Poisson's ratio, $\nu$ .
K	$k$ , strength coefficient, see notes below.
EO	$\varepsilon_0$ , strain corresponding to the initial yield, see notes below.
N	$n$ , hardening exponent for yield strength.
M	$m$ , flow potential exponent in Barlat's Model.
A	$a$ , anisotropy coefficient in Barlat's Model.
B	$b$ , anisotropy coefficient in Barlat's Model.
C	$c$ , anisotropy coefficient in Barlat's Model.
F	$f$ , anisotropy coefficient in Barlat's Model.
G	$g$ , anisotropy coefficient in Barlat's Model.
H	$h$ , anisotropy coefficient in Barlat's Model.
LCID	Option load curve ID defining effective stress versus effective plastic strain. If nonzero, this curve will be used to define the yield stress. The load curve is implemented for solid elements only.

VARIABLE	DESCRIPTION
AOPT	<p>Material axes option:</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA..</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center, this is the <i>a</i>-direction.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by offsetting the material axes by an angle, BETA, from a line determined by taking the cross product of the vector <i>v</i> with the normal to the plane of the element.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</p>
BETA	<p>Material angle in degrees for AOPT = 1 (shells only) and AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO.</p>
MACF	<p>Material axes change flag for brick elements:</p> <p>EQ.1: No change, default,</p> <p>EQ.2: switch material axes <i>a</i> and <i>b</i>,</p> <p>EQ.3: switch material axes <i>a</i> and <i>c</i>,</p> <p>EQ.4: switch material axes <i>b</i> and <i>c</i>.</p>
XP, YP, ZP	<p>Coordinates of point <i>p</i> for AOPT = 1.</p>
A1, A2, A3	<p>Components of vector <i>a</i> for AOPT = 2.</p>
V1, V2, V3	<p>Components of vector <i>v</i> for AOPT = 3.</p>
D1, D2, D3	<p>Components of vector <i>d</i> for AOPT = 2.</p>

**Remarks:**

The yield function  $\Phi$  is defined as:

$$\Phi = |S_1 - S_2|^m + |S_2 - S_3|^m + |S_3 - S_1|^m = 2\bar{\sigma}^m$$

where  $\bar{\sigma}$  is the effective stress and  $S_{i=1,2,3}$  are the principal values of the symmetric matrix  $S_{\alpha\beta}$ ,

$$S_{xx} = [c(\sigma_{xx} - \sigma_{yy}) - b(\sigma_{zz} - \sigma_{xx})]/3$$

$$S_{yy} = [a(\sigma_{yy} - \sigma_{zz}) - c(\sigma_{xx} - \sigma_{yy})]/3$$

$$S_{zz} = [b(\sigma_{zz} - \sigma_{xx}) - a(\sigma_{yy} - \sigma_{zz})]/3$$

$$S_{yz} = f\sigma_{yz}$$

$$S_{zx} = g\sigma_{zx}$$

$$S_{xy} = h\sigma_{xy}$$

The material constants  $a, b, c, f, g$  and  $h$  represent anisotropic properties. When

$$a = b = c = f = g = h = 1,$$

the material is isotropic and the yield surface reduces to the Tresca yield surface for  $m = 1$  and von Mises yield surface for  $m = 2$  or  $4$ .

For face centered cubic (FCC) materials  $m = 8$  is recommended and for body centered cubic (BCC) materials  $m = 6$  is used. The yield strength of the material is

$$\sigma_y = k(\varepsilon^p + \varepsilon_0)^n$$

where  $\varepsilon_0$  is the strain corresponding to the initial yield stress and  $\varepsilon^p$  is the plastic strain.

\*MAT\_BARLAT\_YLD96

This is Material Type 33. This model was developed by Barlat, Maeda, Chung, Yanagawa, Brem, Hayashida, Lege, Matsui, Murtha, Hattori, Becker, and Makosey [1997] for modeling anisotropic material behavior in forming processes in particular for aluminum alloys. This model is available for shell elements only.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	K			
Type	A8	F	F	F	F			

Card 2	1	2	3	4	5	6	7	8
Variable	E0	N	ESR0	M	HARD	A		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	AX	AY	AZ0	AZ1
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT	BETA						
Type	F	F						

Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus, $E$ .
PR	Poisson's ratio, $\nu$ .
K	$k$ , strength coefficient or $a$ in Voce, see notes below.
EO	$\epsilon_0$ , strain corresponding to the initial yield or $b$ in Voce, see notes below.
N	$n$ , hardening exponent for yield strength or $c$ in Voce.
ESR0	$\epsilon_{SR0}$ , in powerlaw rate sensitivity.
M	$m$ , exponent for strain rate effects
HARD	Hardening option: LT.0.0: absolute value defines the load curve ID. EQ.1.0: powerlaw EQ.2.0: Voce
A	Flow potential exponent.
C1	$c_1$ , see equations below.



VARIABLE	DESCRIPTION
C2	$c_2$ , see equations below.
C3	$c_3$ , see equations below.
C4	$c_4$ , see equations below.
AX	$a_x$ , see equations below.
AY	$a_y$ , see equations below.
AZ0	$a_{z_0}$ , see equations below.
AZ1	$a_{z_1}$ , see equations below.
AOPT	Material axes option: EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA. EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: locally orthotropic material axes determined by offsetting the material axes by an angle, BETA, from a line determined by taking the cross product of the vector $\mathbf{v}$ with the normal to the plane of the element. LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA..
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.

**Remarks:**

The yield stress  $\sigma_y$  is defined three ways. The first, the Swift equation, is given in terms of the input constants as:

$$\sigma_y = k(\varepsilon_0 + \varepsilon^p)^n \left( \frac{\dot{\varepsilon}}{\varepsilon_{SR0}} \right)^m$$

The second, the Voce equation, is defined as:

$$\sigma_y = a - be^{-c\varepsilon^p}$$

and the third option is to give a load curve ID that defines the yield stress as a function of effective plastic strain. The yield function  $\Phi$  is defined as:

$$\Phi = \alpha_1 |s_1 - s_2|^a + \alpha_2 |s_2 - s_3|^a + \alpha_3 |s_3 - s_1|^a = 2\sigma_y^a$$

where  $s_i$  is a principle component of the deviatoric stress tensor where in vector notation:

$$\mathbf{s} = \mathbf{L}\boldsymbol{\sigma}$$

and  $\mathbf{L}$  is given as

$$\mathbf{L} = \begin{bmatrix} \frac{c_2 + c_3}{3} & \frac{-c_3}{3} & \frac{-c_2}{3} & 0 \\ \frac{-c_3}{3} & \frac{c_3 + c_1}{3} & \frac{-c_1}{3} & 0 \\ \frac{-c_2}{3} & \frac{-c_1}{3} & \frac{c_1 + c_2}{3} & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix}$$

A coordinate transformation relates the material frame to the principle directions of  $\mathbf{s}$  is used to obtain the  $\alpha_k$  coefficients consistent with the rotated principle axes:

$$\alpha_k = \alpha_x p_{1k}^2 + \alpha_y p_{2k}^2 + \alpha_z p_{3k}^2$$

$$\alpha_z = \alpha_{z0} \cos^2(2\beta) + \alpha_{z1} \sin^2(2\beta)$$

where  $p_{ij}$  are components of the transformation matrix. The angle  $\beta$  defines a measure of the rotation between the frame of the principal value of  $\mathbf{s}$  and the principal anisotropy axes.

\*MAT\_FABRIC

This is Material Type 34. This material is especially developed for airbag materials. The fabric model is a variation on the layered orthotropic composite model of material 22 and is valid for 3 and 4 node membrane elements only.

In addition to being a constitutive model, this model also invokes a special membrane element formulation which is more suited to the deformation experienced by fabrics under large deformation. For thin fabrics, buckling can result in an inability to support compressive stresses; thus a flag is included for this option. A linearly elastic liner is also included which can be used to reduce the tendency for these elements to be crushed when the no-compression option is invoked. In LS-DYNA versions after 931 the isotropic elastic option is available.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB		PRBA	PRAB	
Type	A8	F	F	F		F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	GAB			CSE	EL	PRL	LRATIO	DAMP
Type	F			F	F	F	F	F
Remarks				1	4	4	4	

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	FLC/X2	FAC/X3	ELA	LNRC	FORM	FVOPT	TSRFAC
Type	F	F	F	F	F	F	F	F
Remarks		2	2	3	4	11	9	10

Additional card for FVOPT &lt; 0.

Card 3a	1	2	3	4	5	6	7	8
Variable	L	R	C1	C2	C3			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable		RGBRTH	AOREF	A1	A2	A3	X0	X1
Type		F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3				BETA	ISREFG
Type	F	F	F				F	I

Additional card for FORM = 4, 14, or -14.

Card 6	1	2	3	4	5	6	7	8
Variable	LCA	LCB	LCAB	LCUA	LCUB	LCUAB	RL	
Type	I	I	I	I	I	I	F	

Additional card for FORM = -14.

Card 7	1	2	3	4	5	6	7	8
Variable	LCAA	LCBB	H	DT		ECOAT	SCOAT	TCOAT
Type	I	I	F	F		F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
EA	Young's modulus - longitudinal direction. For an isotropic elastic fabric material only EA and PRBA are defined and are used as the isotropic Young's modulus and Poisson's ratio, respectively. The input for the fiber directions and liner should be input as zero for the isotropic elastic fabric
EB	Young's modulus - transverse direction, set to zero for isotropic elastic material.
PRBA	$\nu_{ba}$ , Minor Poisson's ratio ba direction.
PRAB	$\nu_{ab}$ , Major Poisson's ratio ab direction. (see Remark 15)
GAB	$G_{ab}$ , shear modulus ab direction, set to zero for isotropic elastic material.
CSE	Compressive stress elimination option (see <a href="#">Remark 1</a> ): EQ.0.0: don't eliminate compressive stresses, ( <i>default</i> ) EQ.1.0: eliminate compressive stresses. This option does not apply to the liner.
EL	Young's modulus for elastic liner (required if LRATIO > 0).
PRL	Poisson's ratio for elastic liner (required if LRATIO > 0).
LRATIO	A non-zero value activates the elastic liner and defines the ratio of liner thickness to total fabric thickness (optional).
DAMP	Rayleigh damping coefficient. A 0.05 coefficient is recommended corresponding to 5% of critical damping. Sometimes larger values are necessary.
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description). Also, please refer to <i>Remark 5</i> for additional information specific to fibre directions for fabrics: EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the element nor-

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	mal by an angle BETA.
	EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.

**If X0 is between 0 and 1 (exclusive) then**

X2 (FLC)	X2 is a coefficient of the porosity from the equation in Anagonye and Wang [1999].
X3 (FAC)	X3 is a coefficient of the porosity equation of Anagonye and Wang [1999].

**Else If X0 = 0 or -1 then (sets meaning of the abscissa for load curve cases)**

FLC (X2)	Optional porous leakage flow coefficient. (See theory manual.) GE.0: Porous leakage flow coefficient. LT.0:  FLC  is interpreted as a load curve ID defining FLC as a function of time.
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**If FVOPT < 7 then (sets meaning of the mantissa for load curve case)**

FAC (X3)	Optional characteristic fabric parameter. (See theory manual.) GE.0: Characteristic fabric parameter LT.0:  FAC  is interpreted as a load curve ID defining FAC as a function of absolute pressure.
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VARIABLE	DESCRIPTION
<b>Else if FVOPT ≥ 7 then</b> ( <i>sets meaning of the mantissa for load curve case</i> )	
FAC (X3)	<p>Optional characteristic fabric parameter. (See theory manual.)</p> <p>GE.0: Characteristic fabric parameter</p> <p>LT.0:  FAC  is interpreted as a load curve ID giving <i>leakage volume flux</i> rate versus absolute pressure. The volume flux (per area) rate (per time) has the unit of</p> $d(\text{vol}_{\text{flux}})/dt \approx [\text{length}]^3 / ([\text{length}]^2 [\text{time}])$ $\approx [\text{length}] / [\text{time}],$ <p>equivalent to relative porous gas speed.</p>
<b>End if</b>	
<b>Else if X0 = 1</b> ( <i>sets meaning of the abscissa for load curve cases</i> )	
FLC (X2)	<p>Optional porous leakage flow coefficient. (See theory manual.)</p> <p>GE.0: Porous leakage flow coefficient.</p> <p>LT.0:  FLC  is interpreted as a load curve curve ID defining FLC versus the stretching ratio defined as <math>r_s = A/A_0</math>. See notes below.</p>
<b>If FVOPT &gt; 7 then</b> ( <i>sets meaning of the mantissa for load curve case</i> )	
FAC (X3)	<p>Optional characteristic fabric parameter. (See theory manual.)</p> <p>GE.0: Characteristic fabric parameter</p> <p>LT.0:  FAC  is interpreted as a load curve defining FAC versus the pressure ratio <math>r_p = P_{\text{air}}/P_{\text{bag}}</math>. See <a href="#">Remark 2</a> below.</p>

VARIABLE

DESCRIPTION

Else if FVOPT ≥ 7 then (sets meaning of the mantissa for load curve case)

FAC (X3)      Optional characteristic fabric parameter. (See theory manual.)

                 GE.0: Characteristic fabric parameter

                 LT.0: |FAC| is interpreted as a load curve defining *leakage volume flux* rate versus the pressure ratio defined as  $r_p = P_{air}/P_{bag}$ . See [Remark 2](#) below. The volume flux (per area) rate (per time) has the unit of

$$d(\text{vol}_{\text{flux}})/dt \approx [\text{length}]^3 / ([\text{length}]^2 [\text{time}])$$

$$\approx [\text{length}] / [\text{time}],$$

equivalent to relative porous gas speed.

End if

End if

ELA            Effective leakage area for blocked fabric, ELA (see [Remark 3](#)):

                 LT.0.0: |ELA| is the load curve ID of the curve defining ELA versus time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas.

LNRC          Flag to turn off compression in liner until the reference geometry is reached, i.e., the fabric element becomes tensile.

                 EQ.0.0: off.

                 EQ.1.0: on.

FORM          Flag to modify membrane formulation for fabric material:

                 EQ.0.0: default. Least costly and very reliable.

                 EQ.1.0: invariant local membrane coordinate system

                 EQ.2.0: Green-Lagrange strain formulation

                 EQ.3.0: large strain with nonorthogonal material angles. See [Remark 5](#).

                 EQ.4.0: large strain with nonorthogonal material angles and nonlinear stress strain behavior. Define optional load curve IDs on optional card.

                 EQ.12.0: Enhanced version of formulation 2. See [Remark 11](#).



VARIABLE	DESCRIPTION
	EQ.13.0: Enhanced version of formulation 3. See <a href="#">Remark 11</a> .
	EQ.14.0: Enhanced version of formulation 4. See <a href="#">Remark 11</a> .
	EQ.-14.0: Same as formulation 14, but invokes reading of card 7. See <a href="#">Remark 14</a> .
	EQ.24.0: Enhanced version of formulation 14. See <a href="#">Remark 11</a> .
FVOPT	Fabric venting option. EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered. EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered. EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered. EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered. EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered. EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered. EQ.7: Leakage is based on gas volume outflow versus pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC in the *MAT_FABRIC card. EQ.8: Leakage is based on gas volume outflow versus pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC in the *MAT_FABRIC card. LT.0:  FVOPT  defines the same fabric venting options as above, but a new formula for the leakage area is used to replace the element area. See Remark 16.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TSRFAC	Strain restoration factor LT.0:  TSRFAC  is the ID of a curve defining TSRFAC versus time.. GT.0 and LT.1: TSRFAC applied from time 0. GE.1: TSRFAC is the ID of a curve that defines TSRFAC versus time using an alternate method (not available for FORM = 0 or 1).
L	Dimension of unit cell (length)
R	Radius of yarn (length)
C1	Pressure coefficient (dependent on unit system)
C2	Pressure exponent
C3	Strain coefficient
RGBRTH	Material dependent birth time of airbag reference geometry. Non-zero RGBRTH overwrites the birth time defined in the *AIRBAG_REFERENCE_GEOMETRY_BIRTH section. RGBRTH also applies to reference geometry defined by *AIRBAG_SHELL_REFERENCE_GEOMETRY.
A0REF	Calculation option of initial area, $A_0$ , used for airbag porosity leakage calculation. EQ.0.: default. Use the initial geometry defined in *NODE. EQ.1.: Use the reference geometry defined in *AIRBAG_REFERENCE_GEOMETRY or *AIRBAG_SHELL_REFERENCE_GEOMETRY.
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.
X0, X1	Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area: $A_{leak} = A_0(X_0 + X_1r_s + X_2r_p + X_3r_s r_p)$ X0.EQ.-1: Compressing seal vent option. The leakage area is evaluated as $A_{leak} = \max(A_{current} - A_0, 0)$ .
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.
ISREFG	Initialize stress by *AIRBAG_REFERENCE_GEOMETRY. This option applies only to FORM = 12. Note that *MAT_FABRIC cannot be initialized using a dynain file because *INITIAL_STRESS_SHELL is not applicable to *MAT_FABRIC.  EQ.0.0: default. Not active.  EQ.1.0: active
LCA	Load curve or table ID. Load curve ID defines the stress versus uniaxial strain along the a-axis fiber. Table ID defines for each strain rate a load curve representing stress versus uniaxial strain along the a-axis fiber. Available for FORM = 4, 14, -14, and 24 only, table allowed only for form = -14. If zero, EA is used. For FORM = 14, -14, and 24, this curve can be defined in both tension and compression, see <a href="#">Remark 6</a> below.
LCB	Load curve or table ID. Load curve ID defines the stress versus uniaxial strain along the b-axis fiber. Table ID defines for each strain rate a load curve representing stress versus uniaxial strain along the b-axis fiber. Available for FORM = 4, 14, -14, and 24 only, table allowed only for form = -14. If zero, EB is used. For FORM = 14, -14, and 24, this curve can be defined in both tension and compression, see <a href="#">Remark 6</a> below.
LCAB	Load curve ID for shear stress versus shear strain in the ab-plane; available for FORM = 4, 14, -14, and 24 only. If zero, GAB is used.
LCUA	Unload/reload curve ID for stress versus strain along the a-axis fiber; available for FORM = 4, 14, -14, and 24 only. If zero, LCA is used.
LCUB	Unload/reload curve ID for stress versus strain along the b-axis fiber; available for FORM = 4, 14, -14, and 24 only. If zero, LCB is used.
LCUAB	Unload/reload curve ID for shear stress versus shear strain in the ab-plane; available for FORM = 4, 14, -14, and 24 only. If zero, LCAB is used.
RL	Optional reloading parameter for FORM = 14 and 24. Values between 0.0 (reloading on unloading curve-default) and 1.0 (reloading on a minimum linear slope between unloading curve and loading curve) are possible.

VARIABLE	DESCRIPTION
LCAA	Load curve or table ID. Load curve ID defines the stress along the a-axis fiber versus biaxial strain. Table ID defines for each directional strain rate a load curve representing stress along the a-axis fiber versus biaxial strain. Available for FORM=-14 only, if zero, LCA is used.
LCBB	Load curve or table ID. Load curve ID defines the stress along the b-axis fiber versus biaxial strain. Table ID defines for each directional strain rate a load curve representing stress along the b-axis fiber versus biaxial strain. Available for FORM=-14 only, if zero, LCB is used.
H	Normalized hysteresis parameter between 0 and 1.
DT	Strain rate averaging option. EQ.0.0: Strain rate is evaluated using a running average. LT.0.0: Strain rate is evaluated using average of last 11 time steps. GT.0.0: Strain rate is averaged over the last DT time units.
ECOAT	Young's modulus of coat material, see <a href="#">Remark 14</a> .
SCOAT	Yield stress of coat material, see <a href="#">Remark 14</a> .
TCOAT	Thickness of coat material, may be positive or negative, see <a href="#">Remark 14</a> .

**Remarks:**

1. **The Compressive Stress Elimination Option for Airbag Wrinkling.** Setting CSE=1 switches off compressive stress in the fabric, thereby eliminating wrinkles. Without this "no compression" option the geometry of the bag's wrinkles control the amount of mesh refinement. In eliminating the wrinkles, this feature reduces the number of elements needed to attain an accurate solution.

The no compression option can allow elements to collapse to a line which can lead to elements becoming tangled. The elastic liner option is one way to add some stiffness in compression to prevent this, see Remark 4. Alternatively, when using fabric formulations 14, -14, or 24 (see FORM) tangling can be reduced by defining stress/strain curves that include negative strain and stress values. See Remark 6.use

2. **Porosity.** The parameters FLC and FAC are optional for the Wang-Nefske and Hybrid inflation models. It is possible for the airbag to be constructed of multiple fabrics having different values for porosity and permeability. Typically, FLC and FAC must be determined experimentally and their variations in time or with pressure are optional to allow for maximum flexibility.
3. **Effects of Airbag-Structure Interaction on Porosity.** To calculate the leakage of gas through the fabric it is necessary to accurately determine the leakage area. The dynamics of the airbag may cause the leakage area to change during the course of the simulation. In particular, the deformation may change the leakage area, but the leakage area may also decrease when the contact between the airbag and the structure blocks the flow. LS-DYNA can check the interaction of the bag with the structure and split the areas into regions that are blocked and unblocked depending on whether the regions are in or not in contact, respectively. Blockage effects may be controlled with the ELA field.
4. **Elastic Liner.** An optional elastic liner can be defined using EL, PRL and LRATIO. The liner is an isotropic layer that acts in both tension and compression. However, setting, LNRC to 1.0 eliminates compressive stress in the liner until both principle stresses are tensile. The compressive stress elimination option, CSE=1, has no influence on the liner behavior.
5. **Fiber Axes.** For formulations 0, 1, and 2, (see [FORM](#)) the *a*-axis and *b*-axis fiber directions *are assumed to be orthogonal* and are completely defined by the material axes option, AOPT=0, 2, or 3. For FORM=3, 4, 13, or 14, the fiber directions *are not assumed orthogonal* and must be specified using the ICOMP=1 option on \*SECTION\_SHELL. Offset angles should be input into the B1 and B2 fields used normally for integration points 1 and 2. The *a*-axis and *b*-axis directions will then be offset from the *a*-axis direction as determined by the material axis option, AOPT=0, 2, or 3.
6. **Stress vs. Strain Curves.** For formulations (see [FORM](#)) 4, 14, -14, and 24, 2<sup>nd</sup> Piola-Kirchhoff stress vs. Green's strain curves may be defined for *a*-axis, *b*-axis, and shear stresses for loading and also for unloading and reloading. Alternatively, the *a*-axis and *b*-axis curves can be input using engineering stress vs. strain by setting DATYP = -2 on \*DEFINE\_CURVE.

Additionally, for formulations 14, -14, and 24, the uniaxial loading curves LCA and LCB may be defined for negative values of strain and stress, i.e., a straightforward extension of the curves into the compressive region. This is available in order to model the compressive stresses resulting from tight folding of airbags.

The *a*-axis and *b*-axis stress follow the curves for the entire defined strain region and if compressive behavior is desired the user should preferably make sure the curve covers all strains of interest. For strains below the first point on the curve, the curve is extrapolated using the stiffness from the constant values, EA or EB.

Shear stress/strain behavior is assumed symmetric and curves should be defined for positive strain only. However, formulations 14, -14, and 24 allow the extending of the curves in the negative strain region to model asymmetric behavior. The asymmetric option cannot be used with a shear stress unload curve. If a load curve is omitted, the stress is calculated from the appropriate constant modulus, EA, EB, or GAB.

- 7. Yield Behavior.** When formulations 4, 14, -14, and 24 (see [FORM](#)) are used with loading and unloading curves the initial yield strain is set equal to the strain of the first point in the load curve having a stress greater than zero. When the current strain exceeds the yield strain, the stress follows the load curve and the yield strain is updated to the current strain. When unloading occurs, the unload/reload curve is shifted along the x-axis until it intersects the load curve at the current yield strain. When using unloading curves, compressive stress elimination should be active to prevent the fibers from developing compressive stress during unloading when the strain remains tensile. *To use this option, the unload curve should have a nonnegative second derivate so that the curve will shift right as the yield stress increases.*

If LCUA, LCUB, or LCUAB are input with negative values, then unloading is handled differently. Instead of shifting the unload curve along the x-axis, the curve is stretched in both the x-direction and y-direction such that the first point remains anchored at (0,0) and the initial intersection point of the curves is moved to the current yield point. This option guarantees the stress remains tensile while the strain is tensile so compressive stress elimination is not necessary. *To use this option the unload curve should have an initial slope less steep than the load curve, and should steepen such that it intersects the load curve at some positive strain value.*

- 8. Shear Unload-Reload, Fabric Formulation, and LS-DYNA version.** With release 6.0.0 of version 971, LS-DYNA changed the way that unload/reload curves for shear stress-strain relations are interpreted. Let  $f$  be the shear stress unload-reload curve LCUAB. Then,

$$\sigma_{ab} = c_2 f(c_1 \varepsilon_{ab})$$

where the scale factors  $c_1$  and  $c_2$  depend on the fabric form (see [FORM](#)) and version of LS-DYNA.

Fabric form	4		14 and -14		24	
	$c_1$	$c_2$	$c_1$	$c_2$	$c_1$	$c_2$
LS971 R5.1.0 and earlier	2	1	2	1	-	-
LS971 R6.0.0 to R7.0	2	1	1	2	-	-
LS-DYNA R7.1 and later	2	1	1	2	1	1

When switching fabric forms or versions, the curve scale factors SFA and SFO on \*DEFINE\_CURVE can be used to offset this behavior.

9. **Per Material Venting Option.** The FVOPT flag allows an airbag fabric venting equation to be assigned to a material. The anticipated use for this option is to allow a vent to be defined using FVOPT=1 or 2 for one material and fabric leakage to be defined for using FVOPT=3, 4, 5, or 6 for other materials. In order to use FVOPT, a venting option must first be defined for the airbag using the OPT parameter on \*AIRBAG\_WANG\_NEFSKE or \*AIRBAG\_HYBRID. If OPT=0, then FVOPT is ignored. If OPT is defined and FVOPT is omitted, then FVOPT is set equal to OPT.
10. **TSRFAC option to restore element strains.** Airbags that use a reference geometry will typically have nonzero strains at the start of the calculation. To prevent such initial strains from prematurely opening an airbag, initial strains are stored and subtracted from the measured strain throughout the calculation.

$$\sigma = f(\epsilon - \epsilon_{\text{initial}})$$

- Fabric formulations 2, 3, and 4 (see [FORM](#)) subtract off only the initial tensile strains so these forms are typically used with CSE = 1 and LNRC = 1.
- Fabric formulations 12, 13, 14, -14, and 24 subtract off the total initial strains so these forms may be used with CSE = 0 or 1 and LNRC = 0 or 1. A side effect of this strain modification is that airbags may not achieve the correct volume when they open. Therefore, the TSRFAC option is implemented to reduce the stored initial strain values over time thereby restoring the total strain which drives the airbag towards the correct volume.

During each cycle, the stored initial strains are scaled by  $(1.0 - \text{TSRFAC})$ . A small value on the order of 0.0001 is typically sufficient to restore the strains in a few milliseconds of simulation time.

$$\sigma = f(\epsilon - \epsilon_{\text{adjustment}})$$

The adjustment to restore initial strain is then,

$$\epsilon_{\text{adjustment}} = \epsilon_{\text{initial}} \prod_i [1 - \text{TSRFAC}].$$

- a) *Time Dependent TSRFAC.* When  $\text{TSRFAC} < 0$ ,  $|\text{TSRFAC}|$  becomes the ID of a curve that defines  $\text{TSRFAC}$  as a function of time. To delay the effect of  $\text{TSRFAC}$ , the curve ordinate value should be initially zero and should ramp up to a small number to restore the strain at an appropriate time during the simulation. The adjustment to restore initial strain is then,

$$\epsilon_{\text{adjustment}}(t_i) = \epsilon_{\text{initial}} \prod_i [1 - \text{TSRFAC}(t_i)].$$

To prevent airbags from opening prematurely, it is recommended to use the load curve option of  $\text{TSRFAC}$  to delay the strain restoration until the airbag is partially opened due to pressure loading.

- b) *Alternate Time Dependent TSRFAC.* For fabric formulations 2 and higher, a second curve option is invoked by setting  $\text{TSRFAC} \geq 1$  where  $\text{TSRFAC}$  is again the ID of a curve that defines  $\text{TSRFAC}$  versus time. Like the first curve option, the stored initial strain values are scaled by  $(1.0 - \text{TSRFAC})$ , but the modified initial strains are not saved, so the effect of  $\text{TSRFAC}$  does not accumulate. In this case the adjustment to eliminate initial strain

$$\epsilon_{\text{adjustment}}(t_i) = [1 - \text{TSRFAC}(t_i)] \epsilon_{\text{initial}}.$$

Therefore, the curve should ramp up from zero to one to fully restore the strain. This option gives the user better control of the rate of restoring the strain as it is a function of time rather than solution time step.

11. **Enhancements to the Material Formulations.** Material formulations (See [FORM](#)) 12, 13, and 14 are enhanced versions of formulations 2, 3, and 4, respectively. The most notable difference in their behavior is apparent when a reference geometry is used for the fabric. As discussed in [Remark 10](#), the strain is modified to prevent initial strains from prematurely opening an airbag at the start of a calculation.

Formulations 2, 3, and 4 subtract the initial tensile strains, while the enhanced formulations subtract the total initial strains. Therefore, the enhanced formulations can be used without setting  $\text{CSE} = 1$  and  $\text{LNRC} = 1$  since compressive stress cutoff is not needed to prevent initial airbag movement. Formulations 2, 3, and 4 need compressive stress cutoff when used with a reference geometry or they can generate compressive stress at the start of a calculation. Available for formulation 12 only, the  $\text{ISREFG}$  parameter activates an option to calculate the initial stress by using a reference geometry.

Material formulation 24 is an enhanced version of formulation 14 implementing a correction for Poisson's effects when stress vs. strain curves are input for the  $a$ -fiber or  $b$ -fiber. Also, for formulation 24, the outputted stress and strain in the



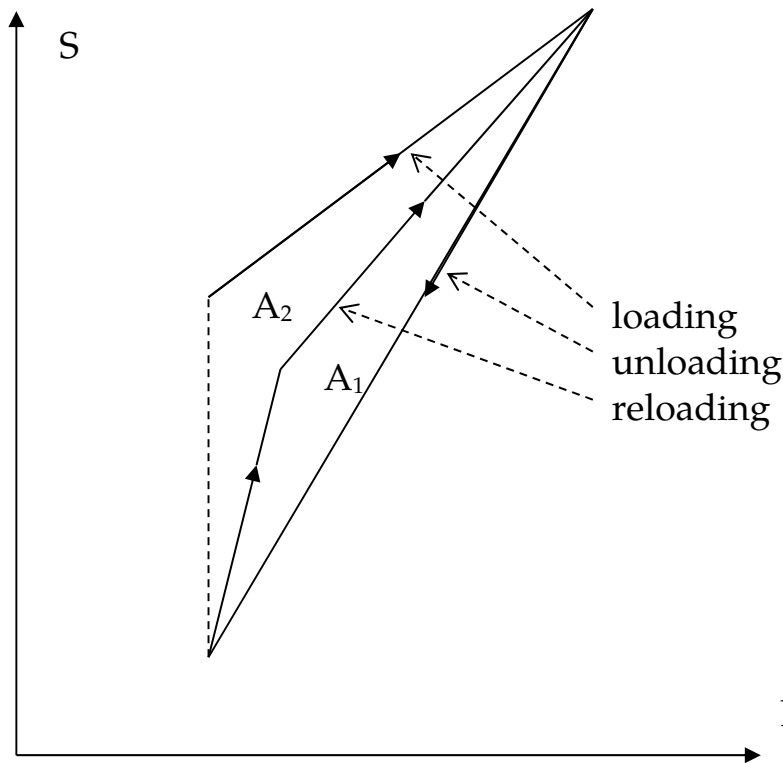


Figure M34-1.

elout or d3plot database files is engineering stress and strain rather than the 2<sup>nd</sup> Piola Kirchoff and Green's strain used by formulations other than 0 and 1.

12. **Noise Reduction for the Strain Rate Measure.** If tables are used, then the strain rate measure is the time derivative of the Green-Lagrange strain in the direction of interest. To suppress noise, the strain rate is averaged according to the value of DT. If  $DT > 0$ , it is recommended to use a large enough value to suppress the noise, while being small enough to not lose important information in the signal.
13. **Hysteresis.** The hysteresis parameter H defines the fraction of dissipated energy during a load cycle in terms of the maximum possible dissipated energy. Referring to the [Figure M34-1](#),

$$H \approx \frac{A_1}{A_1 + A_2}$$

14. **Coating Feature for Additional Rotational Resistance.** It is possible to model coating of the fabric using a sheet of elastic-ideal-plastic material where the

Young’s modulus, yield stress and thickness is specified for the coat material. This will add rotational resistance to the fabric for a more realistic behavior of coated fabrics. To read this parameters set FORM=-14, which adds an extra card containing the three parameters ECOAT, SCOAT and TCOAT, corresponding to the three coat material properties mentioned above.

The thickness, TCOAT, applies to both sides of the fabric. The coat material for a certain fabric element deforms along with this and all elements connected to this element, which is how the rotations are "captured". Note that unless TCOAT is set to a negative value, the coating will add to the membrane stiffness. For negative values of TCOAT the thickness is set to |TCOAT| and the membrane contribution from the coating is suppressed. For this feature to work, the fabric parts must not include any T-intersections, and all of the surface normal vectors of connected fabric elements must point in the same direction. This option increases the computational complexity of this material.

- 15. **Poisson’s Ratios.** Fabric forms 12, 13, 14, -14, and 24 allow input of both the minor Poisson’s ratio,  $\nu_{ba}$ , and the major Poisson’s ratio,  $\nu_{ab}$ . This allows asymmetric Poisson’s behavior to be modelled. If the major Poisson’s ratio is left blank or input as zero, then it will be calculated using  $\nu_{ab} = \nu_{ba} \frac{E_a}{E_b}$ .
- 16. **St. Venant-Wantzel Leakage.** If a negative value for the fabric venting option FVOPT is used (only -1 and -2 are supported), the mass flux through a fabric membrane is calculated according St.Venant-Wantzel by

$$\dot{m} = A_{eff} \Psi \sqrt{2p_i \rho_i}$$

where  $p_i$  describes the internal pressure,  $\rho_i$  the density of the outlet gas and the effluence function  $\psi$  depends on the character of the flow, the adiabatic exponent  $\kappa$  and the pressure difference between the inside ( $p_i$ ) and the outside ( $p_a$ ) of the membrane. For subsonic flow it is formulated as:

$$\Psi = \sqrt{\frac{\kappa}{\kappa - 1} \left[ \left(\frac{p_a}{p_i}\right)^{\frac{2}{\kappa}} - \left(\frac{p_a}{p_i}\right)^{\frac{\kappa+1}{\kappa}} \right]}$$

and for sonic or critical flow as:

$$\Psi = \sqrt{\frac{\kappa}{2} \left(\frac{2}{\kappa + 1}\right)^{\frac{\kappa+1}{\kappa-1}}}$$

The effective venting area of the membrane is determined according M. Schlenger:

$$A_{eff} = \frac{A_0}{L^2} \left[ (C_1 \Delta p^{C_2}) (L - 2r)^2 + C_3 \left( L\lambda_1 - \frac{2r}{\sqrt{\lambda_2}} \right) \left( L\lambda_2 - \frac{2r}{\sqrt{\lambda_1}} \right) \right] \sin(\alpha_{12})$$

where  $\lambda_i$  are the stretches in fiber direction  $i$  and  $\alpha_{12}$  is the angle between the fibers. The initial membrane area is equal to  $A_0$ .  $r$  and  $L$  represent the radius of the fabric fiber and the edge length of the fabric set, respectively. The coefficients  $L, r, C_1, C_2$ , and  $C_3$  have to be defined on additional card 3a. This option is supported for \*AIRBAG\_WANG\_NESFKE and \*AIRBAG\_HYBRID. No additional input in \*AIRBAG cards is needed. All FORM options are supported, whereas  $\alpha_{12}$  can only be different from 90 degree for FORM=3, 4, 13, or 14.



Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
PXX	Table giving engineering local XX-stress as function of engineering local XX-strain and YY-strain.
PYY	Table giving engineering local YY-stress as function of engineering local YY-strain and XX-strain.
SXY	Curve giving local 2 <sup>nd</sup> Piola-Kirchhoff XY-stress as function of local Green XY-strain.
DAMP	Damping coefficient for numerical stability.
TH	Table giving hysteresis factor $0 \leq H < 1$ as function of engineering local XX-strain and YY-strain. GT.0.0: TH is table ID LE.0.0: -TH is used as constant value for hysteresis factor
FVOPT	Fabric venting option, see *MAT_FABRIC.
X0, X1	Fabric venting option parameters, see *MAT_FABRIC.
FLC/X2	Fabric venting option parameter, see *MAT_FABRIC.
FAC/X3	Fabric venting option parameter, see *MAT_FABRIC.
ELA	Fabric venting option parameter, see *MAT_FABRIC.
ISREFG	Initial stress by reference geometry. EQ.0.0: Not active. EQ.1.0: Active

<b>VARIABLE</b>	<b>DESCRIPTION</b>
CSE	Compressive stress elimination option. EQ.0.0: Don't eliminate compressive stresses, EQ.1.0: Eliminate compressive stresses.
SRFAC	Load curve ID for smooth stress initialization when using a reference geometry.
BULKC	Bulk modulus for fabric compaction.
JACC	Jacobian for the onset of fabric compaction.
FXX	Load curve giving scale factor of uniaxial stress in first material direction as function of engineering strain rate.
FYY	Load curve giving scale factor of uniaxial stress in second material direction as function of engineering strain rate.
DT	Time window for smoothing strain rates used for FXX and FYY.
AOPT	Material axes option, see *MAT_FABRIC.
ECOAT	Young's modulus of coat material to include bending properties. This together with the following two parameters (SCOAT and TCOAT) encompass the same coating/bending feature as in *MAT_FABRIC. Please refer to these manual pages and associated remarks.
SCOAT	Yield stress of coat material, see *MAT_FABRIC.
TCOAT	Thickness of coat material, may be positive or negative, see *MAT_FABRIC.
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3.
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

**Remarks:**

This material model invokes a special membrane element formulation regardless of the element choice. It is an anisotropic hyperelastic model where the 2<sup>nd</sup> Piola-Kirchhoff stress

$\mathbf{S}$  is a function of the Green-Lagrange strain  $\mathbf{E}$  and possibly its history. Due to anisotropy, this strain is transformed to obtain the strains in each of the fiber directions  $E_{XX}$  and  $E_{YY}$  together with the shear strain  $E_{XY}$ . The associated stress components in the local system are given as functions of the strain components

$$\begin{aligned} S_{XX} &= \gamma S_{XX}(E_{XX}, E_{YY}) \vartheta \\ S_{YY} &= \gamma S_{YY}(E_{YY}, E_{XX}) \vartheta \\ S_{XY} &= \gamma S_{XY}(E_{XY}) \vartheta. \end{aligned}$$

The factor  $\gamma$  is used for dissipative effects and is described in more detail later, for  $\text{TH} = 0$ ,  $\gamma = 1$ , and the function  $\vartheta$  represents a strain rate scale factor also described below, for  $\text{FXX} = \text{FYY} = 0$  this factor is 1. While the shear relation is given directly through the curve  $S_{XY}$ , the tabular input of the fiber stress components  $P_{XX}$  and  $P_{YY}$  is for the sake of convenience the engineering stress as function of engineering strain, i.e.,

$$\begin{aligned} P_{XX} &= P_{XX}(e_{XX}, e_{YY}) \\ P_{YY} &= P_{YY}(e_{YY}, e_{XX}). \end{aligned}$$

To this end, the following conversion formulae are used between stresses and strains

$$\begin{aligned} e &= \sqrt{1 + 2E} - 1 \\ S &= \frac{P}{1 + e} \end{aligned}$$

these being applied in each of the two fiber directions.

Compressive stress elimination is optional through the CSE parameter, and when activated the principal components of the 2<sup>nd</sup> Piola-Kirchhoff stress is restricted to positive values.

If a reference geometry is used, then SRFAC is the identity of a curve that is a function  $\alpha(t)$  that should increase from zero to unity during a short time span, during which the Green-Lagrange strain used in the formulae above is substituted for

$$\tilde{\mathbf{E}} = \mathbf{E} - [1 - \alpha(t)] \mathbf{E}_0,$$

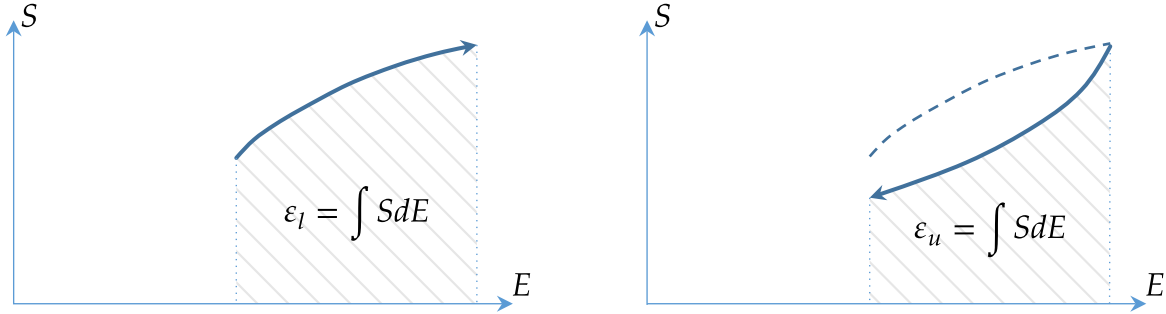
where  $\mathbf{E}_0$  is the strain at time zero. This is done in order to smoothly initialize the stress resulting from using a reference geometry different from the geometry at time zero.

The factor  $\gamma$  is a function of the strain history and is initially set to unity, and depends, more specifically, on the internal work  $\epsilon$  given by the stress power

$$\dot{\epsilon} = \mathbf{S} : \dot{\mathbf{E}}.$$

The evolution of  $\gamma$  is related to the stress power in the sense that it will increase on loading and decrease on unloading, and in this way introduce dissipation. The exact mathematical formula is too complicated to reveal, but in essence the function looks like

$$\gamma = \begin{cases} 1 - H(\bar{e}_{XX}, \bar{e}_{YY}) + H(\bar{e}_{XX}, \bar{e}_{YY}) \exp[\beta(\epsilon - \bar{\epsilon})] & \dot{\epsilon} < 0 \\ 1 - H(\bar{e}_{XX}, \bar{e}_{YY}) \exp[-\beta(\epsilon - \underline{\epsilon})] & \dot{\epsilon} \geq 0 \end{cases}$$



**Figure M34M-1.** Cyclic loading model for hysteresis model  $H$

Here  $\bar{\epsilon}$  is the maximum attained internal work up to this point in time,  $\bar{\epsilon}_{XX}$  and  $\bar{\epsilon}_{YY}$  are the engineering strain values associated with value.  $H(\bar{\epsilon}_{XX}, \bar{\epsilon}_{YY})$  is the hysteresis factor defined by the user through the input parameter TH, it may or may not depend on the strains.  $\beta$  is a decay constant that depends on  $\bar{\epsilon}_{XX}$  and  $\bar{\epsilon}_{YY}$ , and  $\underline{\epsilon}$  is the minimum attained internal work at any point in time after  $\bar{\epsilon}$  was attained. In other words, on unloading  $\gamma$  will exponentially decay to  $1 - H$  and on loading it will exponentially grow to 1 and always be restricted by the lower and upper bounds,  $1 - H < \gamma \leq 1$ . The only thing the user needs to care about is to input a proper hysteresis factor  $H$ , and with reference to a general loading/unloading cycle illustrated in figure M34M-1 below the relation  $1 - H = \epsilon_u / \epsilon_l$  should hold.

To account for the packing of yarns in compression, a compaction effect is modeled by adding a term to the strain energy function on the form

$$W_c = K_c J \left\{ \ln \left( \frac{J}{J_c} \right) - 1 \right\}, \text{ for } J \leq J_c$$

where  $K_c$  (BULKC) is a physical bulk modulus,  $J = \det(\mathbf{F})$  is the jacobian of the deformation and  $J_c$  (JACC) is the critical jacobian for when the effect commences. Here  $\mathbf{F}$  is the deformation gradient. This gives a contribution to the pressure given by

$$p = K_c \ln \left( \frac{J_c}{J} \right), \text{ for } J \leq J_c$$

and thus prevents membrane elements from collapsing or inverting when subjected to compressive loads. The bulk modulus  $K_c$  should be selected with the slopes in the stress map tables in mind, presumably some order of magnitude(s) smaller.

As an option, the local membrane stress can be scaled based on the engineering strain rates via the function  $\vartheta = \vartheta(\dot{\epsilon}, \mathbf{S})$ . We set

$$\dot{\epsilon} = \max \left( \frac{\dot{\epsilon}}{\|\mathbf{F}\mathbf{S}\|}, 0 \right)$$

to be the equivalent engineering strain rate in the direction of loading and define

$$\vartheta(\dot{\epsilon}, \mathbf{S}) = \frac{F_{XX}(\dot{\epsilon})|S_{XX}| + F_{YY}(\dot{\epsilon})|S_{YY}| + 2|S_{XY}|}{|S_{XX}| + |S_{YY}| + 2|S_{XY}|},$$



meaning that the strain rate scale factor defaults to the user input data FXX and FYY for uniaxial loading in the two material directions, respectively. Note that we only consider strain rate scaling in loading and not in unloading, and furthermore that the strain rates used in evaluating the curves are pre-filtered using the time window DT to avoid excessive numerical noise. To this end, it is recommended to set DT to a time corresponding to at least hundred time steps or so.

**\*MAT\_PLASTIC\_GREEN-NAGHDI\_RATE**

This is Material Type 35. This model is available only for brick elements and is similar to model 3, but uses the Green-Naghdi Rate formulation rather than the Jaumann rate for the stress update. For some cases this might be helpful. This model also has a strain rate dependency following the Cowper-Symonds model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A8	F	F	F				

Card 2	1	2	3	4	5	6	7	8
Variable	SIGY	ETAN	SRC	SRP	BETA			
Type	F	F	F	F	F			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Density
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
ETAN	Plastic hardening modulus
SRC	Strain rate parameter, C
SRP	Strain rate parameter, P
BETA	Hardening parameter, $0 < \beta' < 1$

**\*MAT\_3-PARAMETER\_BARLAT\_{OPTION}**

This is Material Type 36. This model was developed by Barlat and Lian [1989] for modeling sheets with anisotropic materials under plane stress conditions. This material allows the use of the Lankford parameters for the definition of the anisotropy. This particular development is due to Barlat and Lian [1989]. A version of this material model which has a flow limit diagram failure option is \*MAT\_FLD\_3-PARAMETER\_BARLAT.

Available options include:

<BLANK>

NLP

The NLP option estimates failure using the Formability Index (F.I.), which accounts for the non-linear strain paths seen in metal forming applications (See the [Remarks](#)). The NLP field in card 3 *must* be defined when using this option. The NLP option is also available in \*MAT\_037, \*MAT\_125 and \*MAT\_226.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	HR	P1	P2	ITER
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	M	R00/AB	R45/CB	R90/HB	LCID	E0	SPI	P3
Type	F	F	F	F	I	F	F	F

Define the following card if and only if  $M < 0$

Card 3	1	2	3	4	5	6	7	8
Variable	CRC1	CRA1	CRC2	CRA2	CRC3	CRA3	CRC4	CRA4
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT	C	P	VLCID		PB	NLP/HTA	HTB
Type	F	F	F	I		F	I/F	F

Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3	HTC	HTD
Type				F	F	F	F	F

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	HTFLAG
Type	F	F	F	F	F	F	F	F

Optional card.

Card 7	1	2	3	4	5	6	7	8
Variable	USRFAIL	LCBI	LCSH					
Type	F	F	F					

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus, $E$ GT.0.0: Constant value. LT.0.0: Load curve ID = $ E $ , which defines Young's Modulus as a function of plastic strain. See <a href="#">Remarks</a> .

VARIABLE	DESCRIPTION
PR	Poisson's ratio, $\nu$
HR	Hardening rule: EQ.1.0: linear (default), EQ.2.0: exponential (Swift) EQ.3.0: load curve or table with strain rate effects EQ.4.0: exponential (Voce) EQ.5.0: exponential (Gosh) EQ.6.0: exponential (Hockett-Sherby) EQ.7.0: load curves in three directions EQ.8.0: table with temperature dependence EQ.9.0: three dimensional table with temperature and strain rate dependence
P1	Material parameter: HR.EQ.1.0: Tangent modulus, HR.EQ.2.0: $k$ , strength coefficient for Swift exponential hardening HR.EQ.4.0: $a$ , coefficient for Voce exponential hardening HR.EQ.5.0: $k$ , strength coefficient for Gosh exponential hardening HR.EQ.6.0: $a$ , coefficient for Hockett-Sherby exponential hardening HR.EQ.7.0: load curve ID for hardening in 45° direction. See <a href="#">Remarks</a> .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
P2	Material parameter: HR.EQ.1.0: Yield stress HR.EQ.2.0: $n$ , exponent for Swift exponential hardening HR.EQ.4.0: $c$ , coefficient for Voce exponential hardening HR.EQ.5.0: $n$ , exponent for Gosh exponential hardening HR.EQ.6.0: $c$ , coefficient for Hocket-Sherby exponential hardening HR.EQ.7.0: load curve ID for hardening in $90^\circ$ direction. See <a href="#">Remarks</a> .
ITER	Iteration flag for speed: EQ.0.0: fully iterative EQ.1.0: fixed at three iterations Generally, ITER= 0 is recommended. However, ITER= 1 is somewhat faster and may give acceptable results in most problems.
M	$m$ , exponent in Barlat's yield surface, absolute value is used if negative.
CRC $n$	Chaboche-Rousselier hardening parameters, see <a href="#">Remarks</a> .
CRA $n$	Chaboche-Rousselier hardening parameters, see <a href="#">Remarks</a> .
R00	$R_{00}$ , Lankford parameter in $0^\circ$ direction GT.0.0: Constant value, LT.0.0: Load curve or Table ID =  R00  which defines $R_{00}$ value as a function of plastic strain (Curve) or as a function of temperature and plastic strain (Table). See <a href="#">Remarks</a> .
R45	$R_{45}$ , Lankford parameter in $45^\circ$ direction GT.0.0: Constant value, LT.0.0: Load curve or Table ID =  R45  which defines $R_{45}$ value as a function of plastic strain (Curve) or as a function of temperature and plastic strain (Table). See <a href="#">Remarks</a> .

VARIABLE	DESCRIPTION
R90	<p><math>R_{90}</math>, Lankford parameter in 90° direction</p> <p>GT.0.0: Constant value,</p> <p>LT.0.0: Load curve or Table ID =  R90  which defines <math>R_{90}</math> value as a function of plastic strain (Curve) or as a function of temperature and plastic strain (Table). See <a href="#">Remarks</a>.</p>
AB	$a$ , Barlat89 parameter, which is read instead of R00 if $PB > 0$ .
CB	$c$ , Barlat89 parameter, which is read instead of R45 if $PB > 0$ .
HB	$h$ , Barlat89 parameter, which is read instead of R90 if $PB > 0$ .
LCID	Load curve/table ID for hardening in the 0 degree direction. See <a href="#">Remarks</a> .
E0	<p>Material parameter</p> <p>HR.EQ.2.0: <math>\epsilon_0</math> for determining initial yield stress for Swift exponential hardening. (Default = 0.0)</p> <p>HR.EQ.4.0: <math>b</math>, coefficient for Voce exponential hardening</p> <p>HR.EQ.5.0: <math>\epsilon_0</math> for determining initial yield stress for Gosh exponential hardening. (Default = 0.0)</p> <p>HR.EQ.6.0: <math>b</math>, coefficient for Hockett-Sherby exponential hardening</p>
SPI	<p><u>Case I: if <math>\epsilon_0</math> is zero above and HR.EQ.2.0. (Default = 0.0)</u></p> <p>EQ.0.0: <math>\epsilon_0 = \left(\frac{E}{k}\right)^{1/(n-1)}</math></p> <p>LE.0.02: <math>\epsilon_0 = SPI</math></p> <p>GT.0.02: <math>\epsilon_0 = \left(\frac{SPI}{k}\right)^{1/n}</math></p> <p><u>Case II: If HR.EQ.5.0</u></p> <p>The strain at plastic yield is determined by an iterative procedure based on the same principles as for HR.EQ.2.0.</p>
P3	<p>Material parameter:</p> <p>HR.EQ.5.0: <math>p</math>, parameter for Gosh exponential hardening</p> <p>HR.EQ.6.0: <math>n</math>, exponent for Hockett-Sherby exponential hardening</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see <code>MAT_OPTION TROPIC_ELASTIC</code> for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <code>*DEFINE_COORDINATE_NODES</code>, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with <code>*DEFINE_COORDINATE_VECTOR</code>.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>v</math> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on <code>*DEFINE_COORDINATE_NODES</code>, <code>*DEFINE_COORDINATE_SYSTEM</code> or <code>*DEFINE_COORDINATE_VECTOR</code>). Available with the R3 release of Version 971 and later.</p>
C	C in Cowper-Symonds strain rate model
P	$p$ in Cowper-Symonds strain rate model, $p = 0.0$ for no strain rate effects
VLCID	Volume correction curve ID defining the relative volume change (change in volume relative to the initial volume) as a function of the effective plastic strain. This is only used when nonzero. See <a href="#">Remarks</a> .
PB	Barlat89 parameter, $p$ . If $PB > 0$ , parameters AB, CB, and HB are read instead of R00, R45 and R90. See <a href="#">Remarks</a> below.
NLP	ID of a load curve of the Forming Limit Diagram (FLD) under linear strain paths. In the load curve, abscissas represent minor strains while ordinates represent major strains. Define only when option NLP is used. See <a href="#">Remarks</a> .
HTA	Load curve/Table ID for postforming parameter $a$ in heat treatment
HTB	Load curve/Table ID for postforming parameter $b$ in heat treatment



<b>VARIABLE</b>	<b>DESCRIPTION</b>
XP, YP, ZP	Coordinates of point <b>p</b> for AOPT = 1.
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.
HTC	Load curve/Table ID for postforming parameter <i>c</i> in heat treatment
HTD	Load curve/Table ID for postforming parameter <i>d</i> in heat treatment
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3.
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT= 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.
HTFLAG	Heat treatment flag (see <a href="#">Remarks</a> ): EQ.0: Preforming stage EQ.1: Heat treatment stage EQ.2: Postforming stage
USRFAIL	User defined failure flag EQ.0: no user subroutine is called EQ.1: user subroutine matusr_24 in dyn21.f is called
LCBI	HR.EQ.7: Load curve defining biaxial stress vs. biaxial strain for hardening rule, see discussion in the formulation section below for a definition. HR.NE.7: Ignored.
LCSH	HR.EQ.7: Load curve defining shear stress vs. shear strain for hardening, see discussion in the formulation section below for a definition. HR.NE.7: Ignored.

**Formulation:**

The effective plastic strain used in this model is defined to be plastic work equivalent. A consequence of this is that for parameters defined as functions of effective plastic strain, the rolling (00) direction should be used as reference direction. For instance, the hardening curve for HR = 3 is the stress as function of strain for uniaxial tension in the rolling

direction, VLCID curve should give the relative volume change as function of strain for uniaxial tension in the rolling direction and load curve given by  $-E$  should give the Young's modulus as function of strain for uniaxial tension in the rolling direction. Optionally, the curve can be substituted for a table defining hardening as function of plastic strain rate (HR = 3) or temperature (HR = 8).

Exceptions from the rule above are curves defined as functions of plastic strain in the  $45^\circ$  and  $90^\circ$  directions, i.e.,  $P1$  and  $P2$  for HR = 7 and negative R45 or R90, see Fleischer et.al. [2007]. The hardening curves are here defined as measured stress as function of measured plastic strain for uniaxial tension in the direction of interest, i.e., as determined from experimental testing using a standard procedure. The optional biaxial and shear hardening curves require some further elaboration, as we assume that a biaxial or shear test reveals that the true stress tensor in the material system expressed as

$$\sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \pm\sigma \end{pmatrix}, \quad \sigma \geq 0,$$

is a function of the (plastic) strain tensor

$$\varepsilon = \begin{pmatrix} \varepsilon_1 & 0 \\ 0 & \pm\varepsilon_2 \end{pmatrix}, \quad \varepsilon_1 \geq 0, \quad \varepsilon_2 \geq 0,$$

The input hardening curves are  $\sigma$  as function of  $\varepsilon_1 + \varepsilon_2$ . The  $\pm$  sign above distinguishes between the biaxial (+) and the shear (-) cases. Moreover, the curves defining the  $R$ -values are as function of the measured plastic strain for uniaxial tension in the direction of interest. These curves are transformed internally to be used with the effective stress and strain properties in the actual model. The effective plastic strain does not coincide with the plastic strain components in other directions than the rolling direction and may be somewhat confusing to the user. Therefore the von Mises work equivalent plastic strain is output as history variable #2 if HR = 7 or if any of the  $R$ -values is defined as function of the plastic strain.

The  $R$ -values in curves are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width  $W$  and thickness  $T$  are measured as function of strain. Then the corresponding  $R$ -values is given by:

$$R = \frac{\frac{dW}{d\varepsilon} / W}{\frac{dT}{d\varepsilon} / T}$$

The anisotropic yield criterion  $\Phi$  for plane stress is defined as:

$$\Phi = a|K_1 + K_2|^m + a|K_1 - K_2|^m + c|2K_2|^m = 2\sigma_Y^m$$

where  $\sigma_Y$  is the yield stress and  $K_{i=1,2}$  are given by:

$$K_1 = \frac{\sigma_x + h\sigma_y}{2}$$

$$K_2 = \sqrt{\left(\frac{\sigma_x - h\sigma_y}{2}\right)^2 + p^2 \tau_{xy}^2}$$

If  $PB = 0$ , the anisotropic material constants  $a, c, h$  and  $p$  are obtained through  $R_{00}, R_{45}$  and  $R_{90}$ :

$$a = 2 - 2\sqrt{\left(\frac{R_{00}}{1 + R_{00}}\right) \left(\frac{R_{90}}{1 + R_{90}}\right)}$$

$$c = 2 - a$$

$$h = \sqrt{\left(\frac{R_{00}}{1 + R_{00}}\right) \left(\frac{1 + R_{90}}{R_{90}}\right)}$$

The anisotropy parameter  $p$  is calculated implicitly. According to Barlat and Lian the  $R$ -value, width to thickness strain ratio, for any angle  $\phi$  can be calculated from:

$$R_\phi = \frac{2m\sigma_Y^m}{\left(\frac{\partial\Phi}{\partial\sigma_x} + \frac{\partial\Phi}{\partial\sigma_y}\right)\sigma_\phi} - 1$$

where  $\sigma_\phi$  is the uniaxial tension in the  $\phi$  direction. This expression can be used to iteratively calculate the value of  $p$ . Let  $\phi = 45$  and define a function  $g$  as:

$$g(p) = \frac{2m\sigma_Y^m}{\left(\frac{\partial\Phi}{\partial\sigma_x} + \frac{\partial\Phi}{\partial\sigma_y}\right)\sigma_\phi} - 1 - R_{45}$$

An iterative search is used to find the value of  $p$ . If  $PB > 0$ , material parameters  $a(AB)$ ,  $c(CB)$ ,  $h(HB)$ , and  $p(PB)$  are used directly.

The effective stress, given as

$$\sigma_{\text{eff}} = \left\{ \frac{1}{2} (a|K_1 + K_2|^m + a|K_1 - K_2|^m + c|2K_2|^m) \right\}^{1/m}$$

can be output to the d3plot database through \*DEFINE\_MATERIAL\_HISTORIES.

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>		
Label	Attributes	Description
Effective Stress	- - - -	Effective stress $\sigma_{\text{eff}}$ , see above

For face centered cubic (FCC) materials  $m = 8$  is recommended and for body centered cubic (BCC) materials  $m = 6$  may be used. The yield strength of the material can be expressed in terms of  $k$  and  $n$ :

$$\sigma_y = k\varepsilon^n = k(\varepsilon_{yp} + \bar{\varepsilon}^p)^n$$

where  $\varepsilon_{yp}$  is the elastic strain to yield and  $\bar{\varepsilon}^p$  is the effective plastic strain (logarithmic). If SIGY is set to zero, the strain to yield is found by solving for the intersection of the linearly elastic loading equation with the strain hardening equation:

$$\sigma = E\varepsilon$$

$$\sigma = k\varepsilon^n$$

which gives the elastic strain at yield as:

$$\varepsilon_{yp} = \left(\frac{E}{k}\right)^{\frac{1}{n-1}}$$

If SIGY yield is nonzero and greater than 0.02 then:

$$\varepsilon_{yp} = \left(\frac{\sigma_y}{k}\right)^{\frac{1}{n}}$$

The other available hardening models include the Voce equation given by:

$$\sigma_Y(\varepsilon_p) = a - be^{-c\varepsilon_p},$$

the Gosh equation given by:

$$\sigma_Y(\varepsilon_p) = k(\varepsilon_0 + \varepsilon_p)^n - p,$$

and finally the Hockett-Sherby equation given by:

$$\sigma_Y(\varepsilon_p) = a - be^{-c\varepsilon_p^n}.$$

For the Gosh hardening law, the interpretation of the variable SPI is the same, i.e., if set to zero the strain at yield is determined implicitly from the intersection of the strain hardening equation with the linear elastic equation.

To include strain rate effects in the model we multiply the yield stress by a factor depending on the effective plastic strain rate. We use the Cowper-Symonds' model; hence the yield stress can be written as:

$$\sigma_Y(\varepsilon_p, \dot{\varepsilon}_p) = \sigma_Y^s(\varepsilon_p) \left\{ 1 + \left(\frac{\dot{\varepsilon}_p}{C}\right)^{1/p} \right\}$$

where  $\sigma_Y^s$  denotes the static yield stress,  $C$  and  $p$  are material parameters, and  $\dot{\varepsilon}_p$  is the effective plastic strain rate. With HR.EQ.3 strain rate effects can be defined using a table, in which each load curve in the table defines the yield stress as function of plastic strain for a given strain rate. In contrast to material 24, when the strain rate is larger than that of any curve in the table, the table is extrapolated in the strain rate direction to find the appropriate yield stress.

A kinematic hardening model is implemented following the works of Chaboche and Roussilier. A back stress  $\alpha$  is introduced such that the effective stress is computed as:

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12})$$

The back stress is the sum of up to four terms according to:

$$\alpha_{ij} = \sum_{k=1}^4 \alpha_{ij}^k$$

and the evolution of each back stress component is as follows:

$$\delta \alpha_{ij}^k = C_k \left( a_k \frac{s_{ij}}{\sigma_{\text{eff}}} - \alpha_{ij}^k \right) \delta \varepsilon_p$$

where  $C_k$  and  $a_k$  are material parameters,  $s_{ij}$  is the deviatoric stress tensor,  $\sigma_{\text{eff}}$  is the effective stress and  $\varepsilon_p$  is the effective plastic strain. The yield condition is for this case modified according to

$$f(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \varepsilon_p) = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12}) - \left\{ \sigma_Y^t(\varepsilon_p, \dot{\varepsilon}_p, 0) - \sum_{k=1}^4 a_k [1 - \exp(-C_k \varepsilon_p)] \right\} \leq 0$$

in order to get the expected stress strain response for uniaxial stress. The calculated effective stress is stored in history variable #7.

### A Failure Criterion for Nonlinear Strain Paths (NLP) in sheet metal forming:

When the option NLP is used, a necking failure criterion is activated to account for the non-linear strain path effect in sheet metal forming. Based on the traditional Forming Limit Diagram (FLD) for the linear strain path, the Formability Index (F.I.) is calculated for every element in the model throughout the simulation duration. The entire F.I. time history for every element is stored in history variable #1 in d3plot files, accessible from *Post/History* menu in *LS-PrePost* v4.0. In addition to the F.I. output, other useful information stored in other history variables can be found as follows,

1. Formability Index: #1
2. Strain ratio (in-plane minor strain/major strain): #2
3. Effective strain from the planar isotropic assumption: #3

To enable the output of these history variables to the d3plot files, NEIPS on the \*DATA-BASE\_EXTENT\_BINARY card must be set to at least 3. The history variables can also be plotted on the formed sheet blank as a color contour map, accessible from *Post/FriComp/Misc* menu. The index value starts from 0.0, with the onset of necking failure when it reaches 1.0. The F.I. is calculated based on critical effect strain method, as explained in manual pages in \*MAT\_037. The theoretical background based on two papers can also be found in manual pages in \*MAT\_037.

When d3plot files are used to plot the history variable #1 (the F.I.) in color contour, the value in the Max pull-down menu in Post/FriComp needs to be set to Min, meaning that the necking failure occurs only when all integration points through the thickness have reached the critical value of 1.0 (refer to Tharrett and Stoughton's paper in 2003 SAE 2003-01-1157). It is also suggested to set the variable "MAXINT" in \*DATABASE\_EXTENT\_BINARY to the same value as the variable "NIP" in \*SECTION\_SHELL. In addition, the value in the Avg pull-down menu in Post/FriRang needs to be set to None. The strain path history (major vs. minor strain) of each element can be plotted with the radial dial button Strain Path in Post/FLD.

An example of a partial input for the material is provided below, where the FLD for the linear strain path is defined by the variable NLP with load curve ID 211, where abscissas represent minor strains and ordinates represent major strains.

```

*MAT_3-PARAMETER_BARLAT_NLP
$-----1-----2-----3-----4-----5-----6-----7-----8
$      MID          RO          E          PR          HR          P1          P2          ITER
$      1 2.890E-09  6.900E04  0.330  3.000
$      M          R00          R45          R90          LCID          E0          SPI          P3
$      8.000      0.800      0.600      0.550      99
$      AOPT          C          P          VLCID          NLP
$      2.000
$
$              A1          A2          A3
$              0.000      1.000      0.000
$      V1          V2          V3          D1          D2          D3          BETA
$-----1-----2-----3-----4-----5-----6-----7-----8
$ Hardening Curve
*DEFINE_CURVE
  99
      0.000          130.000
      0.002          134.400
      0.006          143.000
      0.010          151.300
      0.014          159.300
      :
      0.900          365.000
      1.000          365.000
$ FLD Definition
*DEFINE_CURVE
211
      -0.2          0.325
      -0.1054      0.2955
      -0.0513      0.2585
      0.0000       0.2054
      0.0488       0.2240
      0.0953       0.2396
      0.1398       0.2523
      0.1823       0.2622
      :

```

Shown in Figures M36-1, M36-2 and M36-3, predictions and validations of forming limit curves (FLC) of various nonlinear strain paths on a single shell element was done using this new option, for an Aluminum alloy with  $R_{00} = 0.8$ ,  $R_{45} = 0.6$  and  $R_{90} = 0.55$  and the yield at 130.0 MPa. In each case, the element is further strained in three different paths (uniaxial stress – U.A., plane strain – P.S., and equi-biaxial strain – E.B.) separately, following a pre-straining in uniaxial, plane strain and equi-biaxial strain state, respectively. The forming

limits are determined at the end of the secondary straining for each path, when the F.I. has reached the value of 1.0. It is seen that the predicted FLCs (dashed curves) in case of the nonlinear strain paths are totally different from the FLCs under the linear strain paths. It is noted that the current predicted FLCs under nonlinear strain path are obtained by connecting the ends of the three distinctive strain paths. More detailed FLCs can be obtained by straining the elements in more paths between U.A. and P.S. and between P.S. and E.B. In [Figure M36-4](#), time-history plots of F.I., strain ratio and effective strain are shown for uniaxial pre-strain followed by equi-biaxial strain path on the same single element.

Typically, to assess sheet formability, F.I. contour of the entire part should be plotted. Based on the contour plot, non-linear strain path and the F.I. time history of a few elements in the area of concern can be plotted for further study. These plots are similar to those shown in manual pages of \*MAT\_037.

### **Smoothing of the Strain Ratio $\beta$ :**

\*CONTROL\_FORMING\_TOLERANC applies a smoothing algorithm to reduce output noise level of the strain ratio  $\beta$  (minor strain/major strain) which is used to calculate the Formability Index.

### **Heat treatment with variable HTFLAG:**

Heat treatment for increasing the formability of prestrained aluminum sheets can be simulated through the use of HTFLAG, where the intention is to run a forming simulation in steps involving preforming, springback, heat treatment and postforming. In each step the history is transferred to the next via the use of dynain (see \*INTERFACE\_SPRINGBACK). The first two steps are performed with HTFLAG = 0 according to standard procedures, resulting in a plastic strain field  $\varepsilon_p^0$  corresponding to the prestrain. The heat treatment step is performed using HTFLAG = 1 in a coupled thermomechanical simulation, where the blank is heated. The coupling between thermal and mechanical is only that the maximum temperature  $T^0$  is stored as a history variable in the material model, this corresponding to the heat treatment temperature. Here it is important to export all history variables to the dynein file for the postforming step. In the final postforming step, HTFLAG = 2, the yield stress is then augmented by the Hockett-Sherby like term:

$$\Delta\sigma = b - (b - a)\exp\left[-c(\varepsilon_p - \varepsilon_p^0)^d\right]$$

where  $a, b, c$  and  $d$  are given as tables as functions of the heat treatment temperature  $T^0$  and prestrain  $\varepsilon_p^0$ . That is, in the table definitions each load curve corresponds to a given prestrain and the load curve value is with respect to the heat treatment temperature,

$$a = a(T^0, \varepsilon_p^0) \quad b = b(T^0, \varepsilon_p^0) \quad c = c(T^0, \varepsilon_p^0) \quad d = d(T^0, \varepsilon_p^0)$$

The effect of heat treatment is that the material strength decreases but hardening increases, thus typically:

$$a \leq 0 \quad b \geq a \quad c > 0 \quad d > 0$$

**Revision information:**

The option NLP is available in explicit dynamic analysis and in SMP and MPP, starting in Revision 95576. Smoothing of  $\beta$  is available starting in Revision 109781.



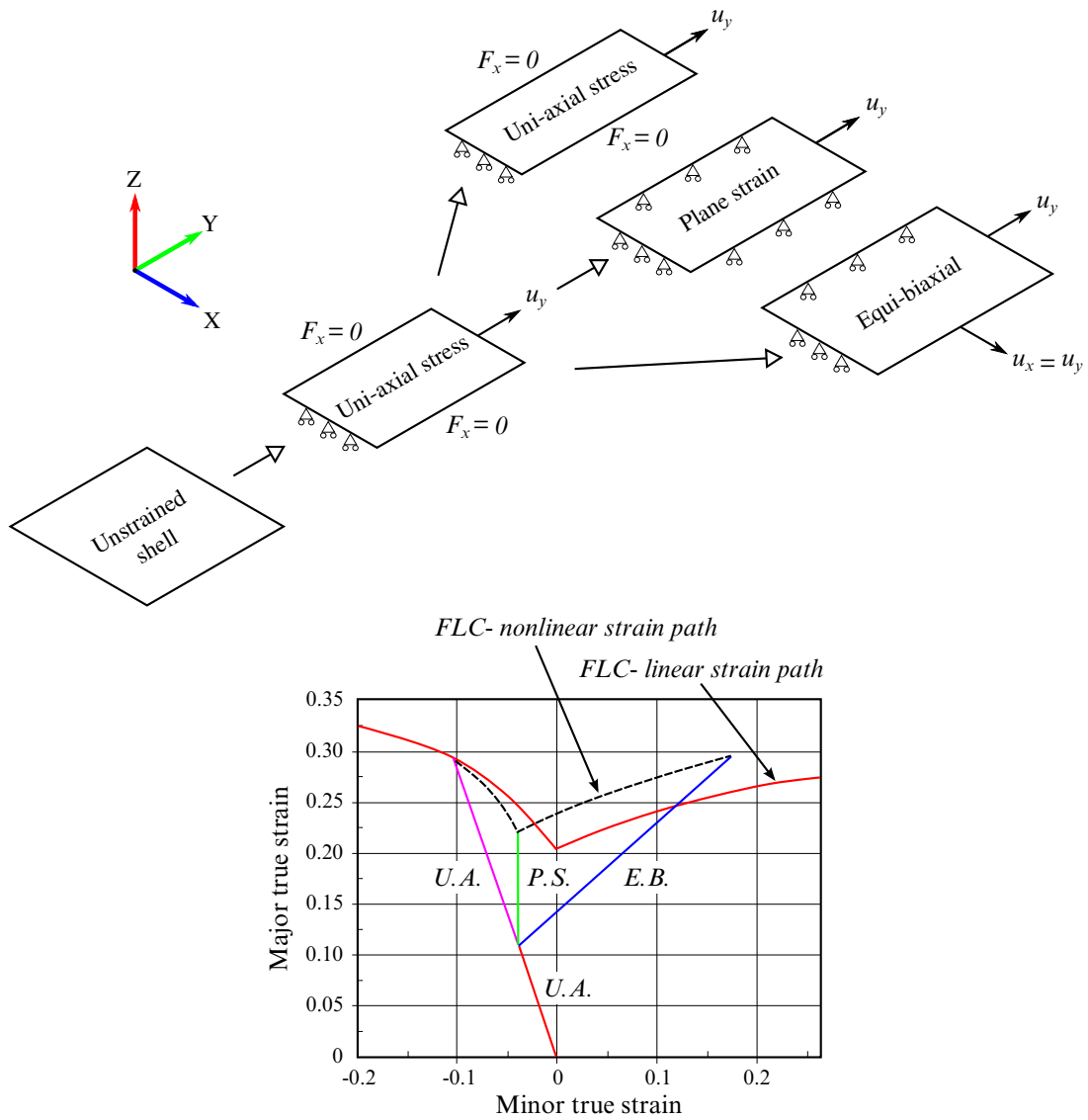


Figure M36-1. Nonlinear FLD prediction with uniaxial pre-straining.

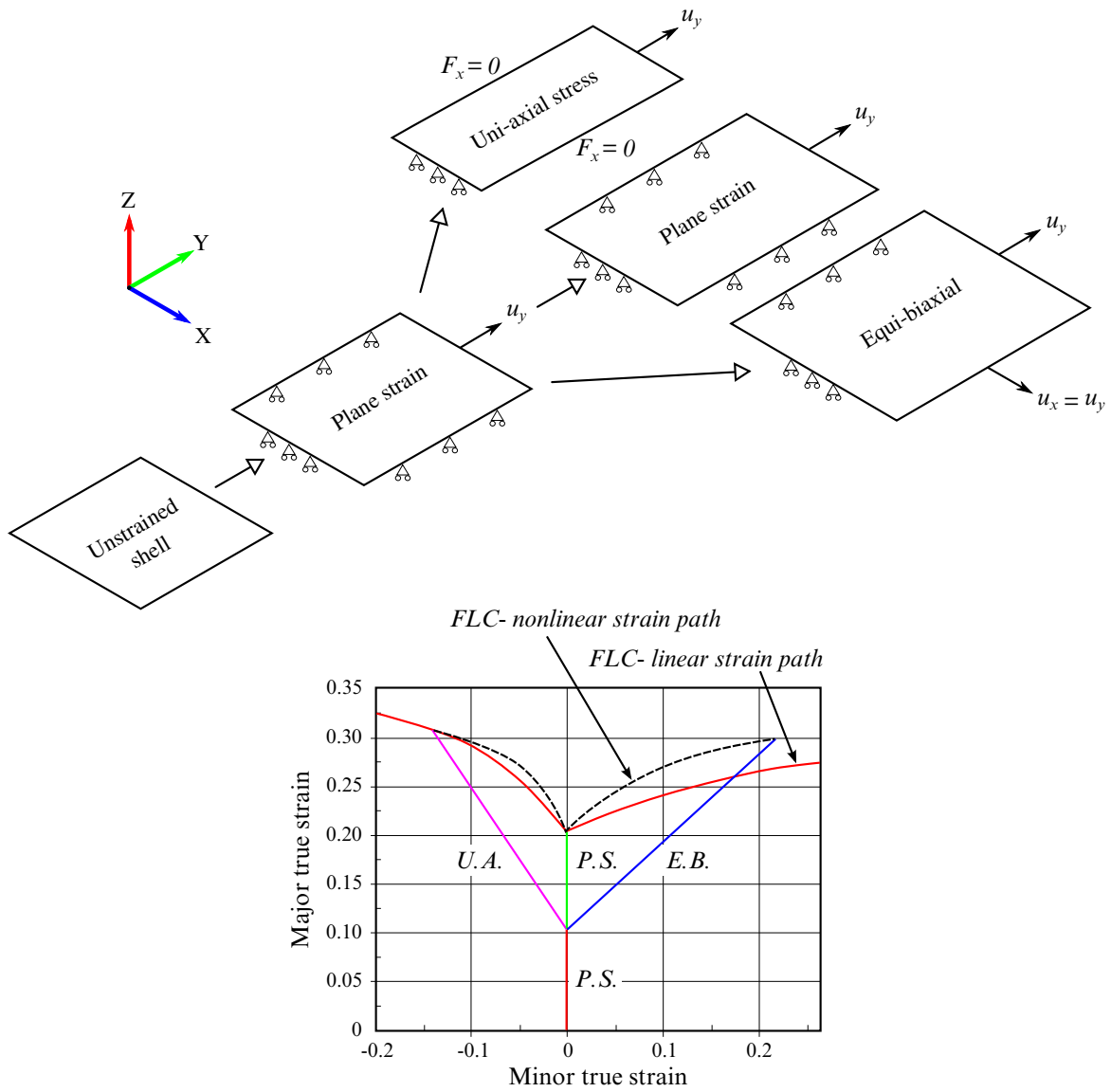


Figure M36-2. Nonlinear FLD prediction with plane strain pre-straining.

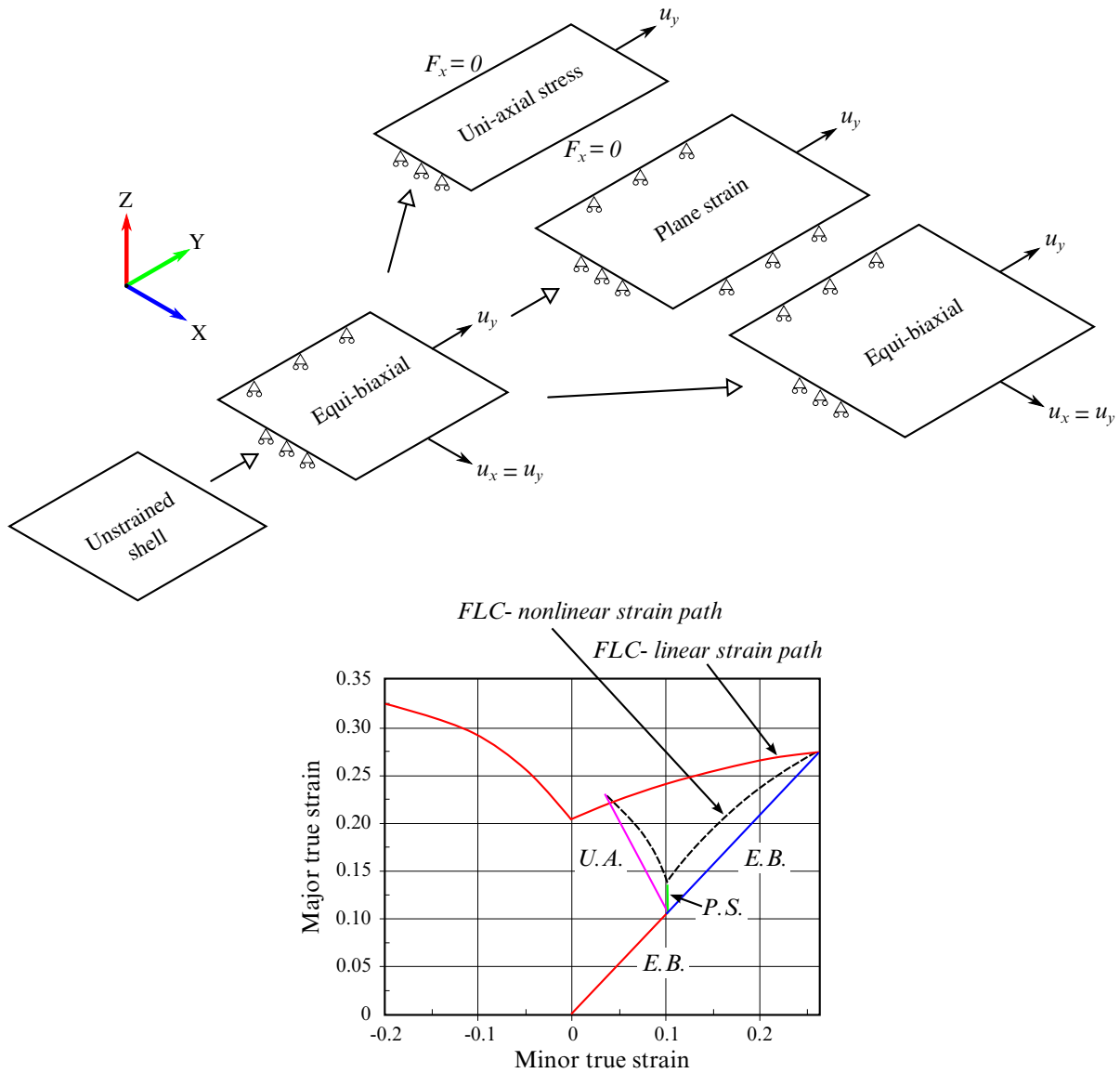


Figure M36-3. Nonlinear FLD prediction with equi-biaxial pre-straining.

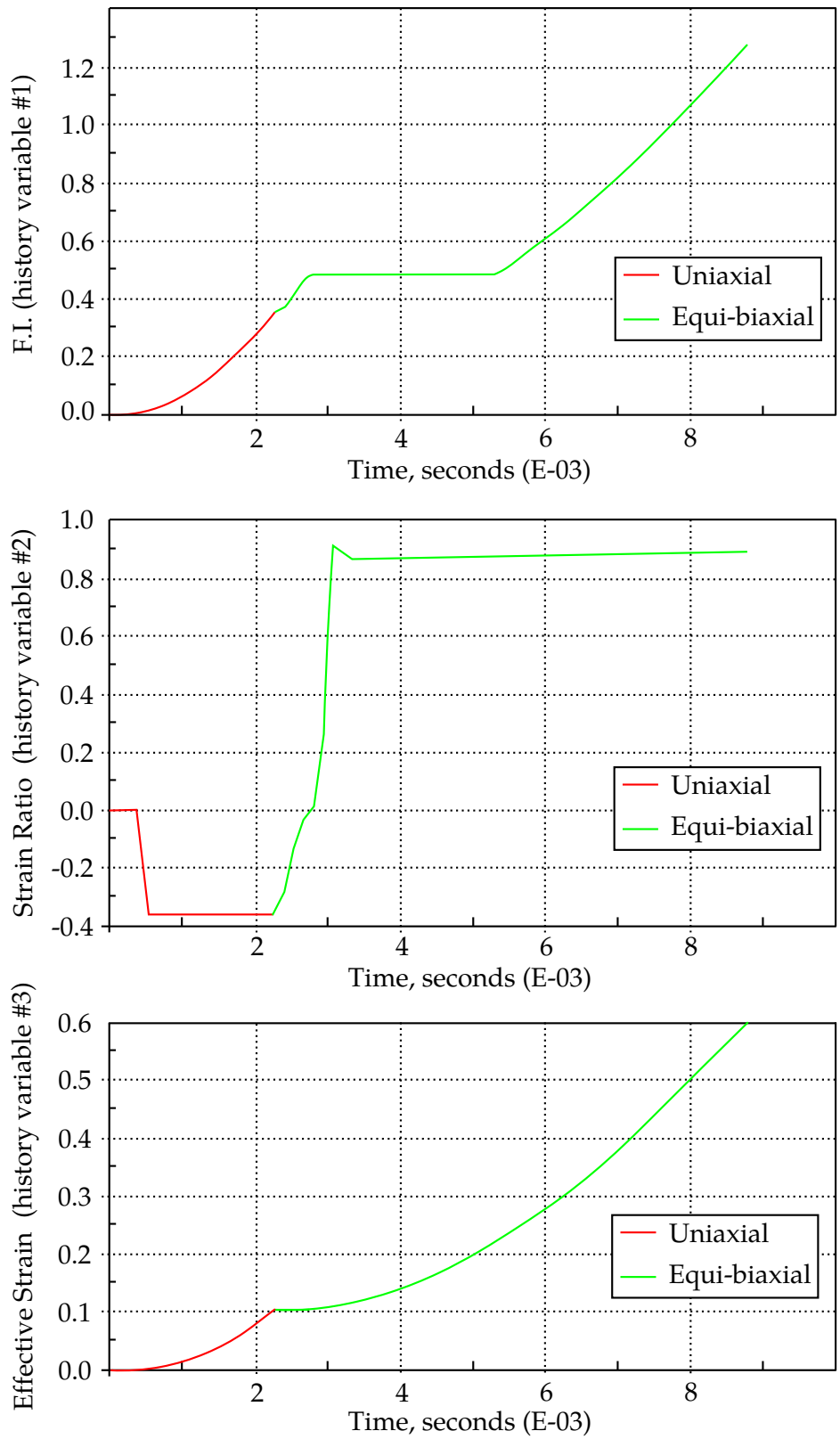


Figure M36-4. Time-history plots of the three history variables.

**\*MAT\_EXTENDED\_3-PARAMETER\_BARLAT**

This is Material Type 36E. This model is an extension to the standard 3-parameter Barlat model and allows for different hardening curves and R-values in different directions, see Fleischer et.al. [2007]. The directions in this context are the three uniaxial directions (0, 45 and 90 degrees) and optionally biaxial and shear.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A8	F	F	F				

Card 2	1	2	3	4	5	6	7	8
Variable	LCH00	LCH45	LCH90	LCHBI	LCHSH			
Type	F	F	F	F	F			

Card 3	1	2	3	4	5	6	7	8
Variable	LCR00	LCR45	LCR90	LCRBI	LCRSH	M		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							

Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus, $E$ .
PR	Poisson's ratio, $\nu$ .
LCHXX	Load curve defining uniaxial stress vs. uniaxial strain in the given direction ( $XX$ is either 00, 45, 90). The exact definition is discussed in the Remarks below. LCH00 must be defined, the other defaults to LCH00 if not defined.
LCHBI	Load curve defining biaxial stress vs. biaxial strain, see discussion in the Remarks below for a definition. If not defined this is determined from LCH00 and the initial R-values to yield a response close to the standard 3-parameter Barlat model.
LCHSH	Load curve defining shear stress vs. shear strain, see discussion in the Remarks below for a definition. If not defined this is ignored to yield a response close to the standard 3-parameter Barlat model.
LCRXX	Load curve defining standard R-value vs. uniaxial strain in the given direction ( $XX$ is either 00, 45, 90). The exact definition is discussed in the Remarks below. Default is a constant R-value of 1.0, a negative input will result in a constant R-value of $-LCRXX$ .
LCRBI	Load curve defining biaxial R-value vs. biaxial strain, see discussion in the Remarks below for a definition. Default is a constant R-value of 1.0, a negative input will result in a constant R-value of $-LCRBI$ .

VARIABLE	DESCRIPTION
LCRSH	Load curve defining shear R-value vs. shear strain, see discussion in the Remarks below for a definition. Default is a constant R-value of 1.0, a negative input will result in a constant R-value of $-LCRSH$ .
M	Barlat flow exponent, $m$ , must be an integer value.
AOPT	Material axes option (see <code>MAT_OPTION TROPIC_ELASTIC</code> for a more complete description): <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <code>*DEFINE_COORDINATE_NODES</code>, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with <code>*DEFINE_COORDINATE_VECTOR</code>.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on <code>*DEFINE_COORDINATE_NODES</code>, <code>*DEFINE_COORDINATE_SYSTEM</code> or <code>*DEFINE_COORDINATE_VECTOR</code>). Available with the R3 release of Version 971 and later.</p>
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see <code>*ELEMENT_SHELL_BETA</code> .

### Formulation:

The hardening curves LCH00, LCH45 and LCH90 are here defined as measured stress as function of measured plastic strain for uniaxial tension in the direction of interest, i.e., as determined from experimental testing using a standard procedure. The optional biaxial

and shear hardening curves LCHBI and LCHSH require some further elaboration, as we assume that a biaxial or shear test reveals that the true stress tensor in the material system expressed as

$$\sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \pm\sigma \end{pmatrix}, \quad \sigma \geq 0,$$

is a function of the (plastic) strain tensor

$$\varepsilon = \begin{pmatrix} \varepsilon_1 & 0 \\ 0 & \pm\varepsilon_2 \end{pmatrix}, \quad \varepsilon_1 \geq 0, \quad \varepsilon_2 \geq 0,$$

The input hardening curves are  $\sigma$  as function of  $\varepsilon_1 + \varepsilon_2$ . The  $\pm$  sign above distinguishes between the biaxial (+) and the shear (-) cases.

Moreover, the curves LCR00, LCR45 and LCR90 defining the R values are as function of the measured plastic strain for uniaxial tension in the direction of interest. The R-values in themselves are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width  $W$  and thickness  $T$  are measured as function of strain. Then the corresponding R-value is given by:

$$R_\varphi = \frac{\frac{dW}{d\varepsilon} / W}{\frac{dT}{d\varepsilon} / T}.$$

These curves are transformed internally to be used with the effective stress and strain properties in the actual model. The effective plastic strain does not coincide with the plastic strain components in other directions than the rolling direction and may be somewhat confusing to the user. Therefore the von Mises work equivalent plastic strain is output as history variable #2. As for hardening, the optional biaxial and shear R-value curves LCRBI and LCRSH are defined in a special way for which we return to the local plastic strain tensor  $\varepsilon$  as defined above. The biaxial and shear R-values are defined as

$$R_{b/s} = \frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2}$$

and again the curves are  $R_{b/s}$  as function of  $\varepsilon_1 + \varepsilon_2$ . Note here that the suffix  $b$  assumes loading biaxially and  $s$  assumes loading in shear, so the R-values to be defined are always positive.



**\*MAT\_TRANSVERSELY\_ANISOTROPIC\_ELASTIC\_PLASTIC\_{OPTION}**

This is Material Type 37. This model is for simulating sheet forming processes with anisotropic material. Only transverse anisotropy can be considered. Optionally an arbitrary dependency of stress and effective plastic strain can be defined with a load curve. This plasticity model is fully iterative and is available only for shell elements.

Available options include:

<BLANK>

ECHANGE

NLP\_FAILURE

NLP2

The ECHANGE option allows the change of Young’s Modulus during the simulation:

The NLP\_FAILURE option estimates failure using the Formability Index (F.I.) which accounts for the non-linear strain paths common in metal forming application (see [Remarks](#)). The option NLP is also available in \*MAT\_036, \*MAT\_125 and \*MAT\_226. A related keyword is \*CONTROL\_FORMING\_TOLERANC, which applies a smoothing algorithm to reduce output noise level of the strain ratio  $\beta$  (minor strain/major strain) in calculating the Formability Index (F.I.), which predicts sheet metal failure under nonlinear strain paths frequently encountered in metal forming application

The NLP\_FAULURE option uses effective plastic strain to calculate the onset of necking, which assumes the necking happens in an instant. Some researchers think it may happen in a longer duration, which can be addressed by the option NLP2, which calculates the damage during forming and accumulates it to predict the sheet metal failure. Compared with NLP\_FAILURE, there is no input change required.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ETAN	R	HLCID
Type	A	F	F	F	F	F	F	F

**\*MAT\_037****\*MAT\_TRANSVERSELY\_ANISOTROPIC\_ELASTIC\_PLASTIC**

Additional card for ECHANGE and/or NLP\_FAILURE keyword options.

Card 2.	1	2	3	4	5	6	7	8
Variable	IDSCALE	EA	COE	ICFLD		STRAINLT		
Type	I	F	F	F		F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
ETAN	Plastic hardening modulus. When this value is negative, normal stresses (either from contact or applied pressure) are considered and *LOAD_SURFACE_STRESS must be used to capture the stresses.

The negative local z-stresses caused by the contact pressure can be viewed from d3plot files after Revision 97158. This data can be viewed in LS-PrePost by selecting z-stress under *FCOMP* → *Stress* and select *local* under *FCOMP* in *LS-PrePost*). Prior to Revision 97158, the negative local z-stresses are stored in *history variable* #5, and can be viewed with menu options *FCOMP* → *Misc* → *history var* #5 in *LS-PrePost*. This feature is applicable to both shell element types 2 and 16. It is found in some cases this inclusion can improve forming simulation accuracy.

VARIABLE	DESCRIPTION
R	<p>Anisotropic parameter <math>\bar{r}</math>, also commonly called r-bar, in sheet metal forming literature. Its interpretation is given <a href="#">here</a>.</p> <p>GT.0: Standard formulation.</p> <p>LT.0: The anisotropic parameter is set to <math> R </math>. When <math>R</math> is set to a negative value the algorithm is modified for better stability in sheet thickness or thinning for sheet metal forming involving high strength steels or in cases when the simulation time is long. This feature is available to both element formulations 2 and 16. An example using this feature is provided in <a href="#">Remarks</a>, and shown in <a href="#">Figure M37-4</a>.</p>
HLCID	Load curve ID expressing effective yield stress as a function of effective plastic strain in uniaxial tension.
IDSCALE	Load curve ID expressing the scale factor for the Young's modulus as a function of effective strain. If the EA and COE fields are specified, this curve is unnecessary. This field is only used and should only be specified when the option ECHANGE is active.
EA, COE	Coefficients defining the Young's modulus with respect to the effective strain, EA is $E^A$ and COE is $\zeta$ . If IDSCALE is defined, these two parameters are not necessary. Input only when the option ECHANGE is used. Also see *MAT_125 for an example to obtain these two coefficients from a test curve.
ICFLD	ID of a load curve of the Forming Limit Diagram (FLD) under linear strain paths (see <a href="#">Remarks</a> ). In the load curve, abscissas represent minor strains while ordinates represent major strains. Define only when the option NLP_FAILURE is active.
STRAINLT	Critical strain value at which strain averaging is activated. Input only when the option NLP_FAILURE is active. See <a href="#">Remarks</a> .

### Formulation:

Consider Cartesian reference axes which are parallel to the three symmetry planes of anisotropic behavior. Then, the yield function suggested by Hill [1948] can be written as:

$$F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 - 1 = 0$$

where  $\sigma_{y1}$ ,  $\sigma_{y2}$ , and  $\sigma_{y3}$ , are the tensile yield stresses and  $\sigma_{y12}$ ,  $\sigma_{y23}$ , and  $\sigma_{y31}$  are the shear yield stresses. The constants  $F, G, H, L, M, N$  are related to the yield stress by:

$$2L = \frac{1}{\sigma_{y23}^2}$$

$$2M = \frac{1}{\sigma_{y31}^2}$$

$$2N = \frac{1}{\sigma_{y12}^2}$$

$$2F = \frac{1}{\sigma_{y2}^2} + \frac{1}{\sigma_{y3}^2} - \frac{1}{\sigma_{y1}^2}$$

$$2G = \frac{1}{\sigma_{y3}^2} + \frac{1}{\sigma_{y1}^2} - \frac{1}{\sigma_{y2}^2}$$

$$2H = \frac{1}{\sigma_{y1}^2} + \frac{1}{\sigma_{y2}^2} - \frac{1}{\sigma_{y3}^2} .$$

The isotropic case of von Mises plasticity can be recovered by setting:

$$F = G = H = \frac{1}{2\sigma_y^2}$$

and

$$L = M = N = \frac{3}{2\sigma_y^2}$$

For the particular case of transverse anisotropy, where properties do not vary in the  $x_1 - x_2$  plane, the following relations hold:

$$2F = 2G = \frac{1}{\sigma_{y3}^2}$$

$$2H = \frac{2}{\sigma_y^2} - \frac{1}{\sigma_{y3}^2}$$

$$N = \frac{2}{\sigma_y^2} - \frac{1}{2\sigma_{y3}^2}$$

where it has been assumed that  $\sigma_{y1} = \sigma_{y2} = \sigma_y$ .

Letting  $K = \frac{\sigma_y}{\sigma_{y3}}$ , the yield criteria can be written as:

$$F(\sigma) = \sigma_e = \sigma_y,$$

where,

$$F(\sigma) \equiv \left[ \sigma_{11}^2 + \sigma_{22}^2 + K^2 \sigma_{33}^2 - K^2 \sigma_{33} (\sigma_{11} + \sigma_{22}) - (2 - K^2) \sigma_{11} \sigma_{22} + 2L\sigma_y^2 (\sigma_{23}^2 + \sigma_{31}^2) + 2 \left( 2 - \frac{1}{2} K^2 \right) \sigma_{12}^2 \right]^{1/2}.$$

The rate of plastic strain is assumed to be normal to the yield surface so  $\dot{\epsilon}_{ij}^p$  is found from:

$$\dot{\epsilon}_{ij}^p = \lambda \frac{\partial F}{\partial \sigma_{ij}}.$$

Now consider the case of plane stress, where  $\sigma_{33} = 0$ . Also, define the anisotropy input parameter,  $R$ , as the ratio of the in-plane plastic strain rate to the out-of-plane plastic strain rate,

$$R = \frac{\dot{\epsilon}_{22}^p}{\dot{\epsilon}_{33}^p}.$$

It then follows that

$$R = \frac{2}{K^2} - 1.$$

Using the plane stress assumption and the definition of  $R$ , the yield function may now be written as:

$$F(\sigma) = \left[ \sigma_{11}^2 + \sigma_{22}^2 - \frac{2R}{R+1} \sigma_{11} \sigma_{22} + 2 \frac{2R+1}{R+1} \sigma_{12}^2 \right]^{1/2}.$$

### Discussion and ECHANGE:

It is noted that there are several differences between this model and other plasticity models for shell elements such as the model, MAT\_PIECEWISE\_LINEAR\_PLASTICITY. First, the yield function for plane stress does not include the transverse shear stress components which are updated elastically, and, secondly, this model is always fully iterative. Consequently, in comparing results for the isotropic case where  $R = 1.0$  with other isotropic model, differences in the results are expected, even though they are usually insignificant.

The Young's modulus has been assumed to be constant. However, some researchers have found that Young's modulus decreases with respect to the increase of effective strain. To accommodate this new observation, a new option of ECHANGE is added. There are two methods defining the change of Young's modulus change:

The first method is to use a curve to define the scale factor with respect to the effective strain. The value of this scale factor should decrease from 1.0 to 0.0 with the increase of effective strain.

The second method is to use a function as proposed by Yoshida [2003]:

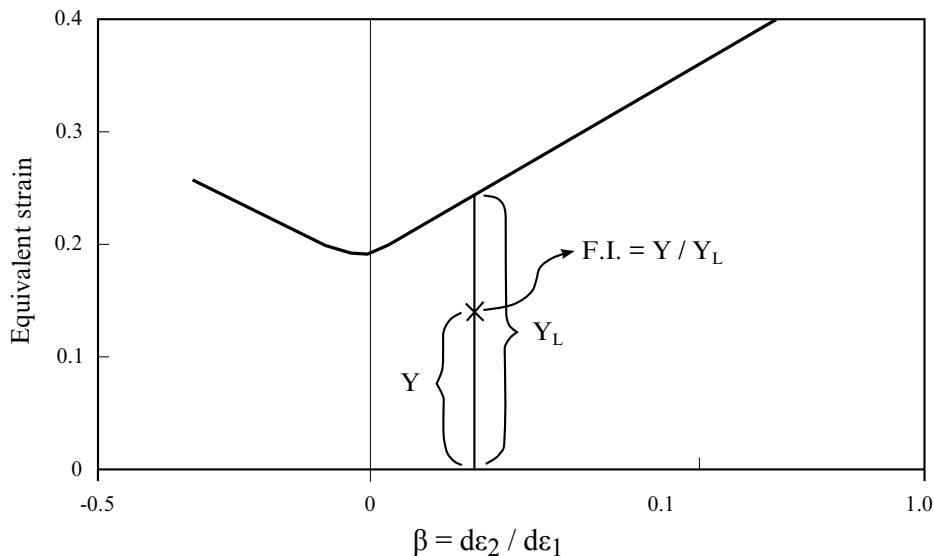
$$E = E^0 - (E^0 - E^A)[1 - \exp(-\zeta\bar{\epsilon})].$$

An example of the option ECHANGE is provided in the Remarks section of the \*MAT\_125 manual pages.

### A Failure Criterion for Nonlinear Strain Paths (NLP):

#### Background and Definition.

When the option **NLP\_FAILURE** is used, a necking failure criterion independent of strain path changes is activated. In sheet metal forming, as strain path history (plotted on in-plane major and minor strain space) of an element becomes non-linear, the position and shape of a traditional strain-based Forming Limit Diagram (FLD) changes. This option provides a simple formability index (F.I.) which remains invariant regardless of the presence of the non-linear strain paths in the model, and can be used to identify if the element has reached its necking limit.



**Figure M37-1.** Calculation of F.I. based on critical effective strain method.

Formability index (F.I) is calculated, as illustrated in [Figure M37-1](#), for every element in the sheet blank throughout the simulation duration. The value of F.I. is 0.0 for virgin material and reaches maximum of 1.0 when the material fails. The theoretical background can be found in two papers: 1) T.B. Stoughton, X. Zhu, "Review of Theoretical Models of the Strain-Based FLD and their Relevance to the Stress-Based FLD, *International Journal of Plasticity*", V. 20, Issues 8-9, P. 1463-1486, 2003; and 2) Danielle Zeng, Xinhai Zhu, Laurent B. Chappuis, Z.

*Cedric Xia, "A Path Independent Forming Limited Criterion for Sheet Metal Forming Simulations", 2008 SAE Proceedings, Detroit MI, April, 2008.*

### **Required Inputs.**

The load curve input for ICFLD follows keyword format in \*DEFINE\_CURVE, with abscissas as minor strains and ordinates as major strains.

ICFLD can also be specified using the \*DEFINE\_CURVE\_FLC keyword where the sheet metal thickness and strain hardening value are used. Detailed usage information can be found in the manual entry for \*DEFINE\_CURVE\_FLC.

The formability index is output as a history variable #1 in d3plot files. In addition to the F.I. values, starting in Revision 95599, the strain ratio  $\beta$  and effective plastic strain  $\bar{\epsilon}$  are written to the d3plot database as history variables #2 and #3, respectively provided NEIPS on the second field of the first card of \*DATABASE\_EXTENT\_BINARY is set to at least 3. The contour map of history variables can be plotted in LS-PrePost, accessible in *Post/FriComp*, under *Misc*, and by *Element*, under *Post/History*.

### **Post-Processing Information.**

When plotting the formability index contour map, first select the *history var #1* from *Misc* in the *FriComp* menu. The pull-down menu under *FriComp* can be used to select the minimum value *Min* for necking failure detection (refer to *Tharrett and Stoughton's paper in 2003 SAE 2003-01-1157*). In the *FriRang* dialog, select *None* on the pull-down menu next to *Avg*. Lastly, set the simulation result to the last state on the *animation tool bar*. The index value ranges from 0.0 to 1.5. The non-linear forming limit is reached at 1.0.

In addition, the evolution of the index throughout the simulation can be plotted in LS-PrePost4.0 under *Post/History* by *Element*. Select the last entry, which is *history var#1*, which may be hidden by a scroll bar. Furthermore, the strain path of an element can be plotted in *Post/FLD*, using the *Tracer* option, by selecting the corresponding integration point representing the *Min* index value in the *Position* pull-down menu.

Similarly contour maps and the evolution of the strain ratio and the effective plastic strain can be plotted in the same way using variables 2 and 3.

By setting the STRAINLT field strains (and strain ratios) can be averaged to reduce noise, which, in turn, affect the calculation of the formability index. The strain STRAINLT causes the formability index calculation to use only time averaged strains. Reasonable STRAINLT values range from  $5 \times 10^{-3}$  to  $10^{-2}$ .

It is suggested that variable MAXINT in \*DATABASE\_EXTENT\_BINARY be set to the same value of as the NIP field for the \*SECTION\_SHELL keyword.

**Input Example.**

An example of a partial keyword input using this non-linear strain path failure criterion is provided below:

```

*KEYWORD
...
*DATABASE_EXTENT_BINARY
$   NEIPH      NEIPS      MAXINT      STRFLG      SIGFLG      EPSFLG      RLTF LG      ENGFLG
      3         &nip         1
$   CMPFLG      IEVERP      BEAMIP      DCOMP      SHGE      STSSZ
      1         2
...
*MAT_TRANSVERSELY_ANISOTROPIC_ELASTIC_PLASTIC_NLP_FAILURE
$   MID      RO      E      PR      SIGY      ETAN      R      HLCID
      1 7.830E-09 2.070E+05      0.28      0.0      0.0      0.864      200
$   IDY      EA      COE      ICFLD      STRAINLT
      891      1.0E-02

*DEFINE_CURVE
891
$ minor, major strains for FLD definition
-3.375000e-01      4.965000e-01
-2.750000e-01      4.340000e-01
-2.250000e-01      3.840000e-01
-1.840909e-01      3.430909e-01
-1.500000e-01      3.090000e-01
-1.211539e-01      2.801539e-01
-9.642858e-02      2.554286e-01
-7.500000e-02      2.340000e-01
-5.625001e-02      2.152500e-01
-3.970589e-02      1.987059e-01
-2.500000e-02      1.840000e-01

$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$ load curve 200: Mat_037 property, DP600 NUMISHEET'05 Xnbr, Power law fit
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
*DEFINE_CURVE
200
0.000,395.000
0.001,425.200
0.003,440.300
0.004,452.000
0.005,462.400
0.006,472.100
    
```

As shown in [Figure M37-2](#), F.I contours can be plotted using *FriComp/Misc*, in LS-PrePost4.0. Strain paths of individual elements, or elements in an area can be plotted ([Figure M37-3](#) left) using the “*Tracer*” feature in the *FLD* menu. Finally, time history plot of the formability index for elements selected can be plotted in *History* menu, [Figure M37-3](#) right.

**Smoothing of the Strain Ratio  $\beta$ :**

Details related to the smoothing of  $\beta$  can be found in manual pages under *\*CONTROL\_FORMING\_TOLERANC*.



**Thickness/Thinning Stabilization for Shell Types 2 and 16:**

When the  $R$  value is set to a negative value, it stabilizes the sheet thickness or thinning in sheet metal forming for some high strength types of steel or in cases where the simulation time is long. In [Figure M37-4](#), a comparison of thinning contours is shown on a U-channel forming (one-half model) using negative and positive  $R$  values. Maximum thinning on the draw wall is slight higher in the negative  $R$  case than that in the positive  $R$  case.

**Revision Information:**

- 1)Revision 60925: the NLP\_FAILURE option is implemented in explicit dynamic and is available.
- 2)Revision 72219: the maximum F.I. value is change from 1.0 to 1.5.
- 3)Revision 73241: the NLP\_FAILURE option is also available for implicit static calculation.
- 4)Revision 95599: history variables #2 and #3 output is available.
- 5)Local z-stress output in d3plot files (when used together with \*LOAD\_SURFACE\_STRESS) is available starting in Revision 97158.
- 6)Negative  $E$  option (contact pressure/normal stress) activated in formability index starts in Revision 97296.
- 7)Numerical material model type with the NLP\_FAILURE option (\*MAT\_037\_NLP\_FAILURE) is available starting in Revision 106898.
- 8)Revision 110928: smoothing of  $\beta$  is available.
- 9)Revision 111547: option NLP2.

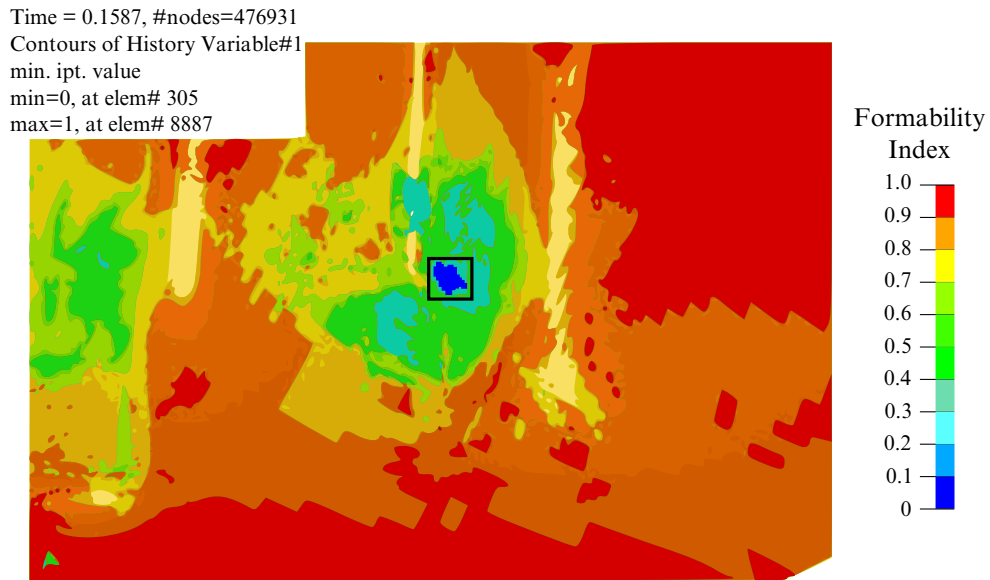
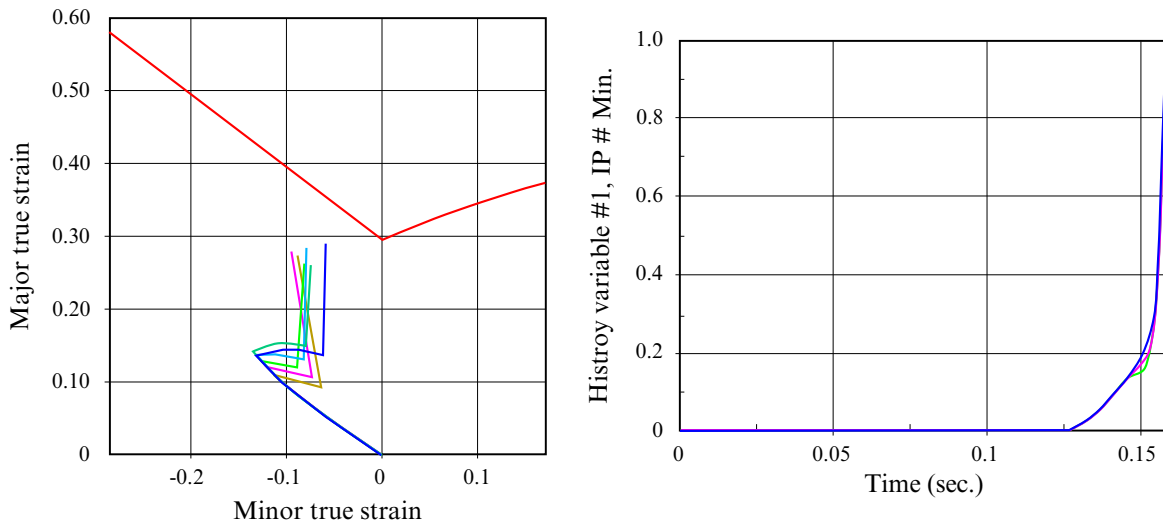


Figure M37-2. F.I. contour plot (min IP value, non-averaged).



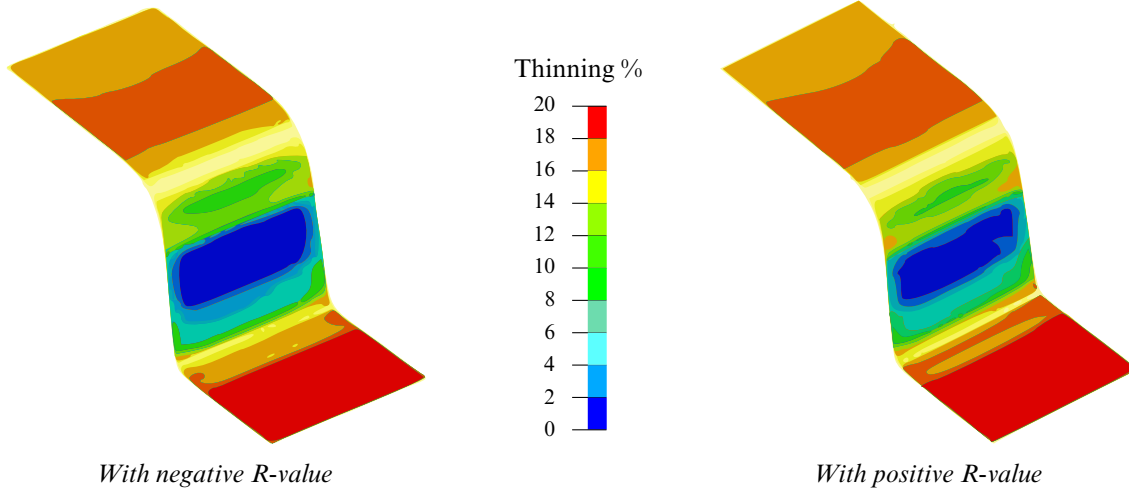
Nonlinear strain paths of a few elements in the box

F.I. time history plots of the elements

Figure M37-3. Strain paths and F.I. history plot for elements in the black square box of Figure M37-2.

Time=0.010271, #nodes=4594, #elem=4349  
Contours of % Thickness Reduction based on current z-strain  
min=0.0093799, at elem#42249  
max=22.1816, at elem#39875

Time=0.010271, #nodes=4594, #elem=4349  
Contours of % Thickness Reduction based on current z-strain  
min=0.0597092, at elem#39814  
max=21.2252, at elem#40457



**Figure M37-4.** Thinning contour comparison.

**\*MAT\_BLATZ-KO\_FOAM**

This is Material Type 38. This model is for the definition of rubber like foams of polyurethane. It is a simple one-parameter model with a fixed Poisson's ratio of .25.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G	REF				
Type	A8	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G	Shear modulus.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.

**Remarks:**

The strain energy functional for the compressible foam model is given by

$$W = \frac{G}{2} \left( \frac{\text{II}}{\text{III}} + 2\sqrt{\text{III}} - 5 \right)$$

Blatz and Ko [1962] suggested this form for a 47 percent volume polyurethane foam rubber with a Poisson's ratio of 0.25. In terms of the strain invariants, I, II, and III, the second Piola-Kirchhoff stresses are given as

$$S^{ij} = G \left[ (I\delta_{ij} - C_{ij}) \frac{1}{\text{III}} + \left( \sqrt{\text{III}} - \frac{\text{II}}{\text{III}} \right) C_{ij}^{-1} \right]$$

where  $C_{ij}$  is the right Cauchy-Green strain tensor. This stress measure is transformed to the Cauchy stress,  $\sigma_{ij}$ , according to the relationship

$$\sigma^{ij} = \text{III}^{-1/2} F_{ik} F_{jl} S_{lk}$$

where  $F_{ij}$  is the deformation gradient tensor.

**\*MAT\_FLD\_TRANSVERSELY\_ANISOTROPIC**

This is Material Type 39. This model is for simulating sheet forming processes with anisotropic material. Only transverse anisotropy can be considered. Optionally, an arbitrary dependency of stress and effective plastic strain can be defined via a load curve. A Forming Limit Diagram (FLD) can be defined using a curve and is used to compute the maximum strain ratio which can be plotted in LS-PrePost. This plasticity model is fully iterative and is available only for shell elements. Also see the notes below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ETAN	R	HLCID
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	LCFLD							
Type	F							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
ETAN	Plastic hardening modulus, see notes for model 37.
R	Anisotropic hardening parameter, see notes for model 37.
HLCID	Load curve ID defining effective stress versus effective plastic strain. The yield stress and hardening modulus are ignored with this option.

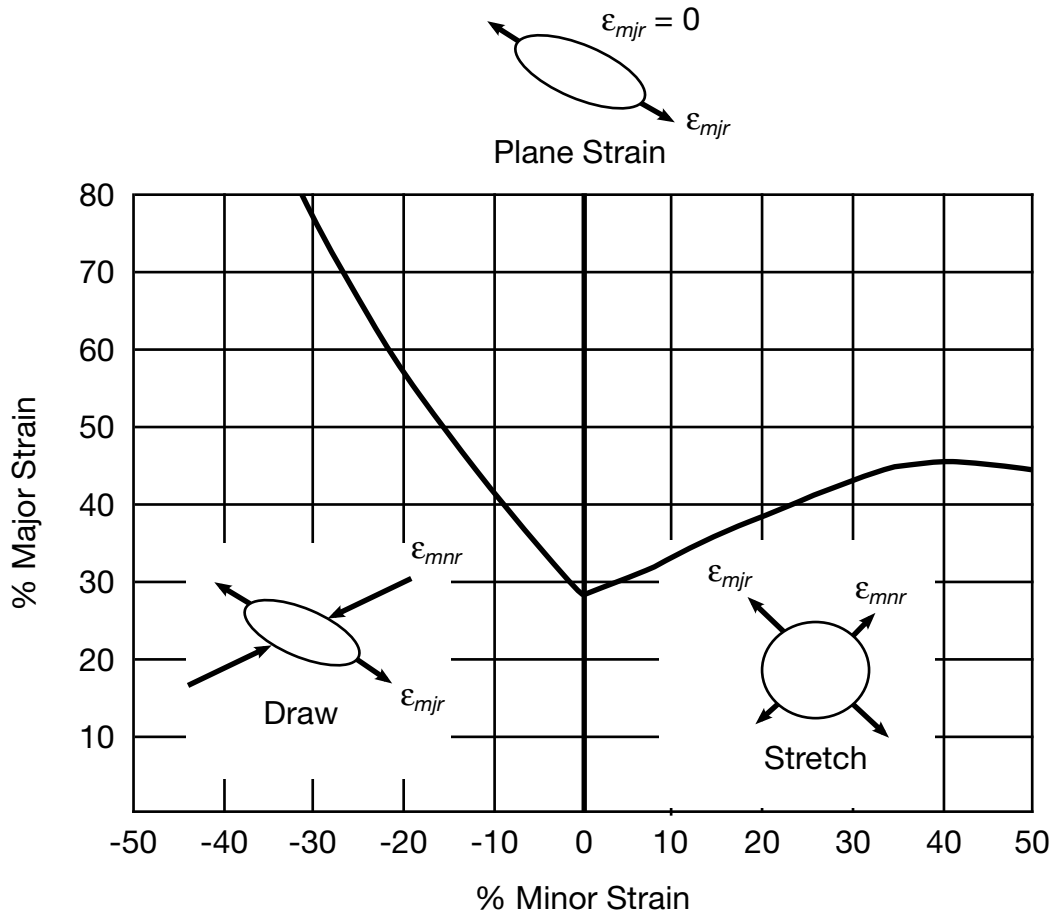


Figure M39-1. Forming limit diagram.

VARIABLE	DESCRIPTION
LCFLD	Load curve ID defining the Forming Limit Diagram. Minor strains in percent are defined as abscissa values and Major strains in percent are defined as ordinate values. The forming limit diagram is shown in Figure M39-1. In defining the curve list pairs of minor and major strains starting with the left most point and ending with the right most point, see *DEFINE_CURVE.

**Remarks:**

See material model 37 for the theoretical basis. The first history variable is the maximum strain ratio:

$$\frac{\epsilon_{\text{major\_workpiece}}}{\epsilon_{\text{major\_fld}}}$$

corresponding to  $\epsilon_{\text{minor\_workpiece}}$ .

**\*MAT\_NONLINEAR\_ORTHOTROPIC**

This is Material Type 40. This model allows the definition of an orthotropic nonlinear elastic material based on a finite strain formulation with the initial geometry as the reference. Failure is optional with two failure criteria available. Optionally, stiffness proportional damping can be defined. In the stress initialization phase, temperatures can be varied to impose the initial stresses. This model is only available for shell and solid elements.

**WARNING:** We do not recommend using this model at this time since it can be unstable especially if the stress-strain curves increase in stiffness with increasing strain.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	EA	EB	EC	PRBA	PRCA	PRCB
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	DT	TRAMP	ALPHA		
Type	F	F	F	F	F	F		
Default	none	none	none	0	0	0		



Card 3	1	2	3	4	5	6	7	8
Variable	LCIDA	LCIDB	EFAIL	DFAIL	CDAMP	AOPT	MACF	
Type	F	F	F	F	F	F	I	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0	

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

Optional Card 6 (Applies to Solid elements only)

Card 6	1	2	3	4	5	6	7	8
Variable	LCIDC	LCIDAB	LCIDBC	LCIDCA				
Type	F	F	F	F				
Default	optional	optional	optional	optional				

**VARIABLE**

**DESCRIPTION**

MID                      Material identification. A unique number or label not exceeding 8 characters must be specified.

RO                        Mass density.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EA	$E_a$ , Young's modulus in $a$ -direction.
EB	$E_b$ , Young's modulus in $b$ -direction.
EC	$E_c$ , Young's modulus in $c$ -direction.
PRBA	$\nu_{ba}$ , Poisson's ratio $ba$ .
PRCA	$\nu_{ba}$ , Poisson's ratio $ca$ .
PRCB	$\nu_{cb}$ , Poisson's ratio $cb$ .
GAB	$G_{ab}$ , shear modulus $ab$ .
GBC	$G_{bc}$ , shear modulus $bc$ .
GCA	$G_{ca}$ , shear modulus $ca$ .
DT	Temperature increment for isotropic stress initialization. This option can be used during dynamic relaxation.
TRAMP	Time to ramp up to the final temperature.
ALPHA	Thermal expansion coefficient.
LCIDA	Optional load curve ID defining the nominal stress versus strain along $a$ -axis. Strain is defined as $\lambda_a - 1$ where $\lambda_a$ is the stretch ratio along the $a$ -axis.
LCIDB	Optional load curve ID defining the nominal stress versus strain along $b$ -axis. Strain is defined as $\lambda_b - 1$ where $\lambda_b$ is the stretch ratio along the $b$ -axis.
EFAIL	Failure strain, $\lambda - 1$ .
DFAIL	Time step for automatic element erosion
CDAMP	Damping coefficient.
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):  EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA.

VARIABLE	DESCRIPTION
	EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <i>a</i> -direction. This option is for solid elements only.
	EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <i>v</i> with the element normal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.
	EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <i>v</i> , and an originating point, <i>p</i> , which define the centerline axis. This option is for solid elements only.
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
MACF	Material axes change flag: EQ.1: No change, default, EQ.2: switch material axes <i>a</i> and <i>b</i> , EQ.3: switch material axes <i>a</i> and <i>c</i> , EQ.4: switch material axes <i>b</i> and <i>c</i> .
XP, YP, ZP	Define coordinates of point <i>p</i> for AOPT = 1 and 4.
A1, A2, A3	( <i>a</i> <sub>1</sub> , <i>a</i> <sub>2</sub> , <i>a</i> <sub>3</sub> ) define components of vector <i>a</i> for AOPT = 2.
D1, D2, D3	( <i>d</i> <sub>1</sub> , <i>d</i> <sub>2</sub> , <i>d</i> <sub>3</sub> ) define components of vector <i>d</i> for AOPT = 2.
V1, V2, V3	( <i>v</i> <sub>1</sub> , <i>v</i> <sub>2</sub> , <i>v</i> <sub>3</sub> ) define components of vector <i>v</i> for AOPT = 3 and 4.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overridden on the element card, see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO..

The following input is optional and applies to **SOLID ELEMENTS** only

LCIDC	Load curve ID defining the nominal stress versus strain along <i>c</i> -axis. Strain is defined as $\lambda_c - 1$ where $\lambda_c$ is the stretch ratio along the <i>c</i> -axis.
LCIDAB	Load curve ID defining the nominal <i>ab</i> shear stress versus <i>ab</i> -strain in the <i>ab</i> -plane. Strain is defined as the $\sin(\gamma_{ab})$ where $\gamma_{ab}$ is the shear angle.
LCIDBC	Load curve ID defining the nominal <i>ab</i> shear stress versus <i>ab</i> -strain in the <i>bc</i> -plane. Strain is defined as the $\sin(\gamma_{bc})$ where $\gamma_{bc}$ is the shear angle.
LCIDCA	Load curve ID defining the nominal <i>ab</i> shear stress versus <i>ab</i> -strain in the <i>ca</i> -plane. Strain is defined as the $\sin(\gamma_{ca})$ where $\gamma_{ca}$ is the shear angle.



Define LMC material parameters using 8 parameters per card.

Card	1	2	3	4	5	6	7	8
Variable	P1	P2	P3	P4	P5	P6	P7	P8
Type	F	F	F	F	F	F	F	F

Define LMCA material parameters using 8 parameters per card.

Card	1	2	3	4	5	6	7	8
Variable	P1	P2	P3	P4	P5	P6	P7	P8
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
MT	User material type (41 - 50 inclusive). A number between 41 and 50 has to be chosen. If $MT < 0$ , subroutine <code>rwumat</code> in <code>dyn21.f</code> is called, where the material parameter reading can be modified.

**WARNING:** If two or more materials in an input deck share the same MT value, those materials also share values of other variables on Cards 1 and 2 excluding MID and RO. Those shared values are taken from the first material where the common MT is encountered.

LMC	Length of material constant array which is equal to the number of material constants to be input. (see <a href="#">Remark 2</a> )
NHV	Number of history variables to be stored, see Appendix A. When the model is to be used with an equation of state, NHV must be increased by 4 to allocate the storage required by the equation of state.

VARIABLE	DESCRIPTION
IORTHO/ ISPOT	EQ.1: if the material is orthotropic. EQ.2: if material is used with spot weld thinning. EQ.3: if material is orthotropic and used with spot weld thinning
IBULK	Address of bulk modulus in material constants array, see Appendix A.
IG	Address of shear modulus in material constants array, see Appendix A.
IVECT	Vectorization flag (on = 1). A vectorized user subroutine must be supplied.
IFAIL	Failure flag. EQ.0: No failure, EQ.1: Allows failure of shell and solid elements, LT.0:  IFAIL  is the address of NUMINT in the material constants array. NUMINT is defined as the number of failed integration points that will trigger element deletion. This option applies only to shell and solid elements (release 5 of version 971).
ITHERM	Temperature flag (on = 1). Compute element temperature.
IHYPHER	Deformation gradient flag (on = 1 or -1, or 3). Compute deformation gradient, see Appendix A.
IEOS	Equation of state (on = 1).
LMCA	Length of additional material constant array.
AOPT	Material axes option (see *MAT_002 for a more complete description): EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA. EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <i>a</i> -direction. This option is for solid elements only.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector $\mathbf{v}$ with the element normal.
	EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector $\mathbf{v}$ , and an originating point, $\mathbf{p}$ , which define the centerline axis. This option is for solid elements only.
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
MACF	Material axes change flag for brick elements for quick changes: EQ.1: No change, default, EQ.2: switch material axes $a$ and $b$ , EQ.3: switch material axes $a$ and $c$ , EQ.4: switch material axes $b$ and $c$ .
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3 and 4.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overridden on the element card, see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO.
IEVTS	Address of $E_a$ for orthotropic material in thick shell formulation 5 (see <a href="#">Remark 4</a> ).
P1	First material parameter.



VARIABLE	DESCRIPTION
P2	Second material parameter.
P3	Third material parameter.
P4	Fourth material parameter.
⋮	⋮
PLMC	LMCth material parameter.

**Remarks:**

1. **Cohesive Elements.** Material models for the cohesive element (solid element type 19) uses the first two *material parameters* to set flags in the *element formulation*.
  - a) **P1.** The P1 field controls how the density is used to calculate the mass when determining the tractions at mid-surface (tractions are calculated on a surface midway between the surfaces defined by nodes 1-2-3-4 and 5-6-7-8). If P1 is set to 1.0, then the density is per unit area of the midsurface instead of per unit volume. Note that the cohesive element formulation permits the element to have zero or negative volume.
  - b) **P2.** The second parameter, P2, specifies the number of integration points (one to four) that are required to fail for the element to fail. If it is zero, the element will not fail regardless of IFAIL. The recommended value for P2 is 1.
  - c) **Other Parameters.** The cohesive element only uses MID, RO, MT, LMC, NHV, IFAIL and IVECT in addition to the material parameters.
  - d) **Appendix R.** See Appendix R for the specifics of the umat subroutine requirements for the cohesive element.
2. **Material Constants.** If IORTHO = 0, LMC must be  $\leq 48$ . If IORTHO = 1, LMC must be  $\leq 40$ . If more material constants are needed, LMCA may be used to create an additional material constant array. There is no limit on the size of LMCA.
3. **Spot weld thinning.** If the user-defined material is used for beam or brick element spot welds that are tied to shell elements, and SPOTHIN > 0 on \*CONTROL\_CONTACT, then spot weld thinning will be done for those shells if ISPOT = 2. Otherwise, it will not be done.
4. **Thick Shell Formulation 5.** IEVTS is optional and is used only by thick shell formulation 5. It points to the position of  $E_a$  in the material constants array. Fol-

Following  $E_a$ , the next 5 material constants must be  $E_b$ ,  $E_c$ ,  $\nu_{ba}$ ,  $\nu_{ca}$ , and  $\nu_{cb}$ . This data enables thick shell formulation 5 to calculate an accurate thickness strain, otherwise the thickness strain will be based on the elastic constants pointed to by IBULK and IG.

\*MAT\_BAMMAN

This is Material Type 51. It allows the modeling of temperature and rate dependent plasticity with a fairly complex model that has many input parameters [Bamman 1989].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	T	HC		
Type	A8	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C9	C10	C11	C12	C13	C14	C15	C16
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	C17	C18	A1	A2	A4	A5	A6	KAPPA
Type	F	F	F	F	F	F	F	F

**VARIABLE**

**DESCRIPTION**

- MID Material identification. A unique number or label not exceeding 8 characters must be specified.
- RO Mass density.
- E Young's modulus (psi)

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
PR	Poisson's ratio
T	Initial temperature (°R, degrees Rankine)
HC	Heat generation coefficient (°R/psi)
C1	Psi
C2	°R
C3	Psi
C4	°R
C5	1/s
C6	°R
C7	1/psi
C8	°R
C9	Psi
C10	°R
C11	1/psi-s
C12	°R
C13	1/psi
C14	°R
C15	psi
C16	°R
C17	1/psi-s
C18	°R
A1	$\alpha_1$ , initial value of internal state variable 1
A2	$\alpha_2$ , initial value of internal state variable 2. Note: $\alpha_3 = -(\alpha_1 + \alpha_2)$

VARIABLE	DESCRIPTION
A3	$\alpha_4$ , initial value of internal state variable 3
A4	$\alpha_5$ , initial value of internal state variable 4
A5	$\alpha_6$ , initial value of internal state variable 5
KAPPA	$\kappa$ , initial value of internal state variable 6

Unit Conversion Table

	Sec $\times$ psi $\times$ $^{\circ}$ R	sec $\times$ MPa $\times$ $^{\circ}$ R	sec $\times$ MPA $\times$ $^{\circ}$ K
C1		$\times 1/145$	$\times 1/145$
C2		—	$\times 5/9$
C3		$\times 1/145$	$\times 1/145$
C4		—	$\times 5/9$
C5		—	—
C6		—	$\times 5/9$
C7		$\times 145$	$\times 145$
C8		—	$\times 5/9$
C9		$\times 1/145$	$\times 1/145$
C10		—	$\times 5/9$
C11		$\times 145$	$\times 145$
C12		—	$\times 5/9$
C13		$\times 145$	$\times 145$
C14		—	$\times 5/9$
C15		$\times 1/145$	$\times 1/145$
C16		—	$\times 5/9$
C17		$\times 145$	$\times 145$
C18		—	$\times 5/9$
C0 = HC		$\times 145$	$\times (145)^{(5/9)}$
E		$\times 1/145$	$\times 1/145$
$\nu$		—	—
T		—	$\times 5/9$

**Remarks:**

The kinematics associated with the model are discussed in references [Hill 1948, Bammann and Aifantis 1987, Bammann 1989]. The description below is taken nearly verbatim from Bammann [1989].

With the assumption of linear elasticity we can write,

$$\overset{\circ}{\sigma} = \lambda \operatorname{tr}(D^e) \mathbf{1} + 2\mu D^e$$

where the Cauchy stress  $\sigma$  is convected with the elastic spin  $W^e$  as,

$$\overset{\circ}{\sigma} = \dot{\sigma} - W^e \sigma + \sigma W^e$$

This is equivalent to writing the constitutive model with respect to a set of directors whose direction is defined by the plastic deformation [Bammann and Aifantis 1987, Bammann and Johnson 1987]. Decomposing both the skew symmetric and symmetric parts of the velocity gradient into elastic and plastic parts we write for the elastic stretching  $D^e$  and the elastic spin  $W^e$ ,

$$D^e = D - D^p - D^{th}, W^e = W = W^p.$$

Within this structure it is now necessary to prescribe an equation for the plastic spin  $W^p$  in addition to the normally prescribed flow rule for  $D^p$  and the stretching due to the thermal expansion  $D^{th}$ . As proposed, we assume a flow rule of the form,

$$D^p = f(T) \sinh \left[ \frac{|\xi| - \kappa - Y(T)}{V(T)} \right] \frac{\xi'}{|\xi'|}$$

where  $T$  is the temperature,  $\kappa$  is the scalar hardening variable, and  $\xi'$  is the difference between the deviatoric Cauchy stress  $\sigma'$  and the tensor variable  $\alpha'$ ,

$$\xi' = \sigma' - \alpha'$$

and  $f(T)$ ,  $Y(T)$ ,  $V(T)$  are scalar functions whose specific dependence upon the temperature is given below. Assuming isotropic thermal expansion and introducing the expansion coefficient  $\dot{A}$ , the thermal stretching can be written,

$$D^{th} = \dot{A} \mathbf{1}$$

The evolution of the internal variables  $\alpha$  and  $\kappa$  are prescribed in a hardening minus recovery format as,

$$\dot{\alpha} = h(T) D^p - [r_d(T) |D^p| + r_s(T)] |\alpha| \alpha,$$

$$\dot{\kappa} = H(T) D^p - [R_d(T) |D^p| + R_s(T)] \kappa^2$$

where  $h$  and  $H$  are the hardening moduli,  $r_s(T)$  and  $R_s(T)$  are scalar functions describing the diffusion controlled 'static' or 'thermal' recovery, and  $r_d(T)$  and  $R_d(T)$  are the functions describing dynamic recovery.

If we assume that  $W^p = 0$ , we recover the Jaumann stress rate which results in the prediction of an oscillatory shear stress response in simple shear when coupled with a Prager kinematic hardening assumption [Johnson and Bammann 1984]. Alternatively we can choose,

$$W^p = R^T \dot{U} U^{-1} R,$$

which recovers the Green-Naghdi rate of Cauchy stress and has been shown to be equivalent to Mandel's isoclinic state [Bammann and Aifantis 1987]. The model employing this rate allows a reasonable prediction of directional softening for some materials, but in

general under-predicts the softening and does not accurately predict the axial stresses which occur in the torsion of the thin walled tube.

The final equation necessary to complete our description of high strain rate deformation is one which allows us to compute the temperature change during the deformation. In the absence of a coupled thermo-mechanical finite element code we assume adiabatic temperature change and follow the empirical assumption that 90 -95% of the plastic work is dissipated as heat. Hence,

$$\dot{T} = \frac{.9}{\rho C_v} (\sigma \cdot D^p),$$

where  $\rho$  is the density of the material and  $C_v$  the specific heat.

In terms of the input parameters the functions defined above become:

$V(T) = C1 \exp(-C2/T)$	$h(T) = C9 \exp(C10/T)$
$Y(T) = C3 \exp(C4/T)$	$rs(T) = C11 \exp(-C12/T)$
$f(T) = C5 \exp(-C6/T)$	$RD(T) = C13 \exp(-C14/T)$
$rd(T) = C7 \exp(-C8/T)$	$H(T) = C15 \exp(C16/T)$
	$RS(T) = C17 \exp(-C18/T)$

and the heat generation coefficient is

$$HC = \frac{0.9}{\rho C_v}.$$

**\*MAT\_BAMMAN\_DAMAGE**

This is Material Type 52. This is an extension of model 51 which includes the modeling of damage. See Bamman et al. [1990].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	T	HC		
Type	A8	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C9	C10	C11	C12	C13	C14	C15	C16
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	C17	C18	A1	A2	A3	A4	A5	A6
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	N	D0	FS					
Type	F	F	F					



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<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus (psi)
PR	Poisson's ratio
T	Initial temperature (°R, degrees Rankine)
HC	Heat generation coefficient ( $^{\circ}\text{R}_{\text{psi}}$ )
C1	Psi
C2	°R
C3	Psi
C4	°R
C5	1/s
C6	°R
C7	1/psi
C8	°R
C9	Psi
C10	°R
C11	1/psi-s
C12	°R
C13	1/psi
C14	°R
C15	psi
C16	°R
C17	1/psi-s

<b>VARIABLE</b>	<b>DESCRIPTION</b>
C18	°R
A1	$\alpha_1$ , initial value of internal state variable 1
A2	$\alpha_2$ , initial value of internal state variable 2
A3	$\alpha_3$ , initial value of internal state variable 3
A4	$\alpha_4$ , initial value of internal state variable 4
A5	$\alpha_5$ , initial value of internal state variable 5
A6	$\alpha_6$ , initial value of internal state variable 6
N	Exponent in damage evolution
D0	Initial damage (porosity)
FS	Failure strain for erosion.

**Remarks:**

The evolution of the damage parameter,  $\phi$  is defined by Bammann et al. [1990]

$$\dot{\phi} = \beta \left[ \frac{1}{(1 - \phi)^N} - (1 - \phi) \right]^{|D^p|}$$

in which

$$\beta = \sinh \left[ \frac{2(2N - 1)p}{(2N - 1)\bar{\sigma}} \right]$$

where  $p$  is the pressure and  $\bar{\sigma}$  is the effective stress.

**\*MAT\_CLOSED\_CELL\_FOAM**

This is Material Type 53. This allows the modeling of low density, closed cell polyurethane foam. It is for simulating impact limiters in automotive applications. The effect of the confined air pressure is included with the air being treated as an ideal gas. The general behavior is isotropic with uncoupled components of the stress tensor.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	A	B	C	P0	PHI
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	GAMA0	LCID						
Type	F	I						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
A	a, factor for yield stress definition, see notes below.
B	b, factor for yield stress definition, see notes below.
C	c, factor for yield stress definition, see notes below.
P0	Initial foam pressure, $P_0$
PHI	Ratio of foam to polymer density, $\phi$
GAMA0	Initial volumetric strain, $\gamma_0$ . The default is zero.

VARIABLE	DESCRIPTION
LCID	Optional load curve defining the von Mises yield stress versus $-\gamma$ . If the load curve ID is given, the yield stress is taken from the curve and the constants a, b, and c are not needed. The load curve is defined in the positive quadrant, i.e., positive values of $\gamma$ are defined as negative values on the abscissa.

**Remarks:**

A rigid, low density, closed cell, polyurethane foam model developed at Sandia Laboratories [Neilsen, Morgan and Krieg 1987] has been recently implemented for modeling impact limiters in automotive applications. A number of such foams were tested at Sandia and reasonable fits to the experimental data were obtained.

In some respects this model is similar to the crushable honeycomb model type 26 in that the components of the stress tensor are uncoupled until full volumetric compaction is achieved. However, unlike the honeycomb model this material possesses no directionality but includes the effects of confined air pressure in its overall response characteristics.

$$\sigma_{ij} = \sigma_{ij}^{sk} - \delta_{ij}\sigma^{air}$$

where  $\sigma_{ij}^{sk}$  is the skeletal stress and  $\sigma^{air}$  is the air pressure computed from the equation:

$$\sigma^{air} = -\frac{p_0\gamma}{1 + \gamma - \phi}$$

where  $p_0$  is the initial foam pressure, usually taken as the atmospheric pressure, and  $\gamma$  defines the volumetric strain

$$\gamma = V - 1 + \gamma_0$$

where  $V$  is the relative volume, defined as the ratio of the current volume to the initial volume, and  $\gamma_0$  is the initial volumetric strain, which is typically zero. The yield condition is applied to the principal skeletal stresses, which are updated independently of the air pressure. We first obtain the skeletal stresses:

$$\sigma_{ij}^{sk} = \sigma_{ij} + \sigma_{ij}\sigma^{air}$$

and compute the trial stress,  $\sigma^{skt}$

$$\sigma_{ij}^{skt} = \sigma_{ij}^{sk} + E \dot{\epsilon}_{ij} \Delta t$$

where  $E$  is Young's modulus. Since Poisson's ratio is zero, the update of each stress component is uncoupled and  $2G = E$  where  $G$  is the shear modulus. The yield condition is applied to the principal skeletal stresses such that, if the magnitude of a principal trial stress component,  $\sigma_i^{skt}$ , exceeds the yield stress,  $\sigma_y$ , then

$$\sigma_i^{\text{sk}} = \min(\sigma_y, |\sigma_i^{\text{skt}}|) \frac{\sigma_i^{\text{skt}}}{|\sigma_i^{\text{skt}}|}$$

The yield stress is defined by

$$\sigma_y = a + b(1 + c\gamma)$$

where a, b, and c are user defined input constants and  $\gamma$  is the volumetric strain as defined above. After scaling the principal stresses they are transformed back into the global system and the final stress state is computed

$$\sigma_{ij} = \sigma_{ij}^{\text{sk}} - \delta_{ij}\sigma^{\text{air}}.$$

**\*MAT\_ENHANCED\_COMPOSITE\_DAMAGE**

These are Material Types 54 - 55 which are enhanced versions of the composite model material type 22. Arbitrary orthotropic materials, e.g., unidirectional layers in composite shell structures can be defined. Optionally, various types of failure can be specified following either the suggestions of [Chang and Chang 1987b] or [Tsai and Wu 1971]. In addition special measures are taken for failure under compression. See [Matzenmiller and Schweizerhof 1991].

By using the user defined integration rule, see \*INTEGRATION\_SHELL, the constitutive constants can vary through the shell thickness. For all shells, except the DKT formulation, laminated shell theory can be activated to properly model the transverse shear deformation. Lamination theory is applied to correct for the assumption of a uniform constant shear strain through the thickness of the shell.

For sandwich shells where the outer layers are much stiffer than the inner layers, the response will tend to be too stiff unless lamination theory is used. To turn on lamination theory see \*CONTROL\_SHELL. A damage model for transverse shear strain to model interlaminar shear failure is available. The definition of minimum stress limits is available for thin/thick shells and solids.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	(KF)	AOPT	2WAY	TI	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable				A1	A2	A3	MANGLE	
Type				F	F	F	F	

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	DFAILM	DFAILS
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	TFAIL	ALPH	SOFT	FBRT	YCFAC	DFAILT	DFAILC	EFS
Type	F	F	F	F	F	F	F	F

Card 6	1	2	3	4	5	6	7	8
Variable	XC	XT	YC	YT	SC	CRIT	BETA	
Type	F	F	F	F	F	F	F	

**Optional Card 7 (only for CRIT = 54)**

Card 7	1	2	3	4	5	6	7	8
Variable	PFL	EPSF	EPSR	TSMD	SOFT2			
Type	F	F	F	F	F			

**Optional Card 8 (only for CRIT = 54)**

Card 8	1	2	3	4	5	6	7	8
Variable	SLIMT1	SLIMC1	SLIMT2	SLIMC2	SLIMS	NCYRED	SOFTG	
Type	F	F	F	F	F	F	F	

**Optional Card 9 (only for CRIT = 54)**

Card 9	1	2	3	4	5	6	7	8
Variable	LCXC	LCXT	LCYC	LCYT	LCSC	DT		
Type	I	I	I	I	I	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
EA	$E_a$ , Young's modulus - longitudinal direction
EB	$E_b$ , Young's modulus - transverse direction
EC	$E_c$ , Young's modulus - normal direction
PRBA	$\nu_{ba}$ , Poisson's ratio $ba$
PRCA	$\nu_{ca}$ , Poisson's ratio $ca$
PRCB	$\nu_{cb}$ , Poisson's ratio $cb$
GAB	$G_{ab}$ , shear modulus $ab$
GBC	$G_{bc}$ , shear modulus $bc$
GCA	$G_{ca}$ , shear modulus $ca$
(KF)	Bulk modulus of failed material (not used)
AOPT	Material axes option (see MAT_{OPTION}/TROPIC_ELASTIC for a more complete description): <ul style="list-style-type: none"> <li>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle MANGLE.</li> <li>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</li> <li>EQ.3.0: locally orthotropic material axes determined by rotating</li> </ul>



VARIABLE	DESCRIPTION
	the material axes about the element normal by an angle (MANGLE) from a line in the plane of the element defined by the cross product of the vector $\mathbf{v}$ with the element normal. EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector $\mathbf{v}$ , and an originating point, $\mathbf{p}$ , which define the centerline axis. This option is for solid elements only. LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
2WAY	Flag to turn on 2-way fiber action. EQ.0.0: Standard unidirectional behavior. EQ.1.0: 2-way fiber behavior. The meaning of the fields DFAILT, DFAILC, YC, YT, SLIMT2 and SLIMC2 are altered if this flag is set. This option is only available for MAT 54 using thin shells.
TI	Flag to turn on transversal isotropic behavior for *MAT_054 solid elements. EQ.0.0: Standard unidirectional behavior. EQ.1.0: Transversal isotropic behavior (see additional remarks 1).
A1, A2, A3	Define components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Define components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Define components of vector $\mathbf{d}$ for AOPT = 2.
MANGLE	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. MANGLE may be overridden on the element card, see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO.
DFAILM	Maximum strain for matrix straining in tension or compression (active only for MAT_054 and only if DFAILT > 0). The layer in the element is completely removed after the maximum strain in the matrix direction is reached. The input value is always positive.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
DFAILS	Maximum tensorial shear strain (active only for MAT_054 and only if DFAILT > 0). The layer in the element is completely removed after the maximum shear strain is reached. The input value is always positive.
TFAIL	Time step size criteria for element deletion: <p><math>t_{fail} \leq 0</math>: no element deletion by time step size. The crashfront algorithm only works if <math>t_{fail}</math> is set to a value above zero.</p> <p><math>0 &lt; t_{fail} \leq 0.1</math>: element is deleted when its time step is smaller than the given value,</p> <p><math>t_{fail} &gt; 0.1</math>: element is deleted when the quotient of the actual time step and the original time step drops below the given value.</p>
ALPH	Shear stress parameter for the nonlinear term, see Material 22.
SOFT	Softening reduction factor for material strength in crashfront elements (default = 1.0). TFAIL must be greater than zero to activate this option. Crashfront elements are elements that are direct neighbors of failed (deleted) elements.
FBRT	Softening for fiber tensile strength: <p>EQ.0.0: tensile strength = XT</p> <p>GT.0.0: tensile strength = XT, reduced to <math>XT \times FBRT</math> after failure has occurred in compressive matrix mode.</p>
YCFAC	Reduction factor for compressive fiber strength after matrix compressive failure (MAT_054 only). The compressive strength in the fiber direction after compressive matrix failure is reduced to: $X_c = YCFAC \times Y_c, \quad (\text{default: } YCFAC = 2.0)$
DFAILT	Maximum strain for fiber tension (MAT_054 only). (Maximum 1 = 100% strain). The layer in the element is completely removed after the maximum tensile strain in the fiber direction is reached. If a nonzero value is given for DFAILT, a nonzero, negative value must also be provided for DFAILC. <p>If the 2-way fiber flag is set then DFAILT is the fiber tensile failure strain in the <math>a</math> and <math>b</math> directions.</p>

VARIABLE	DESCRIPTION
DFAILC	<p>Maximum strain for fiber compression (MAT_054 only). (Maximum -1 = 100% compression). The layer in the element is completely removed after the maximum compressive strain in the fiber direction is reached. The input value should be negative and is required if DFAILT &gt; 0.</p> <p>If the 2-way fiber flag is set then DFAILC is the fiber compressive failure strain in the <i>a</i> and <i>b</i> directions.</p>
EFS	Effective failure strain (MAT_054 only).
XC	<p>Longitudinal compressive strength (absolute value is used).</p> <p>GE.0.0: Poisson effect (PRBA) after failure is active.</p> <p>LT.0.0: Poisson effect after failure is not active, i.e. PRBA = 0.</p>
XT	Longitudinal tensile strength, see below.
YC	<p>Transverse compressive strength, <i>b</i>-axis (positive value), see below.</p> <p>If the 2-way fiber flag is set then YC is the fiber compressive failure stress in the <i>b</i> direction.</p>
YT	<p>Transverse tensile strength, <i>b</i>-axis, see below.</p> <p>If the 2-way fiber flag is set then YT is the fiber tensile failure stress in the <i>b</i> direction.</p>
SC	Shear strength, <i>ab</i> plane, see below.
CRIT	<p>Failure criterion (material number):</p> <p>EQ.54.0: Chang criterion for matrix failure (as Material 22) (default),</p> <p>EQ.55.0: Tsai-Wu criterion for matrix failure.</p>
BETA	Weighting factor for shear term in tensile fiber mode (MAT_054 only). ( $0.0 \leq \text{BETA} \leq 1.0$ )
PFL	<p>Percentage of layers which must fail until crashfront is initiated. E.g.  PFL  = 80.0, then 80% of layers must fail until strengths are reduced in neighboring elements. Default: all layers must fail. A single layer fails if 1 in-plane IP fails (PFL &gt; 0) or if 4 in-plane IPs fail (PFL &lt; 0). (MAT_054 only, thin and thick shells).</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EPSF	Damage initiation transverse shear strain. (MAT_054 only).
EPSR	Final rupture transverse shear strain. (MAT_054 only).
TSMD	Transverse shear maximum damage, default = 0.90. (MAT_054 only,).
SOFT2	Optional “orthogonal” softening reduction factor for material strength in crashfront elements (default = 1.0). See remarks (MAT_054 only, thin and thick shells).
SLIMT1	Factor to determine the minimum stress limit after stress maximum (fiber tension). Similar to *MAT_058 (MAT_054 only).
SLIMC1	Factor to determine the minimum stress limit after stress maximum (fiber compression). Similar to *MAT_058 (MAT_054 only).
SLIMT2	Factor to determine the minimum stress limit after stress maximum (matrix tension). Similar to *MAT_058 (MAT_054 only).  If the 2-way fiber flag is set then SLIMT2 is the factor to determine the minimum stress limit after tensile failure stress is reached in the <i>b</i> fiber direction.
SLIMC2	Factor to determine the minimum stress limit after stress maximum (matrix compression). Similar to *MAT_058 (MAT_054 only).  If the 2-way fiber flag is set then SLIMC2 is the factor to determine the minimum stress limit after compressive failure stress is reached in the <i>b</i> fiber direction.
SLIMS	Factor to determine the minimum stress limit after stress maximum (shear). Similar to *MAT_058 (MAT_054 only).
NCYRED	Number of cycles for stress reduction from maximum to minimum (MAT_054 only).
SOFTG	Softening reduction factor for transverse shear moduli GBC and GCA in crashfront elements (default = 1.0) (MAT_054 only, thin and thick shells).
LCXC	Load curve ID for XC vs. strain rate (XC is ignored with that option)
LCXT	Load curve ID for XT vs. strain rate (XT is ignored with that option)

VARIABLE	DESCRIPTION
LCYC	Load curve ID for YC vs. strain rate (YC is ignored with that option)
LCYT	Load curve ID for YT vs. strain rate (YT is ignored with that option)
LCSC	Load curve ID for SC vs. strain rate (SC is ignored with that option)
DT	Strain rate averaging option. EQ.0.0: Strain rate is evaluated using a running average. LT.0.0: Strain rate is evaluated using average of last 11 time steps. GT.0.0: Strain rate is averaged over the last DT time units.

**Material Formulation:**

The Chang/Chang (MAT\_54) criteria is given as follows:

for the tensile fiber mode,

$$\sigma_{aa} > 0 \Rightarrow e_f^2 = \left(\frac{\sigma_{aa}}{X_t}\right)^2 + \beta \left(\frac{\sigma_{ab}}{S_c}\right)^2 - 1, \quad \begin{array}{l} e_f^2 \geq 0 \Rightarrow \text{failed} \\ e_f^2 < 0 \Rightarrow \text{elastic} \end{array}$$

$$E_a = E_b = G_{ab} = \nu_{ba} = \nu_{ab} = 0$$

for the compressive fiber mode,

$$\sigma_{aa} < 0 \Rightarrow e_c^2 = \left(\frac{\sigma_{aa}}{X_c}\right)^2 - 1, \quad \begin{array}{l} e_c^2 \geq 0 \Rightarrow \text{failed} \\ e_c^2 < 0 \Rightarrow \text{elastic} \end{array}$$

$$E_a = \nu_{ba} = \nu_{ab} = 0$$

for the tensile matrix mode,

$$\sigma_{bb} > 0 \Rightarrow e_m^2 = \left(\frac{\sigma_{bb}}{Y_t}\right)^2 + \left(\frac{\sigma_{ab}}{S_c}\right)^2 - 1, \quad \begin{array}{l} e_m^2 \geq 0 \Rightarrow \text{failed} \\ e_m^2 < 0 \Rightarrow \text{elastic} \end{array}$$

$$E_b = \nu_{ba} = 0 \Rightarrow G_{ab} = 0,$$

and for the compressive matrix mode,

$$\sigma_{bb} < 0 \Rightarrow e_d^2 = \left(\frac{\sigma_{bb}}{2S_c}\right)^2 + \left[\left(\frac{Y_c}{2S_c}\right)^2 - 1\right] \frac{\sigma_{bb}}{Y_c} + \left(\frac{\sigma_{ab}}{S_c}\right)^2 - 1, \quad \begin{array}{l} e_d^2 \geq 0 \Rightarrow \text{failed} \\ e_d^2 < 0 \Rightarrow \text{elastic} \end{array}$$

$$E_b = \nu_{ba} = \nu_{ab} = 0 \Rightarrow G_{ab} = 0$$

$$X_c = 2Y_c, \text{ for 50\% fiber volume}$$

If the 2-way fiber flag is set then the failure criteria for tensile and compressive fiber failure in the local X direction are unchanged. For the local y-direction, the same failure criteria as for the x-direction fibers are used.

Tension, y-direction,

$$\sigma_{bb} > 0 \Rightarrow e_f^2 = \left(\frac{\sigma_{bb}}{Y_t}\right)^2 + \beta \left(\frac{\sigma_{ab}}{S_c}\right)^2 - 1, \quad \begin{array}{l} e_f^2 \geq 0 \Rightarrow \text{failed} \\ e_f^2 < 0 \Rightarrow \text{elastic} \end{array}$$

Compressive y-direction,

$$\sigma_{bb} < 0 \Rightarrow e_c^2 = \left(\frac{\sigma_{bb}}{Y_c}\right)^2 - 1, \quad \begin{array}{l} e_c^2 \geq 0 \Rightarrow \text{failed} \\ e_c^2 < 0 \Rightarrow \text{elastic} \end{array}$$

Matrix failure criterion,

$$e_f^2 = \left(\frac{\sigma_{ab}}{S_c}\right)^2 - 1$$

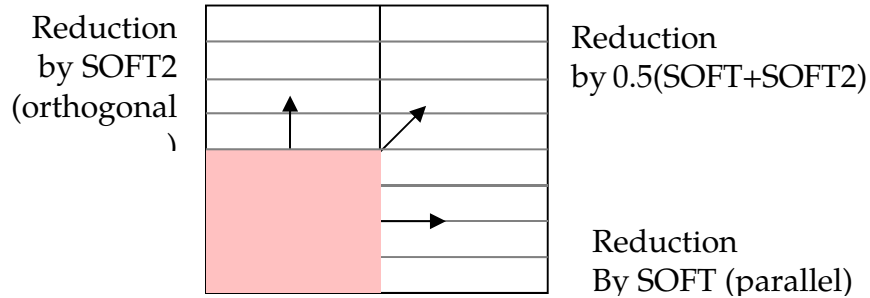
In the Tsai-Wu (MAT\_055) criteria the tensile and compressive fiber modes are treated as in the Chang-Chang criteria. The failure criterion for the tensile and compressive matrix mode is given as:

$$e_{md}^2 = \frac{\sigma_{bb}^2}{Y_c Y_t} + \left(\frac{\sigma_{ab}}{S_c}\right)^2 + \frac{(Y_c - Y_t) \sigma_{bb}}{Y_c Y_t} - 1, \quad \begin{array}{l} e_{md}^2 \geq 0 \Rightarrow \text{failed} \\ e_{md}^2 < 0 \Rightarrow \text{elastic} \end{array}$$

For  $\beta = 1$  we get the original criterion of Hashin [1980] in the tensile fiber mode. For  $\beta = 0$  we get the maximum stress criterion which is found to compare better to experiments.

In MAT\_054, failure can occur in any of four different ways:

1. If DFAILT is zero, failure occurs if the Chang-Chang failure criterion is satisfied in the tensile fiber mode.
2. If DFAILT is greater than zero, failure occurs if:
  - the fiber strain is greater than DFAILT or less than DFAILC
  - if absolute value of matrix strain is greater than DFAILM
  - if absolute value of tensorial shear strain is greater than DFAILS
3. If EFS is greater than zero, failure occurs if the effective strain is greater than EFS.
4. If TFAIL is greater than zero, failure occurs according to the element timestep as described in the definition of TFAIL above.



**Figure M54-1.** Direction dependent softening

When failure has occurred in all the composite layers (through-thickness integration points), the element is deleted. For bricks, the element is deleted after one integration point has met the failure criteria. Elements which share nodes with the deleted element become “crashfront” elements and can have their strengths reduced by using the SOFT parameter with TFAIL greater than zero. An earlier initiation of crashfront elements is possible by using parameter PFL.

An optional direction dependent strength reduction can be invoked by setting  $0 < \text{SOFT2} < 1$ . Then, SOFT equals a strength reduction factor for fiber parallel failure and SOFT2 equals a strength reduction factor for fiber orthogonal failure. Linear interpolation is used for angles in between. See [Figure M54-1](#).

Information about the status in each layer (integration point) and element can be plotted using additional integration point variables. The number of additional integration point variables for shells written to the LS-DYNA database is input by the \*DATABASE\_EXTENT\_BINARY definition as variable NEIPS. For Models 54 and 55 these additional variables are tabulated below ( $i$  = shell integration point):

History Variable	Description	Value	LS-PrePost History Variable
1 ef( $i$ )	tensile fiber mode	1 – elastic  0 – failed	1
2 ec( $i$ )	compressive fiber mode		2
3 em( $i$ )	tensile matrix mode		3
4 ed( $i$ )	compressive matrix mode		4
5 efail	max[ef(ip)]		5
6 dam	damage parameter	-1 – element intact $10^{-8}$ – element in crashfront +1 – element failed	6

These variables can be plotted in LS-PrePost element history variables 1 to 6. The following components, defined by the sum of failure indicators over all through-thickness integration points, are stored as element component 7 instead of the effective plastic strain.

Description	Integration point
$\frac{1}{\text{nip}} \sum_{i=1}^{\text{nip}} ef(i)$	1
$\frac{1}{\text{nip}} \sum_{i=1}^{\text{nip}} ec(i)$	2
$\frac{1}{\text{nip}} \sum_{i=1}^{\text{nip}} em(i)$	3

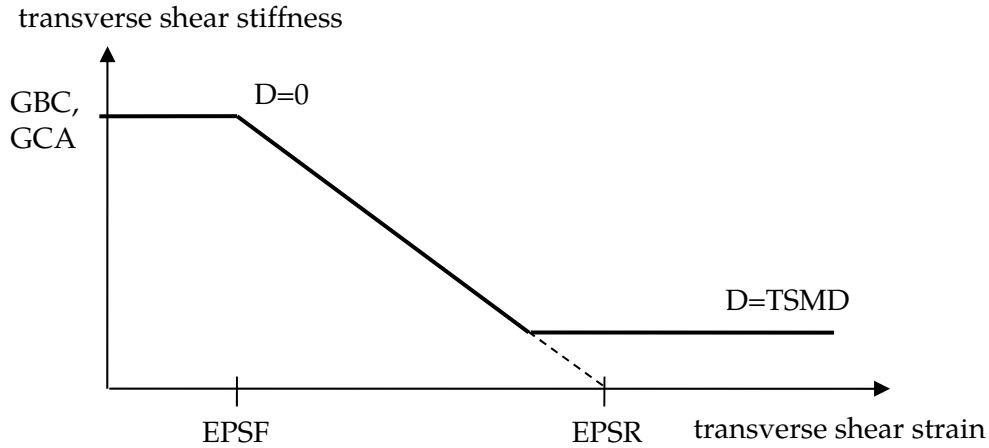
In an optional damage model for transverse shear strain, out-of-plane stiffness (GBC and GCA) can get linearly decreased to model interlaminar shear failure. Damage starts when effective transverse shear strain

$$\varepsilon_{56}^{\text{eff}} = \sqrt{\varepsilon_{yz}^2 + \varepsilon_{zx}^2}$$

reaches EPSF. Final rupture occurs when effective transverse shear strain reaches EPSR. A maximum damage of TSMD ( $0.0 < \text{TSMD} < 0.99$ ) cannot be exceeded. See [Figure M54-2](#).

**Additional Remarks:**





**Figure M54-2.** Linear Damage for transverse shear behavior

1. **TI-Flag**

(Applies only to Transversal Isotropic behavior for \*MAT\_054 solid elements.)

The behavior in the b-c-plane is assumed to be isotropic, thus the elastic constants EC, PRCA and GCA are ignored and set according to the given values EA, EB, PRAB, GAB. Damage in transverse shear (EPSF, EPSR, TSMD, SOFTG) is ignored. The failure criterion is evaluated by replacing  $\sigma_{bb}$  and  $\sigma_{ab}$  with the corresponding stresses  $\sigma_{11}$  and  $\sigma_{a1}$  in a principal stress frame rotated around the local a-axis. The principal axes 1 and 2 in the b-c plane are chosen such that  $|\sigma_{11}| \geq |\sigma_{22}|$  is fulfilled.

**\*MAT\_LOW\_DENSITY\_FOAM**

This is Material Type 57 for modeling highly compressible low density foams. Its main applications are for seat cushions and padding on the Side Impact Dummies (SID). Optionally, a tension cut-off failure can be defined. A table can be defined if thermal effects are considered in the nominal stress versus strain behavior. Also, see the notes below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	LCID	TC	HU	BETA	DAMP
Type	A8	F	F	F	F	F	F	F
Default					10 <sup>20</sup>	1.		0.05
Remarks						3	1	

Card 2	1	2	3	4	5	6	7	8
Variable	SHAPE	FAIL	BVFLAG	ED	BETA1	KCON	REF	
Type	F	F	F	F	F	F	F	
Default	1.0	0.0	0.0	0.0	0.0	0.0	0.0	
Remarks	3		2	5	5	6		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus used in tension. For implicit problems E is set to the initial slope of load curve LCID.

VARIABLE	DESCRIPTION
LCID	Load curve or table ID, see *DEFINE_CURVE, for the nominal stress versus strain curve definition. If a table is used, a family of curves is defined each corresponding to a discrete temperature, see *DEFINE_TABLE.
TC	Cut-off for the nominal tensile stress $\tau_i$
HU	Hysteretic unloading factor between 0 and 1 (default = 1, i.e., no energy dissipation), see also <a href="#">Figure M57-1</a> .
BETA	$\beta$ , decay constant to model creep in unloading
DAMP	Viscous coefficient (.05 < recommended value <.50) to model damping effects. <p style="margin-left: 40px;">LT.0.0:  DAMP  is the load curve ID, which defines the damping constant as a function of the maximum strain in compression defined as:</p>
	$\varepsilon_{\max} = \max(1 - \lambda_1, 1 - \lambda_2, 1 - \lambda_3).$
	In tension, the damping constant is set to the value corresponding to the strain at 0. The abscissa should be defined from 0 to 1.
SHAPE	Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor. Values less than one reduces the energy dissipation and greater than one increases dissipation, see also <a href="#">Figure M57-1</a> .
FAIL	Failure option after cutoff stress is reached: <p style="margin-left: 40px;">EQ.0.0: tensile stress remains at cut-off value,  EQ.1.0: tensile stress is reset to zero.</p>
BVFLAG	Bulk viscosity activation flag, see remark below: <p style="margin-left: 40px;">EQ.0.0: no bulk viscosity (recommended),  EQ.1.0: bulk viscosity active.</p>
ED	Optional Young's relaxation modulus, $E_d$ , for rate effects. See Remark 5.
BETA1	Optional decay constant, $\beta_1$ .

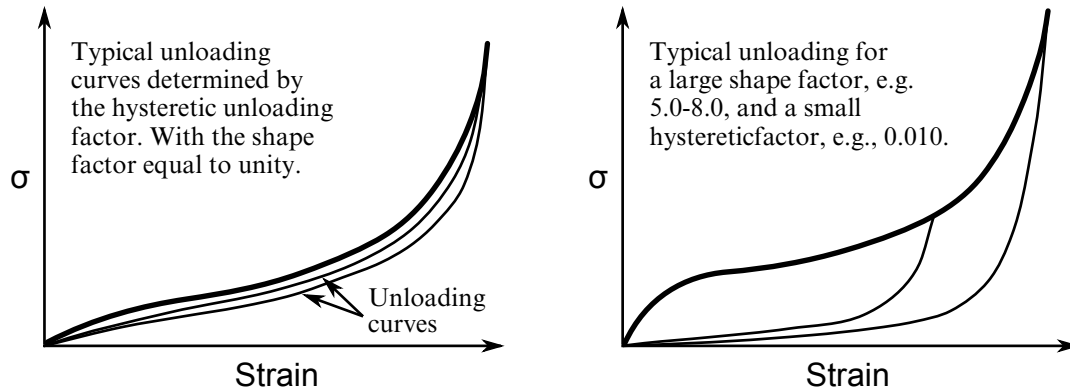


Figure M57-1. Behavior of the low density urethane foam model

VARIABLE	DESCRIPTION
KCON	Stiffness coefficient for contact interface stiffness. If undefined the maximum slope in stress vs. strain curve is used. When the maximum slope is taken for the contact, the time step size for this material is reduced for stability. In some cases $\Delta t$ may be significantly smaller, and defining a reasonable stiffness is recommended.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.

**Material Formulation:**

The compressive behavior is illustrated in Figure M57-1 where hysteresis on unloading is shown. This behavior under uniaxial loading is assumed not to significantly couple in the transverse directions. In tension the material behaves in a linear fashion until tearing occurs. Although our implementation may be somewhat unusual, it was motivated by Storakers [1986].

The model uses tabulated input data for the loading curve where the nominal stresses are defined as a function of the elongations,  $\epsilon_i$ , which are defined in terms of the principal stretches,  $\lambda_i$ , as:

$$\epsilon_i = \lambda_i - 1$$

The principal stretches are stored as extra history variables 16, 17, and 18 if ED = 0 and as extra history variables 28, 29, and 30 if ED > 0. (See NEIPH in \*DATABASE\_EXTENT\_BINARY for output of extra history variables.) The stretch ratios are found by solving for the

eigenvalues of the left stretch tensor,  $V_{ij}$ , which is obtained via a polar decomposition of the deformation gradient matrix,  $F_{ij}$ . Recall that,

$$F_{ij} = R_{ik}U_{kj} = V_{ik}R_{kj}$$

The update of  $V_{ij}$  follows the numerically stable approach of Taylor and Flanagan [1989]. After solving for the principal stretches, we compute the elongations and, if the elongations are compressive, the corresponding values of the nominal stresses,  $\tau_i$  are interpolated. If the elongations are tensile, the nominal stresses are given by

$$\tau_i = E\varepsilon_i$$

and the Cauchy stresses in the principal system become

$$\sigma_i = \frac{\tau_i}{\lambda_j\lambda_k}$$

The stresses can now be transformed back into the global system for the nodal force calculations.

#### Remarks:

1. When hysteretic unloading is used the reloading will follow the unloading curve if the decay constant,  $\beta$ , is set to zero. If  $\beta$  is nonzero the decay to the original loading curve is governed by the expression:

$$1 - e^{-\beta t}$$

2. The bulk viscosity, which generates a rate dependent pressure, may cause an unexpected volumetric response and, consequently, it is optional with this model.
3. The hysteretic unloading factor results in the unloading curve to lie beneath the loading curve as shown in [Figure M57-1](#) This unloading provides energy dissipation which is reasonable in certain kinds of foam.
4. Note that since this material has no effective plastic strain, the internal energy per initial volume is written into the output databases.
5. Rate effects are accounted for through linear viscoelasticity by a convolution integral of the form

$$\sigma_{ij}^r = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  is the relaxation function. The stress tensor,  $\sigma_{ij}^r$ , augments the stresses determined from the foam,  $\sigma_{ij}^f$ ; consequently, the final stress,  $\sigma_{ij}$ , is taken as the summation of the two contributions:

$$\sigma_{ij} = \sigma_{ij}^f + \sigma_{ij}^r.$$

Since we wish to include only simple rate effects, the relaxation function is represented by one term from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta t}$$

given by,

$$g(t) = E_d e^{-\beta_1 t}$$

This model is effectively a Maxwell fluid which consists of a damper and spring in series. We characterize this in the input by a Young's modulus,  $E_d$ , and decay constant,  $\beta_1$ . The formulation is performed in the local system of principal stretches where only the principal values of stress are computed and triaxial coupling is avoided. Consequently, the one-dimensional nature of this foam material is unaffected by this addition of rate effects. The addition of rate effects necessitates twelve additional history variables per integration point. The cost and memory overhead of this model comes primarily from the need to "remember" the local system of principal stretches.

6. The time step size is based on the current density and the maximum of the instantaneous loading slope,  $E$ , and KCON. If KCON is undefined the maximum slope in the loading curve is used instead.



**\*MAT\_058****\*MAT\_LAMINATED\_COMPOSITE\_FABRIC**

Card 6	1	2	3	4	5	6	7	8
Variable	E11C	E11T	E22C	E22T	GMS			
Type	F	F	F	F	F			

Card 7	1	2	3	4	5	6	7	8
Variable	XC	XT	YC	YT	SC			
Type	F	F	F	F	F			

**First Optional Strain Rate Dependence Card.**

Card 8	1	2	3	4	5	6	7	8
Variable	LCXC	LCXT	LCYC	LCYT	LCSC	LCTAU	LCGAM	DT
Type	I	I	I	I	I	I	I	F

**Second Optional Strain Rate Dependence Card.**

Card 9	1	2	3	4	5	6	7	8
Variable	LCE11C	LCE11T	LCE22C	LCE22T	LCGMS	LCEFS		
Type	I	I	I	I	I	I		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density



VARIABLE	DESCRIPTION
EA	<p>GT.0.0: <math>E_a</math>, Young's modulus - longitudinal direction</p> <p>LT.0.0: Load curve ID or Table ID = (-EA)</p> <p><b>Load Curve.</b> When (-EA) is equal to a Load curve ID, it is taken as defining the uniaxial elastic stress vs. strain behavior in longitudinal direction.</p> <p><b>Tabular Data.</b> When (-EA) is equal to a Table ID, it defines for each strain rate value a Load curve ID giving the uniaxial elastic stress vs. strain behavior in longitudinal direction.</p> <p><b>Logarithmically Defined Tables.</b> If the <i>first</i> uniaxial elastic stress vs. strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for <i>all</i> stress-strain curves.</p>
EB	<p>GT.0.0: <math>E_b</math>, Young's modulus - transverse direction</p> <p>LT.0.0: Load curve ID or Table ID = (-EB)</p> <p><b>Load Curve.</b> When (-EB) is equal to a Load curve ID, it is taken as defining the uniaxial elastic stress vs. strain behavior in transverse direction.</p> <p><b>Tabular Data.</b> When (-EB) is equal to a Table ID, it defines for each strain rate value a Load curve ID giving the uniaxial elastic stress vs. strain behavior in transverse direction.</p> <p><b>Logarithmically Defined Tables.</b> If the <i>first</i> uniaxial elastic stress vs. strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for <i>all</i> stress-strain curves.</p>
(EC)	$E_c$ , Young's modulus - normal direction (not used)
PRBA	$\nu_{ba}$ , Poisson's ratio ba
PRCA	$\nu_{ca}$ , Poisson's ratio ca, can be defined in card 4, col. 7, default = PRBA
PRCB	$\nu_{cb}$ , Poisson's ratio cb, can be defined in card 4, col. 8, default = PRBA

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TAU1	$\tau_1$ , stress limit of the first slightly nonlinear part of the shear stress versus shear strain curve. The values $\tau_1$ and $\gamma_1$ are used to define a curve of shear stress versus shear strain. These values are input if FS, defined below, is set to a value of -1.
GAMMA1	$\gamma_1$ , strain limit of the first slightly nonlinear part of the shear stress versus engineering shear strain curve.
GAB	<p>GT.0.0: <math>G_{ab}</math>, shear modulus ab direction</p> <p>LT.0.0: Load curve ID or Table ID = (-GAB)</p> <p><b>Load Curve.</b> When (-GAB) is equal to a Load curve ID, it is taken as defining the elastic shear stress vs. shear strain behavior in ab direction.</p> <p><b>Tabular Data.</b> When (-GAB) is equal to a a Table ID, it defines for each strain rate value a Load curve ID giving the elastic shear stress vs. shear strain behavior in ab direction.</p> <p><b>Logarithmically Defined Tables.</b> If the <i>first</i> elastic shear stress vs. shear strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for <i>all</i> shear stress-shear strain curves.</p>
GBC	$G_{bc}$ , shear modulus <i>bc</i>
GCA	$G_{ca}$ , shear modulus <i>ca</i>
SLIMT1	Factor to determine the minimum stress limit after stress maximum (fiber tension).
SLIMC1	Factor to determine the minimum stress limit after stress maximum (fiber compression).
SLIMT2	Factor to determine the minimum stress limit after stress maximum (matrix tension).
SLIMC2	Factor to determine the minimum stress limit after stress maximum (matrix compression).
SLIMS	Factor to determine the minimum stress limit after stress maximum (shear).
AOPT	Material axes option (see MAT_OPTION TROPIC_ELASTIC for a

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	more complete description):
	EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.
	EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle (BETA) from a line in the plane of the element defined by the cross product of the vector $\mathbf{v}$ with the element normal.
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
TSIZE	Time step for automatic element deletion.
ERODS	Maximum effective strain for element layer failure. A value of unity would equal 100% strain. GT.0.0: fails when effective strain calculated assuming material is volume preserving exceeds ERODS (old way). LT.0.0: fails when effective strain calculated from the full strain tensor exceeds  ERODS .
SOFT	Softening reduction factor for strength in the crashfront.

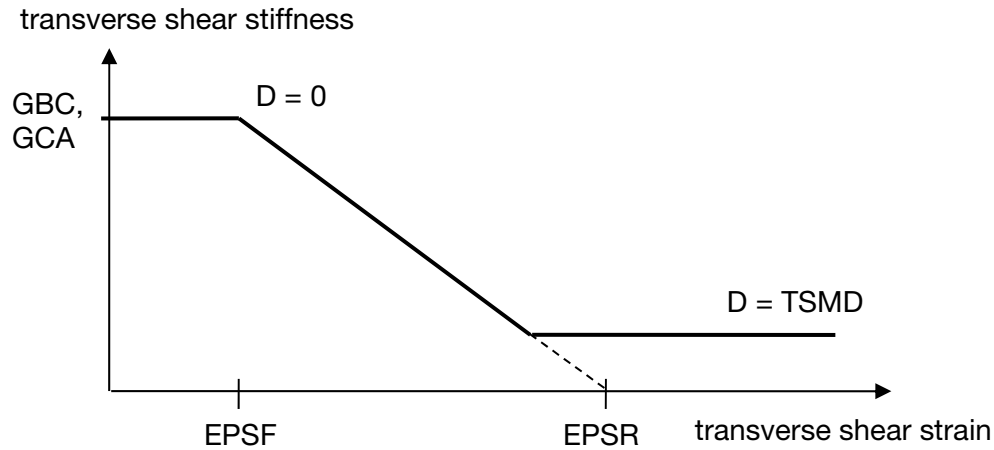


Figure M58-1. Linear Damage for transverse shear behavior

VARIABLE	DESCRIPTION
FS	<p>Failure surface type:</p> <p>EQ.1.0: smooth failure surface with a quadratic criterion for both the fiber (a) and transverse (b) directions. This option can be used with complete laminates and fabrics.</p> <p>EQ.0.0: smooth failure surface in the transverse (b) direction with a limiting value in the fiber (a) direction. This model is appropriate for unidirectional (UD) layered composites only.</p> <p>EQ.-1.: faceted failure surface. When the strength values are reached then damage evolves in tension and compression for both the fiber and transverse direction. Shear behavior is also considered. This option can be used with complete laminates and fabrics.</p>
EPSF	Damage initiation transverse shear strain.
EPSR	Final rupture transverse shear strain.
TSMD	Transverse shear maximum damage, default = 0.90.
XP, YP, ZP	Define coordinates of point <b>p</b> for AOPT = 1.
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2.
V1, V2 V3	Define components of vector <b>v</b> for AOPT = 3.
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	Material angle in degrees for AOPT = 0 and AOPT = 3. BETA may be overridden on the element card, see *ELEMENT_SHELL_BETA.
E11C	Strain at longitudinal compressive strength, <i>a</i> -axis (positive).
E11T	Strain at longitudinal tensile strength, <i>a</i> -axis.
E22C	Strain at transverse compressive strength, <i>b</i> -axis.
E22T	Strain at transverse tensile strength, <i>b</i> -axis.
GMS	Engineering shear strain at shear strength, <i>ab</i> plane.
XC	Longitudinal compressive strength (positive value).
XT	Longitudinal tensile strength, see below.
YC	Transverse compressive strength, <i>b</i> -axis (positive value), see below.
YT	Transverse tensile strength, <i>b</i> -axis, see below.
SC	Shear strength, <i>ab</i> plane, see below.
LCXC	Load curve ID defining longitudinal compressive strength XC vs. strain rate (XC is ignored with that option). If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
LCXT	Load curve ID defining longitudinal tensile strength XT vs. strain rate (XT is ignored with that option) If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
LCYC	Load curve ID defining transverse compressive strength YC vs. strain rate (YC is ignored with that option) If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
LCYT	Load curve ID defining transverse tensile strength YT vs. strain rate (YT is ignored with that option) If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCSC	Load curve ID defining shear strength SC vs. strain rate (SC is ignored with that option) If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
LCTAU	Load curve ID defining TAU1 vs. strain rate (TAU1 is ignored with that option). This value is only used for FS = -1. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
LCGAM	Load curve ID defining GAMMA1 vs. strain rate (GAMMA1 is ignored with that option). This value is only used for FS = -1. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
DT	Strain rate averaging option. EQ.0.0: Strain rate is evaluated using a running average. LT.0.0: Strain rate is evaluated using average of last 11 time steps. GT.0.0: Strain rate is averaged over the last DT time units.
LCE11C	Load curve ID defining E11C vs. strain rate (E11C is ignored with that option) If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
LCE11T	Load curve ID defining E11T vs. strain rate (E11T is ignored with that option) If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
LCE22C	Load curve ID defining E22C vs. strain rate (E22C is ignored with that option) If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
LCE22T	Load curve ID defining E22T vs. strain rate (E22T is ignored with that option) If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.

VARIABLE	DESCRIPTION
LCGMS	Load curve ID defining GMS vs. strain rate (GMS is ignored with that option) If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
LCEFS	Load curve ID defining ERODS vs. strain rate (ERODS is ignored with that option). The full strain tensor is used to compute the equivalent strain (new option). If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.

**Remarks:**

Parameters to control failure of an element layer are: ERODS, the maximum effective strain, i.e., maximum 1 = 100% straining. The layer in the element is completely removed after the maximum effective strain (compression/tension including shear) is reached.

The stress limits are factors used to limit the stress in the softening part to a given value,

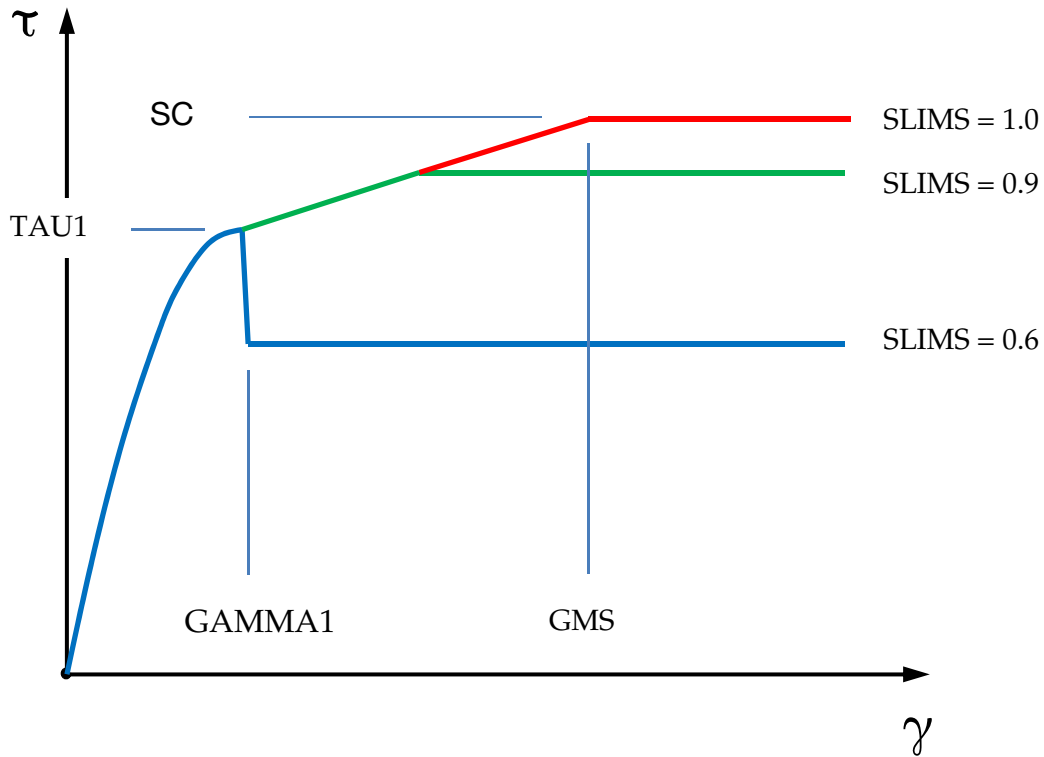
$$\sigma_{\min} = \text{SLIM}_{xx} \times \text{strength},$$

thus, the damage value is slightly modified such that elastoplastic like behavior is achieved with the threshold stress. The SLIM<sub>xx</sub> fields may range between 0.0 and 1.0. With a factor of 1.0, the stress remains at a maximum value identical to the strength, which is similar to ideal elastoplastic behavior. For tensile failure a small value for SLIM<sub>Tx</sub> is often reasonable; however, for compression SLIM<sub>Cx</sub> = 1.0 is preferred. This is also valid for the corresponding shear value.

If SLIM<sub>xx</sub> is smaller than 1.0, then localization can be observed depending on the total behavior of the lay-up. If the user is intentionally using SLIM<sub>xx</sub> < 1.0, it is generally recommended to avoid a drop to zero and set the value to something in between 0.05 and 0.10. Then elastoplastic behavior is achieved in the limit which often leads to less numerical problems. Defaults for SLIM<sub>xx</sub> = 10<sup>-8</sup>.

The crashfront-algorithm is started if and only if a value for TSIZE is input. Note that time step size, with element elimination after the actual time step becomes smaller than TSIZE.

The damage parameters can be written to the post processing database for each integration point as the first three additional element variables and can be visualized.



**Figure M58-2.** Stress-strain diagram for shear

Material models with FS = 1 or FS = -1 are favorable for complete laminates and fabrics, as all directions are treated in a similar fashion.

For material model FS = 1 an interaction between normal stresses and the shear stresses is assumed for the evolution of damage in the a and b-directions. For the shear damage is always the maximum value of the damage from the criterion in a or b-direction is taken.

For material model FS = -1 it is assumed that the damage evolution is independent of any of the other stresses. A coupling is only present via the elastic material parameters and the complete structure.

In tensile and compression directions and in a as well as in b- direction different failure surfaces can be assumed. The damage values, however, increase only also when the loading direction changes.

**Special control of shear behavior of fabrics:**

For fabric materials a nonlinear stress strain curve for the shear part for failure surface FS = -1 can be assumed as given below. This is not possible for other values of FS.

The curve, shown in [Figure M58-2](#) is defined by three points:



1. the origin (0,0) is assumed,
2. the limit of the first slightly nonlinear part (must be input), stress (TAU1) and strain (GAMMA1), see below.
3. the shear strength at failure and shear strain at failure.

In addition a stress limiter can be used to keep the stress constant via the SLIMS parameter. This value must be less or equal 1.0 but positive, and leads to an elastoplastic behavior for the shear part. The default is  $10^{-8}$ , assuming almost brittle failure once the strength limit SC is reached.



**Card 5 for SHELL Keyword Option.**

Card 5	1	2	3	4	5	6	7	8
Variable	TSIZE	ALP	SOFT	FBRT	SR	SF		
Type	F	F	F	F	F	F		

**Card 6 for SHELL Keyword Option.**

Card 6	1	2	3	4	5	6	7	8
Variable	XC	XT	YC	YT	SC			
Type	F	F	F	F	F			

**Card 5 for SPH and SOLID Keyword Options.**

Card 5	1	2	3	4	5	6	7	8
Variable	SBA	SCA	SCB	XXC	YYC	ZZC		
Type	F	F	F	F	F	F		

**Card 6 for SPH and SOLID Keyword Options.**

Card 6	1	2	3	4	5	6	7	8
Variable	XXT	YYT	ZZT					
Type	F	F	F					

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Density
EA	$E_a$ , Young's modulus - longitudinal direction

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EB	$E_b$ , Young's modulus - transverse direction
EC	$E_c$ , Young's modulus - normal direction
PRBA	$\nu_{ba}$ , Poisson's ratio ba
PRCA	$\nu_{ca}$ , Poisson's ratio ca
PRCB	$\nu_{cb}$ , Poisson's ratio cb
GAB	$G_{ab}$ , Shear Modulus
GBC	$G_{bc}$ , Shear Modulus
GCA	$G_{ca}$ , Shear Modulus
KF	Bulk modulus of failed material
AOPT	<p>Material axes option (see MAT_{OPTION}TROPIC_ELASTIC for a more complete description) (SPH particles only support AOPT = 2.0):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <math>a</math>-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>\mathbf{p}</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
MACF	Material axes change flag for brick elements. EQ.1: No change, default, EQ.2: switch material axes $a$ and $b$ , EQ.3: switch material axes $a$ and $c$ , EQ.4: switch material axes $b$ and $c$ .
XP YP ZP	Define coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1 A2 A3	Define components of vector $\mathbf{a}$ for AOPT = 2.
V1 V2 V3	Define components of vector $\mathbf{v}$ for AOPT = 3 and 4.
D1 D2 D3	Define components of vector $\mathbf{d}$ for AOPT = 2:
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO.
TSIZE	Time step for automatic element deletion
ALP	Nonlinear shear stress parameter
SOFT	Softening reduction factor for material strength in crashfront elements
FBRT	Softening of fiber tensile strength
SR	$s_r$ , reduction factor (default = 0.447)
SF	$s_f$ , softening factor (default = 0.0)
XC	Longitudinal compressive strength, $a$ -axis (positive value).
XT	Longitudinal tensile strength, $a$ -axis
YC	Transverse compressive strength, $b$ -axis (positive value).
YT	Transverse tensile strength, $b$ -axis

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
SC	Shear strength, <i>ab</i> plane: GT.0.0: faceted failure surface theory, LT.0.0: ellipsoidal failure surface theory.
SBA	In plane shear strength.
SCA	Transverse shear strength.
SCB	Transverse shear strength.
XXC	Longitudinal compressive strength <i>a</i> -axis (positive value).
YYC	Transverse compressive strength <i>b</i> -axis (positive value).
ZZC	Normal compressive strength <i>c</i> -axis (positive value).
XXT	Longitudinal tensile strength <i>a</i> -axis.
YYT	Transverse tensile strength <i>b</i> -axis.
ZZT	Normal tensile strength <i>c</i> -axis.

**\*MAT\_ELASTIC\_WITH\_VISCOSITY**

This is Material Type 60 which was developed to simulate forming of glass products (e.g., car windshields) at high temperatures. Deformation is by viscous flow but elastic deformations can also be large. The material model, in which the viscosity may vary with temperature, is suitable for treating a wide range of viscous flow problems and is implemented for brick and shell elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	V0	A	B	C	LCID	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	PR1	PR2	PR3	PR4	PR5	PR6	PR7	PR8
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	T1	T2	T3	T4	T5	T6	T7	T8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	V4	V5	V6	V7	V8
Type	F	F	F	F	F	F	F	F

**\*MAT\_060****\*MAT\_ELASTIC\_WITH\_VISCOSITY**

Card 5	1	2	3	4	5	6	7	8
Variable	E1	E2	E3	E4	E5	E6	E7	E8
Type	F	F	F	F	F	F	F	F

Card 6	1	2	3	4	5	6	7	8
Variable	ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
V0	Temperature independent dynamic viscosity coefficient, $V_0$ . If defined, the temperature dependent viscosity defined below is skipped, see type (i) and (ii) definitions for viscosity below.
A	Dynamic viscosity coefficient, see type (i) and (ii) definitions below.
B	Dynamic viscosity coefficient, see type (i) and (ii) definitions below.
C	Dynamic viscosity coefficient, see type (i) and (ii) definitions below.
LCID	Load curve (see *DEFINE_CURVE) defining viscosity versus temperature, see type (iii). (Optional)
T1, T2, ..., TN	Temperatures, define up to 8 values
PR1, PR2, ..., PRN	Poisson's ratios for the temperatures $T_i$
V1, V2, ..., VN	Corresponding dynamic viscosity coefficients (define only one if not varying with temperature)
E1, E2, ..., EN	Corresponding Young's moduli coefficients (define only one if not varying with temperature)



VARIABLE	DESCRIPTION
ALPHA1, ..., ALPHAN.	Corresponding thermal expansion coefficients

**Remarks:**

Volumetric behavior is treated as linear elastic. The deviatoric strain rate is considered to be the sum of elastic and viscous strain rates:

$$\dot{\boldsymbol{\epsilon}}'_{\text{total}} = \dot{\boldsymbol{\epsilon}}'_{\text{elastic}} + \dot{\boldsymbol{\epsilon}}'_{\text{viscous}} = \frac{\dot{\boldsymbol{\sigma}}'}{2G} + \frac{\boldsymbol{\sigma}'}{2\nu}$$

where  $G$  is the elastic shear modulus,  $\nu$  is the viscosity coefficient, and bold indicates a tensor. The stress increment over one timestep  $dt$  is

$$d\boldsymbol{\sigma}' = 2G\dot{\boldsymbol{\epsilon}}'_{\text{total}}dt - \frac{G}{\nu}dt\boldsymbol{\sigma}'$$

The stress before the update is used for  $\boldsymbol{\sigma}'$ . For shell elements the through-thickness strain rate is calculated as follows.

$$d\sigma_{33} = 0 = K(\dot{\epsilon}_{11} + \dot{\epsilon}_{22} + \dot{\epsilon}_{33})dt + 2G\dot{\epsilon}'_{33}dt - \frac{G}{\nu}dt\sigma'_{33}$$

where the subscript  $ij = 33$  denotes the through-thickness direction and  $K$  is the elastic bulk modulus. This leads to:

$$\dot{\epsilon}_{33} = -a(\dot{\epsilon}_{11} + \dot{\epsilon}_{22}) + bp$$

$$a = \frac{\left(K - \frac{2}{3}G\right)}{\left(K + \frac{4}{3}G\right)}$$

$$b = \frac{Gdt}{\nu\left(K + \frac{4}{3}G\right)}$$

in which  $p$  is the pressure defined as the negative of the hydrostatic stress.

The variation of viscosity with temperature can be defined in any one of the 3 ways.

- (i) Constant,  $V = V_0$  Do not define constants,  $A$ ,  $B$ , and  $C$  or the piecewise curve.(leave card 4 blank)
- (ii)  $V = V_0 \times 10^{(A/(T-B) + C)}$
- (iii) Piecewise curve: define the variation of viscosity with temperature.

**NOTE:** Viscosity is inactive during dynamic relaxation.

**\*MAT\_ELASTIC\_WITH\_VISCOSITY\_CURVE**

This is Material Type 60 which was developed to simulate forming of glass products (e.g., car windshields) at high temperatures. Deformation is by viscous flow but elastic deformations can also be large. The material model, in which the viscosity may vary with temperature, is suitable for treating a wide range of viscous flow problems and is implemented for brick and shell elements. Load curves are used to represent the temperature dependence of Poisson's ratio, Young's modulus, the coefficient of expansion, and the viscosity.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	V0	A	B	C	LCID	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	PR_LC	YM_LC	A_LC	V_LC	V_LOG			
Type	F	F	F	F	F			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
V0	Temperature independent dynamic viscosity coefficient, $V_0$ . If defined, the temperature dependent viscosity defined below is skipped, see type (i) and (ii) definitions for viscosity below.
A	Dynamic viscosity coefficient, see type (i) and (ii) definitions below.
B	Dynamic viscosity coefficient, see type (i) and (ii) definitions below.
C	Dynamic viscosity coefficient, see type (i) and (ii) definitions below.
LCID	Load curve (see *DEFINE_CURVE) defining factor on dynamic viscosity versus temperature, see type (iii). (Optional).

VARIABLE	DESCRIPTION
PR_LC	Load curve (see *DEFINE_CURVE) defining Poisson's ratio as a function of temperature.
YM_LC	Load curve (see *DEFINE_CURVE) defining Young's modulus as a function of temperature.
A_LC	Load curve (see *DEFINE_CURVE) defining the coefficient of thermal expansion as a function of temperature.
V_LC	Load curve (see *DEFINE_CURVE) defining the dynamic viscosity as a function of temperature.
V_LOG	Flag for the form of V_LC. If V_LOG = 1.0, the value specified in V_LC is the natural logarithm of the viscosity, ln(V). The value interpolated from the curve is then exponentiated to obtain the viscosity. If V_LOG = 0.0, the value is the viscosity. The logarithmic form is useful if the value of the viscosity changes by orders of magnitude over the temperature range of the data.

### Remarks:

Volumetric behavior is treated as linear elastic. The deviatoric strain rate is considered to be the sum of elastic and viscous strain rates:

$$\dot{\boldsymbol{\epsilon}}'_{\text{total}} = \dot{\boldsymbol{\epsilon}}'_{\text{elastic}} + \dot{\boldsymbol{\epsilon}}'_{\text{viscous}} = \frac{\dot{\boldsymbol{\sigma}}'}{2G} + \frac{\dot{\boldsymbol{\sigma}}'}{2\nu}$$

where G is the elastic shear modulus,  $\nu$  is the viscosity coefficient, and bold~ indicates a tensor. The stress increment over one timestep dt is

$$d\boldsymbol{\sigma}' = 2G\dot{\boldsymbol{\epsilon}}'_{\text{total}} dt - \frac{G}{\nu} dt \boldsymbol{\sigma}'$$

The stress before the update is used for  $\boldsymbol{\sigma}'$ . For shell elements the through-thickness strain rate is calculated as follows.

$$d\sigma_{33} = 0 = K(\dot{\epsilon}_{11} + \dot{\epsilon}_{22} + \dot{\epsilon}_{33})dt + 2G\dot{\epsilon}'_{33}dt - \frac{G}{\nu}dt\sigma'_{33}$$

where the subscript ij = 33 denotes the through-thickness direction and K is the elastic bulk modulus. This leads to:

$$\dot{\epsilon}_{33} = -a(\dot{\epsilon}_{11} + \dot{\epsilon}_{22}) + bp$$

$$a = \frac{\left(K - \frac{2}{3}G\right)}{\left(K + \frac{4}{3}G\right)}$$

$$b = \frac{Gdt}{v\left(K + \frac{4}{3}G\right)}$$

in which  $p$  is the pressure defined as the negative of the hydrostatic stress.

The variation of viscosity with temperature can be defined in any one of the 3 ways.

- (i) Constant,  $V = V_0$  Do not define constants, A, B, and C or the piecewise curve.(leave card 4 blank)
- (ii)  $V = V_0 \times 10^{(A/(T-B) + C)}$
- (iii) Piecewise curve: define the variation of viscosity with temperature.

**Note:** Viscosity is inactive during dynamic relaxation.

**\*MAT\_KELVIN-MAXWELL\_VISCOELASTIC**

This is Material Type 61. This material is a classical Kelvin-Maxwell model for modeling viscoelastic bodies, e.g., foams. This model is valid for solid elements only. See also notes below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	G0	GI	DC	FO	S0
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
BULK	Bulk modulus (elastic)
G0	Short-time shear modulus, $G_0$
GI	Long-time (infinite) shear modulus, $G_\infty$
DC	Maxwell decay constant, $\beta$ [FO = 0.0] or Kelvin relaxation constant, $\tau$ [FO = 1.0]
FO	Formulation option: EQ.0.0: Maxwell, EQ.1.0: Kelvin.

VARIABLE	DESCRIPTION
SO	<p>Strain (logarithmic) output option to control what is written as component 7 to the d3plot database. (LS-PrePost always blindly labels this component as effective plastic strain.) The maximum values are updated for each element each time step:</p> <p>EQ.0.0: maximum principal strain that occurs during the calculation,</p> <p>EQ.1.0: maximum magnitude of the principal strain values that occurs during the calculation,</p> <p>EQ.2.0: maximum effective strain that occurs during the calculation.</p>

**Remarks:**

The shear relaxation behavior is described for the Maxwell model by:

$$G(t) = G + (G_0 - G_\infty)e^{-\beta t}$$

A Jaumann rate formulation is used

$$\overset{\nabla}{\sigma}'_{ij} = 2 \int_0^t G(t - \tau) D'_{ij}(\tau) dt$$

where the prime denotes the deviatoric part of the stress rate,  $\overset{\nabla}{\sigma}'_{ij}$ , and the strain rate  $D_{ij}$ . For the Kelvin model the stress evolution equation is defined as:

$$\dot{s}_{ij} + \frac{1}{\tau} s_{ij} = (1 + \delta_{ij}) G_0 \dot{e}_{ij} + (1 + \delta_{ij}) \frac{G_\infty}{\tau} \dot{e}_{ij}$$

The strain data as written to the LS-DYNA database may be used to predict damage, see [Bandak 1991].

\*MAT\_VISCOUS\_FOAM

This is Material Type 62. It was written to represent the Confor Foam on the ribs of EuroSID side impact dummy. It is only valid for solid elements, mainly under compressive loading.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E1	N1	V2	E2	N2	PR
Type	A8	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E1	Initial Young's modulus ( $E_1$ )
N1	Exponent in power law for Young's modulus ( $n_1$ )
V2	Viscous coefficient ( $V_2$ )
E2	Elastic modulus for viscosity ( $E_2$ ), see notes below.
N2	Exponent in power law for viscosity ( $n_2$ )
PR	Poisson's ratio, $\nu$

**Remarks:**

The model consists of a nonlinear elastic stiffness in parallel with a viscous damper. The elastic stiffness is intended to limit total crush while the viscosity absorbs energy. The stiffness  $E_2$  exists to prevent timestep problems. It is used for time step calculations as long as  $E_1^t$  is smaller than  $E_2$ . It has to be carefully chosen to take into account the stiffening effects of the viscosity. Both  $E_1$  and  $V_2$  are nonlinear with crush as follows:

$$E_1^t = E_1(V^{-n_1})$$

$$V_2^t = V_2|1 - V|^{n_2}$$

where viscosity generates a shear stress given by

$$\tau = V_2\dot{\gamma}$$

$\dot{\gamma}$  is the engineering shear strain rate, and  $V$  is the relative volume defined by the ratio of the current to initial volume.

Table showing typical values (units of N, mm, s):

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E1	N1	V2	E2	N2	PR
Value			0.0036	4.0	0.0015	100.0	0.2	0.05



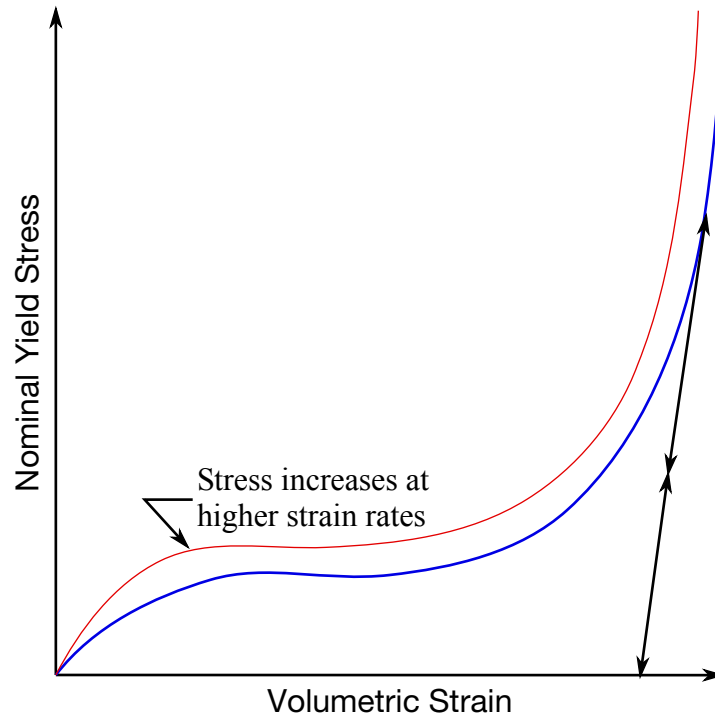
**\*MAT\_CRUSHABLE\_FOAM**

This is Material Type 63 which is dedicated to modeling crushable foam with optional damping and tension cutoff. Unloading is fully elastic. Tension is treated as elastic-perfectly-plastic at the tension cut-off value. A modified version of this model, \*MAT\_MODIFIED\_CRUSHABLE\_FOAM includes strain rate effects.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	LCID	TSC	DAMP	
Type	A8	F	F	F	F	F	F	
Default	none	none	none	none	none	0.0	0.10	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
LCID	Load curve ID defining yield stress versus volumetric strain, $\gamma$ , see <a href="#">Figure M63-1</a> .
TSC	Tensile stress cutoff. A nonzero, positive value is strongly recommended for realistic behavior.
DAMP	Rate sensitivity via damping coefficient ( $.05 <$ recommended value $< .50$ ).



**Figure M63-1.** Behavior of strain rate sensitive crushable foam. Unloading is elastic to the tension cutoff. Subsequent reloading follows the unloading curve.

**Remarks:**

The volumetric strain is defined in terms of the relative volume,  $V$ , as:

$$\gamma = 1 - V$$

The relative volume is defined as the ratio of the current to the initial volume. In place of the effective plastic strain in the d3plot database, the integrated volumetric strain (natural logarithm of the relative volume) is output.

**\*MAT\_RATE\_SENSITIVE\_POWERLAW\_PLASTICITY**

This is Material Type 64 which will model strain rate sensitive elasto-plastic material with a power law hardening. Optionally, the coefficients can be defined as functions of the effective plastic strain.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	K	M	N	E0
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0001	none	0.0002

Card 2	1	2	3	4	5	6	7	8
Variable	VP	EPS0						
Type	F	F						
Default	0.0	1.0						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus of elasticity
PR	Poisson's ratio
K	Material constant, k. If $k < 0$ the absolute value of k is taken as the load curve number that defines k as a function of effective plastic strain.
M	Strain hardening coefficient, m. If $m < 0$ the absolute value of m is taken as the load curve number that defines m as a function of effective plastic strain.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
N	Strain rate sensitivity coefficient, $n$ . If $n < 0$ the absolute value of $n$ is taken as the load curve number that defines $n$ as a function of effective plastic strain.
E0	Initial strain rate (default = 0.0002)
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default) EQ.1.0: Viscoplastic formulation
EPS0	Quasi-static threshold strain rate. See description under *MAT_015.

**Remarks:**

This material model follows a constitutive relationship of the form:

$$\sigma = k\varepsilon^m \dot{\varepsilon}^n$$

where  $\sigma$  is the yield stress,  $\varepsilon$  is the effective plastic strain,  $\dot{\varepsilon}$  is the effective total strain rate ( $VP = 0$ ), respectively the effective plastic strain rate ( $VP = 1$ ), and the constants  $k$ ,  $m$ , and  $n$  can be expressed as functions of effective plastic strain or can be constant with respect to the plastic strain. The case of no strain hardening can be obtained by setting the exponent of the plastic strain equal to a very small positive value, i.e. 0.0001.

This model can be combined with the superplastic forming input to control the magnitude of the pressure in the pressure boundary conditions in order to limit the effective plastic strain rate so that it does not exceed a maximum value at any integration point within the model.

A fully viscoplastic formulation is optional. An additional cost is incurred but the improvement in results can be dramatic.

**\*MAT\_MODIFIED\_ZERILLI\_ARMSTRONG**

This is Material Type 65 which is a rate and temperature sensitive plasticity model which is sometimes preferred in ordnance design calculations.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G	E0	N	TROOM	PC	SPALL
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	EFAIL	VP
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	B1	B2	B3	G1	G2	G3	G4	BULK
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
G	Shear modulus
E0	$\dot{\epsilon}_0$ , factor to normalize strain rate
N	n, exponent for bcc metal
TROOM	$T_r$ , room temperature
PC	$p_c$ , Pressure cutoff

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SPALL	Spall Type: EQ.1.0: minimum pressure limit, EQ.2.0: maximum principal stress, EQ.3.0: minimum pressure cutoff.
C1	C <sub>1</sub> , coefficients for flow stress, see notes below.
C2	C <sub>2</sub> , coefficients for flow stress, see notes below.
C3	C <sub>3</sub> , coefficients for flow stress, see notes below.
C4	C <sub>4</sub> , coefficients for flow stress, see notes below.
C5	C <sub>5</sub> , coefficients for flow stress, see notes below.
C6	C <sub>6</sub> , coefficients for flow stress, see notes below.
EFAIL	Failure strain for erosion
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default) EQ.1.0: Viscoplastic formulation
B1	B <sub>1</sub> , coefficients for polynomial to represent temperature dependency of flow stress yield.
B2	B <sub>2</sub>
B3	B <sub>3</sub>
G1	G <sub>1</sub> , coefficients for defining heat capacity and temperature dependency of heat capacity.
G2	G <sub>2</sub>
G3	G <sub>3</sub>
G4	G <sub>4</sub>
BULK	Bulk modulus defined for shell elements only. Do not input for solid elements.

**Remarks:**

The Armstrong-Zerilli Material Model expresses the flow stress as follows.

For fcc metals ( $n = 0$ ),

$$\sigma = C_1 + \left\{ C_2(\varepsilon^p)^{1/2} [e^{[-C_3 + C_4 \ln(\dot{\varepsilon}^*)]T}] + C_5 \right\} \left[ \frac{\mu(T)}{\mu(293)} \right]$$

where,

$\varepsilon^p$  = effective plastic strain

$\dot{\varepsilon}^*$  = effective plastic strain rate

$$= \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}$$

and  $\dot{\varepsilon}_0 = 1, 1e-3, 1e-6$  for time units of seconds, milliseconds, and microseconds, respectively.

For bcc metals ( $n > 0$ ),

$$\sigma = C_1 + C_2 e^{[-C_3 + C_4 \ln(\dot{\varepsilon}^*)]T} + [C_5(\varepsilon^p)^n + C_6] \left[ \frac{\mu(T)}{\mu(293)} \right]$$

where

$$\frac{\mu(T)}{\mu(293)} = B_1 + B_2 T + B_3 T^2.$$

The relationship between heat capacity (specific heat) and temperature may be characterized by a cubic polynomial equation as follows:

$$C_p = G_1 + G_2 T + G_3 T^2 + G_4 T^3$$

A fully viscoplastic formulation is optional. An additional cost is incurred but the improvement in results can be dramatic.

**\*MAT\_LINEAR\_ELASTIC\_DISCRETE\_BEAM**

This is Material Type 66. This material model is defined for simulating the effects of a linear elastic beam by using six springs each acting about one of the six local degrees-of-freedom. The two nodes defining a beam may be coincident to give a zero length beam, or offset to give a finite length beam. For finite length discrete beams the absolute value of the variable SCOOR in the SECTION\_BEAM input should be set to a value of 2.0, which causes the local r-axis to be aligned along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this model. A triad is used to orient the beam for the directional springs. Translational/rotational stiffness and viscous damping effects are considered for a local cartesian system, see notes below. Applications for this element include the modeling of joint stiffnesses.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	TKR	TKS	TKT	RKR	RKS	RKT
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	TDR	TDS	TDT	RDR	RDS	RDT		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	FOR	FOS	FOT	MOR	MOS	MOT		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also "volume" in the *SECTION_BEAM definition.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
TKR	Translational stiffness along local r-axis, see notes below.
TKS	Translational stiffness along local s-axis.
TKT	Translational stiffness along local t-axis.
RKR	Rotational stiffness about the local r-axis.
RKS	Rotational stiffness about the local s-axis.
RKT	Rotational stiffness about the local t-axis.
TDR	Translational viscous damper along local r-axis. (Optional)
TDS	Translational viscous damper along local s-axis. (Optional)
TDT	Translational viscous damper along local t-axis. (Optional)
RDR	Rotational viscous damper about the local r-axis. (Optional)
RDS	Rotational viscous damper about the local s-axis. (Optional)
RDT	Rotational viscous damper about the local t-axis. (Optional)
FOR	Preload force in r-direction. (Optional)
FOS	Preload force in s-direction. (Optional)
FOT	Preload force in t-direction. (Optional)
MOR	Preload moment about r-axis. (Optional)
MOS	Preload moment about s-axis. (Optional)
MOT	Preload moment about t-axis. (Optional)

**Remarks:**

The formulation of the discrete beam (type 6) assumes that the beam is of zero length and requires no orientation node. A small distance between the nodes joined by the beam is permitted. The local coordinate system which determines (r,s,t) is given by the coordinate ID, see \*DEFINE\_COORDINATE\_OPTION, in the cross sectional input, see \*SECTION\_BEAM, where the global system is the default. The local coordinate system axes can rotate with either node of the beam or an average rotation of both nodes (see SCOOR variable in \*SECTION\_BEAM).

**\*MAT\_066**

**\*MAT\_LINEAR\_ELASTIC\_DISCRETE\_BEAM**

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For null stiffness coefficients, no forces corresponding to these null values will develop.  
The viscous damping coefficients are optional.



**Optional Failure Cards.** Cards 4 and 5 must be defined to consider failure; otherwise, they are optional.

Card 4	1	2	3	4	5	6	7	8
Variable	FFAILR	FFAILS	FFAILT	MFAILR	MFAILS	MFAILT		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

Card 5	1	2	3	4	5	6	7	8
Variable	UFAILR	UFAILS	UFAILT	TFAILR	TFAILS	TFAILT		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
LCIDTR	Load curve ID defining translational force resultant along local r-axis versus relative translational displacement, see Remarks and <a href="#">Figure M67-1</a> .
LCIDTS	Load curve ID defining translational force resultant along local s-axis versus relative translational displacement.
LCIDTT	Load curve ID defining translational force resultant along local t-axis versus relative translational displacement.
LCIDRR	Load curve ID defining rotational moment resultant about local r-axis versus relative rotational displacement.
LCIDRS	Load curve ID defining rotational moment resultant about local s-axis versus relative rotational displacement.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDRT	Load curve ID defining rotational moment resultant about local t-axis versus relative rotational displacement.
LCIDTDR	Load curve ID defining translational damping force resultant along local r-axis versus relative translational velocity.
LCIDTDS	Load curve ID defining translational damping force resultant along local s-axis versus relative translational velocity.
LCIDTDT	Load curve ID defining translational damping force resultant along local t-axis versus relative translational velocity.
LCIDRDR	Load curve ID defining rotational damping moment resultant about local r-axis versus relative rotational velocity.
LCIDRDS	Load curve ID defining rotational damping moment resultant about local s-axis versus relative rotational velocity.
LCIDRDT	Load curve ID defining rotational damping moment resultant about local t-axis versus relative rotational velocity.
FOR	Preload force in r-direction. (Optional)
FOS	Preload force in s-direction. (Optional)
FOT	Preload force in t-direction. (Optional)
MOR	Preload moment about r-axis. (Optional)
MOS	Preload moment about s-axis. (Optional)
MOT	Preload moment about t-axis. (Optional)
FFAILR	Optional failure parameter. If zero, the corresponding force, $F_r$ , is not considered in the failure calculation.
FFAILS	Optional failure parameter. If zero, the corresponding force, $F_s$ , is not considered in the failure calculation.
FFAILT	Optional failure parameter. If zero, the corresponding force, $F_t$ , is not considered in the failure calculation.
MFAILR	Optional failure parameter. If zero, the corresponding moment, $M_r$ , is not considered in the failure calculation.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MFAILS	Optional failure parameter. If zero, the corresponding moment, $M_s$ , is not considered in the failure calculation.
MFAILT	Optional failure parameter. If zero, the corresponding moment, $M_t$ , is not considered in the failure calculation.
UFAILR	Optional failure parameter. If zero, the corresponding displacement, $u_r$ , is not considered in the failure calculation.
UFAILS	Optional failure parameter. If zero, the corresponding displacement, $u_s$ , is not considered in the failure calculation.
UFAILT	Optional failure parameter. If zero, the corresponding displacement, $u_t$ , is not considered in the failure calculation.
TFAILR	Optional failure parameter. If zero, the corresponding rotation, $\theta_r$ , is not considered in the failure calculation.
TFAILS	Optional failure parameter. If zero, the corresponding rotation, $\theta_s$ , is not considered in the failure calculation.
TFAILT	Optional failure parameter. If zero, the corresponding rotation, $\theta_t$ , is not considered in the failure calculation.

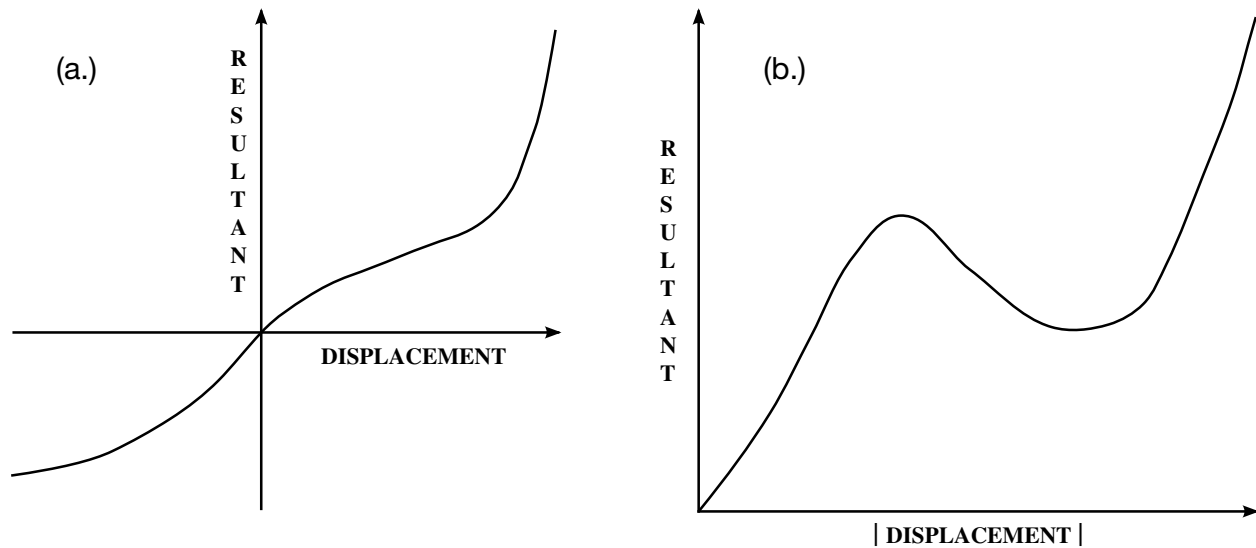
**Remarks:**

For null load curve ID's, no forces are computed.

The formulation of the discrete beam (type 6) assumes that the beam is of zero length and requires no orientation node. A small distance between the nodes joined by the beam is permitted. The local coordinate system which determines (r,s,t) is given by the coordinate ID, see *\*DEFINE\_COORDINATE\_OPTION*, in the cross sectional input, see *\*SECTION\_BEAM*, where the global system is the default. The local coordinate system axes can rotate with either node of the beam or an average rotation of both nodes (see *SCOOR* variable in *\*SECTION\_BEAM*).

If different behavior in tension and compression is desired in the calculation of the force resultants, the load curve(s) must be defined in the negative quadrant starting with the most negative displacement then increasing monotonically to the most positive. If the load curve behaves similarly in tension and compression, define only the positive quadrant. Whenever displacement values fall outside of the defined range, the resultant forces will be extrapolated. [Figure M67-1](#) depicts a typical load curve for a force resultant. Load curves used for determining the damping forces and moment resultants always act identically in

tension and compression, since only the positive quadrant values are considered, i.e., start the load curve at the origin [0,0].



**Figure M67-1.** The resultant forces and moments are determined by a table lookup. If the origin of the load curve is at [0,0] as in (b.) and tension and compression responses are symmetric.

Catastrophic failure based on force resultants occurs if the following inequality is satisfied.

$$\left(\frac{F_r}{F_r^{\text{fail}}}\right)^2 + \left(\frac{F_s}{F_s^{\text{fail}}}\right)^2 + \left(\frac{F_t}{F_t^{\text{fail}}}\right)^2 + \left(\frac{M_r}{M_r^{\text{fail}}}\right)^2 + \left(\frac{M_s}{M_s^{\text{fail}}}\right)^2 + \left(\frac{M_t}{M_t^{\text{fail}}}\right)^2 - 1. \geq 0.$$

After failure the discrete element is deleted. Likewise, catastrophic failure based on displacement resultants occurs if the following inequality is satisfied:

$$\left(\frac{u_r}{u_r^{\text{fail}}}\right)^2 + \left(\frac{u_s}{u_s^{\text{fail}}}\right)^2 + \left(\frac{u_t}{u_t^{\text{fail}}}\right)^2 + \left(\frac{\theta_r}{\theta_r^{\text{fail}}}\right)^2 + \left(\frac{\theta_s}{\theta_s^{\text{fail}}}\right)^2 + \left(\frac{\theta_t}{\theta_t^{\text{fail}}}\right)^2 - 1. \geq 0.$$

After failure the discrete element is deleted. If failure is included either one or both of the criteria may be used.





Card 4	1	2	3	4	5	6	7	8
Variable	FFAILR	FFAILS	FFAILT	MFAILR	MFAILS	MFAILT		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

Card 5	1	2	3	4	5	6	7	8
Variable	UFAILR	UFAILS	UFAILT	TFAILR	TFAILS	TFAILT		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

Card 6	1	2	3	4	5	6	7	8
Variable	FOR	FOS	FOT	MOR	MOS	MOT		
Type	F	F	F	F	F	F		

**VARIABLE**

**DESCRIPTION**

- MID Material identification. A unique number or label not exceeding 8 characters must be specified.
- RO Mass density, see also volume on \*SECTION\_BEAM definition.
- TKR Translational stiffness along local r-axis
- TKS Translational stiffness along local s-axis
- TKT Translational stiffness along local t-axis
- RKR Rotational stiffness about the local r-axis
- RKS Rotational stiffness about the local s-axis

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RKT	Rotational stiffness about the local t-axis
TDR	Translational viscous damper along local r-axis
TDS	Translational viscous damper along local s-axis
TDT	Translational viscous damper along local t-axis
RDR	Rotational viscous damper about the local r-axis
RDS	Rotational viscous damper about the local s-axis
RDT	Rotational viscous damper about the local t-axis
LCPDR	Load curve ID-yield force versus plastic displacement r-axis. If the curve ID is zero, and if TKR is nonzero, then elastic behavior is obtained for this component.
LCPDS	Load curve ID-yield force versus plastic displacement s-axis. If the curve ID is zero, and if TKS is nonzero, then elastic behavior is obtained for this component.
LCPDT	Load curve ID-yield force versus plastic displacement t-axis. If the curve ID is zero, and if TKT is nonzero, then elastic behavior is obtained for this component.
LCPMR	Load curve ID-yield moment versus plastic rotation r-axis. If the curve ID is zero, and if RKR is nonzero, then elastic behavior is obtained for this component.
LCPMS	Load curve ID-yield moment versus plastic rotation s-axis. If the curve ID is zero, and if RKS is nonzero, then elastic behavior is obtained for this component.
LCPMT	Load curve ID-yield moment versus plastic rotation t-axis. If the curve ID is zero, and if RKT is nonzero, then elastic behavior is obtained for this component.
FFAILR	Optional failure parameter. If zero, the corresponding force, $F_r$ , is not considered in the failure calculation.
FFAILS	Optional failure parameter. If zero, the corresponding force, $F_s$ , is not considered in the failure calculation.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FFAILT	Optional failure parameter. If zero, the corresponding force, $F_t$ , is not considered in the failure calculation.
MFAILR	Optional failure parameter. If zero, the corresponding moment, $M_r$ , is not considered in the failure calculation.
MFAILS	Optional failure parameter. If zero, the corresponding moment, $M_s$ , is not considered in the failure calculation.
MFAILT	Optional failure parameter. If zero, the corresponding moment, $M_t$ , is not considered in the failure calculation.
UFAILR	Optional failure parameter. If zero, the corresponding displacement, $u_r$ , is not considered in the failure calculation.
UFAILS	Optional failure parameter. If zero, the corresponding displacement, $u_s$ , is not considered in the failure calculation.
UFAILT	Optional failure parameter. If zero, the corresponding displacement, $u_t$ , is not considered in the failure calculation.
TFAILR	Optional failure parameter. If zero, the corresponding rotation, $\theta_r$ , is not considered in the failure calculation.
TFAILS	Optional failure parameter. If zero, the corresponding rotation, $\theta_s$ , is not considered in the failure calculation.
TFAILT	Optional failure parameter. If zero, the corresponding rotation, $\theta_t$ , is not considered in the failure calculation.
FOR	Preload force in r-direction. (Optional)
FOS	Preload force in s-direction. (Optional)
FOT	Preload force in t-direction. (Optional)
MOR	Preload moment about r-axis. (Optional)
MOS	Preload moment about s-axis. (Optional)
MOT	Preload moment about t-axis. (Optional)

**Remarks:**

For the translational and rotational degrees of freedom where elastic behavior is desired, set the load curve ID to zero.

The plastic displacement for the load curves is defined as:

$$\text{plastic displacement} = \text{total displacement} - \text{yield force/elastic stiffness}$$

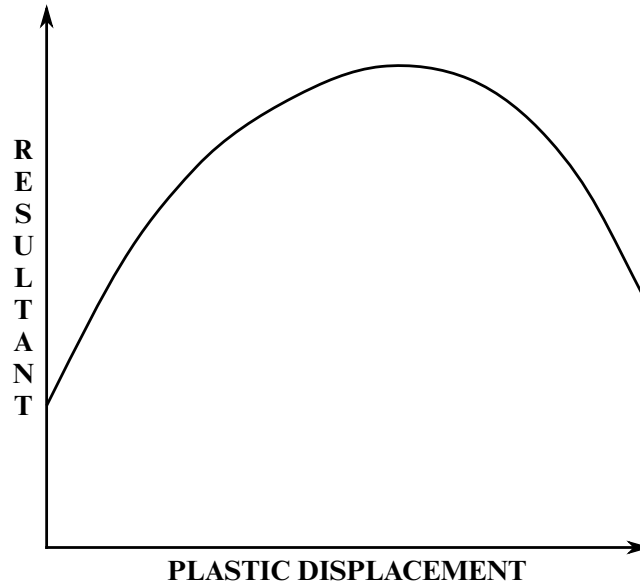
The formulation of the discrete beam (type 6) assumes that the beam is of zero length and requires no orientation node. A small distance between the nodes joined by the beam is permitted. The local coordinate system which determines (r,s,t) is given by the coordinate ID (see \*DEFINE\_COORDINATE\_OPTION) in the cross sectional input, see \*SECTION\_BEAM, where the global system is the default. The local coordinate system axes can rotate with either node of the beam or an average rotation of both nodes (see SCOOR variable in \*SECTION\_BEAM).

Catastrophic failure based on force resultants occurs if the following inequality is satisfied.

$$\left(\frac{F_r}{F_r^{\text{fail}}}\right)^2 + \left(\frac{F_s}{F_s^{\text{fail}}}\right)^2 + \left(\frac{F_t}{F_t^{\text{fail}}}\right)^2 + \left(\frac{M_r}{M_r^{\text{fail}}}\right)^2 + \left(\frac{M_s}{M_s^{\text{fail}}}\right)^2 + \left(\frac{M_t}{M_t^{\text{fail}}}\right)^2 - 1. \geq 0.$$

After failure the discrete element is deleted. Likewise, catastrophic failure based on displacement resultants occurs if the following inequality is satisfied:

$$\left(\frac{u_r}{u_r^{\text{fail}}}\right)^2 + \left(\frac{u_s}{u_s^{\text{fail}}}\right)^2 + \left(\frac{u_t}{u_t^{\text{fail}}}\right)^2 + \left(\frac{\theta_r}{\theta_r^{\text{fail}}}\right)^2 + \left(\frac{\theta_s}{\theta_s^{\text{fail}}}\right)^2 + \left(\frac{\theta_t}{\theta_t^{\text{fail}}}\right)^2 - 1. \geq 0.$$



**Figure M68-1.** The resultant forces and moments are limited by the yield definition. The initial yield point corresponds to a plastic displacement of zero.

After failure the discrete element is deleted. If failure is included either one or both of the criteria may be used.

**\*MAT\_SID\_DAMPER\_DISCRETE\_BEAM**

This is Material Type 69. The side impact dummy uses a damper that is not adequately treated by the nonlinear force versus relative velocity curves since the force characteristics are dependent on the displacement of the piston. See also notes below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ST	D	R	H	K	C
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	C3	STF	RHOF	C1	C2	LCIDF	LCIDD	S0
Type	F	F	F	F	F	F	F	F

**Orifice Cards.** Include on card per orifice. Read in up to 15 orifice locations. Input is terminated when a "\*" card is found. On the first card below the optional input parameters SF and DF may be specified.

Cards 3	1	2	3	4	5	6	7	8
Variable	ORFLOC	ORFRAD	SF	DC				
Type	F	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume on *SECTION_BEAM definition.
ST	$S_t$ , piston stroke. $S_t$ must equal or exceed the length of the beam element, see <a href="#">Figure M69-1</a> below.
D	d, piston diameter
R	R, default orifice radius

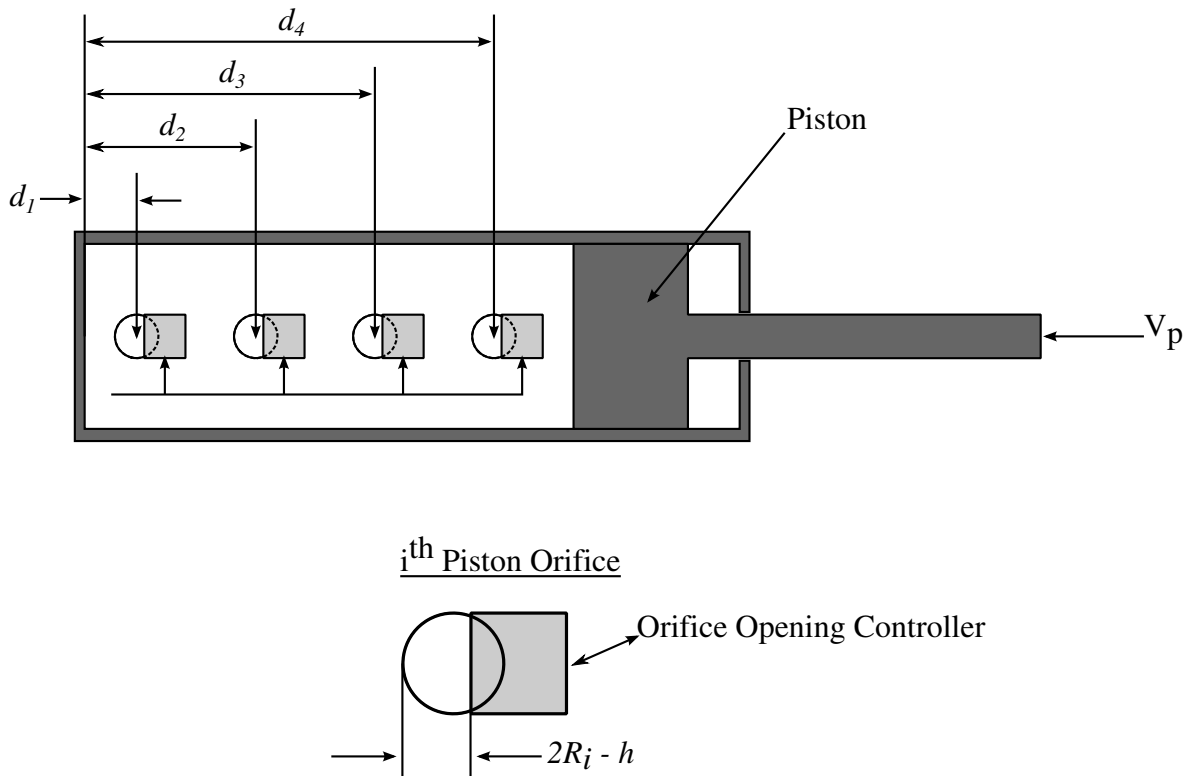
VARIABLE	DESCRIPTION
H	h, orifice controller position
K	K, damping constant LT.0.0: $ K $ is the load curve number ID, see *DEFINE_CURVE, defining the damping coefficient as a function of the absolute value of the relative velocity.
C	C, discharge coefficient
C3	Coefficient for fluid inertia term
STF	k, stiffness coefficient if piston bottoms out
RHOF	$\rho_{fluid}$ , fluid density
C1	$C_1$ , coefficient for linear velocity term
C2	$C_2$ , coefficient for quadratic velocity term
LCIDF	Load curve number ID defining force versus piston displacement, $s$ , i.e., term $f(s + s_0)$ . Compressive behavior is defined in the positive quadrant of the force displacement curve. Displacements falling outside of the defined force displacement curve are extrapolated. Care must be taken to ensure that extrapolated values are reasonable.
LCIDD	Load curve number ID defining damping coefficient versus piston displacement, $s$ , i.e., $g(s + s_0)$ . Displacements falling outside the defined curve are extrapolated. Care must be taken to ensure that extrapolated values are reasonable.
S0	Initial displacement $s_0$ , typically set to zero. A positive displacement corresponds to compressive behavior.
ORFLOC	$d_i$ , orifice location of $i$ th orifice relative to the fixed end.
ORFRAD	$r_i$ , orifice radius of $i$ th orifice, if zero the default radius is used.
SF	Scale factor on calculated force. The default is set to 1.0
DC	$c$ , linear viscous damping coefficient used after damper bottoms out either in tension or compression.

**Remarks:**

As the damper moves, the fluid flows through the open orifices to provide the necessary damping resistance. While moving as shown in Figure M69-1 the piston gradually blocks off and effectively closes the orifices. The number of orifices and the size of their opening control the damper resistance and performance. The damping force is computed from,

$$F = SF \times \left\{ KA_p V_p \left\{ \frac{C_1}{A_0^t} + C_2 |V_p| \rho_{\text{fluid}} \left[ \left( \frac{A_p}{CA_0^t} \right)^2 - 1 \right] \right\} - f(s + s_0) + V_p g(s + s_0) \right\}$$

where K is a user defined constant or a tabulated function of the absolute value of the relative velocity,  $V_p$  is the piston velocity, C is the discharge coefficient,  $A_p$  is the piston area,  $A_0^t$  is the total open areas of orifices at time t,  $\rho_{\text{fluid}}$  is the fluid density,  $C_1$  is the coefficient for the linear term, and  $C_2$  is the coefficient for the quadratic term.



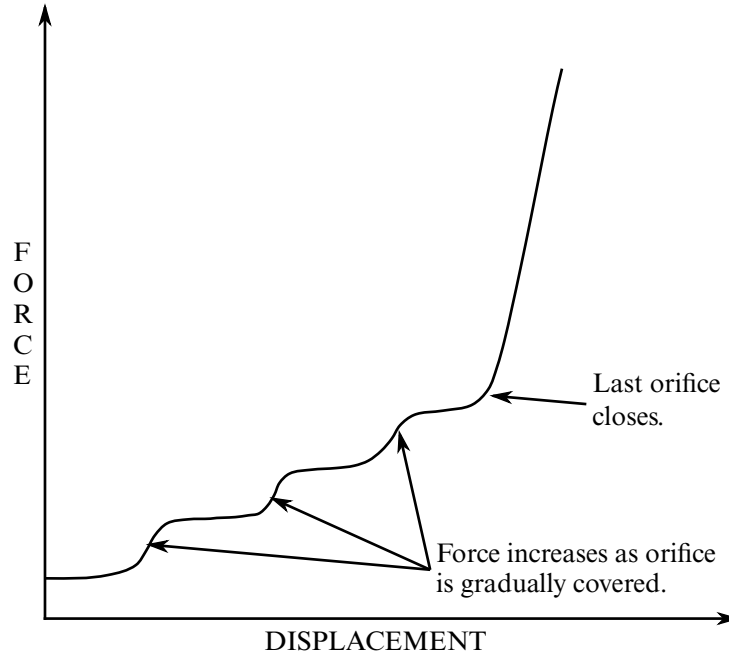
**Figure M69-1.** Mathematical model for the Side Impact Dummy damper.

In the implementation, the orifices are assumed to be circular with partial covering by the orifice controller. As the piston closes, the closure of the orifice is gradual. This gradual closure is properly taken into account to insure a smooth response. If the piston stroke is exceeded, the stiffness value, k, limits further movement, i.e., if the damper bottoms out in tension or compression the damper forces are calculated by replacing the damper by a bottoming out spring and damper, k and c, respectively. The piston stroke must exceed the initial length of the beam element. The time step calculation is based in part on the stiffness



value of the bottoming out spring. A typical force versus displacement curve at constant relative velocity is shown in [Figure M69-2](#).

The factor, SF, which scales the force defaults to 1.0 and is analogous to the adjusting ring on the damper.



**Figure M69-2.** Force versus displacement as orifices are covered at a constant relative velocity. Only the linear velocity term is active.

**\*MAT\_070****\*MAT\_HYDRAULIC\_GAS\_DAMPER\_DISCRETE\_BEAM****\*MAT\_HYDRAULIC\_GAS\_DAMPER\_DISCRETE\_BEAM**

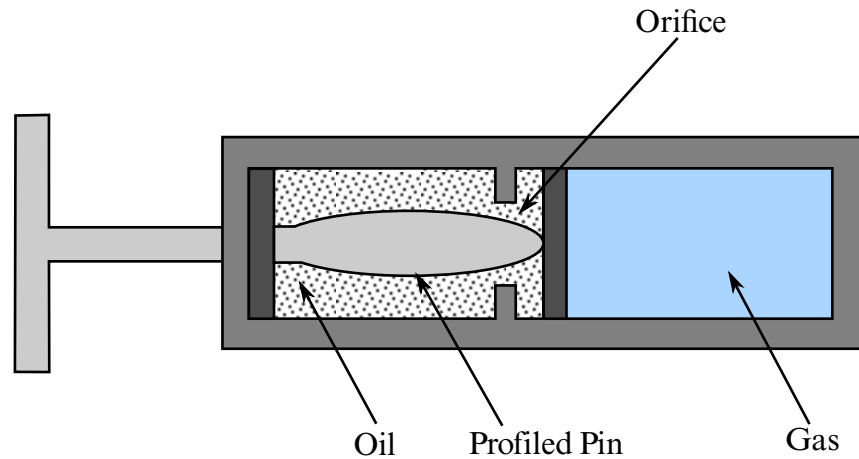
This is Material Type 70. This special purpose element represents a combined hydraulic and gas-filled damper which has a variable orifice coefficient. A schematic of the damper is shown in [Figure M70-1](#). Dampers of this type are sometimes used on buffers at the end of railroad tracks and as aircraft undercarriage shock absorbers. This material can be used only as a discrete beam element. See also notes below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	CO	N	P0	PA	AP	KH
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	FR	SCLF	CLEAR				
Type	F	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
CO	Length of gas column, $C_0$
N	Adiabatic constant
P0	Initial gas pressure, $P_0$
PA	Atmospheric pressure, $P_a$
AP	Piston cross sectional area, $A_p$
KH	Hydraulic constant, $K$
LCID	Load curve ID, see *DEFINE_CURVE, defining the orifice area, $a_0$ , versus element deflection.



**Figure M70-1.** Schematic of Hydraulic/Gas damper.

VARIABLE	DESCRIPTION
FR	Return factor on orifice force. This acts as a factor on the hydraulic force only and is applied when unloading. It is intended to represent a valve that opens when the piston unloads to relieve hydraulic pressure. Set it to 1.0 for no such relief.
SCLF	Scale factor on force. (Default = 1.0)
CLEAR	Clearance (if nonzero, no tensile force develops for positive displacements and negative forces develop only after the clearance is closed.

### Remarks:

As the damper is compressed two actions contribute to the force which develops. First, the gas is adiabatically compressed into a smaller volume. Secondly, oil is forced through an orifice. A profiled pin may occupy some of the cross-sectional area of the orifice; thus, the orifice area available for the oil varies with the stroke. The force is assumed proportional to the square of the velocity and inversely proportional to the available area.

The equation for this element is:

$$F = \text{SCLF} \times \left\{ K_h \left( \frac{V}{a_0} \right)^2 + \left[ P_0 \left( \frac{C_0}{C_0 - S} \right)^n - P_a \right] A_p \right\}$$

where S is the element deflection and V is the relative velocity across the element.

# \*MAT\_071

# \*MAT\_CABLE\_DISCRETE\_BEAM

## \*MAT\_CABLE\_DISCRETE\_BEAM

This is Material Type 71. This model permits elastic cables to be realistically modeled; thus, no force will develop in compression.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	LCID	F0	TMAXF0	TRAMP	IREAD
Type	A8	F	F	F	F	F	F	I
Default	none	none	none	none	0	0	0	0

Additional card for IREAD > 1.

Card 2	1	2	3	4	5	6	7	8
Variable	OUTPUT	TSTART	FRACLO	MXEPS	MXFRC			
Type	I	F	F	F	F			
Default	0	0	0	1.0E+20	1.0E+20			

### VARIABLE

### DESCRIPTION

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
E	GT.0.0: Young's modulus LT.0.0: Stiffness
LCID	Load curve ID, see *DEFINE_CURVE, defining the stress versus engineering strain. (Optional).
F0	Initial tensile force. If F0 is defined, an offset is not needed for an initial tensile force.
TMAXF0	Time for which pre-tension force will be held

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TRAMP	Ramp-up time for pre-tension force
IREAD	Set to 1 to read second line of input
OUTPUT	Flag = 1 to output axial strain (see note below concerning OUTPUT)
TSTART	Time at which the ramp-up of pre-tension begins
FRACL0	Fraction of initial length that should be reached over time period of TRAMP. Corresponding tensile force builds up as necessary to reach cable length = FRACL0 × L0 at time t = TRAMP.
MXEPS	Maximum strain at failure
MXFRC	Maximum force at failure

**Remarks:**

The force,  $F$ , generated by the cable is nonzero if and only if the cable is tension. The force is given by:

$$F = \max(F_0 + K\Delta L, 0.)$$

where  $\Delta L$  is the change in length

$$\Delta L = \text{current length} - (\text{initial length} - \text{offset})$$

and the stiffness ( $E > 0.0$  only) is defined as:

$$K = \frac{E \times \text{area}}{(\text{initial length} - \text{offset})}$$

Note that a constant force element can be obtained by setting:

$$F_0 > 0 \text{ and } K = 0$$

although the application of such an element is unknown.

The area and offset are defined on either the cross section or element cards. For a slack cable the offset should be input as a negative length. For an initial tensile force the offset should be positive.

If a load curve is specified the Young's modulus will be ignored and the load curve will be used instead. The points on the load curve are defined as engineering stress versus engineering strain, i.e., the change in length over the initial length. The unloading behavior follows the loading.

By default, cable pretension is applied only at the start of the analysis. If the cable is attached to flexible structure, deformation of the structure will result in relaxation of the cables, which will therefore lose some or all of the intended preload.

This can be overcome by using TMAXF0. In this case, it is expected that the structure will deform under the loading from the cables and that this deformation will take time to occur during the analysis. The unstressed length of the cable will be continuously adjusted until time TMAXF0 such that the force is maintained at the user-defined pre-tension force – this is analogous to operation of the pre-tensioning screws in real cables. After time TMAXF0, the unstressed length is fixed and the force in the cable is determined in the normal way using the stiffness and change of length.

Sudden application of the cable forces at time zero may result in an excessively dynamic response during pre-tensioning. A ramp-up time TRAMP may optionally be defined. The cable force ramps up from zero at time TSTART to the full pre-tension F0 at time TSTART + TRAMP. TMAXF0, if set less than TSTART + TRAMP by the user, will be internally reset to TSTART + TRAMP.

If the model does not use dynamic relaxation, it is recommended that damping be applied during pre-tensioning so that the structure reaches a steady state by time TMAXF0.

If the model uses dynamic relaxation, TSTART, TRAMP, and TMAXF0 apply only during dynamic relaxation. The cable preload at the end of dynamic relaxation carries over to the start of the subsequent transient analysis.

The cable mass will be calculated from length  $\times$  area  $\times$  density if VOL is set to zero on \*SECTION\_BEAM. Otherwise, VOL  $\times$  density will be used.

If OUTPUT is set to 1, one additional history variable representing axial strain is output to d3plot for the cable elements. This axial strain can be plotted by LS-PrePost by selecting the beam component labeled as “axial stress”. Though the label says “axial stress”, it is actually axial strain.

If the stress-strain load curve option, LCID, is combined with preload, two types of behavior are available:

1. If the preload is applied using the TMAXF0/TRAMP method, the initial strain is calculated from the stress-strain curve to achieve the desired preload.
2. If TMAXF0/TRAMP are not used, the preload force is taken as additional to the force calculated from the stress/strain curve. Thus, the total stress in the cable will be higher than indicated by the stress/strain curve.







Card 8	1	2	3	4	5	6	7	8
Variable	$\eta_9$	$\eta_{10}$	$\eta_{11}$	$\eta_{12}$	$\eta_{13}$			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
PR	Poisson's ratio.
SIGF	Maximum principal stress for failure.
A0	Cohesion.
A1	Pressure hardening coefficient.
A2	Pressure hardening coefficient.
A0Y	Cohesion for yield
A1Y	Pressure hardening coefficient for yield limit
A2Y	Pressure hardening coefficient for yield limit
A1F	Pressure hardening coefficient for failed material.
A2F	Pressure hardening coefficient for failed material.
B1	Damage scaling factor.
B2	Damage scaling factor for uniaxial tensile path.
B3	Damage scaling factor for triaxial tensile path.
PER	Percent reinforcement.
ER	Elastic modulus for reinforcement.

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PRR	Poisson's ratio for reinforcement.
SIGY	Initial yield stress.
ETAN	Tangent modulus/plastic hardening modulus.
LCP	Load curve ID giving rate sensitivity for principal material, see *DEFINE_CURVE.
LCR	Load curve ID giving rate sensitivity for reinforcement, see *DEFINE_CURVE.
$\lambda_1 - \lambda_{13}$	Tabulated damage function
$\eta_1 - \eta_{13}$	Tabulated scale factor.

**Remarks:**

1. Cohesion for failed material  $a_{0f} = 0$ .
2. B3 must be positive or zero.
3.  $\lambda_n \leq \lambda_{n+1}$ . The first point must be zero.

\*MAT\_CONCRETE\_DAMAGE\_REL3

This is Material Type 72R3. The Karagozian & Case (K&C) Concrete Model - Release III is a three-invariant model, uses three shear failure surfaces, includes damage and strain-rate effects, and has origins based on the Pseudo-TENSOR Model (Material Type 16). The most significant user improvement provided by Release III is a model parameter generation capability, based solely on the unconfined compression strength of the concrete. The implementation of Release III significantly changed the user input, thus previous input files using Material Type 72, i.e. prior to LS-DYNA Version 971, are not compatible with the present input format.

An open source reference, that precedes the parameter generation capability, is provided in Malvar et al. [1997]. A workshop proceedings reference, Malvar et al. [1996], is useful, but may be difficult to obtain. More recent, but *limited distribution* reference materials, e.g. Malvar et al. [2000], may be obtained by contacting Karagozian & Case.

Seven card images are required to define the *complete* set of model parameters for the K&C Concrete Model. An Equation-of-State is also required for the pressure-volume strain response. Brief descriptions of all the input parameters are provided below, however it is expected that this model will be used primarily with the option to automatically generate the model parameters based on the unconfined compression strength of the concrete. These generated material parameters, along with the generated parameters for \*EOS\_TABULATED\_COMPACTION, are written to the d3hsp file.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	PR					
Type	A8	F	F					
Default	none	none	none					

Card 2	1	2	3	4	5	6	7	8
Variable	FT	A0	A1	A2	B1	OMEGA	A1F	
Type	F	F	F	F	F	F	F	
Default	none	0.0	0.0	0.0	0.0	none	0.0	



Card 7	1	2	3	4	5	6	7	8
Variable	$\eta_{09}$	$\eta_{10}$	$\eta_{11}$	$\eta_{12}$	$\eta_{13}$	B2	A2F	A2Y
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
PR	Poisson's ratio, $\nu$ .
FT	Uniaxial tensile strength, $f_t$ .
A0	Maximum shear failure surface parameter, $a_0$ or $-f'_c$ for <b>parameter generation</b> (recommended).
A1	Maximum shear failure surface parameter, $a_1$ .
A2	Maximum shear failure surface parameter, $a_2$ .
B1	Compressive damage scaling parameter, $b_1$
OMEGA	Fractional dilatancy, $\omega$ .
A1F	Residual failure surface coefficient, $a_{1f}$ .
S $\lambda$	$\lambda$ stretch factor, $s$ .
NOUT	Output selector for effective plastic strain (see table).
EDROP	Post peak dilatancy decay, $N^\alpha$ .
RSIZE	Unit conversion factor for length (inches/user-unit), e.g. 39.37 if user length unit in meters.
UCF	Unit conversion factor for stress (psi/user-unit), e.g. 145 if $f'_c$ in MPa.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCRATE	Define (load) curve number for strain-rate effects; effective strain rate on abscissa (negative = tension) and strength enhancement on ordinate. If LCRATE is set to -1, strain rate effects are automatically included, based on equations provided in Wu, Crawford, Lan, and Magallanes [2014].
LOCWID	Three times the maximum aggregate diameter (input in user length units).
NPTS	Number of points in $\lambda$ versus $\eta$ damage relation; must be 13 points.
$\lambda 01$	1 <sup>st</sup> value of damage function, (a.k.a., 1 <sup>st</sup> value of “modified” effective plastic strain; see references for details).
$\lambda 02$	2 <sup>nd</sup> value of damage function,
$\lambda 03$	3 <sup>rd</sup> value of damage function,
$\lambda 04$	4 <sup>th</sup> value of damage function,
$\lambda 05$	5 <sup>th</sup> value of damage function,
$\lambda 06$	6 <sup>th</sup> value of damage function,
$\lambda 07$	7 <sup>th</sup> value of damage function,
$\lambda 08$	8 <sup>th</sup> value of damage function,
$\lambda 09$	9 <sup>th</sup> value of damage function,
$\lambda 10$	10 <sup>th</sup> value of damage function,
$\lambda 11$	11 <sup>th</sup> value of damage function,
$\lambda 12$	12 <sup>th</sup> value of damage function,
$\lambda 13$	13 <sup>th</sup> value of damage function.
B3	Damage scaling coefficient for triaxial tension, $b_3$ .
A0Y	Initial yield surface cohesion, $a_{0y}$ .
A1Y	Initial yield surface coefficient, $a_{1y}$ .
$\eta 01$	1 <sup>st</sup> value of scale factor,
$\eta 02$	2 <sup>nd</sup> value of scale factor,

VARIABLE	DESCRIPTION
η03	3 <sup>rd</sup> value of scale factor,
η04	4 <sup>th</sup> value of scale factor,
η05	5 <sup>th</sup> value of scale factor,
η06	6 <sup>th</sup> value of scale factor,
η07	7 <sup>th</sup> value of scale factor,
η08	8 <sup>th</sup> value of scale factor,
η09	9 <sup>th</sup> value of scale factor,
η10	10 <sup>th</sup> value of scale factor,
η11	11 <sup>th</sup> value of scale factor,
η12	12 <sup>th</sup> value of scale factor,
η13	13 <sup>th</sup> value of scale factor.
B2	Tensile damage scaling exponent, $b_2$ .
A2F	Residual failure surface coefficient, $a_{2f}$ .
A2Y	Initial yield surface coefficient, $a_{2y}$ .

$\lambda$ , sometimes referred to as “modified” effective plastic strain, is computed internally as a function of effective plastic strain, strain rate enhancement factor, and pressure.  $\eta$  is a function of  $\lambda$  as specified by the  $\eta$  vs.  $\lambda$  curve. The  $\eta$  value, which is always between 0 and 1, is used to interpolate between the yield failure surface and the maximum failure surface, or between the maximum failure surface and the residual failure surface, depending on whether  $\lambda$  is to the left or right of the first peak in the the  $\eta$  vs.  $\lambda$  curve. The “scaled damage measure” ranges from 0 to 1 as the material transitions from the yield failure surface to the maximum failure surface, and thereafter ranges from 1 to 2 as the material ranges from the maximum failure surface to the residual failure surface. See the references for details.

**Output of Selected Variables:**

The quantity labeled as “plastic strain” by LS-PrePost is actually the quantity described in [Table M72-1](#), in accordance with the input value of NOUT (see Card 3 above).

NOUT	Function	Description
1		Current shear failure surface radius
2	$\delta = 2\lambda / (\lambda + \lambda_m)$	Scaled damage measure
3	$\dot{\sigma}_{ij} \dot{\epsilon}_{ij}$	Strain energy (rate)
4	$\dot{\sigma}_{ij} \dot{\epsilon}_{ij}^p$	Plastic strain energy (rate)

**Table M72-1.** Description of quantity labeled “plastic strain” by LS-PrePost.

An additional six extra history variables as shown in [Table M72-2](#) may be written by setting NEIPH = 6 on the keyword \*DATABASE\_EXTENT\_BINARY. The extra history variables are labeled as "history var#1" through "history var#6" in LS-PrePost.

Label	Description
history var#1	Internal energy
history var#2	Pressure from bulk viscosity
history var#3	Volume in previous time step
history var#4	Plastic volumetric strain
history var#5	Slope of damage evolution ( $\eta$ vs. $\lambda$ ) curve
history var#6	“Modified” effective plastic strain ( $\lambda$ )

**Table M72-2.** Extra History Variables for \*MAT\_072R3

**Sample Input for Concrete:**

As an example of the K&C Concrete Model material parameter generation, the following sample input for a 45.4 MPa (6,580 psi) unconfined compression strength concrete is provided. The basic units for the provided parameters are length in millimeters (mm), time in milliseconds (msec), and mass in grams (g). This base unit set yields units of force in Newtons (N) and pressure in Mega-Pascals (MPa).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	PR					
Type	72	2.3E-3						





**\*MAT\_072R3****\*MAT\_CONCRETE\_DAMAGE\_REL3**

Card 7	1	2	3	4	5	6	7	8
Variable	$\eta_{09}$	$\eta_{10}$	$\eta_{11}$	$\eta_{12}$	$\eta_{13}$	B2	A2F	A2Y
Type								

Shear strength enhancement factor versus effective strain rate is given by a curve (\*DEFINE\_CURVE) with LCID 723. The sample input values, see Malvar & Ross [1998], are given in [Table M72-3](#).

Strain-Rate (1/ms)	Enhancement
-3.0E+01	9.70
-3.0E-01	9.70
-1.0E-01	6.72
-3.0E-02	4.50
-1.0E-02	3.12
-3.0E-03	2.09
-1.0E-03	1.45
-1.0E-04	1.36
-1.0E-05	1.28
-1.0E-06	1.20
-1.0E-07	1.13
-1.0E-08	1.06
0.0E+00	1.00
3.0E-08	1.00
1.0E-07	1.03
1.0E-06	1.08
1.0E-05	1.14
1.0E-04	1.20
1.0E-03	1.26
3.0E-03	1.29
1.0E-02	1.33
3.0E-02	1.36
1.0E-01	2.04
3.0E-01	2.94
3.0E+01	2.94

**Table M72-3.** Enhancement versus effective strain rate for 45.4 MPa concrete (sample). When defining curve LCRATE, input negative (tensile) values of effective strain rate first. The enhancement should be positive and should be 1.0 at a strain rate of zero.

# \*MAT\_073

# \*MAT\_LOW\_DENSITY\_VISCOUS\_FOAM

## \*MAT\_LOW\_DENSITY\_VISCOUS\_FOAM

This is Material Type 73 for Modeling Low Density Urethane Foam with high compressibility and with rate sensitivity which can be characterized by a relaxation curve. Its main applications are for seat cushions, padding on the Side Impact Dummies (SID), bumpers, and interior foams. Optionally, a tension cut-off failure can be defined. Also, see the notes below and the description of material 57: \*MAT\_LOW\_DENSITY\_FOAM.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	LCID	TC	HU	BETA	DAMP
Type	A8	F	F	F	F	F	F	F
Default					1.E+20	1.		0.05
Remarks						3	1	

Card 2	1	2	3	4	5	6	7	8
Variable	SHAPE	FAIL	BVFLAG	KCON	LCID2	BSTART	TRAMP	NV
Type	F	F	F	F	F	F	F	I
Default	1.0	0.0	0.0	0.0	0	0.0	0.0	6

**Relaxation Constant Cards.** If  $LCID2 = 0$  then include the following viscoelastic constants. Up to 6 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 6 cards are used.

Card 3	1	2	3	4	5	6	7	8
Variable	G1	BETA1	REF					
Type	F	F	F					

**Frequency Dependence Card.** If LCID2 = -1 then include the following frequency dependent viscoelastic data.

Card 4	1	2	3	4	5	6	7	8
Variable	LCID3	LCID4	SCALEW	SCALEA				
Type	I	I	I	I				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young’s modulus used in tension. For implicit problems E is set to the initial slope of load curve LCID.
LCID	Load curve ID, see *DEFINE_CURVE, for nominal stress versus strain.
TC	Tension cut-off stress
HU	Hysteretic unloading factor between 0 and 1 (default = 1, i.e., no energy dissipation), see also <a href="#">Figure M57-1</a>
BETA	β, decay constant to model creep in unloading. EQ.0.0: No relaxation.
DAMP	Viscous coefficient (.05 < recommended value <.50) to model damping effects. LT.0.0:  DAMP  is the load curve ID, which defines the damping constant as a function of the maximum strain in compression defined as: $\epsilon_{max} = \max(1 - \lambda_1, 1 - \lambda_2, 1 - \lambda_3)$ In tension, the damping constant is set to the value corresponding to the strain at 0. The abscissa should be defined from 0 to 1.
SHAPE	Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor. Values less than one reduces the energy dissipation and greater than one increases dissipation, see also <a href="#">Figure M57-1</a> .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FAIL	Failure option after cutoff stress is reached: EQ.0.0: tensile stress remains at cut-off value, EQ.1.0: tensile stress is reset to zero.
BVFLAG	Bulk viscosity activation flag, see remark below: EQ.0.0: no bulk viscosity (recommended), EQ.1.0: bulk viscosity active.
KCON	Stiffness coefficient for contact interface stiffness. Maximum slope in stress vs. strain curve is used. When the maximum slope is taken for the contact, the time step size for this material is reduced for stability. In some cases $\Delta t$ may be significantly smaller, and defining a reasonable stiffness is recommended.
LCID2	Load curve ID of relaxation curve. If constants $\beta_t$ are determined via a least squares fit. This relaxation curve is shown in <a href="#">Figure M76-1</a> . This model ignores the constant stress.
BSTART	Fit parameter. In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$ , $\beta_4$ is 10 times greater than $\beta_3$ , and so on. If zero, BSTART = .01.
TRAMP	Optional ramp time for loading.
NV	Number of terms in fit. If zero, the default is 6. Currently, the maximum number is set to 6. Values of 2 or 3 are recommended, since each term used adds significantly to the cost. Caution should be exercised when taking the results from the fit. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once a satisfactory fit has been achieved it is recommended that the coefficients which are written into the output file be input in future runs.
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETA $_i$	Optional decay constant if $i^{\text{th}}$ term

VARIABLE	DESCRIPTION
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.
LCID3	Load curve ID giving the magnitude of the shear modulus as a function of the frequency. LCID3 must use the same frequencies as LCID4.
LCID4	Load curve ID giving the phase angle of the shear modulus as a function of the frequency. LCID4 must use the same frequencies as LCID3.
SCALEW	Flag for the form of the frequency data. EQ.0.0: Frequency is in cycles per unit time. EQ.1.0: Circular frequency.
SCALEA	Flag for the units of the phase angle. EQ.0.0: Degrees. EQ.1.0: Radians.

### Material Formulation:

This viscoelastic foam model is available to model highly compressible viscous foams. The hyperelastic formulation of this model follows that of Material 57.

Rate effects are accounted for through linear viscoelasticity by a convolution integral of the form

$$\sigma_{ij}^r = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  is the relaxation function. The stress tensor,  $\sigma_{ij}^r$ , augments the stresses determined from the foam,  $\sigma_{ij}^f$ ; consequently, the final stress,  $\sigma_{ij}$ , is taken as the summation of the two contributions:

$$\sigma_{ij} = \sigma_{ij}^f + \sigma_{ij}^r.$$

Since we wish to include only simple rate effects, the relaxation function is represented by up to six terms of the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta_m t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. The formulation is performed in the local system of principal stretches where only the principal values of stress are computed and triaxial coupling is avoided. Consequently, the one-dimensional nature of this foam material is unaffected by this addition of rate effects. The addition of rate effects necessitates 42 additional history variables per integration point. The cost and memory overhead of this model comes primarily from the need to “remember” the local system of principal stretches and the evaluation of the viscous stress components.

Frequency data can be fit to the Prony series. Using Fourier transforms the relationship between the relaxation function and the frequency dependent data is

$$G_s(\omega) = \alpha_0 + \sum_{m=1}^N \frac{\alpha_m (\omega/\beta_m)^2}{1 + (\omega/\beta_m)^2}$$

$$G_l(\omega) = \sum_{m=1}^N \frac{\alpha_m \omega/\beta_m}{1 + \omega/\beta_m}$$

where the storage modulus and loss modulus are defined in terms of the frequency dependent magnitude  $G$  and phase angle  $\phi$  given by load curves LCID3 and LCID4 respectively,

$$G_s(\omega) = G(\omega) \cos[\phi(\omega)] , \text{ and}$$

$$G_l(\omega) = G(\omega) \sin[\phi(\omega)]$$

### Remarks:

When hysteretic unloading is used the reloading will follow the unloading curve if the decay constant,  $\beta$ , is set to zero. If  $\beta$  is nonzero the decay to the original loading curve is governed by the expression:

$$1 - e^{-\beta t}$$

The bulk viscosity, which generates a rate dependent pressure, may cause an unexpected volumetric response and, consequently, it is optional with this model.

The hysteretic unloading factor results in the unloading curve to lie beneath the loading curve as shown in [Figure M57-1](#). This unloading provides energy dissipation which is reasonable in certain kinds of foam.



**\*MAT\_ELASTIC\_SPRING\_DISCRETE\_BEAM**

This is Material Type 74. This model permits elastic springs with damping to be combined and represented with a discrete beam element type 6. Linear stiffness and damping coefficients can be defined, and, for nonlinear behavior, a force versus deflection and force versus rate curves can be used. Displacement based failure and an initial force are optional.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	F0	D	CDF	TDF	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	FLCID	HLCID	C1	C2	DLE	GLCID		
Type	F	F	F	F	F	I		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
K	Stiffness coefficient.
F0	Optional initial force. This option is inactive if this material is referenced in a part referenced by material type *MAT_ELASTIC_6DOF_SPRING
D	Viscous damping coefficient.
CDF	Compressive displacement at failure. Input as a positive number. After failure, no forces are carried. This option does not apply to zero length springs. EQ.0.0: inactive.
TDF	Tensile displacement at failure. After failure, no forces are carried.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FLCID	Load curve ID, see *DEFINE_CURVE, defining force versus deflection for nonlinear behavior.
HLCID	Load curve ID, see *DEFINE_CURVE, defining force versus relative velocity for nonlinear behavior (optional). If the origin of the curve is at (0,0) the force magnitude is identical for a given magnitude of the relative velocity, i.e., only the sign changes.
C1	Damping coefficient for nonlinear behavior (optional).
C2	Damping coefficient for nonlinear behavior (optional).
DLE	Factor to scale time units. The default is unity.
GLCID	Optional load curve ID, see *DEFINE_CURVE, defining a scale factor versus deflection for load curve ID, HLCID. If zero, a scale factor of unity is assumed.

**Remarks:**

If the linear spring stiffness is used, the force,  $F$ , is given by:

$$F = F_0 + K\Delta L + D\Delta\dot{L}$$

but if the load curve ID is specified, the force is then given by:

$$F = F_0 + Kf(\Delta L) \left\{ 1 + C1 \times \Delta\dot{L} + C2 \times \text{sgn}(\Delta\dot{L}) \ln \left[ \max \left( 1, \frac{\Delta\dot{L}}{DLE} \right) \right] \right\} + D\Delta\dot{L} + g(\Delta L)h(\Delta\dot{L})$$

In these equations,  $\Delta L$  is the change in length

$$\Delta L = \text{current length} - \text{initial length}$$

The cross sectional area is defined on the section card for the discrete beam elements, See \*SECTION\_BEAM. The square root of this area is used as the contact thickness offset if these elements are included in the contact treatment.

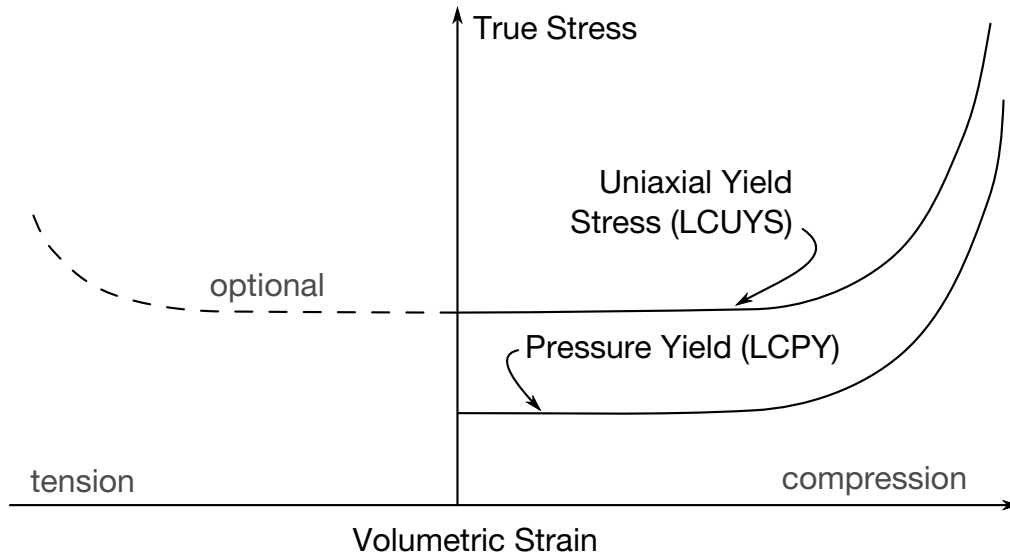
## \*MAT\_BILKHU/DUBOIS\_FOAM

This is Material Type 75. This model is for the simulation of isotropic crushable foams. Uniaxial and triaxial test data are used to describe the behavior.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	YM	LCPY	LCUYS	VC	PC	VPC
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	TSC	VTSC	LCRATE	PR	KCON	ISFLG	NCYCLE	
Type	I	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
YM	Young's modulus $E$
LCPY	Load curve ID giving pressure for plastic yielding versus volumetric strain, see <a href="#">Figure M75-1</a> .
LCUYS	Load curve ID giving uniaxial yield stress versus volumetric strain, see <a href="#">Figure M75-1</a> , all abscissa values should be positive if only the results of a compression test are included, optionally the results of a tensile test can be added (corresponding to negative values of the volumetric strain), in the latter case PC, VPC, TC and VTC will be ignored.
VC	Viscous damping coefficient ( $0.05 < \text{recommended value} < 0.50$ ; default is 0.05).
PC	Pressure cutoff for hydrostatic tension. If zero, the default is set to one-tenth of $p_0$ , the yield pressure corresponding to a volumetric strain of zero. PC will be ignored if TC is non zero.



**Figure M75-1.** Behavior of crushable foam. Unloading is elastic.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
VPC	Variable pressure cutoff for hydrostatic tension as a fraction of pressure yield value. If non-zero this will override the pressure cutoff value PC.
TC	Tension cutoff for uniaxial tensile stress. Default is zero. A nonzero value is recommended for better stability.
VTC	Variable tension cutoff for uniaxial tensile stress as a fraction of the uniaxial compressive yield strength, if non-zero this will override the tension cutoff value TC.
LCRATE	Load curve ID giving a scale factor for the previous yield curves, dependent upon the volumetric strain rate.
PR	Poisson coefficient, which applies to both elastic and plastic deformations, must be smaller than 0.5
KCON	Stiffness coefficient for contact interface stiffness. If undefined one-third of Young's modulus, YM, is used. KCON is also considered in the element time step calculation; therefore, large values may reduce the element time step size.

VARIABLE	DESCRIPTION
ISFLG	Flag for tensile response (active only if negative abscissa are present in load curve LCUYS) EQ.0: load curve abscissa in tensile region correspond to volumetric strain EQ.1: load curve abscissa in tensile region correspond to effective strain (for large PR, when volumetric strain vanishes)
NCYCLE	Number of cycles to determine the average volumetric strain rate. NCYCLE is 1 by default (no smoothing) and cannot exceed 100.

**Remarks:**

The logarithmic volumetric strain is defined in terms of the relative volume,  $V$ , as:

$$\gamma = -\ln(V)$$

If option ISFLG = 1 is used, the effective strain is defined in the usual way:

$$\varepsilon_{\text{eff}} = \sqrt{\frac{2}{3} \text{tr}(\varepsilon^t \varepsilon)}$$

In defining the load curve LCPY the stress and strain pairs should be positive values starting with a volumetric strain value of zero.

The load curve LCUYS can optionally contain the results of the tensile test (corresponding to negative values of the volumetric strain), if so, then the load curve information will override PC, VPC, TC and VTC. This is the recommended approach, because the necessary continuity between tensile and compressive regime becomes obvious (see Figure above).

The yield surface is defined as an ellipse in the equivalent pressure and von Mises stress plane. This ellipse is characterized by three points:

1. the hydrostatic compression limit (LCPY),
2. the uniaxial compression limit (LCUYS), and
3. either the pressure cutoff for hydrostatic stress (PC,VPC), the tension cutoff for uniaxial tension (TC,VTC), or the optional tensile part of LCUYS.

To prevent high frequency oscillations in the strain rate from causing similar high frequency oscillations in the yield stress, a modified volumetric strain rate is used obtain the scaled yield stress. The modified strain rate is obtained as follows. If NYCLE is > 1, then the modified strain rate is obtained by a time average of the actual strain rate over NCYCLE solution cycles. The averaged strain rate is stored on history variable #3.

**\*MAT\_GENERAL\_VISCOELASTIC\_{OPTION}**

The available options include:

<BLANK>

MOISTURE

This is Material Type 76. This material model provides a general viscoelastic Maxwell model having up to 18 terms in the Prony series expansion and is useful for modeling dense continuum rubbers and solid explosives. Either the coefficients of the Prony series expansion or a relaxation curve may be specified to define the viscoelastic deviatoric and bulk behavior.

The material model can also be used with laminated shell. Either an elastic or viscoelastic layer can be defined with the laminated formulation. To activate laminated shell you need the laminated formulation flag on \*CONTROL\_SHELL. With the laminated option a user defined integration rule is needed.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	PCF	EF	TREF	A	B
Type	A8	F	F	F	F	F	F	F

**Relaxation Curve Card.** Leave blank if the *Prony Series Cards* are used below. Also, leave blank if an elastic layer is defined in a laminated shell.

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	NT	BSTART	TRAMP	LCIDK	NTK	BSTARTK	TRAMPK
Type	F	I	F	F	F	I	F	F

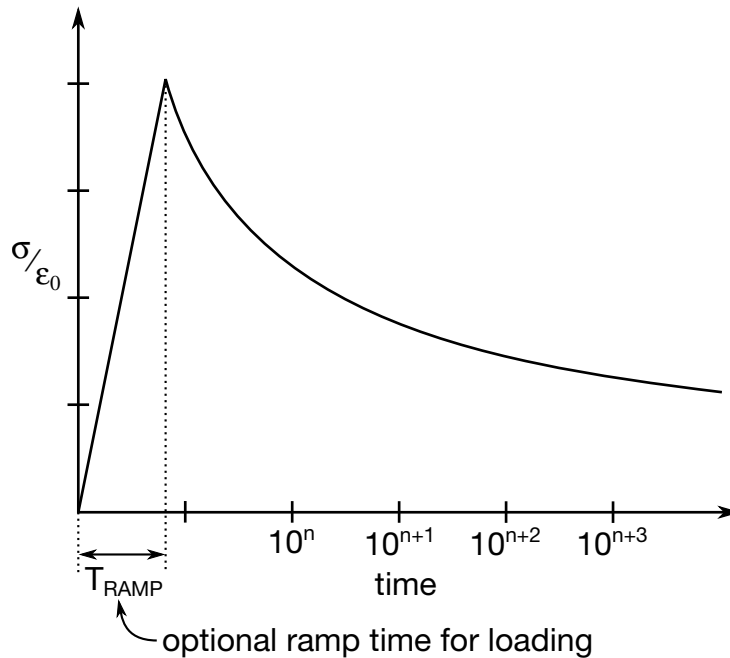
**Moisture Card.** Additional card for MOISTURE keyword option.

Card 3	1	2	3	4	5	6	7	8
Variable	M0	ALPHA	BETA	GAMMA	M			
Type	F	F	F	F	F			

**Prony Series cards.** Card Format for viscoelastic constants. Up to 18 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 18 cards are used. These cards are not needed if relaxation data is defined. The number of terms for the shear behavior may differ from that for the bulk behavior: insert zero if a term is not included. If an elastic layer is defined you only need to define GI and KI (note in an elastic layer only one card is needed)

Card 4	1	2	3	4	5	6	7	8
Variable	GI	BETAI	KI	BETAKI				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
BULK	Elastic bulk modulus.
PCF	Tensile pressure elimination flag for solid elements only. If set to unity tensile pressures are set to zero.
EF	Elastic flag (if equal 1, the layer is elastic. If 0 the layer is viscoelastic).
TREF	Reference temperature for shift function (must be greater than zero).
A	Coefficient for the Arrhenius and the Williams-Landel-Ferry shift functions.
B	Coefficient for the Williams-Landel-Ferry shift function.
LCID	Load curve ID for deviatoric relaxation behavior. If LCID is given, constants $G_i$ , and $\beta_i$ are determined via a least squares fit. See <a href="#">Figure M76-1</a> for an example relaxation curve.
NT	Number of terms in shear fit. If zero the default is 6. Fewer than NT terms will be used if the fit produces one or more negative shear moduli. Currently, the maximum number is set to 18.
BSTART	In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$ , $\beta_4$ is 10 times $\beta_3$ , and so on. If zero, BSTART is determined by an iterative trial and error scheme.



**Figure M76-1.** Relaxation curves for deviatoric behavior and bulk behavior. The ordinate of LCID is the deviatoric stress divided by (2 times the constant value of deviatoric strain) where the stress and strain are in the direction of the prescribed strain, or in non-directional terms, the effective stress divided by (3 times the effective strain). LCIDK defines the mean stress divided by the constant value of volumetric strain imposed in a hydrostatic stress relaxation experiment, versus time. For best results, the points defined in the curve should be equally spaced on the logarithmic scale. Note the values for the abscissa are input as time, not log(time). Furthermore, the curve should be smooth and defined in the positive quadrant. If nonphysical values are determined by least squares fit, LS-DYNA will terminate with an error message after the initialization phase is completed. If the ramp time for loading is included, then the relaxation which occurs during the loading phase is taken into account. This effect may or may not be important.

VARIABLE	DESCRIPTION
TRAMP	Optional ramp time for loading.
LCIDK	Load curve ID for bulk relaxation behavior. If LCIDK is given, constants $K_i$ , and $\beta_{ki}$ are determined via a least squares fit. See <a href="#">Figure M76-1</a> for an example relaxation curve.
NTK	Number of terms desired in bulk fit. If zero the default is 6. Currently, the maximum number is set to 18.
BSTARTK	In the fit, $\beta_{k1}$ , is set to zero, $\beta_{k2}$ is set to BSTARTK, $\beta_{k3}$ is 10 times $\beta_{k2}$ , $\beta_{k4}$ is 100 times greater than $\beta_{k3}$ , and so on. If zero, BSTARTK is determined by an iterative trial and error scheme.



VARIABLE	DESCRIPTION
TRAMPK	Optional ramp time for bulk loading.
MST	Moisture, $M$ . If the moisture is 0.0, the moisture option is disabled. GT.0.0: Specifies a curve ID to make moisture a function of time. LT.0.0: Specifies the negative of a constant value of moisture.
MO	Initial moisture, $M_0$ . Defaults to zero.
ALPHA	Specifies $\alpha$ as a function of moisture. GT.0.0: Specifies a curve ID. LT.0.0: Specifies the negative of a constant value.
BETA	Specifies $\beta$ as a function of moisture. GT.0.0: Specifies a curve ID. LT.0.0: Specifies the negative of a constant value.
GAMMA	Specifies $\gamma$ as a function of moisture. GT.0.0: Specifies a curve ID. LT.0.0: Specifies the negative of a constant value.
GI	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETAI	Optional shear decay constant for the $i^{\text{th}}$ term
KI	Optional bulk relaxation modulus for the $i^{\text{th}}$ term
BETAKI	Optional bulk decay constant for the $i^{\text{th}}$ term

**Remarks:**

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  is the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by 18 terms from the Prony series:

$$g(t) = \sum_{m=1}^N G_m e^{-\beta_m t}$$

We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . An arbitrary number of terms, up to 18, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk moduli:

$$k(t) = \sum_{m=1}^N K_m e^{-\beta_{k_m} t}$$

The Arrhenius and Williams-Landel-Ferry (WLF) shift functions account for the effects of the temperature on the stress relaxation. A scaled time,  $t'$ ,

$$t' = \int_0^t \Phi(T) dt$$

is used in the relaxation function instead of the physical time. The Arrhenius shift function is

$$\Phi(T) = \exp \left[ -A \left( \frac{1}{T} - \frac{1}{T_{\text{REF}}} \right) \right]$$

and the Williams-Landel-Ferry shift function is

$$\Phi(T) = \exp \left( -A \frac{T - T_{\text{REF}}}{B + T - T_{\text{REF}}} \right)$$

If all three values ( $T_{\text{REF}}$ ,  $A$ , and  $B$ ) are not zero, the WLF function is used; the Arrhenius function is used if  $B$  is zero; and no scaling is applied if all three values are zero.

The moisture model allows the scaling of the material properties as a function of the moisture content of the material. The shear and bulk moduli are scaled by  $\alpha$ , the decay constants are scaled by  $\beta$ , and a moisture strain,  $\gamma(M)[M - M_O]$  is introduced analogous to the thermal strain.

\*MAT\_HYPERELASTIC\_RUBBER

This is Material Type 77. This material model provides a general hyperelastic rubber model combined optionally with linear viscoelasticity as outlined by Christensen [1980].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	PR	N	NV	G	SIGF	REF
Type	A8	F	F	I	I	F	F	F

**Hysteresis Card.** Additional card read in when PR < 0 (Mullins Effect).

Card 2	1	2	3	4	5	6	7	8
Variable	TBHYS							
Type	F							

**Card 3 for N > 0.** For N > 0 a least squares fit is computed from uniaxial data.

Card 3	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LCID1	DATA	LCID2	BSTART	TRAMP
Type	F	F	F	F	F	F	F	F

**Card 3 for N = 0.** Set the material parameters directly.

Card 3	1	2	3	4	5	6	7	8
Variable	C10	C01	C11	C20	C02	C30	THERML	
Type	F	F	F	F	F	F	F	

**Optional Viscoelastic Constants & Frictional Damping Constant Cards.** Up to 12 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 12 cards are used.

Card 4	1	2	3	4	5	6	7	8
Variable	Gi	BETAi	Gj	SIGFj				
Type	F	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
PR	Poisson's ratio (> .49 is recommended, smaller values may not work and should not be used). If this is set to a negative number, then the absolute value is used and an extra card is read for Mullins effect.
TBHYS	Table ID for hysteresis, could be positive or negative, see <a href="#">Remarks 1</a> and <a href="#">2</a> .
N	Number of constants to solve for: EQ.1: Solve for C10 and C01 EQ.2: Solve for C10, C01, C11, C20, and C02 EQ.3: Solve for C10, C01, C11, C20, C02, and C30
NV	Number of Prony series terms in fit. If zero, the default is 6. Currently, the maximum number is set to 12. Values less than 12, possibly 3 - 5 are recommended, since each term used adds significantly to the cost. Caution should be exercised when taking the results from the fit. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once a satisfactory fit has been achieved it is recommended that the coefficients which are written into the output file be input in future runs.

VARIABLE	DESCRIPTION
G	Shear modulus for frequency independent damping. Frequency independent damping is based of a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250 - 1000 times greater than SIGF.
SIGF	Limit stress for frequency independent frictional damping.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.

**If N>0 test information from a uniaxial test are used.**

SGL	Specimen gauge length
SW	Specimen width
ST	Specimen thickness
LCID1	Load curve ID giving the force versus actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), then curve LCID1 is also engineering stress versus engineering strain.
DATA	Type of experimental data. EQ.0.0: uniaxial data (Only option for this model)
LCID2	Load curve ID of the deviatoric stress relaxation curve, neglecting the long term deviatoric stress. If LCID2 is given, constants $G_i$ and $\beta_i$ are determined via a least squares fit. See M76-1 for an example relaxation curve. The ordinate of the curve is the viscoelastic deviatoric stress divided by (2 times the constant value of deviatoric strain) where the stress and strain are in the direction of the prescribed strain, or in non-directional terms, the effective stress divided by (3 times the effective strain).
BSTART	In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$ , $\beta_4$ is 10 times $\beta_3$ , and so on. If zero, BSTART is determined by an iterative trial and error scheme.
TRAMP	Optional ramp time for loading.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
<b>If N=0, the following constants have to be defined:</b>	
C10	$C_{10}$
C01	$C_{01}$
C11	$C_{11}$
C20	$C_{20}$
C02	$C_{02}$
C30	$C_{30}$
THERML	Flag for the thermal option. If THERML > 0.0, then G, SIGF, C10 and C01 specify curve IDs giving the values as functions of temperature, otherwise they specify the constants. This option is available only for solid elements.
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETA $i$	Optional decay constant if $i^{\text{th}}$ term
$G_j$	Optional shear modulus for frequency independent damping represented as the $j^{\text{th}}$ spring and slider in series in parallel to the rest of the stress contributions.
SIGF $j$	Limit stress for frequency independent, frictional, damping represented as the $j^{\text{th}}$ spring and slider in series in parallel to the rest of the stress contributions.

**Background:**

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material a hydrostatic work term,  $W_H(J)$ , is included in the strain energy functional which is function of the relative volume,  $J$ , [Ogden 1984]:

$$W(J_1, J_2, J) = \sum_{p,q=0}^n C_{pq} (J_1 - 3)^p (J_2 - 3)^q + W_H(J)$$

$$J_1 = I_1 I_3^{-1/3}$$

$$J_2 = I_2 I_3^{-2/3}$$

In order to prevent volumetric work from contributing to the hydrostatic work the first and second invariants are modified as shown. This procedure is described in more detail by Sussman and Bathe [1987].

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \epsilon_{kl}}{\partial \tau} d\tau$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl}(t - \tau) \frac{\partial E_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta t}$$

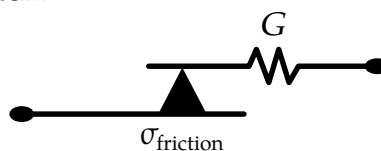
given by,

$$g(t) = \sum_{i=1}^n G_i e^{-\beta_i t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . The viscoelastic behavior is optional and an arbitrary number of terms may be used. In order to avoid a constant shear modulus from this visco-elastic formulation, a term in the series is included only when  $\beta_i > 0$ .

The Mooney-Rivlin rubber model (model 27) is obtained by specifying  $n = 1$ . In spite of the differences in formulations with model 27, we find that the results obtained with this model are nearly identical with those of material 27 as long as large values of Poisson's ratio are used.

The frequency independent damping is obtained by the having a spring and slider in series as shown in the following sketch:



Several springs and sliders in series can be defined that are put in parallel to the rest of the stress contributions of this material model.

**Remarks:**

1. **Hysteresis (TBHYS > 0).** If a positive table ID for hysteresis is defined, then TBHYS is a table having curves that are functions of strain-energy density. Let  $W_{\text{dev}}$  be the current value of the deviatoric strain energy density as calculated above. Furthermore, let  $\bar{W}_{\text{dev}}$  be the peak strain energy density reached up to this point in time. It is then assumed that the resulting stress is reduced by a damage factor according to

$$\mathbf{S} = D(W_{\text{dev}}, \bar{W}_{\text{dev}}) \frac{\partial W_{\text{dev}}}{\partial \mathbf{E}} + \frac{\partial W_{\text{vol}}}{\partial \mathbf{E}}.$$

where  $D(W_{\text{dev}}, \bar{W}_{\text{dev}})$  is the damage factor which is input as the table, TBHYS. This table consists of curves giving stress reduction (between 0 and 1) as a function of  $W_{\text{dev}}$  indexed by  $\bar{W}_{\text{dev}}$ .

Each  $\bar{W}_{\text{dev}}$  curve must be valid for strain energy densities between 0 and  $\bar{W}_{\text{dev}}$ . It is *recommended* that each curve be monotonically increasing, and it is *required* that each curve equals 1 when  $W_{\text{dev}} > \bar{W}_{\text{dev}}$ . Additionally, \*DEFINE\_TABLE *requires* that each curve have the same beginning and end point and, furthermore, that they not cross except at the boundaries, although they are not required to cross.

This table can be estimated from a uniaxial quasistatic compression test as follows:

- a) Load the specimen to a maximum displacement  $\bar{d}$  and measure the force as function of displacement:  $f_{\text{load}}(\bar{d})$ .
- b) Unload the specimen again measuring the force as a function of displacement:  $f_{\text{unload}}(d)$ .
- c) The strain energy density is, then, given as a function of the loaded displacement as

$$W_{\text{dev}}(d) = \frac{1}{V} \int_0^d f_{\text{load}}(s) ds.$$

- i) The peak energy, which is used to index the data set, is given by
 
$$\bar{W}_{\text{dev}} = W_{\text{dev}}(\bar{d}).$$
- ii) From this energy curve we can also determine the inverse:  $d(W_{\text{dev}})$ . Using this inverse the load curve for LS-DYNA is then given by:

$$D(W_{\text{dev}}, \bar{W}_{\text{dev}}) = \frac{f_{\text{unload}}[d(W_{\text{dev}})]}{f_{\text{load}}[d(\bar{W}_{\text{dev}})]}.$$

- d) This procedure is repeated for different values of  $\bar{d}$  (or equivalently  $\bar{W}_{\text{dev}}$ ).



2. **Hysteresis (TBHYS < 0).** If a negative table ID for hysteresis is defined, then all of the above holds. The difference being that the load curves comprising table, |TBHYS|, must give the strain-energy density,  $W_{\text{dev}}$ , as a function of the stress reduction factor. *This scheme guarantees that all curves have the same beginning point, 0, and the same end point, 1.* For negative TBHYS the user provides  $W_{\text{dev}}(D, \bar{W}_{\text{dev}})$  instead of  $D(W_{\text{dev}}, \bar{W}_{\text{dev}})$ . In practice, this case corresponds to swapping the load curve axes.

**\*MAT\_OGDEN\_RUBBER**

This is also Material Type 77. This material model provides the Ogden [1984] rubber model combined optionally with linear viscoelasticity as outlined by Christensen [1980].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PR	N	NV	G	SIGF	REF
Type	A8	F	F	I	I	F	F	F

**Hysteresis Card.** Additional card read in when  $PR < 0$  (Mullins Effect).

Card 2	1	2	3	4	5	6	7	8
Variable	TBHYS							
Type	F							

**Card 3 for  $N > 0$ .** For  $N > 0$  a least squares fit is computed from uniaxial data.

Card 3	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LCID1	DATA	LCID2	BSTART	TRAMP
Type	F	F	F	F	F	F		F

**Card 3 for  $N = 0$ .** Set the material parameters directly.

Card 3	1	2	3	4	5	6	7	8
Variable	MU1	MU2	MU3	MU4	MU5	MU6	MU7	MU8
Type	F	F	F	F	F	F	F	F

Card 4 for N = 0. Set the material parameters directly.

Card 4	1	2	3	4	5	6	7	8
Variable	ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8
Type	F	F	F	F	F	F	F	F

Optional Viscoelastic Constants Cards. Up to 12 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 12 cards are used.

Card 5	1	2	3	4	5	6	7	8
Variable	GI	BETA1	VFLAG					
Type	F	F	I					
Default			0					

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
PR	Poisson's ratio ( $\geq 49$ is recommended; smaller values may not work and should not be used). If this is set to a negative number, then the absolute value is used and an extra card is read for Mullins effect.
N	Order of fit to the Ogden model, (currently $< 9$ , 2 generally works okay). The constants generated during the fit are printed in the output file and can be directly input in future runs, thereby, saving the cost of performing the nonlinear fit. The users need to check the correction of the fit results before proceeding to compute.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
NV	Number of Prony series terms in fit. If zero, the default is 6. Currently, the maximum number is set to 12. Values less than 12, possibly 3-5 are recommended, since each term used adds significantly to the cost. Caution should be exercised when taking the results from the fit. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once a satisfactory fit has been achieved it is recommended that the coefficients which are written into the output file be input in future runs.
G	Shear modulus for frequency independent damping. Frequency independent damping is based on a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250-1000 times greater than SIGF.
SIGF	Limit stress for frequency independent frictional damping.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.
TBHYS	Table ID for hysteresis, could be positive or negative, see Remarks on *MAT_HYPERELASTIC_RUBBER

**If N > 0 test information from a uniaxial test are used:**

SGL	Specimen gauge length
SW	Specimen width
ST	Specimen thickness
LCID1	Load curve ID giving the force versus actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), then curve LCID1 is also engineering stress versus engineering strain.

VARIABLE	DESCRIPTION
DATA	Type of experimental data. EQ.1.0: uniaxial data (default) EQ.2.0: biaxial data EQ.3.0: pure shear data
LCID2	Load curve ID of the deviatoric stress relaxation curve, neglecting the long term deviatoric stress. If LCID2 is given, constants $G_i$ and $\beta_i$ are determined via a least squares fit. See M76-1 for an example relaxation curve. The ordinate of the curve is the viscoelastic deviatoric stress divided by (2 times the constant value of deviatoric strain) where the stress and strain are in the direction of the prescribed strain, or in non-directional terms, the effective stress divided by (3 times the effective strain).
BSTART	In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$ , $\beta_4$ is 10 times $\beta_3$ , and so on. If zero, BSTART is determined by an iterative trial and error scheme.
TRAMP	Optional ramp time for loading.
MU $i$	$\mu_i$ , the $i^{\text{th}}$ shear modulus, $i$ varies up to 8. See discussion below.
ALPHA $i$	$\alpha_i$ , the $i^{\text{th}}$ exponent, $i$ varies up to 8. See discussion below.
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETA $i$	Optional decay constant if $i^{\text{th}}$ term
VFLAG	Flag for the viscoelasticity formulation. This appears only on the first line defining $G_i$ , BETA $i$ , and VFLAG. If VFLAG = 0, the standard viscoelasticity formulation is used (the default), and if VFLAG = 1, the viscoelasticity formulation using the instantaneous elastic stress is used.

### Remarks:

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material a hydrostatic work term is included in the strain energy functional which is function of the relative volume,  $J$ , [Ogden 1984]:

$$W^* = \sum_{i=1}^3 \sum_{j=1}^n \frac{\mu_j}{\alpha_j} (\lambda_i^{*\alpha_j} - 1) + K(J - 1 - \ln J)$$

The asterisk (\*) indicates that the volumetric effects have been eliminated from the principal stretches,  $\lambda_j^*$ . The number of terms,  $n$ , may vary from 1 to 8 inclusive, and  $K$  is the bulk modulus.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl}(t - \tau) \frac{\partial E_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta t}$$

given by,

$$g(t) = \sum_{i=1}^n G_i e^{-\beta_i t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . The viscoelastic behavior is optional and an arbitrary number of terms may be used. In order to avoid a constant shear modulus from this viscoelastic formulation, a term in the series is included only when  $\beta_i > 0$ .

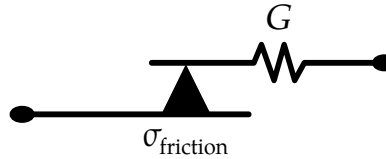
For  $VFLAG = 1$ , the viscoelastic term is

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \sigma_{kl}^E}{\partial \tau} d\tau$$

where  $\sigma_{kl}^E$  is the instantaneous stress evaluated from the internal energy functional. The coefficients in the Prony series therefore correspond to normalized relaxation moduli instead of elastic moduli.

The Mooney-Rivlin rubber model (model 27) is obtained by specifying  $n = 1$ . In spite of the differences in formulations with Model 27, we find that the results obtained with this model are nearly identical with those of Material 27 as long as large values of Poisson's ratio are used.

The frequency independent damping is obtained by the having a spring and slider in series as shown in the following sketch:



# \*MAT\_078

# \*MAT\_SOIL\_CONCRETE

## \*MAT\_SOIL\_CONCRETE

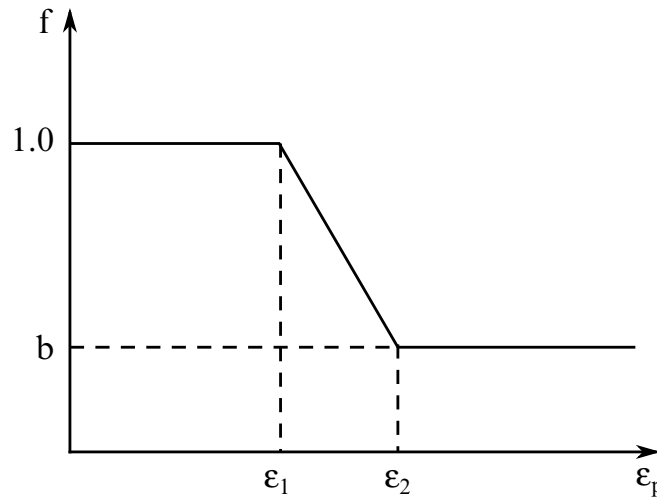
This is Material Type 78. This model permits concrete and soil to be efficiently modeled. See the explanations below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G	K	LCPV	LCYP	LCFP	LCRP
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	PC	OUT	B	FAIL				
Type	F	F	F	F				

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
G	Shear modulus
K	Bulk modulus
LCPV	Load curve ID for pressure versus volumetric strain. The pressure versus volumetric strain curve is defined in compression only. The sign convention requires that both pressure and compressive strain be defined as positive values where the compressive strain is taken as the negative value of the natural logarithm of the relative volume.
LCYP	Load curve ID for yield versus pressure: GT.0: von Mises stress versus pressure, LT.0: Second stress invariant, $J_2$ , versus pressure. This curve must be defined.
LCFP	Load curve ID for plastic strain at which fracture begins versus pressure. This load curve ID must be defined if $B > 0.0$ .



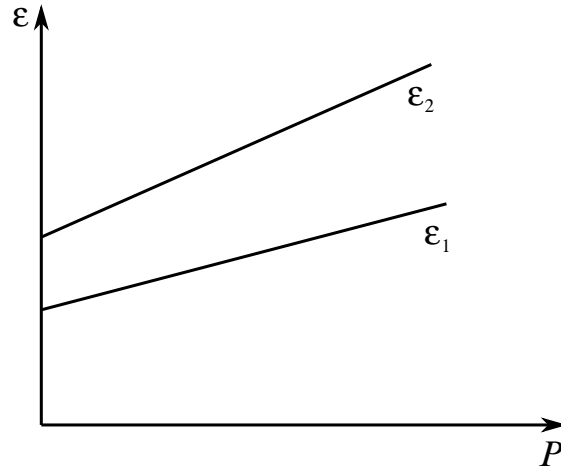


**Figure M78-1.** Strength reduction factor.

VARIABLE	DESCRIPTION
LCRP	Load curve ID for plastic strain at which residual strength is reached versus pressure. This load curve ID must be defined if $B > 0.0$ .
PC	Pressure cutoff for tensile fracture
OUT	Output option for plastic strain in database: EQ.0: volumetric plastic strain, EQ.1: deviatoric plastic strain.
B	Residual strength factor after cracking, see <a href="#">Figure M78-1</a> .
FAIL	Flag for failure: EQ.0: no failure, EQ.1: When pressure reaches failure pressure element is eroded, EQ.2: When pressure reaches failure pressure element loses its ability to carry tension.

### Remarks:

Pressure is positive in compression. Volumetric strain is defined as the natural log of the relative volume and is *positive* in compression where the relative volume,  $V$ , is the ratio of the current volume to the initial volume. The tabulated data should be given in order of increasing compression. If the pressure drops below the cutoff value specified, it is reset to that value and the deviatoric stress state is eliminated.



**Figure M78-2.** Cracking strain versus pressure.

If the load curve ID (LCYP) is provided as a positive number, the deviatoric, perfectly plastic, pressure dependent, yield function  $\phi$ , is given as

$$\phi = \sqrt{3J_2} - F(p) = \sigma_y - F(p)$$

where  $F(p)$  is a tabulated function of yield stress versus pressure, and the second invariant,  $J_2$ , is defined in terms of the deviatoric stress tensor as:

$$J_2 = \frac{1}{2} S_{ij} S_{ij}$$

assuming that if the ID is given as negative then the yield function becomes:

$$\phi = J_2 - F(p)$$

being the deviatoric stress tensor.

If cracking is invoked by setting the residual strength factor,  $B$ , on card 2 to a value between 0.0 and 1.0, the yield stress is multiplied by a factor  $f$  which reduces with plastic strain according to a trilinear law as shown in [Figure M78-1](#).

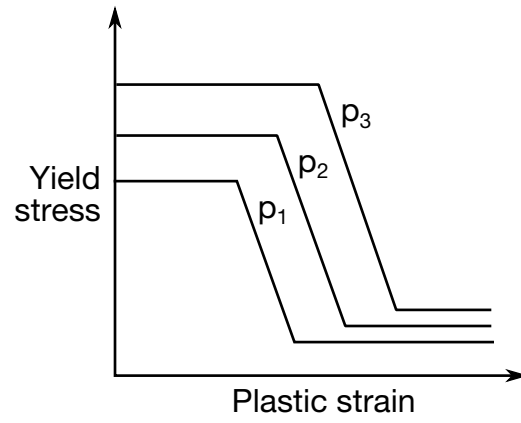
$b$  = residual strength factor

$\epsilon_1$  = plastic stain at which cracking begins.

$\epsilon_2$  = plastic stain at which residual strength is reached.

$\epsilon_1$  and  $\epsilon_2$  are tabulated functions of pressure that are defined by load curves, see [Figure M78-2](#). The values on the curves are pressure versus strain and should be entered in order of increasing pressure. The strain values should always increase monotonically with pressure.

By properly defining the load curves, it is possible to obtain the desired strength and ductility over a range of pressures, see [Figure M78-3](#).



**Figure M78-3.** Yield stress as a function of plastic strain.

**\*MAT\_HYSTERETIC\_SOIL**

This is Material Type 79. This model is a nested surface model with up to ten superposed “layers” of elastic perfectly plastic material, each with its own elastic moduli and yield values. Nested surface models give hysteric behavior, as the different “layers” yield at different stresses. See Remarks below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K0	P0	B	A0	A1	A2
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	DF	RP	LCID	SFLC	DIL_A	DIL_B	DIL_C	DIL_D
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	GAM1	GAM2	GAM3	GAM4	GAM5	LCD	LCSR	PINIT
Type	F	F	F	F	F	I	I	I

Card 4	1	2	3	4	5	6	7	8
Variable	TAU1	TAU2	TAU3	TAU4	TAU5			
Type	F	F	F	F	F			

**VARIABLE****DESCRIPTION**

MID                      Material identification. A unique number or label not exceeding 8 characters must be specified.

RO                        Mass density

<b>VARIABLE</b>	<b>DESCRIPTION</b>
K0	Bulk modulus at the reference pressure
P0	Cut-off/datum pressure (must be $0 \leq$ i.e. tensile). Below this pressure, stiffness and strength disappears; this is also the “zero” pressure for pressure-varying properties.
B	B is the exponent for the pressure-sensitive elastic moduli. <a href="#">See remarks</a> . B must be in the range $0 \leq B < 1$ , and values too close to 1 are not recommended because the pressure becomes indeterminate.
A0	Yield function constant $a_0$ (Default = 1.0), see Material Type 5.
A1	Yield function constant $a_1$ (Default = 0.0), see Material Type 5.
A2	Yield function constant $a_2$ (Default = 0.0), see Material Type 5.
DF	Damping factor. Must be in the range $0 \leq df \leq 1$ : EQ.0: no damping, EQ.1: maximum damping.
RP	Reference pressure for following input data.
LCID	Load curve ID defining shear stress versus shear strain. Up to ten points may be defined in the load curve. See *DEFINE_CURVE.
SFLD	Scale factor to apply to shear stress in LCID.
DIL_A	Dilation parameter A
DIL_B	Dilation parameter B
DIL_C	Dilation parameter C
DIL_D	Dilation parameter D
GAM1	$\gamma_1$ , shear strain (ignored if LCID is non zero).
GAM2	$\gamma_2$ , shear strain (ignored if LCID is non zero).
GAM3	$\gamma_3$ , shear strain (ignored if LCID is non zero).
GAM4	$\gamma_4$ , shear strain (ignored if LCID is non zero).
GAM5	$\gamma_5$ , shear strain (ignored if LCID is non zero).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCD	Optional Load Curve ID defining damping ratio of hysteresis at different strain amplitudes (overrides Masing rules for un-load/reload). The x-axis is shear strain, the y-axis is the damping ratio (e.g., 0.05 for 5% damping). The strains (x-axis values) of curve LCD must be identical to those of curve LCID.
LCSR	Load curve ID defining plastic strain rate scaling effect on yield stress. See *DEFINE_CURVE. The x-axis is plastic strain rate, the y-axis is the yield enhancement factor.
PINIT	Flag for pressure sensitivity. Positive values apply to both B (elastic stiffness scaling) and the A0, A1, A2 equations (strength scaling). Negative values apply only to B, while the A0, A1, A2 equations use the current pressure like PINIT = 0. EQ.0: Use current pressure (will vary during the analysis) EQ.1, -1: Use pressure from initial stress state EQ.2, -2: Use initial "plane stress" pressure $(\sigma_v + \sigma_h)/2$ EQ.3, -3: User (compressive) initial vertical stress
TAU1	$\tau_1$ , shear stress at $\gamma_1$ (ignored if LCID is non zero).
TAU2	$\tau_2$ , shear stress at $\gamma_2$ (ignored if LCID is non zero).
TAU3	$\tau_3$ , shear stress at $\gamma_3$ (ignored if LCID is non zero).
TAU4	$\tau_4$ , shear stress at $\gamma_4$ (ignored if LCID is non zero).
TAU5	$\tau_5$ , shear stress at $\gamma_5$ (ignored if LCID is non zero).

**Remarks:**

The elastic moduli G and K are pressure sensitive:

$$G(p) = \frac{G_0(p - p_0)^b}{(p_{\text{ref}} - p_0)^b}$$

$$K(p) = \frac{K_0(p - p_0)^b}{(p_{\text{ref}} - p_0)^b}$$

where  $G_0$  and  $K_0$  are the input values,  $p$  is the current pressure,  $p_0$  the cut-off or datum pressure (must be zero or negative). If  $p$  attempts to fall below  $p_0$  (i.e., more tensile) the shear stresses are set to zero and the pressure is set to  $p_0$ . Thus, the material has no stiffness or strength in tension. The pressure in compression is calculated as follows:

$$p = p_{ref} \left[ -\frac{K_0}{p_{ref}} \ln(V) \right]^{1/(1-b)}$$

where V is the relative volume, i.e., the ratio between the original and current volume. This formula results in an instantaneous bulk modulus proportional to  $p^b$  whose value at the reference pressure is equal to  $K_0/(1-b)$ .

The constants  $a_0, a_1, a_2$  govern the pressure sensitivity of the yield stress. Only the ratios between these values are important - the absolute stress values are taken from the stress-strain curve.

The stress-strain pairs define a shear stress versus shear strain curve. The first point on the curve is assumed by default to be (0,0) and does not need to be entered. The slope of the curve must decrease with increasing  $\gamma$ . This curves applies at the reference pressure; at other pressures the curve is scaled by

$$\frac{\tau(p, \gamma)}{\tau(p_{ref}, \gamma)} = \sqrt{\frac{[a_0 + a_1(p - p_0) + a_2(p - p_0)^2]}{[a_0 + a_1(p_{ref} - p_0) + a_2(p_{ref} - p_0)^2]}}$$

The shear stress-strain curve (with points  $(\tau_1, \gamma_1), (\tau_2, \gamma_2) \dots (\tau_N, \gamma_N)$ ) is converted into a series of N elastic perfectly plastic curves such that  $\sum(\tau_i, (\gamma)) = \tau(\gamma)$ , as shown in the figure below.

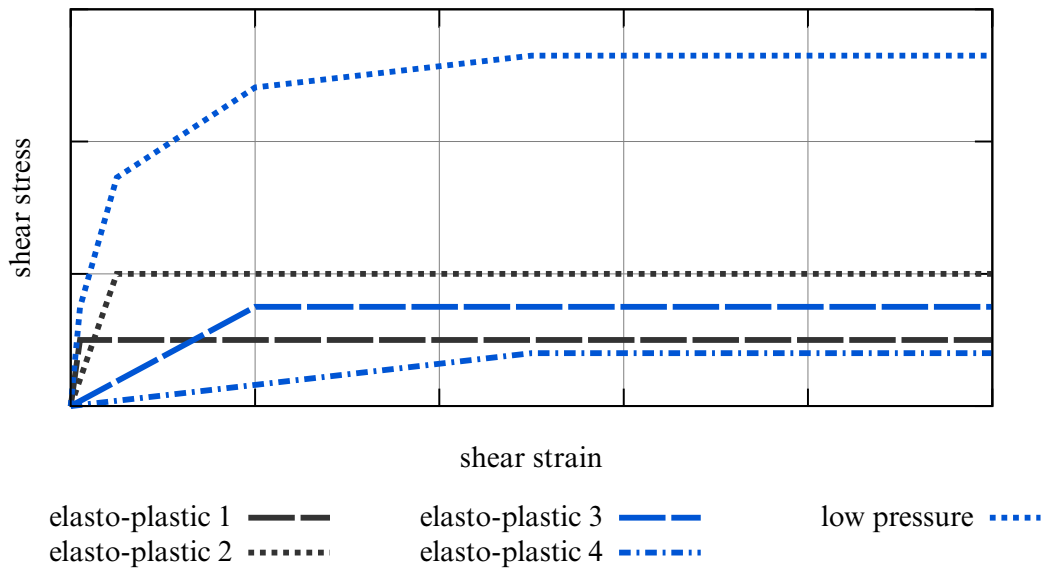
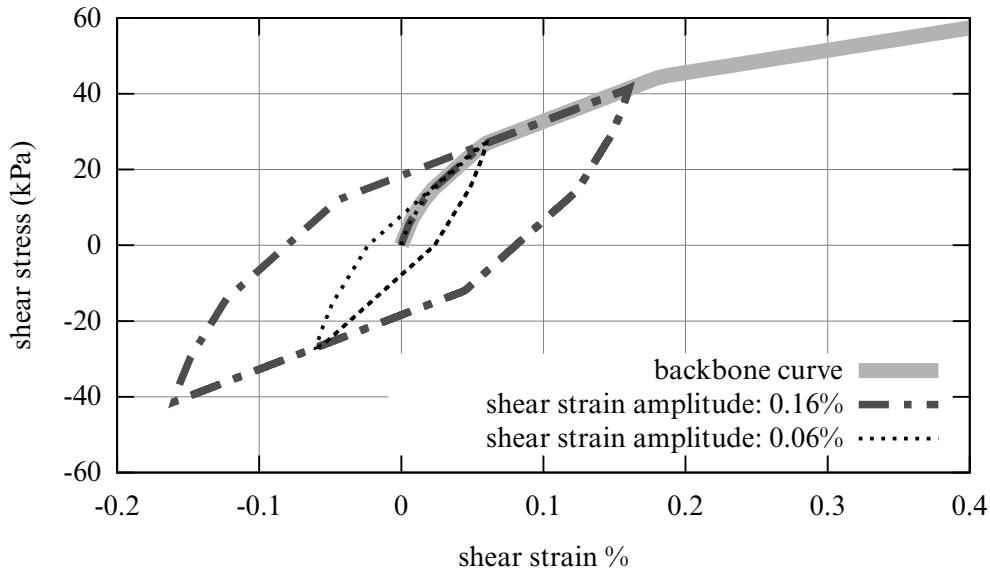


Figure M79-1.

Each elastic perfectly plastic curve represents one “layer” in the material model. Deviatoric stresses are stored and calculated separately for each layer. The total deviatoric stress is the sum of the deviatoric stresses in each layer. By this method, hysteretic (energy-absorbing) stress-strain curves are generated in response to any strain cycle of amplitude greater than

the lowest yield strain of any layer. The example below shows response to small and large strain cycles (blue and pink lines) superposed on the input curve (thick red line).



**Figure M79-2.**

**Definition of shear strain and shear stress:**

Different definitions of “shear strain” and “shear stress” are possible when applied to the three-dimensional stress states. MAT\_079 uses the following definitions:

Input shear stress is treated by the material model as,

$$0.5 \times \text{Von Mises Stress} = \sqrt{(3\sigma'_i : \sigma'_i / 8)}.$$

Input shear strain is treated by the material model as

$$1.5 \times \text{Von Mises Strain} = \sqrt{(3\varepsilon'_i : \varepsilon'_i / 2)}.$$

For a particular stress or strain state (defined by the relationship between the three principal stresses or strains), a scaling factor may be needed in order to convert between the definitions given above and the shear stress or strain that an engineer would expect. The MAT\_079 definitions of shear stress and shear strain are derived from triaxial testing in which one principal stress is applied, while the other two principal stresses are equal to a confining stress which is held constant, i.e. principal stresses and strains have the form (a, b, b). If instead the user wishes the input curve to represent a test in which a pure shear strain is applied over a hydrostatic pressure, such as a shear-box test, then it is recommended to scale both the x-axis and the y-axis of the curve by 0.866. This factor



assumes principal stresses of the form  $(p+t, p-t, p)$  where  $t$  is the applied shear stress, and similar for the principal strains.

### Pressure Sensitivity:

The yield stresses of the layers, and hence the stress at each point on the shear stress-strain input curve, vary with pressure according to constants  $A_0, A_1$  and  $A_2$ . The elastic moduli, and hence also the slope of each section of shear stress-strain curve, vary with pressure according to constant  $B$ . These effects combine to modify the shear stress-strain curve according to pressure:

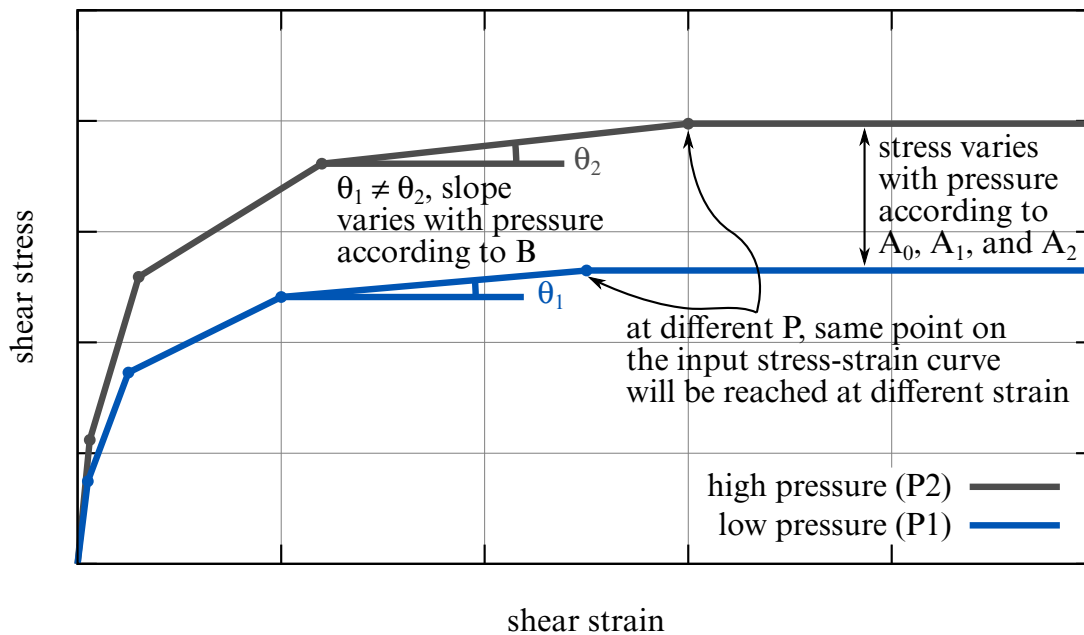


Figure M79-3.

Pressure sensitivity can make the solution sensitive to numerical noise. In cases where the expected pressure changes are small compared to the initial stress state, it may be preferable to use pressure from the initial stress state instead of current pressure as the basis for the pressure sensitivity (option PINIT). This causes the bulk modulus and shear stress-strain curve to be calculated once for each element at the start of the analysis and to remain fixed thereafter. Positive settings of PINIT affect both stiffness scaling (calculated using  $B$ ) and strength scaling (calculated using  $A_0, A_1$  and  $A_2$ ). If PINIT options 2 ("plane stress" pressure) or 3 (vertical stress) are used, these quantities substitute for pressure  $p$  in the equations above. Input values of  $p_{ref}$  and  $p_0$  should then also be "plane stress" pressure or vertical stress, respectively. Negative settings of PINIT have these effects only on stiffness scaling ( $B$ ), while the strength scaling is calculated from the current pressure as for  $PINIT = 0$ .

If PINIT is used, B is allowed to be as high as 1.0 (stiffness proportional to initial pressure); otherwise, values of B higher than about 0.5 are not recommended.

### **Dilatancy:**

Parameters DIL\_A, DIL\_B, DIL\_C and DIL\_D control the compaction and dilatancy that occur in sandy soils as a result of shearing motion. If this facility is used together with pore water pressure (see \*CONTROL\_PORE\_FLUID), liquefaction can be modeled. However, note that the compaction/dilatancy algorithm used in this material model is very unsophisticated compared to recently published research findings.

The dilatancy is expressed as a volume strain  $\gamma_v$ :

$$\begin{aligned}\varepsilon_v &= \varepsilon_r + \varepsilon_g \\ \varepsilon_r &= \text{DIL\_A}(\Gamma)^{\text{DIL\_B}} \\ \varepsilon_g &= \frac{\int (d\gamma_{xz}^2 + d\gamma_{yz}^2)^{1/2}}{\text{DIL\_C} + \text{DIL\_D} \times \int (d\gamma_{xz}^2 + d\gamma_{yz}^2)^{1/2}} \\ \Gamma &= (\gamma_{xz}^2 + \gamma_{yz}^2)^{1/2} \\ \gamma_{xz} &= 2\varepsilon_{xz} \\ \gamma_{yz} &= 2\varepsilon_{yz}\end{aligned}$$

$\gamma_r$  describes the dilation of the soil due to the magnitude of the shear strains; this is caused by the soil particles having to climb over each other to develop shear strain.

$\gamma_g$  describes compaction of the soil due to collapse of weak areas and voids, caused by continuous shear straining.

Recommended inputs for sandy soil if dilatancy is to be modelled:

DIL_A	-	10
DIL_B	-	1.6
DIL_C	-	100
DIL_D	-	2.5

DIL\_A and DIL\_B may cause instabilities in some models.

### **Strain rate sensitivity:**

Strain rate effect is accounted for by scaling the yield stress of each layer as the user-specified function of plastic strain rate. The stress-strain curve defined by LCID is for

quasi-static loading. Scale factor (on the y-axis) versus strain rate (on the x-axis) is defined in curve LCSR. All scale factors must be equal to or larger than 1.0. Because the rate enhancement factor applies to the strength but not the stiffness, and is calculated separately for each layer, an overall enhancement factor between 1.0 and the value in LCSR will be observed in situations where not all the layers are yielding.

**\*MAT\_RAMBERG-OSGOOD**

This is Material Type 80. This model is intended as a simple model of shear behavior and can be used in seismic analysis.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	GAMY	TAUY	ALPHA	R	BULK	
Type	A8	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
GAMY	Reference shear strain ( $\gamma_y$ )
TAUY	Reference shear stress ( $\tau_y$ )
ALPHA	Stress coefficient ( $\alpha$ )
R	Stress exponent (r)
BULK	Elastic bulk modulus

**Remarks:**

The Ramberg-Osgood equation is an empirical constitutive relation to represent the one-dimensional elastic-plastic behavior of many materials, including soils. This model allows a simple rate independent representation of the hysteretic energy dissipation observed in soils subjected to cyclic shear deformation. For monotonic loading, the stress-strain relationship is given by:

$$\frac{\gamma}{\gamma_y} = \frac{\tau}{\tau_y} + \alpha \left| \frac{\tau}{\tau_y} \right|^r, \text{ for } \gamma \geq 0$$

$$\frac{\gamma}{\gamma_y} = \frac{\tau}{\tau_y} - \alpha \left| \frac{\tau}{\tau_y} \right|^r, \text{ for } \gamma < 0$$

where  $\gamma$  is the shear and  $\tau$  is the stress. The model approaches perfect plasticity as the stress exponent  $r \rightarrow \infty$ . These equations must be augmented to correctly model unloading and reloading material behavior. The first load reversal is detected by  $\gamma\dot{\gamma} < 0$ . After the first reversal, the stress-strain relationship is modified to

$$\frac{(\gamma - \gamma_0)}{2\tau_y} = \frac{(\tau - \tau_0)}{2\tau_y} + \alpha \left| \frac{(\tau - \tau_0)}{2\tau_y} \right|^r, \text{ for } \gamma \geq 0$$
$$\frac{(\gamma - \gamma_0)}{2\tau_y} = \frac{(\tau - \tau_0)}{2\tau_y} - \alpha \left| \frac{(\tau - \tau_0)}{2\tau_y} \right|^r, \text{ for } \gamma < 0$$

where  $\gamma_0$  and  $\tau_0$  represent the values of strain and stress at the point of load reversal. Subsequent load reversals are detected by  $(\gamma - \gamma_0)\dot{\gamma} < 0$ .

The Ramberg-Osgood equations are inherently one-dimensional and are assumed to apply to shear components. To generalize this theory to the multidimensional case, it is assumed that each component of the deviatoric stress and deviatoric tensorial strain is independently related by the one-dimensional stress-strain equations. A projection is used to map the result back into deviatoric stress space if required. The volumetric behavior is elastic, and, therefore, the pressure  $p$  is found by

$$p = -K\varepsilon_v$$

where  $\varepsilon_v$  is the volumetric strain.

**\*MAT\_PLASTICITY\_WITH\_DAMAGE\_{OPTION}**

This manual entry apply to *both* types 81 and 82. Materials 81 and 82 model an elasto-visco-plastic material with user-defined *isotropic* stress versus strain curves, which, themselves, may be strain-rate dependent. This model accounts for the effects of damage prior to rupture based on an effective plastic-strain measure. Additionally, failure can be triggered when the time step drops below some specified value.

Available options include:

<BLANK>

ORTHO

ORTHO\_RCDC

ORTHO\_RCDC1980

STOCHAS

The ORTHO option invokes an orthotropic damage model, an extension that was first added as for modelling failure in aluminum panels. Directional damage begins after a defined failure strain is reached in tension and continues to evolve until a tensile rupture strain is reached in either one of the two orthogonal directions. After rupture is detected at *all* integration points, the element is deleted.

The ORTHO\_RCDC option invokes the damage model developed by Wilkins [Wilkins, et al. 1977]. The ORTHO\_RCDC1980 option invokes a damage model based on strain invariants as developed by Wilkins [Wilkins, et al. 1980]. A nonlocal formulation, which requires additional storage, is used if a characteristic length is defined. The RCDC option, which was added at the request of Toyota, works well in predicting failure in cast aluminum; see Yamasaki, et al., [2006].

<p><b>NOTE:</b> This keyword, in its long form, *MAT_PLASTICITY_WITH_DAMAGE, with no options invokes material type 81. Adding an orthotropic damage option will invoke material type 82. Since type 82 must track directional strains it is, computationally, more expensive.</p>
---



**Ortho RCDC Card.** Additional card for keyword option ORTHO\_RCDC.

Card 5	1	2	3	4	5	6	7	8
Variable	ALPHA	BETA	GAMMA	D0	B	LAMBDA	DS	L
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
ETAN	Tangent modulus, ignored if (LCSS.GT.0) is defined.
EPPF	$\epsilon_{failure}^p$ , effective plastic strain at which material softening begins.
TDEL	Minimum time step size for automatic element deletion.
C	Strain rate parameter, $C$ , see formula below.
P	Strain rate parameter, $P$ , see formula below.
LCSS	Load curve ID or Table ID. <ol style="list-style-type: none"> <li>1. <i>Case 1: LCSS is a load curve ID.</i> The load curve LCSS maps effective plastic strain to effective stress. If the fields EPS1 - EPS8 and ES1 - ES8 are defined, they are ignored.</li> <li>2. <i>Case 2: LCSS is a Table ID.</i> Each strain rate value is associated to a load curve ID giving the stress as a function of effective plastic strain for that rate, See <a href="#">Figure M24-1</a>. The stress versus effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress versus effective plas-</li> </ol>



VARIABLE	DESCRIPTION
	<p>tic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters: C and P; the curve ID, LCSR; EPS1 - EPS8 and ES1 - ES8 are ignored if a Table ID is defined.</p> <p>The strain rate values defined in the table may be given as the natural logarithm of the strain rate. If the first stress-strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used. Since the tables are internally discretized to equally space the points, natural logarithms are necessary, for example, if the curves correspond to rates from <math>10^{-4}</math> to <math>10^4</math>.</p>
LCSR	Load curve ID defining strain rate scaling effect on yield stress.
EPPFR	$\epsilon_{rupture}^p$ , effective plastic strain at which material ruptures.
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default), EQ.1.0: Viscoplastic formulation.
LCDM	Optional curve ID defining nonlinear damage curve. To activate the damage curve either the EPPF or EPPFR fields must contain nonzero values.
NUMINT	Number of through thickness integration points which must fail before a shell element is deleted. (If zero, all points must fail.) The default of all integration points is not recommended since shells undergoing large strain are often not deleted due to nodal fiber rotations which limit strains at active integration points after most points have failed. Better results are obtained if NUMINT is set to 1 or a number less than one half of the number of through thickness points. For example, if four through thickness points are used, NUMINT should not exceed 2, even for fully integrated shells which have 16 integration points.
EPS1 - EPS8	Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8.
ALPHA	Parameter $\alpha$ . for the Rc-Dc model

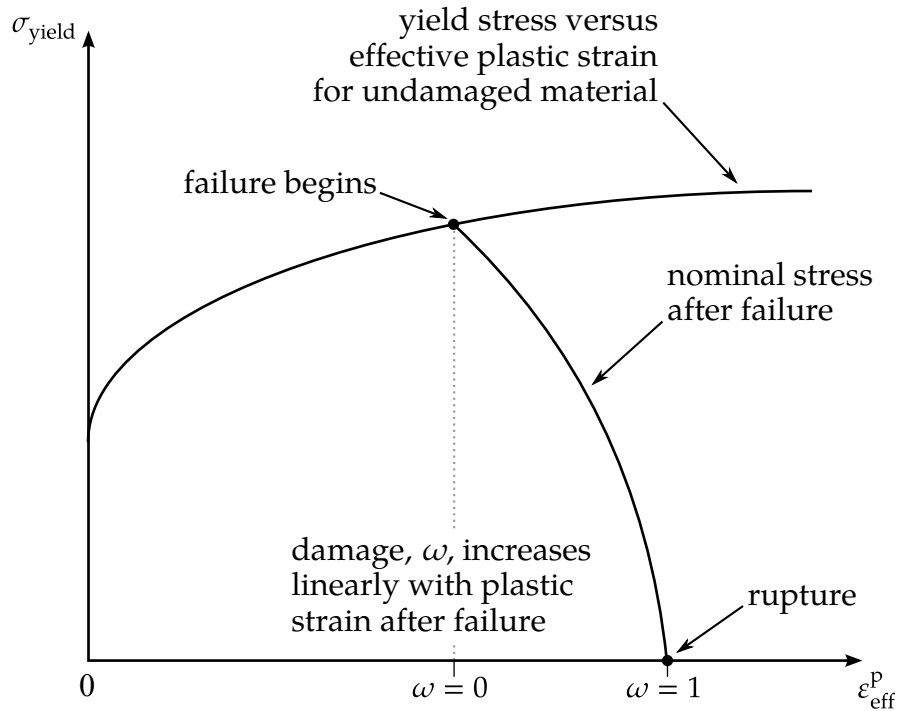


Figure M81-1. Stress strain behavior when damage is included

VARIABLE	DESCRIPTION
BETA	Parameter $\beta$ . for the Rc-Dc model
GAMMA	Parameter $\gamma$ . for the Rc-Dc model
D0	Parameter $D_0$ . for the Rc-Dc model
B	Parameter $b$ . for the Rc-Dc model
LAMBDA	Parameter $\lambda$ . for the Rc-Dc model
DS	Parameter $D_s$ . for the Rc-Dc model
L	Optional characteristic element length for this material. We recommend that the default of 0 always be used, especially in parallel runs. If zero, nodal values of the damage function are used to compute the damage gradient. See discussion below.

**Remarks:**

The stress strain behavior may be treated by a bilinear stress strain curve by defining the tangent modulus, ETAN. Alternately, a curve similar to that shown in [Figure M24-1](#) is expected to be defined by (EPS1, ES1) - (EPS8, ES8); however, an effective stress versus effective plastic strain curve (LCSS) may be input instead if eight points are insufficient.

The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible:

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/6}$$

where  $\dot{\epsilon}$  is the strain rate,  $\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$ .

If the viscoplastic option is active, VP = 1.0, and if SIGY is > 0 then the dynamic yield stress is computed from the sum of the static stress,  $\sigma_y^s(\epsilon_{eff}^p)$ , which is typically given by a load curve ID, and the initial yield stress, SIGY, multiplied by the Cowper-Symonds rate term as follows:

$$\sigma_y(\epsilon_{eff}^p, \dot{\epsilon}_{eff}^p) = \sigma_y^s(\epsilon_{eff}^p) + SIGY \times \left(\frac{\dot{\epsilon}_{eff}^p}{C}\right)^{1/p}$$

where the plastic strain rate is used. With this latter approach similar results can be obtained between this model and material model: \*MAT\_ANISOTROPIC\_VISCOPLASTIC. If SIGY = 0, the following equation is used instead where the static stress,  $\sigma_y^s(\epsilon_{eff}^p)$ , must be defined by a load curve:

$$\sigma_y(\epsilon_{eff}^p, \dot{\epsilon}_{eff}^p) = \sigma_y^s(\epsilon_{eff}^p) \left[ 1 + \left(\frac{\dot{\epsilon}_{eff}^p}{C}\right)^{1/p} \right]$$

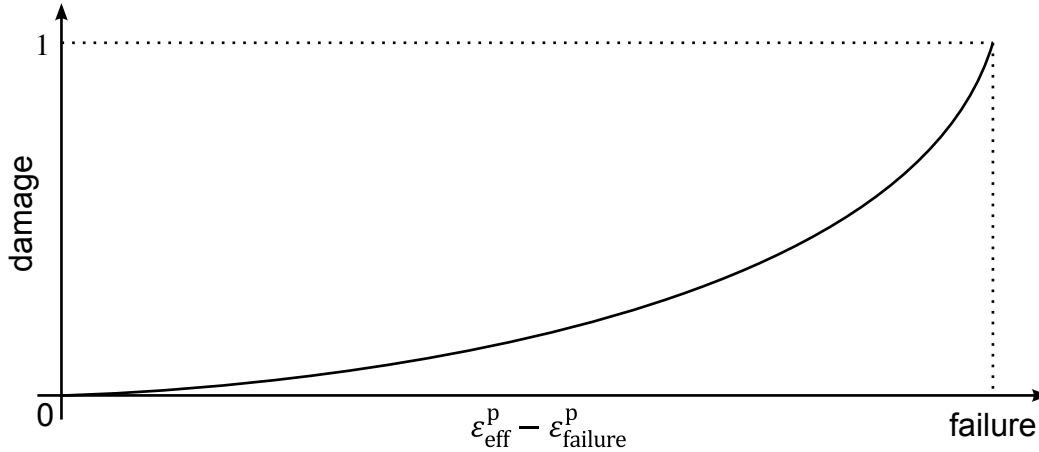
This latter equation is always used if the viscoplastic option is off.

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor versus strain rate is defined.
3. If different stress versus strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in \*DEFINE\_TABLE is expected, see [Figure M24-1](#).

The constitutive properties for the damaged material are obtained from the undamaged material properties. The amount of damage evolved is represented by the constant,  $\omega$ , which varies from zero if no damage has occurred to unity for complete rupture. For uniaxial loading, the nominal stress in the damaged material is given by

$$\sigma_{nominal} = \frac{P}{A}$$

where P is the applied load and A is the surface area. The true stress is given by:



**Figure M81-2.** A nonlinear damage curve is optional. Note that the origin of the curve is at (0,0). It is permissible to input the failure strain EPPF as zero for this option. The nonlinear damage curve is useful for controlling the softening behavior after the failure strain is reached.

$$\sigma_{\text{true}} = \frac{P}{A - A_{\text{loss}}}$$

where  $A_{\text{loss}}$  is the void area. The damage variable can then be defined:

$$\omega = \frac{A_{\text{loss}}}{A}$$

such that

$$0 \leq \omega \leq 1.$$

In this model, unless LCDM is defined by the user, damage is defined in terms of effective plastic strain after the failure strain is exceeded as follows:

$$\omega = \frac{\varepsilon_{\text{eff}}^p - \varepsilon_{\text{failure}}^p}{\varepsilon_{\text{rupture}}^p - \varepsilon_{\text{failure}}^p}, \quad \varepsilon_{\text{failure}}^p \leq \varepsilon_{\text{eff}}^p \leq \varepsilon_{\text{rupture}}^p$$

After exceeding the failure strain softening begins and continues until the rupture strain is reached.

**The Rc-Dc model is defined as:**

The damage D is given by

$$D = \int \omega_1 \omega_2 d\varepsilon^p$$

where  $\varepsilon^p$  is the effective plastic strain,

$$\omega_1 = \left( \frac{1}{1 - \gamma \sigma_m} \right)^\alpha$$

is a triaxial stress weighting term and

$$\omega_2 = (2 - A_D)^\beta$$

is a asymmetric strain weighting term. In the above  $\sigma_m$  is the mean stress. For  $A_D$  we use

$$A_D = \min \left( \left| \frac{\sigma_2}{\sigma_3} \right|, \left| \frac{\sigma_3}{\sigma_2} \right| \right)$$

where  $\sigma_i$  are the principal stresses and  $\sigma_1 > \sigma_2 > \sigma_3$ . Fracture is initiated when the accumulation of damage is

$$\frac{D}{D_c} > 1$$

where  $D_c$  is the a critical damage given by

$$D_c = D_0(1 + b|\nabla D|^\lambda)$$

A fracture fraction,

$$F = \frac{D - D_c}{D_s}$$

defines the degradations of the material by the Rc-Dc model.

For the Rc-Dc model the gradient of damage needs to be estimated. The damage is connected to the integration points, and, thus, the computation of the gradient requires some manipulation of the LS-DYNA source code. Provided that the damage is connected to nodes, it can be seen as a standard bilinear field and the gradient is easily obtained. To enable this, the damage at the integration points are transferred to the nodes as follows. Let  $E_n$  be the set of elements sharing node  $n$ ,  $E_n$  the number of elements in that set,  $P_e$  the set of integration points in element  $e$  and  $|P_e|$  the number of points in that set. The average damage  $\bar{D}_e$  in element  $e$  is computed as

$$\bar{D}_e = \frac{\sum_{p \in P_e} D_p}{|P_e|}$$

where  $D_p$  is the damage in integration point  $p$ . Finally, the damage value in node  $n$  is estimated as

$$D_n = \frac{\sum_{e \in E_n} \bar{D}_e}{|E_n|}$$

This computation is performed in each time step and requires additional storage. Currently we use three times the total number of nodes in the model for this calculation, but this could be reduced by a considerable factor if necessary. There is an Rc-Dc option for the Gurson dilatational-plastic model. In the implementation of this model, the norm of the gradient is computed differently. Let  $E_f^l$  be the set of elements from within a distance  $l$  of element,  $f$  not including the element itself, and let  $|E_f^l|$  be the number of elements in that set. The norm of the gradient of damage is estimated roughly as

$$\|\nabla D\|_f \approx \frac{1}{|E_f^l|} \sum_{e \in E_f^l} \frac{|D_e - D_f|}{d_{ef}}$$

where  $d_{ef}$  is the distance between element  $f$  and  $e$ .

The reason for taking the first approach is that it should be a better approximation of the gradient, it can for one integration point in each element be seen as a weak gradient of an elementwise constant field. The memory consumption as well as computational work should not be much higher than for the other approach.

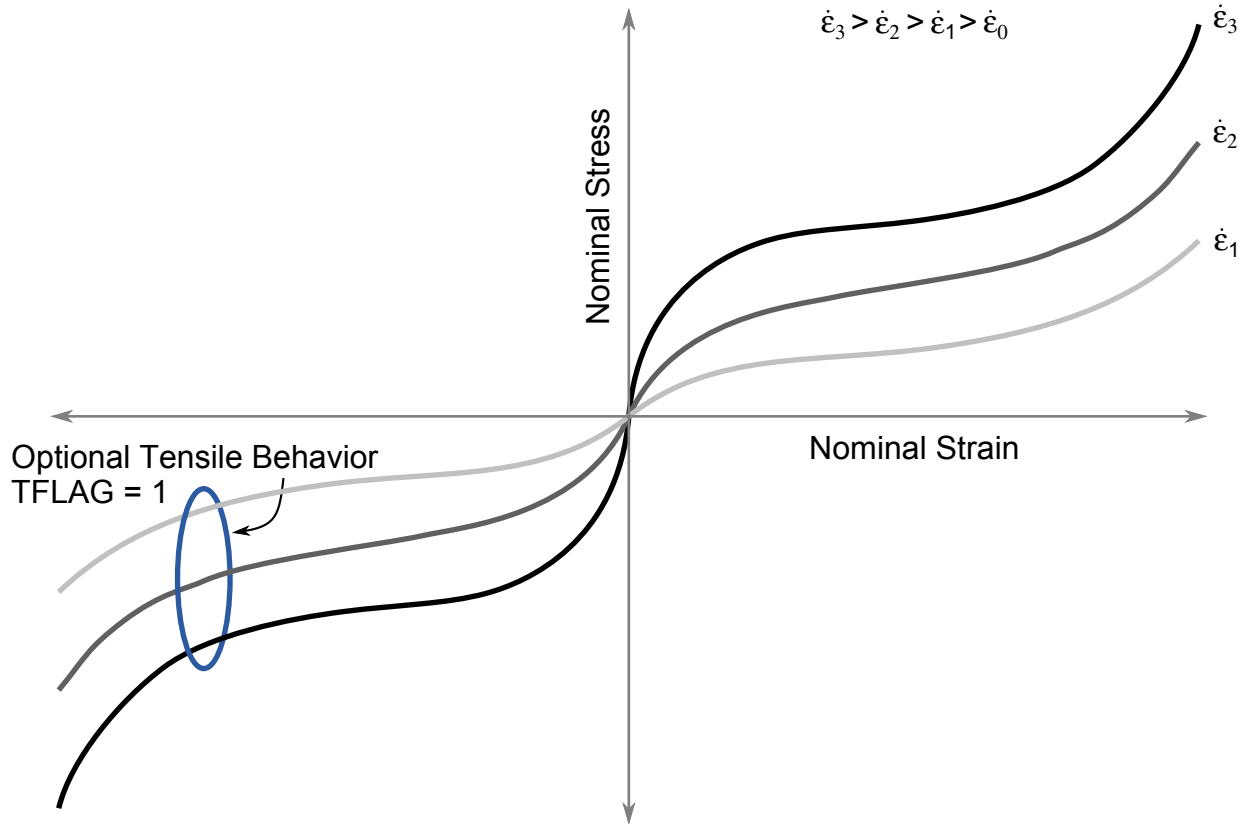
The RCDC1980 model is identical to the RCDC model except the expression for  $A_D$  is in terms of the principal stress deviators and takes the form

$$A_D = \max \left( \left| \frac{S_2}{S_3} \right|, \left| \frac{S_2}{S_1} \right| \right)$$

The STOCHASTIC option allows spatially varying yield and failure behavior. See \*DEFINE\_STOCHASTIC\_VARIATION for additional information.

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>		
<b>Label</b>	<b>Attributes</b>	<b>Description</b>
Instability	- - - -	Failure indicator $\varepsilon_{\text{eff}}^p / \varepsilon_{\text{fail}}^p$ , see EPPF
Plastic Strain Rate	- - - -	Effective plastic strain rate $\dot{\varepsilon}_{\text{eff}}^p$
Damage	- - - -	Damage $\omega$





**Figure M83-1.** Rate effects in the nominal stress versus engineering strain curves, which are used to model rate effects in Fu Chang's foam model.

Card 2	1	2	3	4	5	6	7	8
Variable	BVFLAG	SFLAG	RFLAG	TFLAG	PVID	SRAF	REF	HU
Type	F	F	F	F	F	F	F	F
Default	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Remarks	1	2	3		4			6





This card is optional.

Card 5	1	2	3	4	5	6	7	8
Variable	EXPON	RIULD						
Type	F	F						
Default	1.0	0.0						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
KCON	Optional Young's modulus used in the computation of sound speed. This will influence the time step, contact forces, hourglass stabilization forces, and the numerical damping (DAMP). EQ.0.0: KCON is set equal to the, $\max(E, \text{current tangent to stress-strain curve}),$ if TBID $\neq$ 0. Otherwise, if TBID = 0, KCON is set equal to the maximum slope of the stress-strain curve.
TC	Tension cut-off stress
FAIL	Failure option after cutoff stress is reached: EQ.0.0: tensile stress remains at cut-off value, EQ.1.0: tensile stress is reset to zero.
DAMP	Viscous coefficient to model damping effects (0.05 < recommended value < 0.50; default is 0.05).
TBID	Table ID, see *DEFINE_TABLE, for nominal stress vs. strain data as a function of strain rate. If the table ID is provided, cards 3 and 4 may be left blank and the fit will be done internally. The Table ID can be positive or negative (see Remark 6 below). If TBID < 0, enter  TBID  on the *DEFINE_TABLE keyword.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BVFLAG	Bulk viscosity activation flag, see <a href="#">Remark 1</a> : EQ.0.0: no bulk viscosity (recommended), EQ.1.0: bulk viscosity active.
SFLAG	Strain rate flag (see <a href="#">Remark 2</a> below): EQ.0.0: true constant strain rate, EQ.1.0: engineering strain rate.
RFLAG	Strain rate evaluation flag see <a href="#">Remark 3</a> : EQ.0.0: first principal direction, EQ.1.0: principal strain rates for each principal direction, EQ.2.0: volumetric strain rate.
TFLAG	Tensile stress evaluation: EQ.0.0: linear (follows E) in tension; EQ.1.0: input via load curves with the tensile response corresponds to negative values of stress and strain.
PVID	Optional load curve ID defining pressure versus volumetric strain. See <a href="#">Remark 4</a> .
SRAF	Strain rate averaging flag. See <a href="#">Remark 5</a> . LT.0.0: use exponential moving average. EQ.0.0: use weighted running average. EQ.1.0: average the last twelve values.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.
HU	Hysteretic unloading factor between 0 and 1 (default = 0). See also <a href="#">Remark 6</a> and <a href="#">Figure M83-4</a> .
D0	material constant, see equations below.

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
N0	material constant, see equations below.
N1	material constant, see equations below.
N2	material constant, see equations below.
N3	material constant, see equations below.
C0	material constant, see equations below.
C1	material constant, see equations below.
C2	material constant, see equations below.
C3	material constant, see equations below.
C4	material constant, see equations below.
C5	material constant, see equations below.
AIJ,	material constant, see equations below.
SIJ	material constant, see equations below.
MINR	Ratemin, minimum strain rate of interest.
MAXR	Ratemax, maximum strain rate of interest.
SHAPE	Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor HU. Values less than one reduces the energy dissipation and greater than one increases dissipation, see also <a href="#">Figure M83-2</a> .
BETAT	Decay constant for damage in tension. The damage decays after loading in ceases according to $e^{-\text{BETAT} \times t}$ .
BETAC	Decay constant for damage in compression. The damage decays after loading in ceases according to $e^{-\text{BETAC} \times t}$ .
EXPON	Exponent for unloading. Active for nonzero values of the hysteretic unloading factor HU. Default is 1.0
RIULD	Flag for rate independent unloading, see Remark 6. EQ.0.0: off, EQ.1.0: on.

**Material Formulation:**

The strain is divided into two parts: a linear part and a non-linear part of the strain

$$\mathbf{E}(t) = \mathbf{E}^L(t) + \mathbf{E}^N(t)$$

and the strain rate becomes

$$\dot{\mathbf{E}}(t) = \dot{\mathbf{E}}^L(t) + \dot{\mathbf{E}}^N(t)$$

where  $\dot{\mathbf{E}}^N$  is an expression for the past history of  $\mathbf{E}^N$ . A postulated constitutive equation

may be written as:

$$\sigma(t) = \int_{\tau=0}^{\infty} [\mathbf{E}_t^N(\tau), \mathbf{S}(t)] d\tau$$

where  $\mathbf{S}(t)$  is the state variable and  $\int_{\tau=0}^{\infty}$  is a functional of all values of  $\tau$  in  $T_{\tau}: 0 \leq \tau \leq \infty$  and

$$\mathbf{E}_t^N(\tau) = \mathbf{E}^N(t - \tau)$$

where  $\tau$  is the history parameter:

$$\mathbf{E}_t^N(\tau = \infty) \Leftrightarrow \text{the virgin material}$$

It is assumed that the material remembers only its immediate past, i.e., a neighborhood about  $\tau = 0$ . Therefore, an expansion of  $\mathbf{E}_t^N(\tau)$  in a Taylor series about  $\tau = 0$  yields:

$$\mathbf{E}_t^N(\tau) = \mathbf{E}^N(0) + \frac{\partial \mathbf{E}_t^N}{\partial t}(0) \tau$$

Hence, the postulated constitutive equation becomes:

$$\sigma(t) = \sigma^*[\mathbf{E}^N(t), \dot{\mathbf{E}}^N(t), \mathbf{S}(t)]$$

where we have replaced  $\frac{\partial \mathbf{E}_t^N}{\partial t}$  by  $\dot{\mathbf{E}}^N$ , and  $\sigma^*$  is a function of its arguments.

For a special case,

$$\sigma(t) = \sigma^*(\mathbf{E}^N(t), \mathbf{S}(t))$$

we may write

$$\dot{\mathbf{E}}_t^N = f(\mathbf{S}(t), \mathbf{s}(t))$$

which states that the nonlinear strain rate is the function of stress and a state variable which represents the history of loading. Therefore, the proposed kinetic equation for foam materials is:

$$\dot{\mathbf{E}}^N = \frac{\sigma}{\|\sigma\|} D_0 \exp \left\{ -c_0 \left[ \frac{\sigma : \mathbf{S}}{(\|\sigma\|)^2} \right]^{2n_0} \right\}$$

where  $D_0$ ,  $c_0$ , and  $n_0$  are material constants, and  $\mathbf{S}$  is the overall state variable. If either  $D_0 = 0$  or  $c_0 \rightarrow \infty$  then the nonlinear strain rate vanishes.

$$\dot{S}_{ij} = [c_1(a_{ij}R - c_2S_{ij})P + c_3W^{n_1}(\|\dot{\mathbf{E}}^N\|)^{n_2}I_{ij}]R$$

$$R = 1 + c_4 \left[ \frac{\|\dot{\mathbf{E}}^N\|}{c_5} - 1 \right]^{n_3}$$

$$P = \sigma : \dot{\mathbf{E}}^N$$

$$W = \int \sigma : (d\mathbf{E})$$

where  $c_1$ ,  $c_2$ ,  $c_3$ ,  $c_4$ ,  $c_5$ ,  $n_1$ ,  $n_2$ ,  $n_3$ , and  $a_{ij}$  are material constants and:

$$\|\sigma\| = (\sigma_{ij}\sigma_{ij})^{\frac{1}{2}}$$

$$\|\dot{\mathbf{E}}\| = (\dot{E}_{ij}\dot{E}_{ij})^{\frac{1}{2}}$$

$$\|\dot{\mathbf{E}}^N\| = (\dot{E}_{ij}^N\dot{E}_{ij}^N)^{\frac{1}{2}}$$

In the implementation by Fu Chang the model was simplified such that the input constants  $a_{ij}$  and the state variables  $S_{ij}$  are scalars.

#### **Additional Remarks:**

1. **Bulk Viscosity.** The bulk viscosity, which generates a rate dependent pressure, may cause an unexpected volumetric response and consequently, it is optional with this model.
2. **Constant Velocity Loading.** Dynamic compression tests at the strain rates of interest in vehicle crash are usually performed with a drop tower. In this test the loading velocity is nearly constant but the true strain rate, which depends on the instantaneous specimen thickness, is not. Therefore, the engineering strain rate input is optional so that the stress strain curves obtained at constant velocity loading can be used directly. See the SFLAG field.

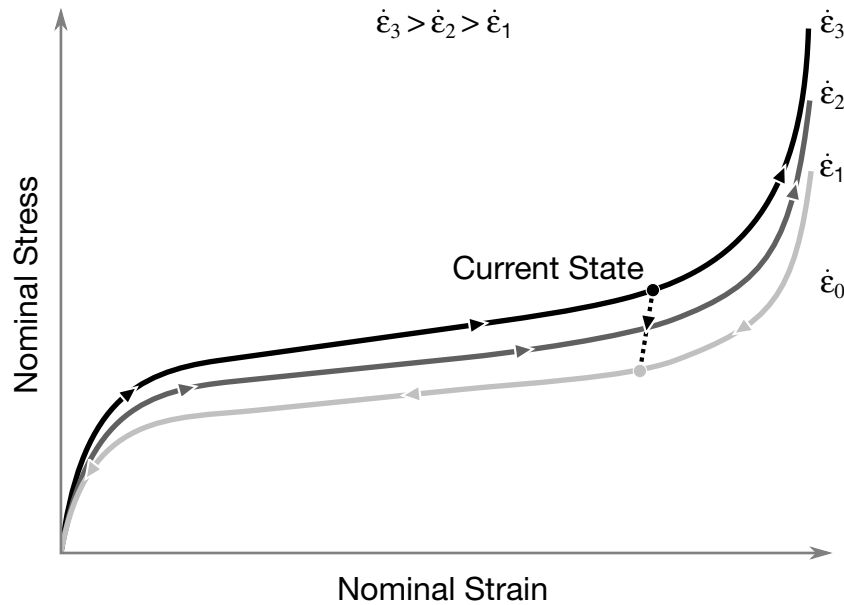


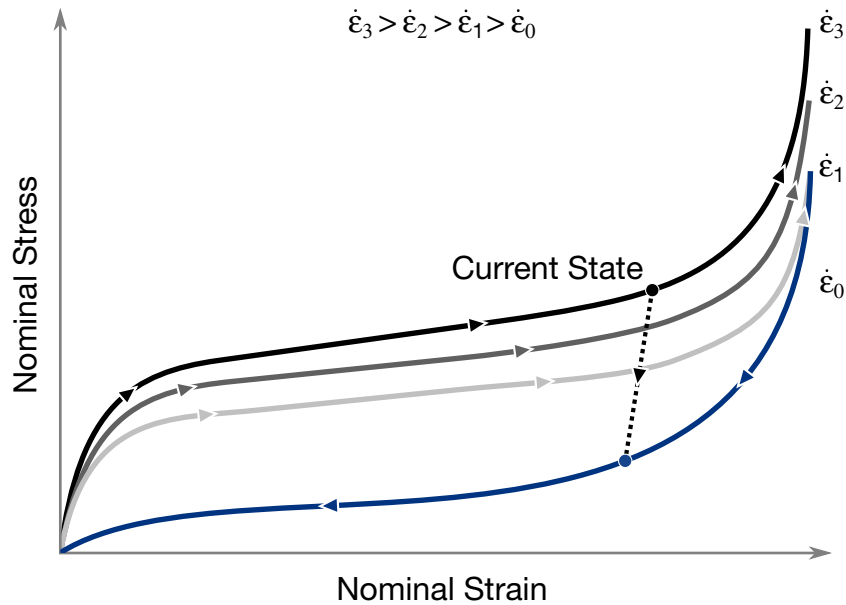
Figure M83-2. HU=0, TBID>0

3. **Strain Rates with Multiaxial Loading.** To further improve the response under multiaxial loading, the strain rate parameter can either be based on the principal strain rates or the volumetric strain rate. See the RFLAG field.
4. **Triaxial Loading.** Correlation under triaxial loading is achieved by directly inputting the results of hydrostatic testing in addition to the uniaxial data. Without this additional information which is fully optional, triaxial response tends to be underestimated. See the PVID field.
5. **Strain Rate Averaging.** Three different options are available. The default, SRAF = 0.0, uses a weighted running average with a weight of 1/12 on the current strain rate. With the second option, SRAF = 1.0, the last twelve strain rates are averaged. The third option, SRAF < 0, uses an exponential moving average with factor |SRAF| representing the degree of weighting decrease ( $-1 \leq \text{SRAF} < 0$ ). The averaged strain rate at time  $t_n$  is obtained by:

$$\dot{\epsilon}_n^{\text{averaged}} = |\text{SRAF}| \dot{\epsilon}_n + (1 - |\text{SRAF}|) \dot{\epsilon}_{n-1}^{\text{averaged}}$$

6. **Unloading Response Options.** Several options are available to control unloading response in MAT\_083:
  - a) HU = 0 and TBID > 0. See [Figure M83-2](#).

This is the old way. In this case the unloading response will follow the curve with the lowest strain rate and is rate-independent. The curve with lowest strain rate value (typically zero) in TBID should correspond to the unloading path of the material as measured in a quasistatic test. The qua-



**Figure M83-3.** HU = 0, TBID < 0

sistatic loading path then corresponds to a realistic (small) value of the strain rate.

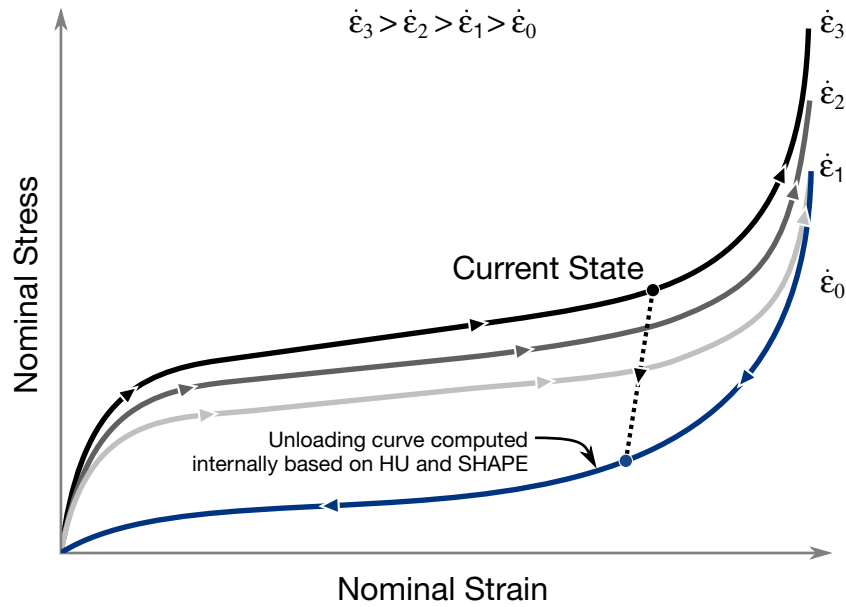
b) HU = 0 and TBID < 0

In this case the curve with lowest strain rate value (typically zero) in TBID must correspond to the unloading path of the material as measured in a quasistatic test. The quasistatic loading path then corresponds to a realistic (small) value of the strain rate. At least three curves should be used in the table (one for unloading, one for quasistatic, and one or more for dynamic response). The quasistatic loading and unloading path (thus the first two curves of the table) should form a closed loop. The unloading response is given by a damage formulation for the principal stresses as follows:

$$\sigma_i = (1 - d)\sigma_i$$

The damage parameter  $d$  is computed internally in such a way that the unloading path under uniaxial tension and compression is fitted exactly in the simulation. The unloading response is rate dependent in this case. In some cases, this rate dependence for loading and unloading can lead to noisy results. To reduce that noise, it is possible to switch to rate independent unloading with RIULD = 1.





**Figure M83-4.** HU > 0, TBID > 0

c) HU > 0 and TBID > 0

No unloading curve should be provided in the table and the curve with the lowest strain rate value in TBID should correspond to the loading path of the material as measured in a quasistatic test. At least two curves should be used in the table (one for quasistatic and one or more for dynamic response). In this case the unloading response is given by a damage formulation for the principal stresses as follows:

$$\sigma_i = (1 - d)\sigma_i$$

$$d = (1 - HU) \left[ 1 - \left( \frac{W_{cur}}{W_{max}} \right)^{SHAPE} \right]^{EXPON}$$

where W corresponds to the current value of the hyperelastic energy per unit undeformed volume. The unloading response is rate dependent in this case. In some cases, this rate dependence for loading and unloading can lead to noisy results. To reduce that noise, it is possible to switch to rate independent unloading with RIULD = 1.

The LOG\_LOG\_INTERPOLATION option uses log-log interpolation for table TBID in the strain rate direction.



Card 4	1	2	3	4	5	6	7	8
Variable	P1	P2	P3	P4	P5	P6	P7	P8
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
TM	Initial tangent modulus of concrete.
PR	Poisson's ratio.
UCS	Uniaxial compressive strength.
UTS	Uniaxial tensile strength.
FE	Depends on value of RATE below. RATE.EQ.0: Fracture energy (energy per unit area dissipated in opening crack). RATE.EQ.1: Crack width at which crack-normal tensile stress goes to zero.
ASIZE	Aggregate size (radius).
E	Young's modulus of rebar.
YS	Yield stress of rebar.
EH	Hardening modulus of rebar
UEONG	Ultimate elongation before rebar fails.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RATE	<p>Rate effects:</p> <p>EQ.0.0: Strain rate effects are included. WARNING: energy may not be conserved using this option.</p> <p>EQ.1.0: Strain rate effects are turned off. Crack widths are stored as extra history variables 30, 31, 32.</p> <p>EQ.2.0: Like RATE = 1 but includes improved crack algorithm (recommended). Crack widths are stored as extra history variables 3, 4, 5.</p>
CONM	<p>GT.0: Factor to convert model mass units to kg.</p> <p>EQ.-1.: Mass, length, time units in model are lbf × sec<sup>2</sup>/in, inch, sec.</p> <p>EQ.-2.: Mass, length, time units in model are g, cm, microsec.</p> <p>EQ.-3.: Mass, length, time units in model are g, mm, msec.</p> <p>EQ.-4.: Mass, length, time units in model are metric ton, mm, sec.</p> <p>EQ.-5.: Mass, length, time units in model are kg, mm, msec.</p>
CONL	<p>CONM.GT.0: CONL is the conversion factor from model length units to meters.</p> <p>CONM.LE.0: CONL is ignored.</p>
CONT	<p>CONM.GT.0: CONL is the conversion factor from time units to seconds</p> <p>CONM.LE.0: CONT is ignored.</p>
EPS1, EPS2, ...	Volumetric strain values (natural logarithmic values), see Remarks below. A maximum of 8 values are allowed.
P1, P2, ...	Pressures corresponding to volumetric strain values given on Card 3.

**Remarks:**

Pressure is positive in compression; volumetric strain is given by the natural log of the relative volume and is negative in compression. The tabulated data are given in order of increasing compression, with no initial zero point.

If the volume compaction curve is omitted, the following scaled curve is automatically used where  $p_1$  is the pressure at uniaxial compressive failure from:

$$p_1 = \frac{\sigma_c}{3}$$

and  $K$  is the bulk unloading modulus computed from

$$K = \frac{E_s}{3(1 - 2\nu)}$$

where  $E_s$  is the input tangent modulus for concrete and  $\nu$  is Poisson's ratio.

Volumetric Strain	Pressure
$-p_1/K$	$1.00p_1$
-0.002	$1.50p_1$
-0.004	$3.00p_1$
-0.010	$4.80p_1$
-0.020	$6.00p_1$
-0.030	$7.50p_1$
-0.041	$9.45p_1$
-0.051	$11.55p_1$
-0.062	$14.25p_1$
-0.094	$25.05p_1$

**Table M84-1.** Default pressure versus volumetric strain curve for concrete if the curve is not defined.

The Winfrith concrete model can generate an additional binary output database containing information on crack locations, directions, and widths. In order to generate the crack database, the LS-DYNA execution line is modified by adding:

$$q=crf$$

where **crf** is the desired name of the crack database, e.g., **q=d3crack**.

LS-PrePost can display the cracks on the deformed mesh plots. To do so, read the d3plot database into LS-PrePost and then select File → Open → Crack from the top menu bar. Or, open the crack database by adding the following to the LS-PrePost execution line:

$$q=crf$$

where **crf** is the name of the crack database, e.g., **q=d3crack**.

By default, all the cracks in visible elements are shown. You can eliminate narrow cracks from the display by setting a minimum crack width for displayed cracks. Do this by choosing Settings → Post Settings → Concrete Crack Width. From the top menu bar of LS-

PrePost, choosing Misc → Model Info will reveal the number of cracked elements and the maximum crack width in a given plot state.

An ASCII “aea\_crack” output file is written if the command \*DATABASE\_BINARY\_D3CRACK command is included in the input deck. This command does not have any bearing on the aforementioned binary crack database.

**\*MAT\_WINFRITH\_CONCRETE\_REINFORCEMENT**

This is \*MAT\_084\_REINF for rebar reinforcement supplemental to concrete defined using Material type 84. Reinforcement may be defined in specific groups of elements, but it is usually more convenient to define a two-dimensional mat in a specified layer of a specified material. Reinforcement quantity is defined as the ratio of the cross-sectional area of steel relative to the cross-sectional area of concrete in the element (or layer). These cards may follow either one of two formats below and may also be defined in any order.

Option 1 (Reinforcement quantities in element groups).

Card 1	1	2	3	4	5	6	7	8
Variable	EID1	EID2	INC	XR	YR	ZR		
Type	I	I	I	F	F	F		

Option 2 (Two dimensional layers by part ID). Option 2 is active when first entry is left blank.

Card 1	1	2	3	4	5	6	7	8
Variable		PID	AXIS	COOR	RQA	RQB		
Type	blank	I	I	F	F	F		

**VARIABLE****DESCRIPTION**

EID1	First element ID in group.
EID2	Last element ID in group
INC	Element increment for generation.
XR	<i>x</i> -reinforcement quantity (for bars running parallel to global <i>x</i> -axis).
YR	<i>y</i> -reinforcement quantity (for bars running parallel to global <i>y</i> -axis).
ZR	<i>z</i> -reinforcement quantity (for bars running parallel to global <i>z</i> -axis).
PID	Part ID of reinforced elements. If PID = 0, the reinforcement is applied to all parts which use the Winfrith concrete model.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AXIS	Axis normal to layer. EQ.1: A and B are parallel to global $y$ and $z$ , respectively. EQ.2: A and B are parallel to global $x$ and $z$ , respectively. EQ.3: A and B are parallel to global $x$ and $y$ , respectively.
COOR	Coordinate location of layer: AXIS.EQ.1: $x$ -coordinate AXIS.EQ.2: $y$ -coordinate AXIS.EQ.3: $z$ -coordinate
RQA	Reinforcement quantity (A).
RQB	Reinforcement quantity (B).

**Remarks:**

1. Reinforcement quantity is the ratio of area of reinforcement in an element to the element's total cross-sectional area in a given direction. This definition is true for both Options 1 and 2. Where the options differ is in the manner in which it is decided which elements are reinforced. In Option 1, the reinforced element IDs are spelled out. In Option 2, elements of part ID PID which are cut by a plane (layer) defined by AXIS and COOR are reinforced.



\*MAT\_ORTHOTROPIC\_VISCOELASTIC

This is Material Type 86. It allows the definition of an orthotropic material with a viscoelastic part. This model applies to shell elements.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	VF	K	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	G0	GINF	BETA	PRBA	PRCA	PRCB		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT	MANGLE			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
EA	Young's Modulus $E_a$
EB	Young's Modulus $E_b$
EC	Young's Modulus $E_c$
VF	Volume fraction of viscoelastic material
K	Elastic bulk modulus
G0	$G_0$ , short-time shear modulus
GINF	$G_\infty$ , long-time shear modulus
BETA	$\beta$ , decay constant
PRBA	Poisson's ratio, $\nu_{ba}$
PRCA	Poisson's ratio, $\nu_{ca}$
PRCB	Poisson's ratio, $\nu_{cb}$
GAB	Shear modulus, $G_{ab}$
GBC	Shear modulus, $G_{bc}$
GCA	Shear modulus, $G_{ca}$
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description): <p style="margin-left: 40px;">EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDI-</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	NATE_NODES, and then rotated about the shell element normal by an angle MANGLE.
	EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, MANGLE, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal.
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
MANGLE	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.
A1 A2 A3	Define components of vector <b>a</b> for AOPT = 2.
V1 V2 V3	Define components of vector <b>v</b> for AOPT = 3.
D1 D2 D3	Define components of vector <b>d</b> for AOPT = 2.

**Remarks:**

For the orthotropic definition it is referred to Material Type 2 and 21.

**\*MAT\_CELLULAR\_RUBBER**

This is Material Type 87. This material model provides a cellular rubber model with confined air pressure combined with linear viscoelasticity as outlined by Christensen [1980]. See [Figure M87-1](#).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PR	N				
Type	A8	F	F	I				

Card 2 if  $N > 0$ , a least squares fit is computed from uniaxial data

Card 2	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LCID				
Type	F	F	F	F				

Card 2 if  $N = 0$ , define the following constants

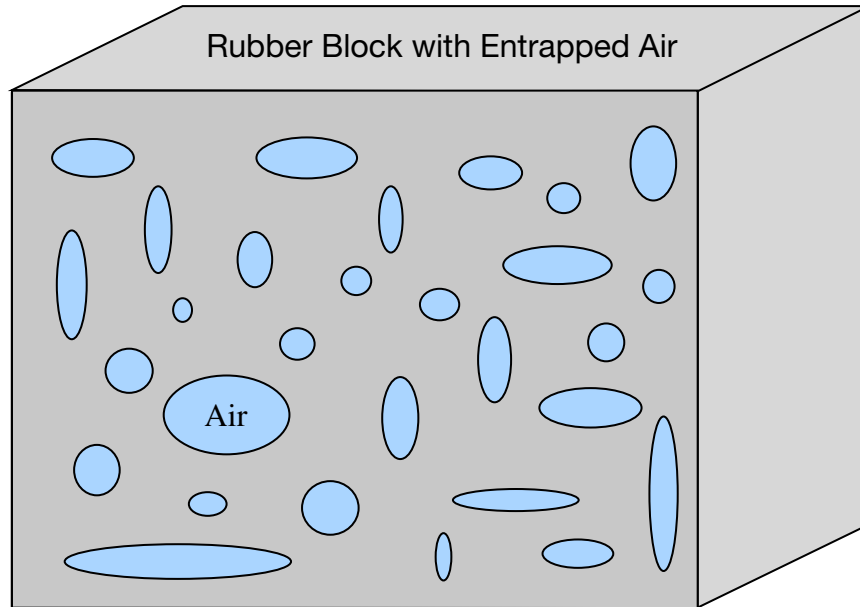
Card 2	1	2	3	4	5	6	7	8
Variable	C10	C01	C11	C20	C02			
Type	F	F	F	F	F			

Card 3	1	2	3	4	5	6	7	8
Variable	P0	PHI	IVS	G	BETA			
Type	F	F	F	F	F			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PR	Poisson's ratio, typical values are between .0 to .2. Due to the large compressibility of air, large values of Poisson's ratio generates physically meaningless results.
N	Order of fit (currently < 3). If $n > 0$ then a least square fit is computed with uniaxial data. The parameters given on card 2 should be specified. Also see *MAT_MOONEY_RIVLIN_RUBBER (material model 27). A Poisson's ratio of .5 is assumed for the void free rubber during the fit. The Poisson's ratio defined on Card 1 is for the cellular rubber. A void fraction formulation is used.
<b>Define, if N &gt; 0:</b>	
SGL	Specimen gauge length $l_0$
SW	Specimen width
ST	Specimen thickness
LCID	Load curve ID giving the force versus actual change $\Delta L$ in the gauge length. If SGL, SW, and ST are set to unity (1.0), then curve LCID is also engineering stress versus engineering strain.
<b>Define, if N = 0:</b>	
C10	Coefficient, $C_{10}$
C01	Coefficient, $C_{01}$
C11	Coefficient, $C_{11}$
C20	Coefficient, $C_{20}$
C02	Coefficient, $C_{02}$
P0	Initial air pressure, $P_0$
PHI	Ratio of cellular rubber to rubber density, $\Phi$
IVS	Initial volumetric strain, $\gamma_0$
G	Optional shear relaxation modulus, $G$ , for rate effects (viscosity)
BETA	Optional decay constant, $\beta_1$



**Figure M87-1.** Cellular rubber with entrapped air. By setting the initial air pressure to zero, an open cell, cellular rubber can be simulated.

#### Remarks:

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material a hydrostatic work term,  $W_H(J)$ , is included in the strain energy functional which is function of the relative volume,  $J$ , [Ogden 1984]:

$$W(J_1, J_2, J) = \sum_{p,q=0}^n C_{pq} (J_1 - 3)^p (J_2 - 3)^q + W_H(J)$$

$$J_1 + I_1 I_3^{-1/3}$$

$$J_2 + I_2 I_3^{-2/3}$$

In order to prevent volumetric work from contributing to the hydrostatic work the first and second invariants are modified as shown. This procedure is described in more detail by Sussman and Bathe [1987].

The effects of confined air pressure in its overall response characteristics is included by augmenting the stress state within the element by the air pressure.

$$\sigma_{ij} = \sigma_{ij}^{sk} - \delta_{ij} \sigma^{air}$$

where  $\sigma_{ij}^{sk}$  is the bulk skeletal stress and  $\sigma^{air}$  is the air pressure computed from the equation:

$$\sigma^{\text{air}} = -\frac{p_0\gamma}{1 + \gamma - \phi}$$

where  $p_0$  is the initial foam pressure usually taken as the atmospheric pressure and  $\gamma$  defines the volumetric strain

$$\gamma = V - 1 + \gamma_0$$

where  $V$  is the relative volume of the voids and  $\gamma_0$  is the initial volumetric strain which is typically zero. The rubber skeletal material is assumed to be incompressible.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

Since we wish to include only simple rate effects, the relaxation function is represented by one term from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta t}$$

given by,

$$g(t) = E_d e^{-\beta_1 t}.$$

This model is effectively a Maxwell fluid which consists of a damper and spring in series. We characterize this in the input by a shear modulus,  $G$ , and decay constant,  $\beta_1$ .

The Mooney-Rivlin rubber model (model 27) is obtained by specifying  $n = 1$  without air pressure and viscosity. In spite of the differences in formulations with Model 27, we find that the results obtained with this model are nearly identical with those of material type 27 as long as large values of Poisson's ratio are used.





Card 5	1	2	3	4	5	6	7	8
Variable	RHOCPR	TEMPRF	ALPHA	EPS0				
Type	F	F						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
SIGA	$\hat{\sigma}_a$ , dislocation interactions with long-range barriers (force/area).
SIGI	$\hat{\sigma}_i$ , dislocation interactions with interstitial atoms (force/area).
SIGS	$\hat{\sigma}_s$ , dislocation interactions with solute atoms (force/area).
SIG0	$\hat{\sigma}_0$ , initial value of $\hat{\sigma}$ at zero plastic strain (force/area) NOT USED.
HF0	$a_0$ , dislocation generation material constant (force/area).
HF1	$a_1$ , dislocation generation material constant (force/area).
HF2	$a_2$ , dislocation generation material constant (force/area).
SIGS0	$\hat{\sigma}_{\text{ES0}}$ , saturation threshold stress at 0° K (force/area).
BULK	Bulk modulus defined for shell elements only. Do not input for solid elements.
EDOTS0	$\dot{\epsilon}_{\text{ES0}}$ , reference strain-rate (time <sup>-1</sup> ).
BURG	Magnitude of Burgers vector (interatomic slip distance), (distance)
CAPA	Material constant, A.
BOLTZ	Boltzmann's constant, k (energy/degree).
SM0	$G_0$ , shear modulus at zero degrees Kelvin (force/area).
SM1	$b_1$ , shear modulus constant (force/area).
SM2	$b_2$ , shear modulus constant (degree).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EDOT0	$\dot{\varepsilon}_{o,r}$ , reference strain-rate (time <sup>-1</sup> ).
G0	$g_{0,r}$ , normalized activation energy for a dislocation/dislocation interaction.
PINV	$\frac{1}{p'}$ , material constant.
QINV	$\frac{1}{q'}$ , material constant.
EDOTI	$\dot{\varepsilon}_{o,i}$ , reference strain-rate (time <sup>-1</sup> ).
G0I	$g_{0,i}$ , normalized activation energy for a dislocation/interstitial interaction.
PINVI	$\frac{1}{p'_i}$ , material constant.
QINVI	$\frac{1}{q'_i}$ , material constant.
EDOTS	$\dot{\varepsilon}_{o,s}$ , reference strain-rate (time <sup>-1</sup> ).
G0S	$g_{0,s}$ , normalized activation energy for a dislocation/solute interaction.
PINVS	$\frac{1}{p'_s}$ , material constant.
QINVS	$\frac{1}{q'_s}$ , material constant.
RHOCPR	$\rho c_p$ , product of density and specific heat.
TEMPRF	$T_{\text{ref}}$ , initial element temperature in degrees K.
ALPHA	$\alpha$ , material constant (typical value is between 0 and 2).
EPS0	$\varepsilon_o$ , factor to normalize strain rate in the calculation of $\Theta_o$ . (time <sup>-1</sup> ).

**Remarks:**

The flow stress  $\sigma$  is given by:

$$\sigma = \hat{\sigma}_a + \frac{G}{G_0} [s_{\text{th}} \hat{\sigma} + s_{\text{th},i} \hat{\sigma}_i + s_{\text{th},s} \hat{\sigma}_s]$$

The first product in the equation for  $\tau$  contains a micro-structure evolution variable, i.e.,  $\hat{\sigma}$ , called the *Mechanical Threshold Stress* (MTS), that is multiplied by a constant-structure

deformation variable  $s_{th}$ :  $s_{th}$  is a function of absolute temperature  $T$  and the plastic strain-rates  $\dot{\varepsilon}^p$ . The evolution equation for  $\hat{\sigma}$  is a differential hardening law representing dislocation-dislocation interactions:

$$\frac{\partial}{\partial \varepsilon^p} \equiv \Theta_o \left[ 1 - \frac{\tanh\left(\alpha \frac{\hat{\sigma}}{\hat{\sigma}_{\varepsilon s}}\right)}{\tanh(\alpha)} \right]$$

The term,  $\frac{\partial \hat{\sigma}}{\partial \varepsilon^p}$ , represents the hardening due to dislocation generation and the stress ratio,  $\frac{\hat{\sigma}}{\hat{\sigma}_{\varepsilon s}}$ , represents softening due to dislocation recovery. The threshold stress at zero strain-hardening  $\hat{\sigma}_{\varepsilon s}$  is called the saturation threshold stress. Relationships for  $\Theta_o$ ,  $\hat{\sigma}_{\varepsilon s}$  are:

$$\Theta_o = a_o + a_1 \ln\left(\frac{\dot{\varepsilon}^p}{\varepsilon_0}\right) + a_2 \sqrt{\frac{\dot{\varepsilon}^p}{\varepsilon_0}}$$

which contains the material constants,  $a_o$ ,  $a_1$ , and  $a_2$ . The constant,  $\hat{\sigma}_{\varepsilon s}$ , is given as:

$$\hat{\sigma}_{\varepsilon s} = \hat{\sigma}_{\varepsilon s0} \left(\frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_{\varepsilon s0}}\right)^{kT/Gb^3A}$$

which contains the input constants:  $\hat{\sigma}_{\varepsilon s0}$ ,  $\dot{\varepsilon}_{\varepsilon s0}$ ,  $b$ ,  $A$ , and  $k$ . The shear modulus  $G$  appearing in these equations is assumed to be a function of temperature and is given by the correlation.

$$G = G_0 - b_1 / (e^{b_2/T} - 1)$$

which contains the constants:  $G_0$ ,  $b_1$ , and  $b_2$ . For thermal-activation controlled deformation  $s_{th}$  is evaluated via an Arrhenius rate equation of the form:

$$s_{th} = \left\{ 1 - \left[ \frac{kT \ln\left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}^p}\right)}{Gb^3 g_0} \right]^{\frac{1}{q}} \right\}^{\frac{1}{p}}$$

The absolute temperature is given as:

$$T = T_{\text{ref}} + \frac{E}{\rho c_p}$$

where  $E$  is the internal energy density per unit initial volume.

**\*MAT\_PLASTICITY\_POLYMER**

This is Material Type 89. An elasto-plastic material with an arbitrary stress versus strain curve and arbitrary strain rate dependency can be defined. It is intended for applications where the elastic and plastic sections of the response are not as clearly distinguishable as they are for metals. Rate dependency of failure strain is included. Many polymers show a more brittle response at high rates of strain.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A8	F	F	F				
Default	none	none	none	none				

Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR				
Type	F	F	F	F				
Default	0	0	0	0				

Card 3	1	2	3	4	5	6	7	8
Variable	EFTX	DAMP	RFAC	LCFAIL				
Type	F	F	F	F				
Default	0	0	0	0				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.

VARIABLE	DESCRIPTION
E	Young's modulus.
PR	Poisson's ratio.
C	Strain rate parameter, <i>C</i> , (Cowper Symonds).
P	Strain rate parameter, <i>P</i> , (Cowper Symonds).
LCSS	Curve ID or Table ID. <ol style="list-style-type: none"><li>1. <i>Case 1: LCSS is a curve ID.</i> The curve defines effective stress as a function of total effective strain.</li><li>2. <i>Case 2: LCSS is a table ID.</i> Each strain rate value in the table is associated to a curve ID giving the stress as a function of effective strain for that rate. <p>The strain rate values defined in the table may be given as the natural logarithm of the strain rate. If the first stress-strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used. Since the tables are internally discretized to equally space the points, natural logarithms are necessary, for example, if the curves correspond to rates from <math>10^{-4}</math> to <math>10^4</math>.</p></li></ol>
LCSR	Load curve ID defining strain rate scaling effect on yield stress. If LCSR is negative, the load curve is evaluated using a binary search for the correct interval for the strain rate. The binary search is slower than the default incremental search, but in cases where large changes in the strain rate may occur over a single time step, it is more robust.
EFTX	Failure flag. EQ.0.0: failure determined by maximum tensile strain (default), EQ.1.0: failure determined only by tensile strain in local <i>x</i> direction, EQ.2.0: failure determined only by tensile strain in local <i>y</i> direction.
DAMP	Stiffness-proportional damping ratio. Typical values are $10^{-3}$ or $10^{-4}$ . If set too high instabilities can result.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RFAC	Filtering factor for strain rate effects. Must be between 0 (no filtering) and 1 (infinite filtering). The filter is a simple low pass filter to remove high frequency oscillation from the strain rates before they are used in rate effect calculations. The cut off frequency of the filter is $[(1 - \text{RFAC}) / \text{timestep}]$ rad/sec.
LCFAIL	Load curve ID giving variation of failure strain with strain rate. The points on the $x$ -axis should be natural log of strain rate, the $y$ -axis should be the true strain to failure. Typically this is measured by uniaxial tensile test, and the strain values converted to true strain.

**Remarks:**

- M89 vs. M24.** MAT\_089 is the same as MAT\_024 except for the following points:
  - Load curve lookup for yield stress is based on equivalent uniaxial strain, not plastic strain ([Remarks 2](#) and [3](#))
  - elastic stiffness is initially equal to  $E$  but will be increased according to the slope of the stress-strain curve ([Remark 7](#))
  - special strain calculation used for failure and damage ([Remark 2](#))
  - failure strain depends on strain rate ([Remark 4](#))
- Strain Calculation for Failure and Damage.** The strain used for failure and damage calculation,  $\epsilon_{\text{pm}}$  is based on an approximation of the greatest value of maximum principal strain encountered during the analysis:

$$\epsilon_{\text{pm}} = \max_{i \leq n} (\epsilon_H^i + \epsilon_{\text{VM}}^i)$$

where

$n$  = current time step index

$\max_{i \leq n}(\dots)$  = maximum value attained by the argument during the calculation

$$\epsilon_H = \frac{\epsilon_x + \epsilon_y + \epsilon_z}{3}$$

$\epsilon_x, \epsilon_y, \epsilon_z$  = cumulative strain in the local  $x, y,$  or  $z$  direction

$$\epsilon_{\text{vm}} = \sqrt{\frac{2}{3} \text{tr}(\epsilon'^T \epsilon')}, \text{ the usual definition of equivalent uniaxial strain}$$

$\epsilon'$  = deviatoric strain tensor, where each  $\epsilon_x, \epsilon_y,$  and  $\epsilon_z$  is cumulative

- Yield Stress Load Curves.** When looking up yield stress from the load curve LCSS, the  $x$ -axis value is  $\epsilon_{\text{vm}}$ .

4. **Failure Strain Load Curves.**

$$\varepsilon_{sr} = \frac{d\varepsilon_{pm}}{dt} = \text{strain rate for failure and damage calculation}$$

$$\varepsilon_F = \text{LCFAIL}(\varepsilon_{sr})$$

= Instantaneous true strain to failure from look-up on the curve LCFAIL

5. **Damage.** A damage approach is used to avoid sudden shocks when the failure strain is reached. Damage begins when the "strain ratio,"  $R$ , reaches 1.0, where

$$R = \int \frac{d\varepsilon_{pm}}{\varepsilon_F}$$

Damage is complete, and the element fails and is deleted, when  $R = 1.1$ . The damage,

$$D = \begin{cases} 1.0 & R < 1.0 \\ 10(1.1 - R) & 1.0 < R < 1.1 \end{cases}$$

is a reduction factor applied to all stresses, for example, when  $R = 1.05$ , then  $D = 0.5$ .

6. **Strain Definitions.** Unlike other LS-DYNA material models, both the input stress-strain curve and the strain to failure are defined as total true strain, not plastic strain. The input can be defined from uniaxial tensile tests; nominal stress and nominal strain from the tests must be converted to true stress and true strain. The elastic component of strain must not be subtracted out.

7. **Elastic Stiffness Scaling.** The stress-strain curve is permitted to have sections steeper (i.e. stiffer) than the elastic modulus. When these are encountered the elastic modulus is increased to prevent spurious energy generation. The elastic stiffness is scaled by a factor  $f_e$ , which is calculated as follows:

$$f_e = \max\left(1.0, \frac{S_{\max}}{3G}\right)$$

where

$G$  = initial shear modulus

$S_{\max}$  = maximum slope of stress-strain curve encountered during the analysis

8. **Precision.** Double precision is recommended when using this material model, especially if the strains become high.

9. **Shell Numbering.** Invariant shell numbering is recommended when using this material model. See \*CONTROL\_ACCURACY.

**\*MAT\_ACOUSTIC**

This is Material Type 90. This model is appropriate for tracking low pressure stress waves in an acoustic media such as air or water and can be used only with the acoustic pressure element formulation. The acoustic pressure element requires only one unknown per node. This element is very cost effective. Optionally, cavitation can be allowed.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	C	BETA	CF	ATMOS	GRAV	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	XN	YN	ZN		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
C	Sound speed
BETA	Damping factor. Recommend values are between 0.1 and 1.0.
CF	Cavitation flag: EQ.0.0: off, EQ.1.0: on.
ATMOS	Atmospheric pressure (optional)
GRAV	Gravitational acceleration constant (optional)
XP	x-coordinate of free surface point
YP	y-coordinate of free surface point



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<b>VARIABLE</b>	<b>DESCRIPTION</b>
ZP	z-coordinate of free surface point
XN	x-direction cosine of free surface normal vector
YN	y-direction cosine of free surface normal vector
ZN	z-direction cosine of free surface normal vector

**\*MAT\_SOFT\_TISSUE\_{OPTION}**

Available options include:

<BLANK>

VISCO

This is Material Type 91 (*OPTION*=<BLANK>) or Material Type 92 (*OPTION* = VISCO). This material is a transversely isotropic hyperelastic model for representing biological soft tissues such as ligaments, tendons, and fascia. The representation provides an isotropic Mooney-Rivlin matrix reinforced by fibers having a strain energy contribution with the qualitative material behavior of collagen. The model has a viscoelasticity option which activates a six-term Prony series kernel for the relaxation function. In this case, the hyperelastic strain energy represents the elastic (long-time) response. See Weiss et al. [1996] and Puso and Weiss [1998] for additional details.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	C1	C2	C3	C4	C5	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	XK	XLAM	FANG	XLAMO	FAILSF	FAILSM	FAILSHR	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	AX	AY	AZ	BX	BY	BZ	
Type	F	F	F	F	F	F	F	

Card 4	1	2	3	4	5	6	7	8
Variable	LA1	LA2	LA3	MACF				
Type	F	F	F	I				

**Prony Series Card 1.** Additional card for VISCO keyword option.

Card 5	1	2	3	4	5	6	7	8
Variable	S1	S2	S3	S4	S5	S6		
Type	F	F	F	F	F	F		

**Prony Series Card 2.** Additional card for VISCO keyword option.

Card 6	1	2	3	4	5	6	7	8
Variable	T1	T2	T3	T4	T5	T6		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
C1 - C5	Hyperelastic coefficients (see equations below)
XK	Bulk Modulus
XLAM	Stretch ratio at which fibers are straightened
FANG	Fiber angle in local shell coordinate system (shells only)
XLAM0	Initial fiber stretch (optional)
FAILSF	Stretch ratio for ligament fibers at failure (applies to shell elements only). If zero, failure is not considered.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FAILSM	Stretch ratio for surrounding matrix material at failure (applies to shell elements only). If zero, failure is not considered.
FAILSHR	Shear strain at failure at a material point (applies to shell elements only). If zero, failure is not considered. This failure value is independent of FAILSF and FAILSM.
AOPT	<p>Material axes option, see <a href="#">Figure M2-1</a> (bricks only):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <i>a</i>-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>\mathbf{p}</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
AX, AY, AZ	<p>Equal to XP, YP, ZP for AOPT = 1,            Equal to A1, A2, A3 for AOPT = 2,            Equal to V1, V2, V3 for AOPT = 3 or 4.</p>

VARIABLE	DESCRIPTION
BX, BY, BZ	Equal to D1, D2, D3 for AOPT = 2 Equal to XP, YP, ZP for AOPT = 4
LAX, LAY, LAZ	Local fiber orientation vector (bricks only)
MACF	Material axes change flag for brick elements: EQ.1: No change, default, EQ.2: switch material axes <i>a</i> and <i>b</i> , EQ.3: switch material axes <i>a</i> and <i>c</i> , EQ.4: switch material axes <i>b</i> and <i>c</i> .
S1 – S6	Factors in the Prony series.
T1 - T6	Characteristic times for Prony series relaxation kernel for VISCO option.

### Remarks:

The overall strain energy  $W$  is "uncoupled" and includes two isotropic deviatoric matrix terms, a fiber term  $F$ , and a bulk term:

$$W = C_1(\tilde{I}_1 - 3) + C_2(\tilde{I}_2 - 3) + F(\lambda) + \frac{1}{2}K[\ln(J)]^2$$

Here,  $\tilde{I}_1$  and  $\tilde{I}_2$  are the deviatoric invariants of the right Cauchy deformation tensor,  $\lambda$  is the deviatoric part of the stretch along the current fiber direction, and  $J = \det\mathbf{F}$  is the volume ratio. The material coefficients  $C_1$  and  $C_2$  are the Mooney-Rivlin coefficients, while  $K$  is the effective bulk modulus of the material (input parameter XK).

The derivatives of the fiber term  $F$  are defined to capture the behavior of crimped collagen. The fibers are assumed to be unable to resist compressive loading - thus the model is isotropic when  $\lambda < 1$ . An exponential function describes the straightening of the fibers, while a linear function describes the behavior of the fibers once they are straightened past a critical fiber stretch level  $\lambda \geq \lambda^*$  (input parameter XLAM):

$$\frac{\partial F}{\partial \lambda} = \begin{cases} 0 & \lambda < 1 \\ \frac{C_3}{\lambda} [\exp(C_4(\lambda - 1)) - 1] & \lambda < \lambda^* \\ \frac{1}{\lambda} (C_5\lambda + C_6) & \lambda \geq \lambda^* \end{cases}$$

Coefficients  $C_3$ ,  $C_4$ , and  $C_5$  must be defined by the user.  $C_6$  is determined by LS-DYNA to ensure stress continuity at  $\lambda = \lambda^*$ . Sample values for the material coefficients  $C_1 - C_5$  and

$\lambda^*$  for ligament tissue can be found in Quapp and Weiss [1998]. The bulk modulus  $K$  should be at least 3 orders of magnitude larger than  $C_1$  to ensure near-incompressible material behavior.

Viscoelasticity is included via a convolution integral representation for the time-dependent second Piola-Kirchoff stress  $\mathbf{S}(\mathbf{C}, t)$ :

$$\mathbf{S}(\mathbf{C}, t) = \mathbf{S}^e(\mathbf{C}) + \int_0^t 2G(t-s) \frac{\partial W}{\partial \mathbf{C}(s)} ds$$

Here,  $\mathbf{S}^e$  is the elastic part of the second PK stress as derived from the strain energy, and  $G(t-s)$  is the reduced relaxation function, represented by a Prony series:

$$G(t) = \sum_{i=1}^6 S_i \exp\left(-\frac{t}{T_i}\right)$$

Puso and Weiss [1998] describe a graphical method to fit the Prony series coefficients to relaxation data that approximates the behavior of the continuous relaxation function proposed by Y-C. Fung, as quasilinear viscoelasticity.

#### **Remarks on Input Parameters:**

Cards 1 through 4 must be included for both shell and brick elements, although for shells cards 3 and 4 are ignored and may be blank lines.

For shell elements, the fiber direction lies in the plane of the element. The local axis is defined by a vector between nodes n1 and n2, and the fiber direction may be offset from this axis by an angle FANG.

For brick elements, the local coordinate system is defined using the convention described previously for \*MAT\_ORTHOTROPIC\_ELASTIC. The fiber direction is oriented in the local system using input parameters LAX, LAY, and LAZ. By default, (LAX, LAY, LAZ) = (1,0,0) and the fiber is aligned with the local x-direction.

An optional initial fiber stretch can be specified using XLAM0. The initial stretch is applied during the first time step. This creates preload in the model as soft tissue contacts and equilibrium is established. For example, a ligament tissue "uncrimping strain" of 3% can be represented with initial stretch value of 1.03.

If the **VISCO** option is selected, at least one Prony series term (S1, T1) must be defined.

**\*MAT\_ELASTIC\_6DOF\_SPRING\_DISCRETE\_BEAM**

This is Material Type 93. This material model is defined for simulating the effects of nonlinear elastic and nonlinear viscous beams by using six springs each acting about one of the six local degrees-of-freedom. The input consists of part ID's that reference material type, \*MAT\_ELASTIC\_SPRING\_DISCRETE\_BEAM above (type 74 above). Generally, these referenced parts are used only for the definition of this material model and are not referenced by any elements. The two nodes defining a beam may be coincident to give a zero length beam, or offset to give a finite length beam. For finite length discrete beams the absolute value of the variable SCOR in the SECTION\_BEAM input should be set to a value of 2.0, which causes the local r-axis to be aligned along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad is used to orient the beam for the directional springs.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	TPIDR	TPIDS	TPIDT	RPIDR	RPIDS	RPIDT
Type	A8	F	I	I	I	I	I	I

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
TPIDR	Translational motion in the local r-direction is governed by part ID TPIDR. If zero, no force is computed in this direction.
TPIDS	Translational motion in the local s-direction is governed by part ID TPIDS. If zero, no force is computed in this direction.
TPIDT	Translational motion in the local t-direction is governed by part ID TPIDT. If zero, no force is computed in this direction.
RPIDR	Rotational motion about the local r-axis is governed by part ID RPIDR. If zero, no moment is computed about this axis.
RPIDS	Rotational motion about the local s-axis is governed by part ID RPIDS. If zero, no moment is computed about this axis.

**\*MAT\_093**

**\*MAT\_ELASTIC\_6DOF\_SPRING\_DISCRETE\_BEAM**

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
RPIDT	Rotational motion about the local t-axis is governed by part ID RPIDT. If zero, no moment is computed about this axis.



**\*MAT\_INELASTIC\_SPRING\_DISCRETE\_BEAM**

This is Material Type 94. This model permits elastoplastic springs with damping to be represented with a discrete beam element type 6. A yield force versus deflection curve is used which can vary in tension and compression.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	F0	D	CDF	TDF	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	FLCID	HLCID	C1	C2	DLE	GLCID		
Type	F	F	F	F	F	I		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
K	Elastic loading/unloading stiffness. This is required input.
F0	Optional initial force. This option is inactive if this material is referenced in a part referenced by material type *MAT_INELASTIC_6DOF_SPRING
D	Optional viscous damping coefficient.
CDF	Compressive displacement at failure. Input as a positive number. After failure, no forces are carried. This option does not apply to zero length springs. EQ.0.0: inactive.
TDF	Tensile displacement at failure. After failure, no forces are carried. EQ.0.0: inactive.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FLCID	Load curve ID, see *DEFINE_CURVE, defining the yield force versus plastic deflection. If the origin of the curve is at (0,0) the force magnitude is identical in tension and compression, i.e., only the sign changes. If not, the yield stress in the compression is used when the spring force is negative. The plastic displacement increases monotonically in this implementation. The load curve is required input.
HLCID	Load curve ID, see *DEFINE_CURVE, defining force versus relative velocity (Optional). If the origin of the curve is at (0,0) the force magnitude is identical for a given magnitude of the relative velocity, i.e., only the sign changes.
C1	Damping coefficient.
C2	Damping coefficient
DLE	Factor to scale time units.
GLCID	Optional load curve ID, see *DEFINE_CURVE, defining a scale factor versus deflection for load curve ID, HLCID. If zero, a scale factor of unity is assumed.

**Remarks:**

The yield force is taken from the load curve:

$$F^Y = F_y(\Delta L^{\text{plastic}})$$

where  $L^{\text{plastic}}$  is the plastic deflection. A trial force is computed as:

$$F^T = F^n + K \times \Delta \dot{L}(\Delta t)$$

and is checked against the yield force to determine  $F$ :

$$F = \begin{cases} F^Y & \text{if } F^T > F^Y \\ F^T & \text{if } F^T \leq F^Y \end{cases}$$

The final force, which includes rate effects and damping, is given by:

$$F^{n+1} = F \times \left[ 1 + C1 \times \Delta \dot{L} + C2 \times \text{sgn}(\Delta \dot{L}) \ln \left( \max \left\{ 1, \frac{|\Delta \dot{L}|}{DLE} \right\} \right) \right] + D \times \Delta \dot{L} + g(\Delta L)h(\Delta \dot{L})$$

Unless the origin of the curve starts at (0,0), the negative part of the curve is used when the spring force is negative where the negative of the plastic displacement is used to interpolate,  $F_y$ . The positive part of the curve is used whenever the force is positive. In these equations,  $\Delta L$  is the change in length

$$\Delta L = \text{current length} - \text{initial length}$$

The cross sectional area is defined on the section card for the discrete beam elements, See \*SECTION\_BEAM. The square root of this area is used as the contact thickness offset if these elements are included in the contact treatment.

**\*MAT\_INELASTIC\_6DOF\_SPRING\_DISCRETE\_BEAM**

This is Material Type 95. This material model is defined for simulating the effects of nonlinear inelastic and nonlinear viscous beams by using six springs each acting about one of the six local degrees-of-freedom. The input consists of part ID's that reference material type, \*MAT\_INELASTIC\_SPRING\_DISCRETE\_BEAM above (type 94). Generally, these referenced parts are used only for the definition of this material model and are not referenced by any elements. The two nodes defining a beam may be coincident to give a zero length beam, or offset to give a finite length beam. For finite length discrete beams the absolute value of the variable SCOR in the SECTION\_BEAM input should be set to a value of 2.0, which causes the local r-axis to be aligned along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad must be used to orient the beam for zero length beams.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	TPIDR	TPIDS	TPIDT	RPIDR	RPIDS	RPIDT
Type	A8	F	I	I	I	I	I	I

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
TPIDR	Translational motion in the local r-direction is governed by part ID TPIDR. If zero, no force is computed in this direction.
TPIDS	Translational motion in the local s-direction is governed by part ID TPIDS. If zero, no force is computed in this direction.
TPIDT	Translational motion in the local t-direction is governed by part ID TPIDT. If zero, no force is computed in this direction.
RPIDR	Rotational motion about the local r-axis is governed by part ID RPIDR. If zero, no moment is computed about this axis.
RPIDS	Rotational motion about the local s-axis is governed by part ID RPIDS. If zero, no moment is computed about this axis.

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
RPIDT	Rotational motion about the local t-axis is governed by part ID RPIDT. If zero, no moment is computed about this axis.

**\*MAT\_BRITTLE\_DAMAGE**

This is Material Type 96.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	TLIMIT	SLIMIT	FTOUGH	SRETEN
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	VISC	FRA_RF	E_RF	YS_RF	EH_RF	FS_RF	SIGY	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
TLIMIT	Tensile limit.
SLIMIT	Shear limit.
FTOUGH	Fracture toughness.
SRETEN	Shear retention.
VISC	Viscosity.
FRA_RF	Fraction of reinforcement in section.
E_RF	Young's modulus of reinforcement.
YS_RF	Yield stress of reinforcement.
EH_RF	Hardening modulus of reinforcement.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FS_RF	Failure strain (true) of reinforcement.
SIGY	Compressive yield stress. EQ.0: no compressive yield

**Remarks:**

A full description of the tensile and shear damage parts of this material model is given in Govindjee, Kay and Simo [1994,1995]. It is an anisotropic brittle damage model designed primarily for concrete though it can be applied to a wide variety of brittle materials. It admits progressive degradation of tensile and shear strengths across smeared cracks that are initiated under tensile loadings. Compressive failure is governed by a simplistic J2 flow correction that can be disabled if not desired. Damage is handled by treating the rank 4 elastic stiffness tensor as an evolving internal variable for the material. Softening induced mesh dependencies are handled by a characteristic length method [Oliver 1989].

Description of properties:

1.  $E$  is the Young's modulus of the undamaged material also known as the virgin modulus.
2.  $\nu$  is the Poisson's ratio of the undamaged material also known as the virgin Poisson's ratio.
3.  $f_n$  is the initial principal tensile strength (stress) of the material. Once this stress has been reached at a point in the body a smeared crack is initiated there with a normal that is co-linear with the 1st principal direction. Once initiated, the crack is fixed at that location, though it will convect with the motion of the body. As the loading progresses the allowed tensile traction normal to the crack plane is progressively degraded to a small machine dependent constant.

The degradation is implemented by reducing the material's modulus normal to the smeared crack plane according to a maximum dissipation law that incorporates exponential softening. The restriction on the normal tractions is given by

$$\phi_t = (\mathbf{n} \otimes \mathbf{n}) : \boldsymbol{\sigma} - f_n + (1 - \varepsilon)f_n(1 - \exp[-H\alpha]) \leq 0$$

where  $\mathbf{n}$  is the smeared crack normal,  $\varepsilon$  is the small constant,  $H$  is the softening modulus, and  $\alpha$  is an internal variable.  $H$  is set automatically by the program; see  $g_c$  below.  $\alpha$  measures the crack field intensity and is output in the equivalent plastic strain field,  $\bar{\varepsilon}^p$ , in a normalized fashion.

The evolution of alpha is governed by a maximum dissipation argument. When the normalized value reaches unity it means that the material's strength has been reduced to 2% of its original value in the normal and parallel directions to the smeared crack. Note that for plotting purposes it is never output greater than 5.

4.  $f_s$  is the initial shear traction that may be transmitted across a smeared crack plane. The shear traction is limited to be less than or equal to  $f_s(1 - \beta)(1 - \exp[-H\alpha])$ , through the use of two orthogonal shear damage surfaces. Note that the shear degradation is coupled to the tensile degradation through the internal variable alpha which measures the intensity of the crack field.  $\beta$  is the shear retention factor defined below. The shear degradation is taken care of by reducing the material's shear stiffness parallel to the smeared crack plane.
5.  $g_c$  is the fracture toughness of the material. It should be entered as fracture energy per unit area crack advance. Once entered the softening modulus is automatically calculated based on element and crack geometries.
6.  $\beta$  is the shear retention factor. As the damage progresses the shear tractions allowed across the smeared crack plane asymptote to the product  $\beta f_s$ .
7.  $\eta$  represents the viscosity of the material. Viscous behavior is implemented as a simple Perzyna regularization method. This allows for the inclusion of first order rate effects. The use of some viscosity is recommended as it serves as regularizing parameter that increases the stability of calculations.
8.  $\sigma_y$  is a uniaxial compressive yield stress. A check on compressive stresses is made using the J2 yield function  $\mathbf{s} : \mathbf{s} - \sqrt{\frac{2}{3}}\sigma_y \leq 0$ , where  $\mathbf{s}$  is the stress deviator. If violated, a J2 return mapping correction is executed. This check is executed when (1) no damage has taken place at an integration point yet, (2) when damage has taken place at a point but the crack is currently closed, and (3) during active damage after the damage integration (i.e. as an operator split). Note that if the crack is open the plasticity correction is done in the plane-stress subspace of the crack plane.

A variety of experimental data has been replicated using this model from quasi-static to explosive situations. Reasonable properties for a standard grade concrete would be  $E = 3.15 \times 10^6$  psi,  $f_n = 450$  psi,  $f_s = 2100$  psi,  $\nu = 0.2$ ,  $g_c = 0.8$  lbs/in,  $\beta = 0.03$ ,  $\eta = 0.0$  psi-sec,  $\sigma_y = 4200$  psi. For stability, values of  $\eta$  between 104 to 106 psi/sec are recommended. Our limited experience thus far has shown that many problems require nonzero values of  $\eta$  to run to avoid error terminations.

Various other internal variables such as crack orientations and degraded stiffness tensors are internally calculated but currently not available for output.



**\*MAT\_GENERAL\_JOINT\_DISCRETE\_BEAM**

This is Material Type 97. This model is used to define a general joint constraining any combination of degrees of freedom between two nodes. The nodes may belong to rigid or deformable bodies. In most applications the end nodes of the beam are coincident and the local coordinate system (r,s,t axes) is defined by CID (see \*SECTION\_BEAM).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	TR	TS	TT	RR	RS	RT
Type	A8	F	I	I	I	I	I	
Remarks	1							

Card 2	1	2	3	4	5	6	7	8
Variable	RPST	RPSR						
Type	F	F						
Remarks	2							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
TR	Translational constraint code along the r-axis (0 ⇒ free, 1 ⇒ constrained)
TS	Translational constraint code along the s-axis (0 ⇒ free, 1 ⇒ constrained)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TT	Translational constraint code along the t-axis (0 ⇒ free, 1 ⇒ constrained)
RR	Rotational constraint code about the r-axis (0 ⇒ free, 1 ⇒ constrained)
RS	Rotational constraint code about the s-axis (0 ⇒ free, 1 ⇒ constrained)
RT	Rotational constraint code about the t-axis (0 ⇒ free, 1 ⇒ constrained)
RPST	Penalty stiffness scale factor for translational constraints.
RPSR	Penalty stiffness scale factor for rotational constraints.

**Remarks:**

1. For explicit calculations, the additional stiffness due to this joint may require addition mass and inertia for stability. Mass and rotary inertia for this beam element is based on the defined mass density, the volume, and the mass moment of inertia defined in the \*SECTION\_BEAM input.
2. The penalty stiffness applies to explicit calculations. For implicit calculations, constraint equations are generated and imposed on the system equations; therefore, these constants, RPST and RPSR, are not used.

**\*MAT\_SIMPLIFIED\_JOHNSON\_COOK\_{OPTION}**

Available options include:

<BLANK>

STOCHASTIC

This is Material Type 98 implementing Johnson/Cook strain sensitive plasticity. It is used for problems where the strain rates vary over a large range. In contrast to the full Johnson/Cook model (material type 15) this model introduces the following simplifications:

1. thermal effects and damage are ignored,
2. and the maximum stress is directly limited since thermal softening which is very significant in reducing the yield stress under adiabatic loading is not available.

An iterative plane stress update is used for the shell elements, but due to the simplifications related to thermal softening and damage, this model is 50% faster than the full Johnson/Cook implementation. To compensate for the lack of thermal softening, limiting stress values are introduced to keep the stresses within reasonable limits.

A resultant formulation for the Belytschko-Tsay, the C0 Triangle, and the fully integrated type 16 shell elements is available and can be activated by specifying either zero or one through thickness integration point on the \*SECTION\_SHELL card. While less accurate than through thickness integration, this formulation runs somewhat faster. Since the stresses are not computed in the resultant formulation, the stresses written to the databases for the resultant elements are set to zero.

This model is also available for the Hughes-Liu beam, the Belytschko-Schwer beam, and for the truss element. For the resultant beam formulation, the rate effects are approximated by the axial rate, since the thickness of the beam about its bending axes is unknown. Because this model is primarily used for structural analysis, the pressure is determined using the linear bulk modulus.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	VP			
Type	A8	F	F	F	F			
Default	none	none	none	none	0.0			

Card 2	1	2	3	4	5	6	7	8
Variable	A	B	N	C	PSFAIL	SIGMAX	SIGSAT	EPSO
Type	F	F	F	F	F	F	F	F
Default	none	0.0	0.0	0.0	1.0E+17	SIGSAT	1.0E+28	1.0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default), EQ.1.0: Viscoplastic formulation.  This option applies only to the 4-node shell and 8-node thick shell if and only if through thickness integration is used.
A	See equations below.
B	See equations below.
N	See equations below.
C	See equations below.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PSFAIL	Effective plastic strain at failure. If zero failure is not considered.
SIGMAX	Maximum stress obtainable from work hardening before rate effects are added (optional). This option is ignored if VP = 1.0
SIGSAT	Saturation stress which limits the maximum value of effective stress which can develop after rate effects are added (optional).
EPS0	Quasi-static threshold strain rate. See description under *MAT_015.

**Remarks:**

Johnson and Cook express the flow stress as

$$\sigma_y = \left( A + B \bar{\epsilon}^{p^n} \right) (1 + C \ln \epsilon^*)$$

where

$A, B, C$  = input constants

$\bar{\epsilon}^p$  = effective plastic strain

$$\epsilon^* = \frac{\dot{\bar{\epsilon}}}{\text{EPS0}}$$

= normalized effective strain rate

The maximum stress is limited by SIGMAX and SIGSAT by:

$$\sigma_y = \min \left\{ \min \left[ A + B \bar{\epsilon}^{p^n}, \text{SIGMAX} \right] (1 + c \ln \epsilon^*), \text{SIGSAT} \right\}$$

Failure occurs when the effective plastic strain exceeds PSFAIL.

If the viscoplastic option is active, VP = 1.0, the parameters SIGMAX and SIGSAT are ignored since these parameters make convergence of the viscoplastic strain iteration loop difficult to achieve. The viscoplastic option replaces the plastic strain in the forgoing equations by the viscoplastic strain and the strain rate by the viscoplastic strain rate. Numerical noise is substantially reduced by the viscoplastic formulation.

The STOCHASTIC option allows spatially varying yield and failure behavior. See \*DEFINE\_STOCHASTIC\_VARIATION for additional information.

# **\*MAT\_099 \*MAT\_SIMPLIFIED\_JOHNSON\_COOK\_ORTHOTROPIC\_DAMAGE**

## **\*MAT\_SIMPLIFIED\_JOHNSON\_COOK\_ORTHOTROPIC\_DAMAGE**

This is Material Type 99. This model, which is implemented with multiple through thickness integration points, is an extension of model 98 to include orthotropic damage as a means of treating failure in aluminum panels. Directional damage begins after a defined failure strain is reached in tension and continues to evolve until a tensile rupture strain is reached in either one of the two orthogonal directions. After rupture is detected at NUMINT integration points, the element is deleted.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	VP	EPPFR	LCDM	NUMINT
Type	A8	F	F	F	F	F	I	I
Default	none	none	none	none	0.0	1.e+16	optional	all points

Card 2	1	2	3	4	5	6	7	8
Variable	A	B	N	C	PSFAIL	SIGMAX	SIGSAT	EPSO
Type	F	F	F	F	F	F	F	F
Default	none	0.0	0.0	0.0	1.0E+17	SIGSAT	1.0E+28	1.0

### **VARIABLE**

### **DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio

<b>VARIABLE</b>	<b>DESCRIPTION</b>
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default), EQ.1.0: Viscoplastic formulation. This option applies only to the 4-node shell and 8-node thick shell if and only if through thickness integration is used.
EPPFR	Plastic strain at which material ruptures (logarithmic).
LCDM	Load curve ID defining nonlinear damage curve. See <a href="#">Figure M81-2</a> .
NUMINT	Number of through thickness integration points which must fail before the element is deleted. (If zero, all points must fail.) The default of all integration points is not recommended since elements undergoing large strain are often not deleted due to nodal fiber rotations which limit strains at active integration points after most points have failed. Better results are obtained if NUMINT is set to 1 or a number less than one half of the number of through thickness points. For example, if four through thickness points are used, NUMINT should not exceed 2, even for fully integrated shells which have 16 integration points.
A	See equations below.
B	See equations below.
N	See equations below.
C	See equations below.
PSFAIL	Principal plastic strain at failure. If zero failure is not considered.
SIGMAX	Maximum stress obtainable from work hardening before rate effects are added (optional). This option is ignored if VP = 1.0
SIGSAT	Saturation stress which limits the maximum value of effective stress which can develop after rate effects are added (optional).
EPS0	Quasi-static threshold strain rate. See description under *MAT_015.

**Remarks:**

See the description for the SIMPLIFIED\_JOHNSON\_COOK model above.

**\*MAT\_SPOTWELD\_{OPTION}**

This is Material Type 100. The material model applies to beam element type 9 and to solid element type 1. The failure models apply to both beam and solid elements.

In the case of solid elements, if hourglass type 4 is specified then hourglass type 4 will be used, otherwise, hourglass type 6 will be automatically assigned. Hourglass type 6 is preferred.

The beam elements, based on the Hughes-Liu beam formulation, may be placed between any two deformable shell surfaces and tied with constraint contact, **\*CONTACT\_SPOTWELD**, which eliminates the need to have adjacent nodes at spot weld locations. Beam spot welds may be placed between rigid bodies and rigid/deformable bodies by making the node on one end of the spot weld a rigid body node which can be an extra node for the rigid body, see **\*CONSTRAINED\_EXTRA\_NODES\_OPTION**. In the same way rigid bodies may also be tied together with this spot weld option. This weld option should not be used with rigid body switching. The foregoing advice is valid if solid element spot welds are used; however, since the solid elements have just three degrees-of-freedom at each node, **\*CONTACT\_TIED\_SURFACE\_TO\_SURFACE** must be used instead of **\*CONTACT\_SPOTWELD**.

In flat topologies the shell elements have an unconstrained drilling degree-of-freedom which prevents torsional forces from being transmitted. If the torsional forces are deemed to be important, brick elements should be used to model the spot welds.

Beam and solid element force resultants for **MAT\_SPOTWELD** are written to the spot weld force file, **swforc**, and the file for element stresses and resultants for designated elements, **elout**.

**It is advisable to include all spot welds, which provide the slave nodes, and spot welded materials, which define the master segments, within a single \*CONTACT\_SPOTWELD interface for beam element spot welds or a \*CONTACT\_TIED\_SURFACE\_TO\_SURFACE interface for solid element spot welds.** As a constraint method these interfaces are treated independently which can lead to significant problems if such interfaces share common nodal points. An added benefit is that memory usage can be substantially less with a single interface.

Available options include:

<BLANK>

DAMAGE-FAILURE

The **DAMAGE-FAILURE** option causes one additional line to be read with the damage parameter and a flag that determines how failure is computed from the resultants. On this line the parameter, **RS**, if nonzero, invokes damage mechanics combined with the plasticity model to achieve a smooth drop off of the resultant forces prior to the removal of the spot



weld. The parameter OPT determines the method used in computing resultant based failure, which is unrelated to damage.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	EH	DT	TFAIL
Type	A8	F	F	F	F	F	F	F

**Card 2 for no failure.** Additional card for <blank> keyword option.

Card 2	1	2	3	4	5	6	7	8
Variable	EFAIL	NRR	NRS	NRT	MRR	MSS	MTT	NF
Type	F	F	F	F	F	F	F	F

**Card 2 for resultant based failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = -1.0 or 0.0.

Card 2	1	2	3	4	5	6	7	8
Variable	EFAIL	NRR	NRS	NRT	MRR	MSS	MTT	NF
Type	F	F	F	F	F	F	F	F

**Card 2 for stress based failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 1.0 and positive values in fields 2 and 3.

Card 2	1	2	3	4	5	6	7	8
Variable	EFAIL	SIGAX	SIGTAU					NF
Type	F	F	F					F

**Card 2 for stress based failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 1.0 and negative values in fields 2 and 3.

Card 2	1	2	3	4	5	6	7	8
Variable	EFAIL	-LCAX	-LCTAU					NF
Type	F	F	F					F

**Card 2 for user subroutine based failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 2.0, 12, or 22.

Card 2	1	2	3	4	5	6	7	8
Variable	EFAIL	USRV1	USRV2	USRV3	USRV4	USRV5	USRV6	NF
Type	F	F	F	F	F	F	F	F

**Card 2 for OPT = 3.0 or 4.0.**

Card 2	1	2	3	4	5	6	7	8
Variable	EFAIL	ZD	ZT	ZALP1	ZALP2	ZALP3	ZRRAD	NF
Type	F	F	F	F	F	F	F	F

**Card 2 for OPT = 5.0.**

Card 2	1	2	3	4	5	6	7	8
Variable	EFAIL	ZD	ZT	ZT2				
Type	F	F	F	F				



**Additional card for OPT = 12 or 22**

Card 5	1	2	3	4	5	6	7	8
Variable	USRV15	USRV16	USRV17	USRV18	USRV19	USRV20	USRV21	USRV22
Type	F	F	F	F	F	F	F	F

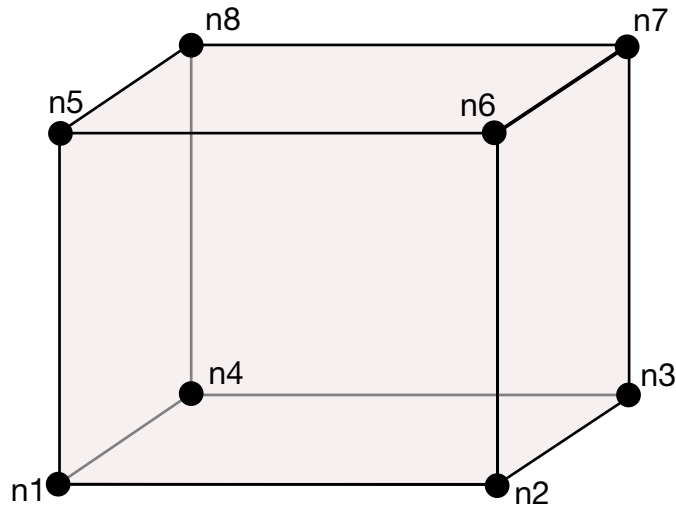
**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus. If input as negative, a uniaxial option for solid spot welds is invoked; see "Uniaxial option" in remarks.
PR	Poisson's ratio
SIGY	GT.0: Initial yield stress. LT.0: A yield curve or table is assigned by  SIGY .
EH	Plastic hardening modulus, $E_h$
DT	Time step size for mass scaling, $\Delta t$
TFAIL	Failure time if nonzero. If zero this option is ignored.
EFAIL	Effective plastic strain in weld material at failure. If the damage option is inactive, the spot weld element is deleted when the plastic strain at each integration point exceeds EFAIL. If the damage option is active, the plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs.
NRR	Axial force resultant $N_{rr_F}$ or maximum axial stress $\sigma_{rr}^F$ at failure depending on the value of OPT (see below). If zero, failure due to this component is not considered. If negative,  NRR  is the load curve ID defining the maximum axial stress at failure as a function of the effective strain rate.
NRS	Force resultant $N_{rs_F}$ or maximum shear stress $\tau^F$ at failure depending on the value of OPT (see below). If zero, failure due to this component is not considered. If negative,  NRS  is the load curve ID

VARIABLE	DESCRIPTION
	defining the maximum shear stress at failure as a function of the effective strain rate.
NRT	Force resultant $N_{rt_F}$ at failure. If zero, failure due to this component is not considered.
MRR	Torsional moment resultant $M_{rr_F}$ at failure. If zero, failure due to this component is not considered.
MSS	Moment resultant $M_{ss_F}$ at failure. If zero, failure due to this component is not considered.
MTT	Moment resultant $M_{tt_F}$ at failure. If zero, failure due to this component is not considered.
NF	Number of force vectors stored for filtering.
SIGAX	Maximum axial stress $\sigma_{rr}^F$ at failure. If zero, failure due to this component is not considered.
SIGTAU	Maximum shear stress $\tau^F$ at failure. If zero, failure due to this component is not considered.
LCAX	Load curve ID defining the maximum axial stress at failure as a function of the effective strain rate. Input as a negative number.
LCTAU	Load curve ID defining the maximum shear stress at failure as a function of the effective strain rate. Input as a negative number.
USRV $n$	Failure constants for user failure subroutine, $n = 1, 2, \dots, 6$
ZD	Notch diameter
ZT	Sheet thickness.
ZALP1	Correction factor alpha1
ZALP2	Correction factor alpha2
ZALP3	Correction factor alpha3
ZRRAD	Notch root radius (OPT = 3.0 only).
ZT2	Second sheet thickness (OPT = 5.0 only)
LCT	Load curve or Table ID. Load curve defines resultant failure force

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	under tension as a function of loading direction (in degree range 0 to 90). Table defines these curves as functions of strain rates. See remarks. (OPT = 11.0 only)
LCC	Load curve or Table ID. Load curve defines resultant failure force under compression as a function of loading direction (in degree range 0 to 90). Table defines these curves as functions of strain rates. See remarks. (OPT = 11.0 only)
RS	Rupture strain. Define if and only if damage is active.
OPT	Failure option: <ul style="list-style-type: none"><li>EQ.-9: OPT = 9 failure is evaluated and written to the swforc file, but element failure is suppressed.</li><li>EQ.-2: same as option -1 but in addition, the peak value of the failure criteria and the time it occurs is stored and is written into the swforc database. This information may be necessary since the instantaneous values written at specified time intervals may miss the peaks. Additional storage is allocated to store this information.</li><li>EQ.-1: OPT = 0 failure is evaluated and written into the swforc file, but element failure is suppressed</li><li>EQ.0: resultant based failure</li><li>EQ.1: stress based failure computed from resultants (Toyota)</li><li>EQ.2: user subroutine uweldfail to determine failure</li><li>EQ.3: notch stress based failure. (beam and hex assembly welds only).</li><li>EQ.4: stress intensity factor at failure. (beam and hex assembly welds only).</li><li>EQ.5: structural stress at failure (beam and hex assembly welds only).</li><li>EQ.6: stress based failure computed from resultants (Toyota). In this option a shell strain rate dependent failure model is used (beam and hex assembly welds only). The static failure stresses are defined by part ID using the keyword *DEFINE_SPOTWELD_RUPTURE_STRESS.</li><li>EQ.7: stress based failure for solid elements (Toyota) with peak stresses computed from resultants, and strength values input for pairs of parts, see *DEFINE_SPOTWELD_FAIL-</li></ul>

VARIABLE	DESCRIPTION
	URE_RESULTANTS. Strain rate effects are optional.
	EQ.8: not used.
	EQ.9: stress based failure from resultants (Toyota). In this option a shell strain rate dependent failure model is used (beam welds only). The static failure stresses are defined by part ID using the keyword *DEFINE_SPOTWELD_RUPTURE_PARAMETER.
	EQ.10: stress based failure with rate effects. Failure data is defined by material using the keyword *DEFINE_SPOWELD_FAILURE.
	EQ.11: resultant based failure (beams only). In this option load curves or tables LCT (tension) and LCC (compression) can be defined as resultant failure force vs. loading direction (curve) or resultant failure force vs. loading direction vs. strain rate (table).
	EQ.12: user subroutine uweldfail12 with 22 material constants to determine damage and failure.
	EQ.22: user subroutine uweldfail22 with 22 material constants to determine failure.
FVAL	Failure parameter. If OPT:
	EQ.-2: Not used.
	EQ.-1: Not used.
	EQ.0: Function ID (*DEFINE_FUNCTION) to define alternative Weld Failure. If this is set, the values given for NRR, NRS, NRT, MRR, MSS and MTT in Card 2 are ignored. (see description of Weld Failure for OPT = 0)
	EQ.1: Not used.
	EQ.2: Not used.
	EQ.3: Notch stress value at failure (KF).
	EQ.4: Stress intensity factor value at failure (KeqF).
	EQ.5: Structural stress value at failure (sF).
	EQ.6: Number of cycles that failure condition must be met to trigger beam deletion.
	EQ.7: Not used.
	EQ.9: Number of cycles that failure condition must be met to



**Figure M100-1.** A solid element used as spot weld is shown. When resultant based failure is used orientation is very important. Nodes n1-n4 attach to the lower shell mid-surface and nodes n5-n8 attach to the upper shell mid-surface. The resultant forces and moments are computed based on the assumption that the brick element is properly oriented.

VARIABLE	DESCRIPTION
	trigger beam deletion. EQ.10: ID of data defined by *DEFINE_SPOTWELD_FAILURE.
TRUE_T	True weld thickness. This optional value is available for solid element failure, and is used to reduce the moment contribution to the failure calculation from artificially thick weld elements under shear loading so shear failure can be modeled more accurately. See comments under the remarks for *MAT_SPOTWELD_DAIMLER CHRYSLER
ASFF	Weld assembly simultaneous failure flag EQ.0: Damaged elements fail individually. EQ.1: Damaged elements fail when first reaches failure criterion.
BETA	Damage model decay rate.
DMGOPT	Damage option flag. If DMGOPT: EQ.0: Plastic strain based damage. EQ.1: Plastic strain based damage with post damage stress limit EQ.2: Time based damage with post damage stress limit EQ.10: Like DMGOPT = 0, but failure option will initiate damage



VARIABLE	DESCRIPTION
	EQ.11: Like DMGOPT = 1, but failure option will initiate damage EQ.12: Like DMGOPT = 2, but failure option will initiate damage
FMODE	Failure surface ratio for damage or failure, for DMGOPT = 10, 11, or 12 EQ.0: Damage initiates GT.0: Damage or failure (see remark)
FFCAP	Failure function limit for OPT = 0 or -1, and DMGOPT = 10, 11, or 12 EQ.0: Damage initiates GT.0: Damage or failure (see remark)
USRV $n$	Failure constants for OPT = 12 or 22 user defined failure, $n = 7, 8, \dots, 22$

### Weld Failure

Spot weld material is modeled with isotropic hardening plasticity coupled to failure models. EFAIL specifies a failure strain which fails each integration point in the spot weld independently. The OPT parameter is used to specify a failure criterion that fails the entire weld element when the criterion is met. Alternatively, EFAIL and OPT option may be used to initiate damage when the DAMAGE-FAILURE option is active using RS, BETA, and DMGOPT as described below.

Beam spot weld elements can use any OPT value except 7. Brick spot weld elements can use any OPT value except 3, 4, 5, 6, 9, and -9. Hex assembly spot welds can use any OPT value except 9 and -9.

### OPT = -1 or 0

OPT = 0 and OPT = -1 invoke a resultant-based failure criterion that fails the weld if the resultants are outside of the failure surface defined by:

$$\left[ \frac{\max(N_{rr}, 0)}{N_{rrF}} \right]^2 + \left[ \frac{N_{rs}}{N_{rsF}} \right]^2 + \left[ \frac{N_{rt}}{N_{rtF}} \right]^2 + \left[ \frac{M_{rr}}{M_{rrF}} \right]^2 + \left[ \frac{M_{ss}}{M_{ssF}} \right]^2 + \left[ \frac{M_{tt}}{M_{ttF}} \right]^2 - 1 = 0$$

where the *numerators* in the equation are the resultants calculated in the local coordinates of the cross section, and the **denominators** are the values specified in the input. If OPT = -1, the failure surface equation is evaluated, but element failure is suppressed. This allows easy identification of vulnerable spot welds when post-processing. Failure is likely to occur if FC > 1.0.

Alternatively a \*DEFINE\_FUNCTION could be used to define the Weld Failure for OPT = 0. Then set FVAL = function ID. Such a function could look like this:

```
*DEFINE_FUNCTION
    100
    func (nrr, nrs, nrt, mrr, mss, mtt) = (nrr/5.0) * (nrr/5.0)
```

The six arguments for this function (nrr, ..., mtt) are the force and moment resultants during the computation.

**OPT = 1:**

OPT = 1 invokes a stress based failure model, which was developed by *Toyota Motor Corporation* and is based on the peak axial and transverse shear stresses. The weld fails if the stresses are outside of the failure surface defined by

$$\left(\frac{\sigma_{rr}}{\sigma_{rr}^F}\right)^2 + \left(\frac{\tau}{\tau^F}\right)^2 - 1 = 0$$

If strain rates are considered then the failure criteria becomes:

$$\left[\frac{\sigma_{rr}}{\sigma_{rr}^F(\dot{\epsilon}_{\text{eff}})}\right]^2 + \left[\frac{\tau}{\tau^F(\dot{\epsilon}_{\text{eff}})}\right]^2 - 1 = 0$$

where  $\sigma_{rr}^F(\dot{\epsilon}_{\text{eff}})$  and  $\tau^F(\dot{\epsilon}_{\text{eff}})$  are defined by load curves LCAX and LCTAU. The peak stresses are calculated from the resultants using simple beam theory.

$$\sigma_{rr} = \frac{N_{rr}}{A} + \frac{\sqrt{M_{ss}^2 + M_{tt}^2}}{Z}$$

$$\tau = \frac{M_{rr}}{2Z} + \frac{\sqrt{N_{rs}^2 + N_{rt}^2}}{A}$$

where the area and section modulus are given by:

$$A = \pi \frac{d^2}{4}$$

$$Z = \pi \frac{d^3}{32}$$

and  $d$  is the equivalent diameter of the beam element or solid element used as a spot weld.

**OPT = 2**

OPT = 2 invokes a user-written subroutine *uweldfail*, documented in Appendix Q.

**OPT = 12 or 22**

OPT = 12 and OPT = 22 invoke similar user-written subroutines, *uweldfail12*, or, *uweldfail22* respectively. Both allow up to 22 failure parameters to be used rather than the 6 allowed with OPT = 2. OPT = 12 also allows user control of weld damage.

**OPT = 3**

OPT = 3 invokes a failure based on notch stress, see Zhang [1999]. Failure occurs when the failure criterion:

$$\sigma_k - \sigma_{kF} \geq 0$$

is satisfied. The notch stress is given by the equation:

$$\sigma_k = \alpha_1 \frac{4F}{\pi dt} \left( 1 + \frac{\sqrt{3} + \sqrt{19}}{8\sqrt{\pi}} \sqrt{\frac{t}{\rho}} \right) + \alpha_2 \frac{6M}{\pi dt^2} \left( 1 + \frac{2}{\sqrt{3\pi}} \sqrt{\frac{t}{\rho}} \right) + \alpha_3 \frac{4F_{rr}}{\pi d^2} \left( 1 + \frac{5}{3\sqrt{2\pi}} \frac{d}{t} \sqrt{\frac{t}{\rho}} \right)$$

Here,

$$F = \sqrt{F_{rs}^2 + F_{rt}^2}$$

$$M = \sqrt{M_{ss}^2 + M_{tt}^2}$$

and  $\alpha_i$   $i = 1,2,3$  are input corrections factors with default values of unity. If spot welds are between sheets of unequal thickness, the minimum thickness of the spot welded sheets may be introduced as a crude approximation.

**OPT = 4**

OPT = 4 invokes failure based on structural stress intensity, see Zhang [1999]. Failure occurs when the failure criterion:

$$K_{eq} - K_{eqF} \geq 0$$

is satisfied where

$$K_{eq} = \sqrt{K_I^2 + K_{II}^2}$$

and

$$K_I = \alpha_1 \frac{\sqrt{3}F}{2\pi d\sqrt{t}} + \alpha_2 \frac{2\sqrt{3}M}{\pi dt\sqrt{t}} + \alpha_3 \frac{5\sqrt{2}F_{rr}}{3\pi d\sqrt{t}}$$

$$K_{II} = \alpha_1 \frac{2F}{\pi d\sqrt{t}}$$

Here, F and M are as defined above for the notch stress formulas and again,  $\alpha_i$   $i = 1,2,3$  are input corrections factors with default values of unity. If spot welds are between sheets

of unequal thickness, the minimum thickness of the spot welded sheets may be used as a crude approximation.

The maximum structural stress at the spot weld was utilized successfully for predicting the fatigue failure of spot welds, see Rupp, et. al. [1994] and Sheppard [1993]. The corresponding results according to Rupp, et. al. are listed below where it is assumed that they may be suitable for crash conditions.

### OPT = 5

OPT = 5 invokes failure by

$$\max(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}) - \sigma_{sF} = 0$$

where  $\sigma_{sF}$  is the critical value of structural stress at failure. It is noted that the forces and moments in the equations below are referred to the beam nodes 1, 2, and to the midpoint, respectively. The three stress values,  $\sigma_{v1}, \sigma_{v2}, \sigma_{v3}$ , are defined by:

$$\sigma_{v1}(\zeta) = \frac{F_{rs1}}{\pi dt_1} \cos\zeta + \frac{F_{rt1}}{\pi dt_1} \sin\zeta - \frac{1.046\beta_1 F_{rr1}}{t_1 \sqrt{t_1}} - \frac{1.123M_{ss1}}{dt_1 \sqrt{t_1}} \sin\zeta + \frac{1.123M_{tt1}}{dt_1 \sqrt{t_1}} \cos\zeta$$

with

$$\beta_1 = \begin{cases} 0 & F_{rr1} \leq 0 \\ 1 & F_{rr1} > 0 \end{cases}$$

$$\sigma_{v2}(\zeta) = \frac{F_{rs2}}{\pi dt_2} \cos\zeta + \frac{F_{rt2}}{\pi dt_2} \sin\zeta - \frac{1.046\beta_1 F_{rr2}}{t_2 \sqrt{t_2}} + \frac{1.123M_{ss2}}{dt_2 \sqrt{t_2}} \sin\zeta - \frac{1.123M_{tt2}}{dt_2 \sqrt{t_2}} \cos\zeta$$

with

$$\beta_2 = \begin{cases} 0 & F_{rr2} \leq 0 \\ 1 & F_{rr2} > 0 \end{cases}$$

$$\sigma_{v3}(\zeta) = 0.5\sigma(\zeta) + 0.5\sigma(\zeta)\cos(2\alpha) + 0.5\tau(\zeta)\sin(2\alpha)$$

where

$$\sigma(\zeta) = \frac{4\beta_3 F_{rr}}{\pi d^2} + \frac{32M_{ss}}{\pi d^3} \sin\zeta - \frac{32M_{tt}}{\pi d^3} \cos\zeta$$

$$\tau(\zeta) = \frac{16F_{rs}}{3\pi d^2} \sin^2\zeta + \frac{16F_{rt}}{3\pi d^2} \cos^2\zeta$$

$$\alpha = \frac{1}{2} \tan^{-1} \frac{2\tau(\zeta)}{\sigma(\zeta)}$$

$$\beta_3 = \begin{cases} 0 & F_{rr} \leq 0 \\ 1 & F_{rr} > 0 \end{cases}$$

The stresses are calculated for all directions,  $0^\circ \leq \zeta \leq 90^\circ$ , in order to find the maximum.

**OPT = 10**

OPT = 10 invokes the failure criterion developed by Lee and Balur (2011). It is available for welds modeled by beam elements, solid elements, or solid assemblies. A detailed discussion of the criterion is given in the user's manual section for \*DEFINE\_SPOTWELD\_FAILURE.

**OPT = 11**

OPT = 11 invokes a resultant force based failure criterion for beams. With corresponding load curves or tables LCT and LCC, resultant force at failure  $F_{fail}$  can be defined as function of loading direction  $\gamma$  (curve) or loading direction  $\gamma$  and effective strain rate  $\dot{\epsilon}$  (table):

$$F_{fail} = f(\gamma) \quad \text{or} \quad F_{fail} = f(\gamma, \dot{\epsilon})$$

with the following definitions for loading direction (in degree) and effective strain rate:

$$\gamma = \tan^{-1} \left( \left| \frac{F_{shear}}{F_{axial}} \right| \right), \quad \dot{\epsilon} = \left[ \frac{2}{3} (\dot{\epsilon}_{axial}^2 + \dot{\epsilon}_{shear}^2) \right]^{1/2}$$

It depends on the sign of the axial beam force, if LCT or LCC are used for failure condition:

$$F_{axial} > 0: \quad [F_{axial}^2 + F_{shear}^2]^{1/2} > F_{fail,LCT} \rightarrow \text{failure}$$

$$F_{axial} < 0: \quad [F_{axial}^2 + F_{shear}^2]^{1/2} > F_{fail,LCC} \rightarrow \text{failure}$$

For all OPT failure criteria, if a zero is input for a failure parameter on card 2, the corresponding term will be omitted from the equation. For example, if for OPT = 0, only  $N_{rrF}$  is nonzero, the failure surface is reduced to  $|N_{rr}| = N_{rrF}$ .

Similarly, if the failure strain EFAIL is set to zero, the failure strain model is not used. Both EFAIL and OPT failure may be active at the same time.

NF specifies the number of terms used to filter the stresses or resultants used in the OPT failure criterion. NF cannot exceed 30. The default value is set to zero which is generally recommended unless oscillatory resultant forces are observed in the time history databases. Although welds should not oscillate significantly, this option was added for consistency with the other spot weld options. NF affects the storage since it is necessary to store the resultant forces as history variables. The NF parameter is available only for beam element welds.

The inertias of the spot welds are scaled during the first time step so that their stable time step size is  $\Delta t$ . A strong compressive load on the spot weld at a later time may reduce the length of the spot weld so that stable time step size drops below  $\Delta t$ . If the value of  $\Delta t$  is zero, mass scaling is not performed, and the spot welds will probably limit the time step size. Under most circumstances, the inertias of the spot welds are small enough that

scaling them will have a negligible effect on the structural response and the use of this option is encouraged.

Spot weld force history data is written into the `swforc` ASCII file. In this database the resultant moments are not available, but they are in the binary time history database and in the ASCII elout file.

## Damage

When the DAMAGE-FAILURE option is invoked, the constitutive properties for the damaged material are obtained from the undamaged material properties. The amount of damage evolved is represented by the constant,  $\omega$ , which varies from zero if no damage has occurred to unity for complete rupture. For uniaxial loading, the nominal stress in the damaged material is given by

$$\sigma_{\text{nominal}} = \frac{P}{A}$$

where  $P$  is the applied load and  $A$  is the surface area. The true stress is given by:

$$\sigma_{\text{true}} = \frac{P}{A - A_{\text{loss}}}$$

where  $A_{\text{loss}}$  is the void area. The damage variable can then be defined:

$$\omega = \frac{A_{\text{loss}}}{A}$$

where,

$$0 \leq \omega \leq 1$$

In this model, damage is initiated when the effective plastic strain in the weld exceeds the failure strain, `EFAIL`. If `DMGOPT = 10, 11, or 12`, damage will initiate when the effective plastic strain exceeds `EFAIL`, or when the failure criterion is met, whichever occurs first. The failure criterion is specified by `OPT` parameter. After damage initiates, the damage variable is evaluated by one of two ways.

For `DMGOPT = 0, 1, 10, or 11`, the damage variable is a function of effective plastic strain in the weld:

$$\varepsilon_{\text{failure}}^p \leq \varepsilon_{\text{eff}}^p \leq \varepsilon_{\text{rupture}}^p \Rightarrow \omega = \frac{\varepsilon_{\text{eff}}^p - \varepsilon_{\text{failure}}^p}{\varepsilon_{\text{rupture}}^p - \varepsilon_{\text{failure}}^p}$$

where  $\varepsilon_{\text{failure}}^p = \text{EFAIL}$  and  $\varepsilon_{\text{rupture}}^p = \text{RS}$ . For `DMGOPT = 2 or 12`, the damage variable is a function of time:

$$t_{\text{failure}} \leq t \leq t_{\text{rupture}} \Rightarrow \omega = \frac{t - t_{\text{failure}}}{t_{\text{rupture}}}$$

where  $t_{\text{failure}}$  is the time at which damage initiates, and  $t_{\text{rupture}} = \text{RS}$ . For this criteria,  $t_{\text{failure}}$  is set to either the time when  $\epsilon_{\text{eff}}^p$  exceeds EFAIL, or the time when the failure criterion is met.

For DMGOPT = 1, the damage behavior is the same as for DMGOPT = 0, but an additional damage variable is calculated to prevent stress growth during softening. Similarly, DMGOPT = 11 behaves like DMGOPT = 10 except for the additional damage variable. This additional function is also used with DMGOPT = 2 and 12. The effect of this additional damage function is noticed only in brick and brick assembly welds in tension loading where it prevents growth of the tensile force in the weld after damage initiates.

For DMGOPT = 10, 11, or 12 an optional FMODE parameter determines whether a weld that reaches the failure surface will fail immediately, or initiate damage. The failure surface calculation has shear terms, which may include the torsional moment, and also normal and bending terms. If FMODE is input with a value between 0 and 1, then when the failure surface is reached, the sum of the square of the shear terms is divided by the sum of the square of all terms. If this ratio exceeds FMODE, then the weld will fail immediately. If the ratio is less than or equal to FMODE, then damage will initiate. The FMODE option is available only for brick and brick assembly welds.

For resultant based failure (OPT = -1 or 0) and DMGOPT = 10, 11, or 12 an optional FFCAP parameter determines whether a weld that reaches the failure surface will fail immediately. After damage initiation, the failure function can reach values above 1.0. This can now be limited by the FFCAP value (should be larger than 1.0):

$$\left( \left[ \frac{\max(N_{rr}, 0)}{N_{rrF}} \right]^2 + \left[ \frac{N_{rs}}{N_{rsF}} \right]^2 + \left[ \frac{N_{rt}}{N_{rtF}} \right]^2 + \left[ \frac{M_{rr}}{M_{rrF}} \right]^2 + \left[ \frac{M_{ss}}{M_{ssF}} \right]^2 + \left[ \frac{M_{tt}}{M_{ttF}} \right]^2 \right)^{\frac{1}{2}} < \text{FFCAP}$$

## BETA

If BETA is specified, the stress is multiplied by an exponential using  $\omega$  defined in the previous equations,

$$\sigma_d = \sigma \exp(-\beta\omega).$$

For weld elements in an assembly (see RPBHX on \*CONTROL\_SPOTWELD\_BEAM or \*DEFINE\_HEX\_SPOTWELD\_ASSEMBLY), the failure criterion is evaluated using the assembly cross section. If damage is not active, all elements will be deleted when the failure criterion is met. If damage is active, then damage is calculated independently in each element of the assembly. By default, elements of the assembly are deleted as damage in each element is complete. If ASFF = 1, then failure and deletion of all elements in the assembly will occur simultaneously when damage is complete in any one of the elements.

**TRUE\_T**

Weld elements and weld assemblies are tied to the mid-plane of shell materials and so typically have a thickness that is half the sum of the thicknesses of the welded shell sections. As a result, a weld under shear loading can be subject to an artificially large moment which will be balanced by normal forces transferred through the tied contact. These normal forces will cause the out-of-plane bending moment used in the failure calculation to be artificially high. Inputting a TRUE\_T that is smaller than the modeled thickness, for example, 10%-30% of true thickness will scale down the moment or stress that results from the balancing moment and provide more realistic failure calculations for solid elements and weld assemblies. TRUE\_T effects only the failure calculation, not the weld element behavior. If TRUE\_T = 0 or data is omitted, the modeled weld element thickness is used. For OPT = 0, the two out-of-plane moments,  $M_{ss}$  and  $M_{tt}$  are replaced by modified terms  $\widehat{M}_{ss}$  and  $\widehat{M}_{tt}$ , as shown below:

$$\left[ \frac{\max(N_{rr}, 0)}{N_{rrF}} \right]^2 + \left[ \frac{N_{rs}}{N_{rsF}} \right]^2 + \left[ \frac{N_{rt}}{N_{rtF}} \right]^2 + \left[ \frac{M_{rr}}{M_{rrF}} \right]^2 + \left[ \frac{\widehat{M}_{ss}}{M_{ssF}} \right]^2 + \left[ \frac{\widehat{M}_{tt}}{M_{ttF}} \right]^2 - 1 = 0$$

$$\widehat{M}_{ss} = M_{ss} - N_{rt}(t - t_{\text{true}})$$

$$\widehat{M}_{tt} = M_{tt} - N_{rs}(t - t_{\text{true}})$$

In the above,  $t$  is the element thickness and  $t_{\text{true}}$  is the TRUE\_T parameter. For OPT = 1, the same modification is done to the moments that contribute to the normal stress, as shown below:

$$\sigma_{rr} = \frac{N_{rr}}{A} + \frac{\sqrt{\widehat{M}_{ss}^2 + \widehat{M}_{tt}^2}}{Z}$$

**Uniaxial option**

A uniaxial stress option is available for solid and solid weld assemblies. It is invoked by defining the elastic modulus,  $E$  as a negative number where the absolute value of  $E$  is the desired value for  $E$ . The uniaxial option causes the two transverse stress terms to be assumed to be zero throughout the calculation. This assumption eliminates parasitic transverse stress that causes slow growth of plastic strain based damage.

The motivation for this option can be explained with a weld loaded in tension. Due to Poisson's effect, an element in tension would be expected to contract in the transverse directions. However, because the weld nodes are constrained to the mid-plane of shell elements, such contraction is only possible to the degree that that shell element contracts. In other words, the uniaxial stress state cannot be represented by the weld. For plastic strain based damage, this effect can be particularly apparent as it causes tensile stress to continue to grow very large as the stress state becomes very nearly triaxial tension. In this stress state, plastic strain grows very slowly so it appears that damage calculation is failing



to knock down the stress. By simply assuming that the transverse stresses are zero, the plastic strain grows as expected and damage is much more effective.

**Material histories**

The probability of failure in solid or beam spotwelds can be estimated by retrieving the corresponding material histories for output to the d3plot database

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>					
<b>Label</b>	<b>Attributes</b>				<b>Description</b>
Instability	-	-	-	-	A measure between 0 and 1 related to how close the spotweld element is to fail
Damage	-	-	-	-	Damage in the spotweld element between 0 and 1

These two labels are supported for all options (OPT and DMGOPT, including assemblies and beams), except for user defined failure. The instability measure is the maximum over time; namely, it gives the maximum value for a given element throughout the simulation. If a damage option is invoked then damage will initiate and increment when the instability reaches unity, and elements are not deleted until the damage value reaches unity. If no damage option is invoked then the damage output is always zero and elements will be deleted at the point when the instability measure reaches unity.

**\*MAT\_SPOTWELD\_DAIMLERCHRYSLER**

This is Material Type 100. The material model applies only to solid element type 1. If hourglass type 4 is specified then hourglass type 4 will be used, otherwise, hourglass type 6 will be automatically assigned. Hourglass type 6 is preferred.

Spot weld elements may be placed between any two deformable shell surfaces and tied with constraint contact, \*CONTACT\_TIED\_SURFACE\_TO\_SURFACE, which eliminates the need to have adjacent nodes at spot weld locations. Spot weld failure is modeled using this card and \*DEFINE\_CONNECTION\_PROPERTIES data. Details of the failure model can be found in Seeger, Feucht, Frank, Haufe, and Keding [2005].

**NOTE:** It is advisable to include all spot welds, which provide the slave nodes, and spot welded materials, which define the master segments, within a single \*CONTACT\_TIED\_SURFACE\_TO\_SURFACE interface. This contact type uses constraint equations. If multiple interfaces are treated independently, significant problems can occur if such interfaces share common nodes. An added benefit is that memory usage can be substantially less with a single interface

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR			DT	TFAIL
Type	A8	F	F	F			F	F

Card 2	1	2	3	4	5	6	7	8
Variable	EFAIL							NF
Type	F							F

Card 3	1	2	3	4	5	6	7	8
Variable	RS	ASFF		TRUE_T	CON_ID		JTOL	
Type	F	I		F	F		F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
DT	Time step size for mass scaling, $\Delta t$ .
TFAIL	Failure time if nonzero. If zero this option is ignored.
EFAIL	Effective plastic strain in weld material at failure. See remark below.
NF	Number of failure function evaluations stored for filtering by time averaging. The default value is set to zero which is generally recommended unless oscillatory resultant forces are observed in the time history databases. Even though these welds should not oscillate significantly, this option was added for consistency with the other spot weld options. NF affects the storage since it is necessary to store the failure terms. When NF is nonzero, the resultants in the output databases are filtered. NF cannot exceed 30.
RS	Rupture strain. See Remarks below.
ASFF	Weld assembly simultaneous failure flag EQ.0: Damaged elements fail individually. EQ.1: Damaged elements fail when first reaches failure criterion.
TRUE_T	True weld thickness for single hexahedron solid weld elements. See comments below.
CON_ID	Connection ID of *DEFINE_CONNECTION card. A negative CON_ID deactivates failure, see comments below.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
JTOL	Tolerance value for relative volume change (default: JTOL = 0.01). Solid element spotwelds with a Jacobian less than JTOL will be eroded.

**Remarks:**

This weld material is modeled with isotropic hardening plasticity. The yield stress and constant hardening modulus are assumed to be those of the welded shell elements as defined in a \*DEFINE\_CONNECTION\_PROPERTIES table. A failure function and damage type is also defined by \*DEFINE\_CONNECTION\_PROPERTIES data. The interpretation of EFAIL and RS is determined by the choice of damage type. This is discussed in remark 4 on \*DEFINE\_CONNECTION\_PROPERTIES.

Solid weld elements are tied to the mid-plane of shell materials and so typically have a thickness that is half the sum of the thicknesses of the welded shell sections. As a result, a weld under shear loading can be subject to an artificially large moment which will be balanced by normal forces transferred through the tied contact. These normal forces will cause the normal term in the failure calculation to be artificially high. Inputting a TRUE\_T that is smaller than the modeled thickness, for example, 10%-30% of true thickness will scale down the normal force that results from the balancing moment and provide more realistic failure calculations. TRUE\_T effects only the failure calculation, not the weld element behavior. If TRUE\_T = 0 or data is omitted, the modeled weld element thickness is used.

For weld elements in an assembly (see RPBHX on \*CONTROL\_SPOTWELD\_BEAM or \*DEFINE\_HEX\_SPOTWELD\_ASSEMBLY), the failure criterion is evaluated using the assembly cross section. If damage is not active, all elements will be deleted when the failure criterion is met. If damage is active, then damage is calculated independently in each element of the assembly. By default, elements of the assembly are deleted as damage in each element is complete. If ASFF = 1, then failure and deletion of all elements in the assembly will occur simultaneously when damage is complete in any one of the elements.

Solid element force resultants for MAT\_SPOTWELD are written to the spot weld force file, **swforc**, and the file for element stresses and resultants for designated elements, **ELOUT**. Also, spot weld failure data is written to the file, **dcfail**.

An option to deactivate weld failure is switched on by setting CON\_ID to a negative value where the absolute value of CON\_ID becomes the connection ID. When weld failure is deactivated, the failure function is evaluated and output to **swforc** and **dcfail** but the weld retains its full strength.

\*MAT\_GEPLASTIC\_SRATE\_2000a

This is Material Type 101. The GEPLASTIC\_SRATE\_2000a material model characterizes General Electric's commercially available engineering thermoplastics subjected to high strain rate events. This material model features the variation of yield stress as a function of strain rate, cavitation effects of rubber modified materials and automatic element deletion of either ductile or brittle materials.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	RATESF	EDOT0	ALPHA	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	LCSS	LCFEPS	LCFSIG	LCE				
Type	F	F	F	F				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's Modulus.
PR	Poisson's ratio.
RATESF	Constant in plastic strain rate equation.
EDOT0	Reference strain rate
ALPHA	Pressure sensitivity factor
LCSS	Load curve ID or Table ID that defines the post yield material behavior. The values of this stress-strain curve are the difference of the yield stress and strain respectively. This means the first values for both stress and strain should be zero. All subsequent values will define softening or hardening.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCFEPS	Load curve ID that defines the plastic failure strain as a function of strain rate.
LCFSIG	Load curve ID that defines the Maximum principal failure stress as a function of strain rate.
LCE	Load curve ID that defines the Unloading moduli as a function of plastic strain.

**Remarks:**

The constitutive model for this approach is:

$$\dot{\epsilon}_p = \dot{\epsilon}_0 \exp\{A[\sigma - S(\epsilon_p)]\} \times \exp(-p\alpha A)$$

where  $\dot{\epsilon}_0$  and A are rate dependent yield stress parameters,  $S(\epsilon_p)$  internal resistance (strain hardening) and  $\alpha$  is a pressure dependence parameter.

In this material the yield stress may vary throughout the finite element model as a function of strain rate and hydrostatic stress. Post yield stress behavior is captured in material softening and hardening values. Finally, ductile or brittle failure measured by plastic strain or maximum principal stress respectively is accounted for by automatic element deletion.

Although this may be applied to a variety of engineering thermoplastics, GE Plastics have constants available for use in a wide range of commercially available grades of their engineering thermoplastics.

**\*MAT\_INV\_HYPERBOLIC\_SIN\_{OPTION}**

This is Material Type 102. It allows the modeling of temperature and rate dependent plasticity, Sheppard and Wright [1979].

Available options include:

<BLANK>

THERMAL

such that the keyword card can appear as:

\*MAT\_INV\_HYPERBOLIC\_SIN or \*MAT\_102

\*MAT\_INV\_HYPERBOLIC\_SIN\_THERMAL or \*MAT\_102\_T

Card 1 for <BLANK> option:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	T	HC	VP	
Type	A8	F	F	F	F	F	F	

Card 2 for <BLANK> option:

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA	N	A	Q	G	EPS0	LCQ	
Type	F	F	F	F	F	F	I	

Card 1 for THERMAL option:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ALPHA	N	A	Q	G	EPS0
Type	A8	F	F	F	F	F	F	F

Card 2 for THERMAL option:

Card 2	1	2	3	4	5	6	7	8
Variable	LCE	LCPR	LCCTE					
Type	F	F	F					

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's Modulus.
PR	Poisson's ratio
T	Initial Temperature.
HC	Heat generation coefficient.
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default) EQ.1.0: Viscoplastic formulation.
ALPHA	$\alpha$ , see Remarks. Not to be confused with coefficient of thermal expansion.
N	See Remarks.
A	See Remarks.
Q	See Remarks.
G	See Remarks.
EPS0	Minimum strain rate considered in calculating Z.
LCQ	ID of curve specifying parameter Q. GT.0: Q as function of plastic strain. LT.0: Q as function of temperature.



VARIABLE	DESCRIPTION
LCE	ID of curve defining Young's Modulus vs. temperature.
LCPR	ID of curve defining Poisson's ratio vs. temperature.
LCCTE	ID of curve defining coefficient of thermal expansion vs. temperature.

**Remarks:**

Resistance to deformation is both temperature and strain rate dependent. The flow stress equation is:

$$\sigma = \frac{1}{\alpha} \sinh^{-1} \left[ \left( \frac{Z}{A} \right)^{\frac{1}{N}} \right]$$

where  $Z$ , the Zener-Holloman temperature compensated strain rate, is:

$$Z = \max(\dot{\epsilon}, EPS0) \times \exp \left( \frac{Q}{GT} \right)$$

The units of the material constitutive constants are as follows:  $A$  (1/sec),  $N$  (dimensionless),  $\alpha$  (1/MPa), the activation energy for flow,  $Q$ (J/mol), and the universal gas constant,  $G$  (J/mol K). The value of  $G$  will only vary with the unit system chosen. Typically it will be either 8.3144 J/mol  $\infty$  K, or 40.8825 lb in/mol  $\infty$  R.

The final equation necessary to complete our description of high strain rate deformation is one that allows us to compute the temperature change during the deformation. In the absence of a couples thermo-mechanical finite element code we assume adiabatic temperature change and follow the empirical assumption that 90-95% of the plastic work is dissipated as heat. Thus the heat generation coefficient is

$$HC \approx \frac{0.9}{\rho C_v}$$

where  $\rho$  is the density of the material and  $C_v$  is the specific heat.

**\*MAT\_ANISOTROPIC\_VISCOPLASTIC**

This is Material Type 103. This anisotropic-viscoplastic material model applies to shell and brick elements. The material constants may be fit directly or, if desired, stress versus strain data may be input and a least squares fit will be performed by LS-DYNA to determine the constants. Kinematic or isotropic or a combination of kinematic and isotropic hardening may be used. A detailed description of this model can be found in the following references: Berstad, Langseth, and Hopperstad [1994]; Hopperstad and Remseth [1995]; and Berstad [1996]. Failure is based on effective plastic strain or by a user defined subroutine.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	FLAG	LCSS	ALPHA
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	QR1	CR1	QR2	CR2	QX1	CX1	QX2	CX2
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	VK	VM	R00 or F	R45 or G	R90 or H	L	M	N
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT	FAIL	NUMINT	MACF				
Type	F	F	F	I				

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus
PR	Poisson's ratio
SIGY	Initial yield stress

VARIABLE	DESCRIPTION
FLAG	<p>Flag</p> <p>EQ.0: Give all material parameters</p> <p>EQ.1: Material parameters parameters <math>Q_{r1}</math>, <math>C_{r1}</math>, <math>Q_{r2}</math>, and <math>C_{r2}</math> for pure isotropic hardening (<math>\alpha = 1</math>) are determined by a least squares fit to the curve or table specified by the variable LCSS. If <math>\alpha</math> is input as less than 1, <math>Q_{r1}</math> and <math>Q_{r2}</math> are then modified by multiplying them by the factor <math>\alpha</math>, while the factors <math>Q_{x1}</math> and <math>Q_{x2}</math> are taken as the product of the original parameters <math>Q_{r1}</math> and <math>Q_{r2}</math>, resp., for pure isotropic hardening and the factor <math>(1 - \alpha)</math>. <math>C_{x1}</math> is set equal to <math>C_{r1}</math> and <math>C_{x2}</math> is set equal to <math>C_{r2}</math>. <math>\alpha</math> is input as variable ALPHA on Card 1 in columns 71-80.</p> <p>EQ.2: Use load curve directly, i.e., no fitting is required for the parameters <math>Q_{r1}</math>, <math>C_{r1}</math>, <math>Q_{r2}</math>, and <math>C_{r2}</math>. A table is not allowed and only isotropic hardening is implemented.</p> <p>EQ.4: Use table definition directly, no fitting is required and the values for <math>Q_{r1}</math>, <math>C_{r1}</math>, <math>Q_{r2}</math>, <math>C_{r2}</math>, <math>V_k</math>, and <math>V_m</math> are ignored. Only isotropic hardening is implemented, and this option is only available for solids.</p>
LCSS	<p>Load curve ID or Table ID.</p> <p><i>Case 1: LCSS is a Load Curve ID.</i> The load curve ID defines effective stress versus effective plastic strain. Card 2 is ignored with this option. For this load curve case viscoplasticity is modeled when the coefficients <math>V_k</math> and <math>V_m</math> are provided.</p> <p><i>Case 2: LCSS is a Table ID.</i> Table consists of stress versus effective plastic strain curves indexed by strain rate. See <a href="#">Figure M24-1</a>.</p> <p>FLAG.EQ.1: Table is used to calculate the coefficients <math>V_k</math> and <math>V_m</math>.</p> <p>FLAG.EQ.4: Table is interpolated and used directly. This option is available only for solid elements.</p>
ALPHA	<p><math>\alpha</math> distribution of hardening used in the curve-fitting. <math>\alpha = 0</math> pure kinematic hardening and <math>\alpha = 1</math> provides pure isotropic hardening</p>
QR1	<p>Isotropic hardening parameter <math>Q_{r1}</math></p>
CR1	<p>Isotropic hardening parameter <math>C_{r1}</math></p>
QR2	<p>Isotropic hardening parameter <math>Q_{r2}</math></p>

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
CR2	Isotropic hardening parameter $C_{r2}$
QX1	Kinematic hardening parameter $Q_{\chi1}$
CX1	Kinematic hardening parameter $C_{\chi1}$
QX2	Kinematic hardening parameter $Q_{\chi2}$
CX2	Kinematic hardening parameter $C_{\chi2}$
VK	Viscous material parameter $V_k$
VM	Viscous material parameter $V_m$
R00	$R_{00}$ for shell (Default = 1.0)
R45	$R_{45}$ for shell (Default = 1.0)
R90	$R_{90}$ for shell (Default = 1.0)
F	$F$ for brick (Default = 1/2)
G	$G$ for brick (Default = 1/2)
H	$H$ for brick (Default = 1/2)
L	$L$ for brick (Default = 3/2)
M	$M$ for brick (Default = 3/2)
N	$N$ for brick (Default = 3/2)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see <i>MAT_OPTION TROPIC_ELASTIC</i> for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <i>*DEFINE_COORDINATE_NODES</i>, and then, for shells only, rotated about the shell element normal by an angle BETA.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with <i>*DEFINE_COORDINATE_VECTOR</i>.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector v, and an originating point, P, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on <i>*DEFINE_COORDINATE_NODES</i>, <i>*DEFINE_COORDINATE_SYSTEM</i> or <i>*DEFINE_COORDINATE_VECTOR</i>). Available in R3 version of 971 and later.</p>
FAIL	<p>Failure flag.</p> <p>LT.0.0: User defined failure subroutine is called to determine failure. This is subroutine named, MATUSR_103, in dyn21.f.</p> <p>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.</p> <p>GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.</p>

VARIABLE	DESCRIPTION
NUMINT	Number of integration points which must fail before element deletion. If zero, all points must fail. This option applies to shell elements only. For the case of one point shells, NUMINT should be set to a value that is less than the number of through thickness integration points. Nonphysical stretching can sometimes appear in the results if all integration points have failed except for one point away from the midsurface. This is due to the fact that unconstrained nodal rotations will prevent strains from developing at the remaining integration point. In fully integrated shells, similar problems can occur.
MACF	Material axes change flag for brick elements: EQ.1: No change, default, EQ.2: switch material axes <i>a</i> and <i>b</i> , EQ.3: switch material axes <i>a</i> and <i>c</i> , EQ.4: switch material axes <i>b</i> and <i>c</i> .
XP, YP, ZP	$x_p, y_p, z_p$ , define coordinates of point <b>p</b> for AOPT = 1 and 4.
A1, A2, A3	$a_1, a_2, a_3$ , define components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	$v_1, v_2, v_3$ define components of vector <b>v</b> for AOPT = 3 and 4.
D1, D2, D3	$d_1, d_2, d_3$ , define components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overridden on the element card, see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO.

### Remarks:

The uniaxial stress-strain curve is given on the following form

$$\sigma(\varepsilon_{\text{eff}}^p, \dot{\varepsilon}_{\text{eff}}^p) = \sigma_0 + Q_{r1}[(1 - \exp(-C_{r1}\varepsilon_{\text{eff}}^p))] + Q_{r2}[1 - \exp(-C_{r2}\varepsilon_{\text{eff}}^p)] \\ + Q_{\chi1}[(1 - \exp(-C_{\chi1}\varepsilon_{\text{eff}}^p))] + Q_{\chi2}[(1 - \exp(-C_{\chi2}\varepsilon_{\text{eff}}^p))] + V_k \dot{\varepsilon}_{\text{eff}}^p V_m$$

For bricks the following yield criteria is used

$$F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 \\ = [\sigma(\varepsilon_{\text{eff}}^p, \dot{\varepsilon}_{\text{eff}}^p)]^2$$

where  $\varepsilon_{\text{eff}}^p$  is the effective plastic strain and  $\dot{\varepsilon}_{\text{eff}}^p$  is the effective plastic strain rate. For shells the anisotropic behavior is given by  $R_{00}$ ,  $R_{45}$  and  $R_{90}$ . The model will work when the three first parameters in card 3 are given values. When  $V_k = 0$  the material will behave elasto-plastically. Default values are given by:

$$F = G = H = \frac{1}{2}$$
$$L = M = N = \frac{3}{2}$$
$$R_{00} = R_{45} = R_{90} = 1$$

Strain rate of accounted for using the Cowper and Symonds model which, e.g., model 3, scales the yield stress with the factor:

$$1 + \left( \frac{\dot{\varepsilon}_{\text{eff}}^p}{C} \right)^{1/p}$$

To convert these constants set the viscoelastic constants,  $V_k$  and  $V_m$ , to the following values:

$$V_k = \left( \frac{1}{C} \right)^{1/p}$$
$$V_m = \frac{1}{p}$$

If LCSS is nonzero, substitute the initial, quasi-static yield stress for SIGY in the equation for  $V_k$  above.

This model properly treats rate effects. The viscoplastic rate formulation is an option in other plasticity models in LS-DYNA, e.g., mat\_3 and mat\_24, invoked by setting the parameter VP to 1.





Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus
PR	Poisson's ratio
SIGY	Initial yield stress
LCSS	Load curve ID. The load curve ID defines effective stress versus effective plastic strain. Card 2 is ignored with this option.
QR1	Isotropic hardening parameter $Q_{r1}$
CR1	Isotropic hardening parameter $C_{r1}$
QR2	Isotropic hardening parameter $Q_{r2}$
CR2	Isotropic hardening parameter $C_{r2}$
R00	$R_{00}$ for anisotropic hardening
R45	$R_{45}$ for anisotropic hardening
R90	$R_{90}$ for anisotropic hardening
S11	Yield stress in local $x$ -direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$ .
S22	Yield stress in local $y$ -direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$ .
S33	Yield stress in local $z$ -direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$ .

VARIABLE	DESCRIPTION
S12	Yield stress in local $x$ -direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$ .
AOPT	Material axes option (see MAT_OPTION TROPIC_ELASTIC for a more complete description): <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
XP, YP, ZP	$x_p, y_p, z_p$ define coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	$a_1, a_2, a_3$ define components of vector $\mathbf{a}$ for AOPT = 2.
D1, D2, D3	$d_1, d_2, d_3$ define components of vector $\mathbf{d}$ for AOPT = 2.
V1, V2, V3	$v_1, v_2, v_3$ define components of vector $\mathbf{v}$ for AOPT = 3 and 4.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

**Remarks:**

If no load curve is defined for the effective stress versus effective plastic strain, the uniaxial stress-strain curve is given on the following form

$$\sigma(\varepsilon_{\text{eff}}^p) = \sigma_0 + Q_{r1}[1 - \exp(-C_{r1}\varepsilon_{\text{eff}}^p)] + Q_{r2}[1 - \exp(-C_{r2}\varepsilon_{\text{eff}}^p)]$$

where  $\epsilon_{\text{eff}}^p$  is the effective plastic strain. For shells the anisotropic behavior is given by  $R_{00}$ ,  $R_{45}$  and  $R_{90}$ , or the yield stress in the different direction. Default values are given by:

$$R_{00} = R_{45} = R_{90} = 1$$

if the variables  $R_{00}$ ,  $R_{45}$ ,  $R_{90}$ ,  $S_{11}$ ,  $S_{22}$ ,  $S_{33}$  and  $S_{12}$  are set to zero.

**\*MAT\_DAMAGE\_1**

This is Material Type 104. This is a continuum damage mechanics (CDM) model which includes anisotropy and viscoplasticity. The CDM model applies to shell, thick shell, and brick elements. A more detailed description of this model can be found in the paper by Berstad, Hopperstad, Lademo, and Malo [1999]. This material model can also model anisotropic damage behavior by setting the FLAG to -1 in Card 2.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	LCSS	LCDS	
Type	A8	F	F	F	F			

Card 2	1	2	3	4	5	6	7	8
Variable	Q1	C1	Q2	C2	EPSD	S or EPSR	DC	FLAG
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	VK	VM	R00 or F	R45 or G	R90 or H	L	M	N
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT		CPH	MACF	Y0	ALPHA	THETA	ETA
Type	F		F	I	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Initial yield stress, $\sigma_0$
LCSS	Load curve ID defining effective stress versus effective plastic strain. Isotropic hardening parameters on Card 2 are ignored with this option.
LCDS	Load curve ID defining nonlinear damage curve. For FLAG = -1.
Q1	Isotropic hardening parameter $Q_1$
C1	Isotropic hardening parameter $C_1$
Q2	Isotropic hardening parameter $Q_2$
C2	Isotropic hardening parameter $C_2$
EPSD	Damage threshold $\varepsilon_{eff,d}^p$ . Damage effective plastic strain when material softening begins. (Default = 0.0)
S	Damage material constant $S$ . (Default = $\frac{\sigma_0}{200}$ ). For FLAG $\geq 0$ .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EPSR	Effective plastic strain at which material ruptures (logarithmic). For FLAG = -1.
DC	Critical damage value $D_C$ . When the damage value $D$ reaches this value, the element is deleted from the calculation. (Default = 0.5) For FLAG $\geq 0$ .
FLAG	Flag EQ.-1: Anisotropic damage check ( <i>only for shell elements</i> ) EQ.0: Standard isotropic damage (default) EQ.1: Standard isotropic damage plus strain localization check ( <i>only for shell elements</i> ) EQ.10: Enhanced isotropic damage EQ.11: Enhanced isotropic damage plus strain localization check ( <i>only for shell elements</i> )
VK	Viscous material parameter $V_k$
VM	Viscous material parameter $V_m$
R00	$R_{00}$ for shell (Default = 1.0)
R45	$R_{45}$ for shell (Default = 1.0)
R90	$R_{90}$ for shell (Default = 1.0)
F	$F$ for brick (Default = 1/2)
G	$G$ for brick (Default = 1/2)
H	$H$ for brick (Default = 1/2)
L	$L$ for brick (Default = 3/2)
M	$M$ for brick (Default = 3/2)
N	$N$ for brick (Default = 3/2)
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description): EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about

VARIABLE	DESCRIPTION
	<p>the shell element normal by an angle BETA.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>v</math> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>v</math>, and an originating point, <math>P</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
CPH	Microdefects closure parameter $h$ for enhanced damage (FLAG $\geq 10$ ).
MACF	<p>Material axes change flag for brick elements:</p> <p>EQ.1: No change, default,</p> <p>EQ.2: switch material axes a and b,</p> <p>EQ.3: switch material axes a and c,</p> <p>EQ.4: switch material axes b and c.</p>
Y0	Initial damage energy release rate $Y_0$ for enhanced damage (FLAG $\geq 10$ ).
ALPHA	Exponent $\alpha$ for enhanced damage (FLAG $\geq 10$ )
THETA	Exponent $\theta$ for enhanced damage (FLAG $\geq 10$ )
ETA	Exponent $\eta$ for enhanced damage (FLAG $\geq 10$ )



VARIABLE	DESCRIPTION
XP, YP, ZP	$x_p, y_p, z_p$ : define coordinates of point <b>p</b> for AOPT = 1 and 4
A1, A2, A3	$a_1, a_2, a_3$ : define components of vector <b>a</b> for AOPT = 2
D1, D2, D3	$d_1, d_2, d_3$ : define components of vector <b>d</b> for AOPT = 2
V1, V2, V3	$v_1, v_2, v_3$ : define components of vector <b>v</b> for AOPT = 3 and 4
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overridden on the element card, see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO.

### Remarks:

**Standard isotropic damage model (FLAG = 0 or 1).** The Continuum Damage Mechanics (CDM) model is based on an approach proposed by Lemaitre [1992]. The effective stress  $\tilde{\sigma}$ , which is the stress calculated over the section that effectively resist the forces, reads

$$\tilde{\sigma} = \frac{\sigma}{1 - D}$$

where  $D$  is the damage variable. The evolution equation for the damage variable is defined as

$$\dot{D} = \begin{cases} 0 & \text{for } \varepsilon_{\text{eff}}^p \leq \varepsilon_{\text{eff,d}}^p \\ \frac{Y}{S} \dot{\varepsilon}_{\text{eff}}^p & \text{for } \varepsilon_{\text{eff}}^p > \varepsilon_{\text{eff,d}}^p \text{ and } \sigma_1 > 0 \end{cases}$$

where  $\varepsilon_{\text{eff,d}}^p$  is the damage threshold,  $S$  is the so-called damage energy release rate and  $\sigma_1$  is the maximum principal stress. The damage energy density release rate is

$$Y = \frac{1}{2} \mathbf{e}_e : \mathbf{C} : \mathbf{e}_e = \frac{\sigma_{vm}^2 R_v}{2E(1 - D)^2}$$

where  $E$  is Young's modulus and  $\sigma_{vm}$  is the equivalent von Mises stress. The triaxiality function  $R_v$  is defined as

$$R_v = \frac{2}{3}(1 + \nu) + 3(1 - 2\nu) \left( \frac{\sigma_H}{\sigma_{vm}} \right)^2$$

with Poisson's ratio  $\nu$  and hydrostatic stress  $\sigma_H$ .

**Enhanced isotropic damage model (FLAG = 10 or 11).** A more sophisticated damage model including crack closure effects (reduced damage under compression) and more flexibility in stress state dependence and functional expressions is invoked by setting FLAG = 10 or 11. The corresponding evolution equation for the damage variable is defined as

$$\dot{D} = \left( \frac{2\tau_{\max}}{\sigma_{vm}} \right)^\eta \left\langle \frac{Y - Y_0}{S} \right\rangle^\alpha (1 - D)^{1-\theta} \dot{\epsilon}_{\text{eff}}^p$$

where  $\tau_{\max}$  is the maximum shear stress,  $Y_0$  is the initial damage energy release rate and  $\eta$ ,  $\alpha$ , and  $\theta$  are additional material constants.  $\langle \rangle$  are the *Macauley brackets*. The damage energy density release rate is

$$Y = \frac{1 + \nu}{2E} \left( \sum_{i=1}^3 (\langle \tilde{\sigma}_i \rangle^2 + h \langle -\tilde{\sigma}_i \rangle^2) \right) - \frac{\nu}{2E} (\langle \tilde{\sigma}_H \rangle^2 + h \langle -\tilde{\sigma}_H \rangle^2)$$

where  $\tilde{\sigma}_i$  are the principal effective stresses and  $h$  is the microdefects closure parameter that accounts for different damage behavior in tension and compression. A value of  $h \approx 0.2$  is typically observed in many experiments as stated in Lemaitre [2000]. A parameter set of  $h = 1$ ,  $Y_0 = 0$ ,  $\alpha = 1$ ,  $\theta = 1$ , and  $\eta = 0$  should give the same results as the standard isotropic damage model (FLAG = 0/1) with  $\epsilon_{\text{eff,d}}^p = 0$  as long as  $\sigma_1 > 0$ .

**Strain localization check (FLAG = 1 or 11).** In order to add strain localization computation to the damage models above, parameter FLAG should be set to 1 (standard damage) or 11 (enhanced damage). An acoustic tensor based bifurcation criterion is checked and history variable no. 4 is set to 1.0 if strain localization is indicated. Only available for shell elements.

**Anisotropic damage model (FLAG = -1).** At each thickness integration points, an anisotropic damage law acts on the plane stress tensor in the directions of the principal total shell strains,  $\epsilon_1$  and  $\epsilon_2$ , as follows:

$$\begin{aligned} \sigma_{11} &= [1 - D_1(\epsilon_1)]\sigma_{110} \\ \sigma_{22} &= [1 - D_2(\epsilon_2)]\sigma_{220} \\ \sigma_{12} &= \left[ 1 - \frac{D_1 + D_2}{2} \right] \sigma_{120} \end{aligned}$$

The transverse plate shear stresses in the principal strain directions are assumed to be damaged as follows:

$$\begin{aligned} \sigma_{13} &= (1 - D_1/2)\sigma_{130} \\ \sigma_{23} &= (1 - D_2/2)\sigma_{230} \end{aligned}$$

In the anisotropic damage formulation,  $D_1(\epsilon_1)$  and  $D_2(\epsilon_2)$  are anisotropic damage functions for the loading directions 1 and 2, respectively. Stresses  $\sigma_{110}$ ,  $\sigma_{220}$ ,  $\sigma_{120}$ ,  $\sigma_{130}$  and  $\sigma_{230}$  are stresses in the principal shell strain directions as calculated from the undamaged elastic-plastic material behavior. The strains  $\epsilon_1$  and  $\epsilon_2$  are the magnitude of the principal strains calculated upon reaching the damage thresholds. Damage can only develop for tensile stresses, and the damage functions  $D_1(\epsilon_1)$  and  $D_2(\epsilon_2)$  are identical to zero for negative strains  $\epsilon_1$  and  $\epsilon_2$ . The principal strain directions are fixed within an integration point as soon as either principal strain exceeds the initial threshold strain in tension. A more detailed description of the damage evolution for this material model is given in the description of Material 81.

**Anisotropic viscoplasticity.** The uniaxial stress-strain curve is given in the following form

$$\sigma(r, \dot{\epsilon}_{\text{eff}}^p) = \sigma_0 + Q_1[1 - \exp(-C_1 r)] + Q_2[1 - \exp(-C_2 r)] + V_k \dot{\epsilon}_{\text{eff}}^p V_m$$

where  $r$  is the damage accumulated plastic strain, which can be calculated by

$$\dot{r} = \dot{\epsilon}_{\text{eff}}^p (1 - D)$$

For bricks the following yield criterion associated with the Hill criterion is used

$$F(\tilde{\sigma}_{22} - \tilde{\sigma}_{33})^2 + G(\tilde{\sigma}_{33} - \tilde{\sigma}_{11})^2 + H(\tilde{\sigma}_{11} - \tilde{\sigma}_{22})^2 + 2L\tilde{\sigma}_{23}^2 + 2M\tilde{\sigma}_{31}^2 + 2N\tilde{\sigma}_{12}^2 = \sigma(r, \dot{\epsilon}_{\text{eff}}^p)$$

where  $r$  is the damage effective viscoplastic strain and  $\dot{\epsilon}_{\text{eff}}^p$  is the effective viscoplastic strain rate. For shells the anisotropic behavior is given by the R-values:  $R_{00}$ ,  $R_{45}$ , and  $R_{90}$ . When  $V_k = 0$  the material will behave as an elastoplastic material without rate effects. Default values for the anisotropic constants are given by:

$$F = G = H = \frac{1}{2}$$

$$L = M = N = \frac{3}{2}$$

$$R_{00} = R_{45} = R_{90} = 1$$

so that isotropic behavior is obtained.

Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p}$$

To convert these constants, set the viscoelastic constants,  $V_k$  and  $V_m$ , to the following values:

$$V_k = \sigma \left(\frac{1}{C}\right)^{\frac{1}{p}}$$

$$V_m = \frac{1}{p}$$

# \*MAT\_105

# \*MAT\_DAMAGE\_2

## \*MAT\_DAMAGE\_2

This is Material Type 105. This is an elastic viscoplastic material model combined with continuum damage mechanics (CDM). This material model applies to shell, thick shell, and brick elements. The elastoplastic behavior is described in the description of material model 24. A more detailed description of the CDM model is given in the description of material model 104 above.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	ETAN	FAIL	TDEL
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	10.E+20	0

Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR				
Type	F	F	F	F				
Default	0	0	0	0				

Card 3	1	2	3	4	5	6	7	8
Variable	EPSD	S	DC					
Type	F	F	F					
Default	none	none	none					

Card 4	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 5	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
ETAN	Tangent modulus, ignored if (LCSS.GT.0) is defined.
FAIL	Failure flag. EQ.0.0: Failure due to plastic strain is not considered. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic element deletion.
C	Strain rate parameter, C, see formula below.
P	Strain rate parameter, P, see formula below.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCSS	Load curve ID or Table ID. Load curve ID defining effective stress versus effective plastic strain. If defined EPS1 - EPS8 and ES1 - ES8 are ignored. The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See <a href="#">Figure M24-1</a> . The stress versus effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress versus effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters: C and P; the curve ID, LCSR; EPS1 - EPS8 and ES1 - ES8 are ignored if a Table ID is defined.
LCSR	Load curve ID defining strain rate scaling effect on yield stress.
EPSD	Damage threshold $r_d$ Damage effective plastic strain when material softening begin. (Default = 0.0)
S	Damage material constant S. (Default = $\frac{\sigma_0}{200}$ )
DC	Critical damage value $D_C$ . When the damage value $D$ reaches this value, the element is deleted from the calculation. (Default = 0.5)
EPS1 - EPS8	Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8.

**Remarks:**

The stress-strain behavior may be treated by a bilinear curve by defining the tangent modulus, ETAN. Alternately, a curve similar to that shown in [Figure M10-1](#) is expected to be defined by (EPS1,ES1) - (EPS8,ES8); however, an effective stress versus effective plastic strain curve ID (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition with table ID, LCSR, discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left( \frac{\dot{\epsilon}}{C} \right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate,  $\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor versus strain rate is defined.
3. If different stress versus strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in \*DEFINE\_TABLE has to be used, see [Figure M24-1](#)

A fully viscoplastic formulation is used in this model.

**\*MAT\_106****\*MAT\_ELASTIC\_VISCOPLASTIC\_THERMAL****\*MAT\_ELASTIC\_VISCOPLASTIC\_THERMAL**

This is Material Type 106. This is an elastic viscoplastic material with thermal effects.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ALPHA	LCSS	FAIL
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	QR1	CR1	QR2	CR2	QX1	CX1	QX2	CX2
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C	P	LCE	LCPR	LCSIGY	LCR	LCX	LCALPH
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	LCC	LCP	TREF	LCFAIL				
Type	F	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus
PR	Poisson's ratio



<b>VARIABLE</b>	<b>DESCRIPTION</b>
SIGY	Initial yield stress
LCSS	Load curve ID or Table ID. The load curve ID defines effective stress versus effective plastic strain. The table ID defines for each temperature value a load curve ID giving the stress versus effective plastic strain for that temperature (DEFINE_TABLE) or it defines for each temperature value a table ID which defines for each strain rate a load curve ID giving the stress versus effective plastic strain (DEFINE_TABLE_3D). The stress versus effective plastic strain curve for the lowest value of temperature or strain rate is used if the temperature or strain rate falls below the minimum value. Likewise, maximum values cannot be exceeded. Card 2 is ignored with this option.
FAIL	Effective plastic failure strain for erosion.
ALPHA	Coefficient of thermal expansion.
QR1	Isotropic hardening parameter $Q_{r1}$
CR1	Isotropic hardening parameter $C_{r1}$
QR2	Isotropic hardening parameter $Q_{r2}$
CR2	Isotropic hardening parameter $C_{r2}$
QX1	Kinematic hardening parameter $Q_{\chi1}$
CX1	Kinematic hardening parameter $C_{\chi1}$
QX2	Kinematic hardening parameter $Q_{\chi2}$
CX2	Kinematic hardening parameter $C_{\chi2}$
C	Viscous material parameter $C$
P	Viscous material parameter $P$
LCE	Load curve defining Young's modulus as a function of temperature. E on card 1 is ignored with this option.
LCPR	Load curve defining Poisson's ratio as a function of temperature. PR on card 1 is ignored with this option.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCSIGY	Load curve defining the initial yield stress as a function of temperature. SIGY on card 1 is ignored with this option.
LCR	Load curve for scaling the isotropic hardening parameters QR1 and QR2 or the stress given by the load curve LCSS as a function of temperature.
LCX	Load curve for scaling the kinematic hardening parameters QX1 and QX2 as a function of temperature.
LCALPH	Load curve ID defining the instantaneous coefficient of thermal expansion as a function of temperature: $d\varepsilon_{ij}^{\text{thermal}} = \alpha(T)dT\delta_{ij}$ <p>ALPHA on card 1 is ignored with this option. If LCALPH is defined as the negative of the load curve ID, the curve is assumed to define the coefficient relative to a reference temperature, TREF below, such that the total thermal strain is give by</p> $\varepsilon_{ij}^{\text{thermal}} = \alpha(T)(T - T_{\text{ref}})\delta_{ij}$
LCC	Load curve for scaling the viscous material parameter C as a function of temperature.
LCP	Load curve for scaling the viscous material parameter P as a function of temperature.
TREF	Reference temperature required if and only if LCALPH is given with a negative curve ID.
LCFAIL	Load curve defining the plastic failure strain as a function of temperature. FAIL on card 1 is ignored with this option.

**Remarks:**

If LCSS is not given any value the uniaxial stress-strain curve has the form

$$\begin{aligned} \sigma(\varepsilon_{\text{eff}}^p) = & \sigma_0 + Q_{r1}[1 - \exp(-C_{r1}\varepsilon_{\text{eff}}^p)] + Q_{r2}[1 - \exp(-C_{r2}\varepsilon_{\text{eff}}^p)] \\ & + Q_{\chi1}[1 - \exp(-C_{\chi1}\varepsilon_{\text{eff}}^p)] + Q_{\chi2}[1 - \exp(-C_{\chi2}\varepsilon_{\text{eff}}^p)] \end{aligned}$$

Viscous effects are accounted for using the Cowper and Symonds model, which scales the yield stress with the factor:

$$1 + \left( \frac{\dot{\epsilon}_{\text{eff}}^p}{C} \right)^{1/p} .$$

# \*MAT\_107

# \*MAT\_MODIFIED\_JOHNSON\_COOK

## \*MAT\_MODIFIED\_JOHNSON\_COOK

This is Material Type 107. Adiabatic heating is included in the material formulation. Material type 107 is not intended for use in a coupled thermal-mechanical analysis or in a mechanical analysis where temperature is prescribed using \*LOAD\_THERMAL.

Define the following two cards with general material parameters

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	BETA	XS1	CP	ALPHA
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	E0DOT	Tr	Tm	T0	FLAG1	FLAG2		
Type	F	F	F	F	F	F		

**Card 3 for Modified Johnson-Cook Constitutive Relation.** This format is used when FLAG1 = 0.

Card 3	1	2	3	4	5	6	7	8
Variable	A	B	N	C	m			
Type	F	F	F	F	F			

**Card 4 for Modified Johnson-Cook Constitutive Relation.** This format is used when FLAG1 = 0.

Card 4	1	2	3	4	5	6	7	8
Variable	Q1	C1	Q2	C2				
Type	F	F	F	F				

**Card 3 for Modified Zerilli-Armstrong Constitutive Relation.** This format is used when FLAG1 = 1.

Card 3	1	2	3	4	5	6	7	8
Variable	SIGA	B	BETA0	BETA1				
Type	F	F	F	F				

**Card 4 for Modified Zerilli-Armstrong Constitutive Relation.** This format is used when FLAG1 = 1.

Card 4	1	2	3	4	5	6	7	8
Variable	A	N	ALPHA0	ALPHA1				
Type	F	F	F	F				

**Card 5 for Modified Johnson-Cook Fracture Criterion.** This format is used when FLAG2 = 0.

Card 5	1	2	3	4	5	6	7	8
Variable	DC	PD	D1	D2	D3	D4	D5	
Type	F	F	F	F	F	F	F	

**Card 5 for Cockcroft Latham Fracture Criterion.** This format is used when FLAG2 = 1.

Card 5	1	2	3	4	5	6	7	8
Variable	DC	WC						
Type	F	F						

**Additional Element Erosion Criteria Card.**

Card 6	1	2	3	4	5	6	7	8
Variable	TC	TAUC						
Type	F	F						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus, E.
PR	Poisson's ratio, $\nu$ .
BETA	Damage coupling parameter; see Eq. (107.3). EQ.0.0: No coupling between ductile damage and the constitutive relation. EQ.1.0: Full coupling between ductile damage and the constitutive relation.
XS1	Taylor-Quinney coefficient $\chi$ , see Eq. (107.20). Gives the portion of plastic work converted into heat (normally taken to be 0.9)
CP	Specific heat $C_p$ , see Eq. (107.20)
ALPHA	Thermal expansion coefficient, $\alpha$ .
EPS0	Quasi-static threshold strain rate ( $\dot{\epsilon}_0 = \dot{p}_0 = \dot{r}_0$ ), see EQ.(107.12).Set description under *MAT_015.
Tr	Room temperature, see Eq. (107.13)
Tm	Melt temperature, see Eq. (107.13)
T0	Initial temperature

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FLAG1	Constitutive relation flag; see Eq. (107.11) and (107.14) EQ.0.0: Modified Johnson-Cook constitutive relation, see Eq. (107.11). EQ.1.0: Zerilli-Armstrong constitutive relation, see Eq. (107.14).
FLAG2	Fracture criterion flag; see Eq. (107.15) and (107.19). EQ.0.0: Modified Johnson-Cook fracture criterion; see Eq. (107.15). EQ.1.0: Cockcroft-Latham fracture criterion; see Eq. (107.19).
K	Bulk modulus
G	Shear modulus
A	Johnson-Cook yield stress A, see Eq. (107.11).
B	Johnson-Cook hardening parameter B, see Eq. (107.11).
N	Johnson-Cook hardening parameter n, see Eq. (107.11).
C	Johnson-Cook strain rate sensitivity parameter C, see Eq. (107.11).
M	Johnson-Cook thermal softening parameter m, see Eq. (107.11).
Q1	Voce hardening parameter $Q_1$ (when $B = n = 0$ ), see Eq. (107.11).
C1	Voce hardening parameter $C_1$ (when $B = n = 0$ ), see Eq. (107.11).
Q2	Voce hardening parameter $Q_2$ (when $B = n = 0$ ), see Eq. (107.11).
C2	Voce hardening parameter $C_2$ (when $B = n = 0$ ), see Eq. (107.11).
SIGA	Zerilli-Armstrong parameter $\alpha_n$ , see Eq. (107.14).
B	Zerilli-Armstrong parameter B, see Eq. (107.14).
BETA0	Zerilli-Armstrong parameter $\beta_0$ , see Eq. (107.14).
BETA1	Zerilli-Armstrong parameter $\beta_1$ , see Eq. (107.14).
A	Zerilli-Armstrong parameter A, see Eq. (107.14).
N	Zerilli-Armstrong parameter n, see Eq. (107.14).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ALPHA0	Zerilli-Armstrong parameter $\alpha_0$ , see Eq. (107.14).
ALPHA1	Zerilli-Armstrong parameter $\alpha_1$ , see Eq. (107.14).
DC	Critical damage parameter $D_c$ , see Eq. (107.15) and (107.21). When the damage value $D$ reaches this value, the element is eroded from the calculation.
PD	Damage threshold, see Eq. (107.15).
D1-D5	Fracture parameters in the Johnson-Cook fracture criterion, see Eq. (107.16).
WC	Critical Cockcroft-Latham parameter $W_c$ , see Eq. (107.19). When the plastic work per volume reaches this value, the element is eroded from the simulation.
TC	Critical temperature parameter $T_c$ , see Eq. (107.23). When the temperature $T$ , reaches this value, the element is eroded from the simulation.
TAUC	Critical shear stress parameter $\tau_c$ . When the maximum shear stress $\tau$ reaches this value, the element is eroded from the simulation.

**Remarks:**

An additive decomposition of the rate-of-deformation tensor  $\mathbf{d}$  is assumed, i.e.

$$\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p + \mathbf{d}^t \quad (107.1)$$

Where  $\mathbf{d}^e$  is the elastic part,  $\mathbf{d}^p$  is the plastic part and  $\mathbf{d}^t$  is the thermal part.

The elastic rate-of-deformation  $\mathbf{d}^e$  is defined by a linear hypo-elastic relation

$$\tilde{\sigma}^{\nabla J} = \left( K - \frac{2}{3}G \right) \text{tr}(\mathbf{d}^e)\mathbf{I} + 2G\mathbf{d}^e \quad (107.2)$$

Where  $\mathbf{I}$  is the unit tensor,  $K$  is the bulk modulus and  $G$  is the shear modulus. The effective stress tensor is defined by

$$\tilde{\sigma} = \frac{\sigma}{1 - \beta D} \quad (107.3)$$

Where  $\sigma$  is the Cauchy-stress and  $D$  is the damage variable, while the Jaumann rate of the effective stress reads

$$\tilde{\sigma}^{\nabla J} = \dot{\tilde{\sigma}} - \mathbf{W} \cdot \tilde{\sigma} - \tilde{\sigma} \cdot \mathbf{W}^T \quad (107.4)$$



Where  $\mathbf{W}$  is the spin tensor. The parameter  $\beta$  is equal to unity for coupled damage and equal to zero for uncoupled damage.

The thermal rate-of-deformation  $\mathbf{d}^T$  is defined by

$$\mathbf{d}^T = \alpha \dot{T} \mathbf{I} \quad (107.5)$$

Where  $\alpha$  is the linear thermal expansion coefficient and  $T$  is the temperature.

The plastic rate-of-deformation is defined by the associated flow rule as

$$\mathbf{d}^p = \dot{r} \frac{\partial f}{\partial \boldsymbol{\sigma}} = \frac{3}{2} \frac{\dot{r}}{1 - \beta D} \frac{\tilde{\boldsymbol{\sigma}}'}{\tilde{\sigma}_{\text{eq}}} \quad (107.6)$$

Where  $(\cdot)'$  means the deviatoric part of the tensor,  $r$  is the damage-equivalent plastic strain,  $f$  is the dynamic yield function, i.e.

$$\mathbf{d}^p = \dot{r} \frac{\partial f}{\partial \boldsymbol{\sigma}} = \frac{3}{2} \frac{\dot{r}}{1 - \beta D} \frac{\tilde{\boldsymbol{\sigma}}'}{\tilde{\sigma}_{\text{eq}}} \quad (107.6)$$

$$f = \sqrt{\frac{3}{2} \tilde{\boldsymbol{\sigma}}' : \tilde{\boldsymbol{\sigma}}'} - \sigma_Y(r, \dot{r}, T) \leq 0, \quad \dot{r} \geq 0, \quad \dot{r} f = 0 \quad (107.7)$$

And  $\tilde{\sigma}_{\text{eq}}$  is the damage-equivalent stress.

$$\tilde{\sigma}_{\text{eq}} = \sqrt{\frac{3}{2} \tilde{\boldsymbol{\sigma}}' : \tilde{\boldsymbol{\sigma}}'} \quad (107.8)$$

The following plastic work conjugate pairs are identified

$$\dot{W}^p = \boldsymbol{\sigma} : \mathbf{d}^p = \tilde{\sigma}_{\text{eq}} \dot{r} = \sigma_{\text{eq}} \dot{p} \quad (107.9)$$

Where  $\dot{W}^p$  is the specific plastic work rate, and the equivalent stress  $\sigma_{\text{eq}}$  and the equivalent plastic strain  $p$  are defined as

$$\sigma_{\text{eq}} = \sqrt{\frac{3}{2} \tilde{\boldsymbol{\sigma}}' : \tilde{\boldsymbol{\sigma}}'} = (1 - \beta D) \tilde{\sigma}_{\text{eq}} \quad \dot{p} = \sqrt{\frac{2}{3} \mathbf{d}^p : \mathbf{d}^p} = \frac{\dot{r}}{(1 - \beta D)} \quad (107.10)$$

The material strength  $\sigma_Y$  is defined by:

1. The modified Johnson-Cook constitutive relation

$$\sigma_Y = \left\{ A + B r^n + \sum_{i=1}^2 Q_i [1 - \exp(-C_i r)] \right\} (1 + \dot{r}^*)^C (1 - T^{*m}) \quad (107.11)$$

Where  $A, B, C, m, n, Q_1, C_1, Q_2, C_2$  are material parameters; the normalized damage-equivalent plastic strain rate  $\dot{r}^*$  is defined by

$$\dot{r}^* = \frac{\dot{r}}{\dot{\epsilon}_0} \quad (107.12)$$

In which  $\dot{\epsilon}_0$  is a user-defined reference strain rate; and the homologous temperature reads

$$T^* = \frac{T - T_r}{T_m - T_r} \quad (107.13)$$

In which  $T_r$  is the room temperature and  $T_m$  is the melting temperature.

2. The Zerilli-Armstrong constitutive relation

$$\sigma_Y = \{\sigma_a + B \exp[-(\beta_0 - \beta_1 \ln \dot{\epsilon})T] + A r^n \exp[-(\alpha_0 - \alpha_1 \ln \dot{\epsilon})T]\} \quad (107.14)$$

Where  $\sigma_a, B, \beta_0, \beta_1, A, n, \alpha_0, \alpha_1$  are material parameters.

Damage evolution is defined by:

1. The extended Johnson-Cook damage evolution rule:

$$\dot{D} = \begin{cases} 0 & p \leq p_d \\ \frac{D_c}{p_f - p_d} & p > p_d \end{cases} \quad (107.15)$$

Where the current equivalent fracture strain  $p_f = p_f(\sigma^*, \dot{p}^*, T^*)$  is defined as

$$p_f = [D_1 + D_2 \exp(D_3 \sigma^*)](1 + \dot{p}^*)^{D_4} (1 + D_5 T^*) \quad (107.16)$$

and  $D_1, D_2, D_3, D_4, D_5, D_c, p_d$  are material parameters; the normalized equivalent plastic strain rate  $\dot{p}^*$  is defined by

$$\dot{p}^* = \frac{\dot{p}}{\dot{\epsilon}_0} \quad (107.17)$$

and the stress triaxiality  $\sigma^*$  reads

$$\sigma^* = \frac{\sigma_H}{\sigma_{eq}}, \quad \sigma_H = \frac{1}{3} \text{tr}(\sigma) \quad (107.18)$$

2. The Cockcroft-Latham damage evolution rule:

$$\dot{D} = \frac{D_C}{W_C} \max(\sigma_1, 0) \dot{p} \quad (107.19)$$

where  $D_C, W_C$  are material parameters.

Adiabatic heating is calculated as

$$\dot{T} = \chi \frac{\sigma: \mathbf{d}^p}{\rho C_p} = \chi \frac{\tilde{\sigma}_{eq} \dot{\epsilon}}{\rho C_p} \quad (107.20)$$

Where  $\chi$  is the Taylor-Quinney parameter,  $\rho$  is the density and  $C_p$  is the specific heat. The initial value of the temperature  $T_0$  may be specified by the user.

Element erosion occurs when one of the following several criteria are fulfilled:

1. The damage is greater than the critical value

$$D \geq D_C \quad (107.21)$$

2. The maximum shear stress is greater than a critical value

$$\tau_{\max} = \frac{1}{2} \max\{|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\} \geq \tau_C \quad (107.22)$$

3. The temperature is greater than a critical value

$$T \geq T_C \quad (107.23)$$

<i>History Variable</i>	<i>Description</i>
1	Evaluation of damage D
2	Evaluation of stress triaxiality $\sigma^*$
3	Evaluation of damaged plastic strain r
4	Evaluation of temperature T
5	Evaluation of damaged plastic strain rate $\dot{r}$
8	Evaluation of plastic work per volume W
9	Evaluation of maximum shear stress $\tau_{\max}$



Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D17	D2	D3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass Density
E11	Young's Modulus in 11-direction
E22	Young's Modulus in 22-direction
G12	Shear modulus in 12-direction
PR12	Poisson's ratio 12
PR23	Poisson's ratio 23
PR31	Poisson's ratio 31
LC	Load curve ID. This curve defines effective stress versus effective plastic strain. QR1, CR1, QR2, and CR2 are ignored if LC is defined.
SIGMA0	Initial yield stress, $\sigma_0$
QR1	Isotropic hardening parameter, $Q_{R1}$
CR1	Isotropic hardening parameter, $C_{R1}$
QR2	Isotropic hardening parameter, $Q_{R2}$
CR2	Isotropic hardening parameter, $C_{R2}$
R11	Yield criteria parameter, $R_{11}$
R22	Yield criteria parameter, $R_{22}$
R33	Yield criteria parameter, $R_{33}$
R12	Yield criteria parameter, $R_{12}$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description)</p> <p>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>. Nodes 1, 2 and 4 of an element are identical to the node used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: Locally orthotropic material axes determined by offsetting the material axes by an angle, BETA, from a line determined by taking the cross product of the vector <math>\mathbf{v}</math> with the normal to the plane of the element.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
BETA	Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card, see *ELEMENT_SHELL_BETA.
XP YP ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1 A2 A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1 V2 V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1 D2 D3	Components of vector $\mathbf{d}$ for AOPT = 2.

**Remarks:**

The yield function is defined as

$$f = \bar{f}(\sigma) - [\sigma_0 + R(\epsilon^p)]$$

where the equivalent stress  $\sigma_{eq}$  is defined as an anisotropic yield criterion

$$\sigma_{eq} = \sqrt{F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2}$$

Where F, G, H, L, M and N are constants obtained by test of the material in different orientations. They are defined as

$$F = \frac{1}{2} \left( \frac{1}{R_{22}^2} + \frac{1}{R_{33}^2} - \frac{1}{R_{11}^2} \right)$$

$$G = \frac{1}{2} \left( \frac{1}{R_{33}^2} + \frac{1}{R_{11}^2} - \frac{1}{R_{22}^2} \right)$$

$$H = \frac{1}{2} \left( \frac{1}{R_{11}^2} + \frac{1}{R_{22}^2} - \frac{1}{R_{33}^2} \right)$$

$$L = \frac{3}{2R_{23}^2}$$

$$M = \frac{3}{2R_{31}^2}$$

$$N = \frac{3}{2R_{12}^2}$$

The yield stress ratios are defined as follows

$$R_{11} = \frac{\bar{\sigma}_{11}}{\sigma_0}$$

$$R_{22} = \frac{\bar{\sigma}_{22}}{\sigma_0}$$

$$R_{33} = \frac{\bar{\sigma}_{33}}{\sigma_0}$$

$$R_{12} = \frac{\bar{\sigma}_{12}}{\tau_0}$$

$$R_{23} = \frac{\bar{\sigma}_{23}}{\tau_0}$$

$$R_{31} = \frac{\bar{\sigma}_{31}}{\tau_0}$$

where  $\sigma_{ij}$  is the measured yield stress values,  $\sigma_0$  is the reference yield stress and  $\tau_0 = \sigma_0/\sqrt{3}$ .

The strain hardening is either defined by the load curve or the strain hardening  $R$  is defined by the extended Voce law,

$$R(\varepsilon^p) = \sum_{i=1}^2 Q_{Ri} [1 - \exp(-C_{Ri} \varepsilon^p)]$$

where  $\varepsilon^p$  is the effective (or accumulated) plastic strain, and  $Q_{Ri}$  and  $C_{Ri}$  are strain hardening parameters.

**\*MAT\_JOHNSON\_HOLMQUIST\_CERAMICS**

This is Material Type 110. This Johnson-Holmquist Plasticity Damage Model is useful for modeling ceramics, glass and other brittle materials. A more detailed description can be found in a paper by Johnson and Holmquist [1993].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G	A	B	C	M	N
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	EPSI	T	SFMAX	HEL	PHEL	BETA		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	D1	D2	K1	K2	K3	FS		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Density
G	Shear modulus
A	Intact normalized strength parameter
B	Fractured normalized strength parameter
C	Strength parameter (for strain rate dependence)
M	Fractured strength parameter (pressure exponent)



VARIABLE	DESCRIPTION
N	Intact strength parameter (pressure exponent).
EPS0	Quasi-static threshold strain rate. See *MAT_015.
T	Maximum tensile pressure strength.
SFMAX	Maximum normalized fractured strength (defaults to $10^{20}$ when set to 0.0).
HEL	Hugoniot elastic limit.
PHEL	Pressure component at the Hugoniot elastic limit.
BETA	Fraction of elastic energy loss converted to hydrostatic energy (affects bulking pressure (history variable 1) that accompanies damage).
D1	Parameter for plastic strain to fracture.
D2	Parameter for plastic strain to fracture (exponent).
K1	First pressure coefficient (equivalent to the bulk modulus).
K2	Second pressure coefficient.
K3	Third pressure coefficient.
FS	Element deletion criterion. FS.LT.0: fail if $p^* + t^* < 0$ (tensile failure). FS.EQ.0: no failure (default). FS.GT.0: fail if the effective plastic strain > FS.

**Remarks:**

The equivalent stress for a ceramic-type material is given by

$$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*)$$

where

$$\sigma_i^* = a(p^* + t^*)^n(1 + \ln \dot{\epsilon}^*)$$

represents the intact, undamaged behavior. The superscript, '\*', indicates a normalized quantity. The stresses are normalized by the equivalent stress at the Hugoniot elastic limit, the pressures are normalized by the pressure at the Hugoniot elastic limit, and the strain

rate by the reference strain rate defined in the input. In this equation  $a$  is the intact normalized strength parameter,  $c$  is the strength parameter for strain rate dependence,  $\dot{\varepsilon}^*$  is the normalized plastic strain rate, and,

$$t^* = \frac{T}{P_{HEL}},$$

$$p^* = \frac{p}{P_{HEL}},$$

where  $T$  is the maximum tensile pressure strength,  $P_{HEL}$  is the pressure component at the Hugoniot elastic limit, and  $p$  is the pressure.

$$D = \sum \frac{\Delta \varepsilon^p}{\varepsilon_f^p}$$

represents the accumulated damage (history variable 2) based upon the increase in plastic strain per computational cycle and the plastic strain to fracture

$$\varepsilon_f^p = d_1 (p^* + t^*)^{d_2}$$

and

$$\sigma_f^* = b (p^*)^m (1 + c \ln \dot{\varepsilon}) \leq SFMAX$$

represents the damaged behavior. The parameter  $d_1$  controls the rate at which damage accumulates. If it is made 0, full damage occurs in one time step i.e. instantaneously. It is also the best parameter to vary if one attempts to reproduce results generated by another finite element program.

In undamaged material, the hydrostatic pressure is given by

$$P = k_1 \mu + k_2 \mu^2 + k_3 \mu^3$$

in compression and

$$P = k_1 \mu$$

in tension where  $\mu = \rho/\rho_0 - 1$ . When damage starts to occur, there is an increase in pressure. A fraction, between 0 and 1, of the elastic energy loss,  $\beta$ , is converted into hydrostatic potential energy (pressure). The details of this pressure increase are given in the reference.

Given HEL and  $G$ ,  $\mu_{hel}$  can be found iteratively from

$$HEL = k_1 \mu_{hel} + k_2 \mu_{hel}^2 + k_3 \mu_{hel}^3 + (4/3)g(\mu_{hel}/(1 + \mu_{hel}))$$

and, subsequently, for normalization purposes,

$$P_{hel} = k_1 \mu_{hel} + k_2 \mu_{hel}^2 + k_3 \mu_{hel}^3$$

and

$$\sigma_{\text{hel}} = 1.5(\text{hel} - p_{\text{hel}})$$

These are calculated automatically by LS-DYNA if  $\rho_{f0}$  is zero on input.

**\*MAT\_JOHNSON\_HOLMQUIST\_CONCRETE**

This is Material Type 111. This model can be used for concrete subjected to large strains, high strain rates and high pressures. The equivalent strength is expressed as a function of the pressure, strain rate, and damage. The pressure is expressed as a function of the volumetric strain and includes the effect of permanent crushing. The damage is accumulated as a function of the plastic volumetric strain, equivalent plastic strain and pressure. A more detailed description of this model can be found in the paper by Holmquist, Johnson, and Cook [1993].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G	A	B	C	N	FC
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	T	EPS0	EFMIN	SFMAX	PC	UC	PL	UL
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	D1	D2	K1	K2	K3	FS		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
G	Shear modulus.
A	Normalized cohesive strength.
B	Normalized pressure hardening.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
C	Strain rate coefficient.
N	Pressure hardening exponent.
FC	Quasi-static uniaxial compressive strength.
T	Maximum tensile hydrostatic pressure.
EPS0	Quasi-static threshold strain rate. See *MAT_015.
EFMIN	Amount of plastic strain before fracture.
SFMAX	Normalized maximum strength.
PC	Crushing pressure.
UC	Crushing volumetric strain.
PL	Locking pressure.
UL	Locking volumetric strain.
D1	Damage constant.
D2	Damage constant.
K1	Pressure constant.
K2	Pressure constant.
K3	Pressure constant.
FS	Failure type: FS.LT.0: fail if damage strength < 0 FS.EQ.0: fail if $P^* + T^* \leq 0$ (tensile failure). FS.GT.0: fail if the effective plastic strain > FS.

**Remarks:**

The normalized equivalent stress is defined as

$$\sigma^* = \frac{\sigma}{f'_c}$$

where  $\sigma$  is the actual equivalent stress, and  $f'$  is the quasi-static uniaxial compressive strength. The expression is defined as:

$$\sigma^* = [A(1 - D) + BP^{*N}][1 + C \ln(\dot{\epsilon}^*)]$$

where  $D$  is the damage parameter,  $P^* = P/f'_c$  is the normalized pressure and  $\dot{\epsilon}^* = \dot{\epsilon}/\dot{\epsilon}_0$  is the dimensionless strain rate. The model incrementally accumulates damage,  $D$ , both from equivalent plastic strain and plastic volumetric strain, and is expressed as

$$D = \sum \frac{\Delta \epsilon_p + \Delta \mu_p}{D_1(P^* + T^*)^{D_2}}$$

where  $\Delta \epsilon_p$  and  $\Delta \mu_p$  are the equivalent plastic strain and plastic volumetric strain,  $D_1$  and  $D_2$  are material constants and  $T^* = T/f'_c$  is the normalized maximum tensile hydrostatic pressure.

The damage strength,  $DS$ , is defined in compression when  $P^* > 0$  as

$$DS = f'_c \min[SFMAX, A(1 - D) + BP^{*N}][1 + C * \ln(\dot{\epsilon}^*)]$$

or in tension if  $P^* < 0$ , as

$$DS = f'_c \max\left[0, A(1 - D) - A\left(\frac{P^*}{T}\right)\right][1 + C * \ln(\dot{\epsilon}^*)]$$

The pressure for fully dense material is expressed as

$$P = K_1 \bar{\mu} + K_2 \bar{\mu}^2 + K_3 \bar{\mu}^3$$

where  $K_1$ ,  $K_2$  and  $K_3$  are material constants and the modified volumetric strain is defined as

$$\bar{\mu} = \frac{\mu - \mu_{\text{lock}}}{1 + \mu_{\text{lock}}}$$

where  $\mu_{\text{lock}}$  is the locking volumetric strain.



Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
ETAN	Tangent modulus, ignored if (LCSS.GT.0) is defined.
C	Strain rate parameter, C, see formula below.
P	Strain rate parameter, P, see formula below.
LCSS	Load curve ID or Table ID. Load curve ID defining effective stress versus effective plastic strain. If defined EPS1 - EPS8 and ES1 - ES8 are ignored. The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See <a href="#">Figure M24-1</a> . The stress versus effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress versus effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters: C and P; the curve ID, LCSR; EPS1 - EPS8 and ES1 - ES8 are ignored if a Table ID is defined.
LCSR	Load curve ID defining strain rate scaling effect on yield stress.



VARIABLE	DESCRIPTION
EPS1 - EPS8	Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. WARNING: If the first point is nonzero the yield stress is extrapolated to determine the initial yield. If this option is used SIGY and ETAN are ignored and may be input as zero.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8.

### Remarks:

The stress strain behavior may be treated by a bilinear stress strain curve by defining the tangent modulus, ETAN. Alternately, a curve similar to that shown in [Figure M10-1](#) is expected to be defined by (EPS1,ES1) - (EPS8,ES8); however, an effective stress versus effective plastic strain curve (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left( \frac{\dot{\epsilon}}{C} \right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate,  $\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$ .

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor versus strain rate is defined.
3. If different stress versus strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in \*DEFINE\_TABLE has to be used, see [Figure M24-1](#).

**\*MAT\_TRIP**

This is Material Type 113. This isotropic elasto-plastic material model applies to shell elements only. It features a special hardening law aimed at modelling the temperature dependent hardening behavior of austenitic stainless TRIP-steels. TRIP stands for Transformation Induced Plasticity. A detailed description of this material model can be found in Hänsel, Hora, and Reissner [1998] and Schedin, Prentzas, and Hilding [2004].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	CP	T0	TREF	TA0
Type	A8	F	F					
Default								

Card 2	1	2	3	4	5	6	7	8
Variable	A	B	C	D	P	Q	EOMART	VM0
Type	F	F						
Default								

Card 3	1	2	3	4	5	6	7	8
Variable	AHS	BHS	M	N	EPS0	HMART	K1	K2
Type								
Default								

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.

VARIABLE	DESCRIPTION
E	Young's modulus.
PR	Poisson's ratio.
CP	Adiabatic temperature calculation option: EQ.0.0: Adiabatic temperature calculation is disabled. GT.0.0: CP is the specific heat $C_p$ . Adiabatic temperature calculation is enabled.
T0	Initial temperature $T_0$ of the material if adiabatic temperature calculation is enabled.
TREF	Reference temperature for output of the yield stress as history variable 1.
TA0	Reference temperature $T_{A0}$ , the absolute zero for the used temperature scale, e.g. -273.15 if the Celsius scale is used and 0.0 if the Kelvin scale is used.
A	Martensite rate equation parameter $A$ , see equations below.
B	Martensite rate equation parameter $B$ , see equations below.
C	Martensite rate equation parameter $C$ , see equations below.
D	Martensite rate equation parameter $D$ , see equations below.
P	Martensite rate equation parameter $p$ , see equations below.
Q	Martensite rate equation parameter $Q$ , see equations below.
E0MART	Martensite rate equation parameter $E_{0(mart)}$ , see equations below.
VM0	The initial volume fraction of martensite $0.0 < V_{m0} < 1.0$ may be initialised using two different methods: GT.0.0: $V_{m0}$ is set to VM0. LT.0.0: Can be used only when there are initial plastic strains $\epsilon^p$ present, e.g. when using *INITIAL_STRESS_SHELL. The absolute value of VM0 is then the load curve ID for a function $f$ that sets $V_{m0} = f(\epsilon^p)$ . The function $f$ must be a monotonically nondecreasing function of $\epsilon^p$ .
AHS	Hardening law parameter $A_{HS}$ , see equations below.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BHS	Hardening law parameter $B_{HS}$ , see equations below.
M	Hardening law parameter $m$ , see equations below.
N	Hardening law parameter $n$ , see equations below.
EPS0	Hardening law parameter $\varepsilon_0$ , see equations below.
HMART	Hardening law parameter $\Delta H_{\gamma \rightarrow \alpha'}$ , see equations below.
K1	Hardening law parameter $K_1$ , see equations below.
K2	Hardening law parameter $K_2$ , see equations below.

**Remarks:**

Here a short description is given of the TRIP-material model. The material model uses the von Mises yield surface in combination with isotropic hardening. The hardening is temperature dependent and therefore this material model must be run either in a coupled thermo-mechanical solution, using prescribed temperatures or using the adiabatic temperature calculation option. Setting the parameter CP to the specific heat  $C_p$  of the material activates the adiabatic temperature calculation that calculates the temperature rate from the equation

$$\dot{T} = \sum_{ij} \frac{\sigma_{ij} D_{ij}^p}{\rho C_p},$$

where  $\sigma : \mathbf{D}^p$  (the numerator) is the plastically dissipated heat. Using the Kelvin scale is recommended, even though other scales may be used without problems.

The hardening behavior is described by the following equations. The Martensite rate equation is

$$\frac{\partial V_m}{\partial \bar{\varepsilon}^p} = \begin{cases} 0 & \varepsilon < E_{0(\text{mart})} \\ \frac{B}{A} V_m^p \left( \frac{1 - V_m}{V_m} \right)^{\frac{B+1}{B}} \frac{[1 - \tanh(C + D \times T)]}{2} \exp\left(\frac{Q}{T - T_{A0}}\right) & \bar{\varepsilon}^p \geq E_{0(\text{mart})} \end{cases}$$

where

$\bar{\varepsilon}^p$  = effective plastic strain and  
 $T$  = temperature.

The martensite fraction is integrated from the above rate equation:

$$V_m = \int_0^\varepsilon \frac{\partial V_m}{\partial \bar{\varepsilon}^p} d\bar{\varepsilon}^p.$$

It always holds that  $0.0 < V_m < 1.0$ . The initial martensite content is  $V_{m0}$  and must be greater than zero and less than 1.0. Note that  $V_{m0}$  is not used during a restart or when initializing the  $V_m$  history variable using \*INITIAL\_STRESS\_SHELL.

The yield stress  $\sigma_y$  is

$$\sigma_y = \{B_{HS} - (B_{HS} - A_{HS})\exp(-m[\bar{\varepsilon}^p + \varepsilon_0]^n)\}(K_1 + K_2T) + \Delta H_{\gamma \rightarrow \alpha'} V_m.$$

The parameters p and B should fulfill the following condition

$$\frac{1 + B}{B} < p$$

if not fulfilled then the martensite rate will approach infinity as  $V_m$  approaches zero. Setting the parameter  $\varepsilon_0$  larger than zero, typical range 0.001-0.02 is recommended. A part from the effective true strain a few additional history variables are output, see below.

**History variables that are output for post-processing:**

<i>Variable</i>	<i>Description</i>
1	Yield stress of material at temperature TREF. Useful to evaluate the strength of the material after e.g., a simulated forming operation.
2	Volume fraction martensite, $V_m$
3	CP.EQ.0.0: Not used CP.GT.0.0: Temperature from adiabatic temperature calculation



Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
ETAN	Tangent modulus, ignored if (LCSS.GT.0) is defined.
FAIL	Failure flag. LT.0.0: User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic element deletion.
C	Strain rate parameter, C, see formula below.
P	Strain rate parameter, P, see formula below.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCSS	Load curve ID or Table ID. Load curve ID defining effective stress versus effective plastic strain. If defined EPS1 - EPS8 and ES1 - ES8 are ignored. The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See <a href="#">Figure M24-1</a> . The stress versus effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress versus effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters: C and P; the curve ID, LCSR; EPS1 - EPS8 and ES1 - ES8 are ignored if a Table ID is defined.
LCSR	Load curve ID defining strain rate scaling effect on yield stress.
EPS1 - EPS8	Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. WARNING: If the first point is nonzero the yield stress is extrapolated to determine the initial yield. If this option is used SIGY and ETAN are ignored and may be input as zero.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8.

**Remarks:**

The stress strain behavior may be treated by a bilinear stress strain curve by defining the tangent modulus, ETAN. Alternately, a curve similar to that shown in [Figure M10-1](#) is expected to be defined by (EPS1, ES1) - (EPS8, ES8); however, an effective stress versus effective plastic strain curve (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left( \frac{\dot{\epsilon}}{C} \right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate,  $\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$ .



2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor versus strain rate is defined.
3. If different stress versus strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in \*DEFINE\_TABLE has to be used, see [Figure M24-1](#).

**\*MAT\_UNIFIED\_CREEP**

This is Material Type 115. This is an elastic creep model for modeling creep behavior when plastic behavior is not considered.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	A	N	M	
Type	A8	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
A	Stress coefficient.
N	Stress exponent.
M	Time exponent.

**Remarks:**

The effective creep strain,  $\bar{\epsilon}^c$ , given as:

$$\bar{\epsilon}^c = A\bar{\sigma}^n\bar{t}^m$$

where  $A$ ,  $n$ , and  $m$  are constants and  $\bar{t}$  is the effective time. The effective stress,  $\bar{\sigma}$ , is defined as:

$$\bar{\sigma} = \sqrt{\frac{3}{2}\sigma_{ij}\sigma_{ij}}$$

The creep strain, therefore, is only a function of the deviatoric stresses. The volumetric behavior for this material is assumed to be elastic. By varying the time constant  $m$  primary

creep ( $m < 1$ ), secondary creep ( $m = 1$ ), and tertiary creep ( $m > 1$ ) can be modeled. This model is described by Whirley and Henshall [1992].



Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
$E_i$	Young's moduli.
$PR_{ij}$	Elastic Poisson's ratios.
$G_{ij}$	Elastic shear moduli.
A	Stress coefficient.
N	Stress exponent.
M	Time exponent.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>\mathbf{p}</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
MACF	<p>Material axes change flag for brick elements:</p> <p>EQ.1: No change, default,</p> <p>EQ.2: switch material axes a and b,</p> <p>EQ.3: switch material axes a and c,</p> <p>EQ.4: switch material axes b and c.</p>
XP, YP, ZP	Define coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	Define components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Define components of vector $\mathbf{v}$ for AOPT = 3 and 4.

VARIABLE	DESCRIPTION
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO.

**Remarks:**

The stress-strain relationship is based on an additive split of the strain,

$$\dot{\epsilon} = \dot{\epsilon}_e + \dot{\epsilon}_c.$$

Here, the multiaxial creep strain is given by

$$\dot{\epsilon}_c = \dot{\bar{\epsilon}}^c \frac{2s}{3\bar{\sigma}},$$

and  $\bar{\epsilon}^c$  is the effective creep strain,  $s$  the deviatoric stress

$$s = \sigma - \frac{1}{3} \text{tr}(\sigma)I.$$

and  $\bar{\sigma}$  the effective stress

$$\bar{\sigma} = \sqrt{\frac{3}{2} s:s}.$$

The effective creep strain is given by

$$\bar{\epsilon}^c = A\bar{\sigma}^N t^M,$$

where  $A$ ,  $N$ , and  $M$  are constants.

The stress increment is given by

$$\Delta\sigma = C\Delta\epsilon_e = C(\Delta\epsilon - \Delta\epsilon_c),$$

where the constitutive matrix  $C$  is taken as orthotropic and can be represented in Voigt notation by its inverse as

$$C^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & & & & \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & & & & \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & & & & \\ & & & \frac{1}{G_{12}} & & & \\ & & & & \frac{1}{G_{23}} & & \\ & & & & & \frac{1}{G_{13}} & \end{bmatrix}.$$



\*MAT\_COMPOSITE\_LAYUP

This is Material Type 116. This material is for modeling the elastic responses of composite layups that have an arbitrary number of layers through the shell thickness. A pre-integration is used to compute the extensional, bending, and coupling stiffness for use with the Belytschko-Tsay resultant shell formulation. The angles of the local material axes are specified from layer to layer in the \*SECTION\_SHELL input. This material model must be used with the user defined integration rule for shells, see \*INTEGRATION\_SHELL, which allows the elastic constants to change from integration point to integration point. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero. Note that this shell *does not use laminated shell theory* and that storage is allocated for just one integration point (as reported in D3HSP) regardless of the layers defined in the integration rule.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT				
Type	F	F	F	F				

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
EA	$E_a$ , Young's modulus in a-direction.
EB	$E_b$ , Young's modulus in b-direction.
EC	$E_c$ , Young's modulus in c-direction.
PRBA	$\nu_{ba}$ , Poisson's ratio ba.
PRCA	$\nu_{ca}$ , Poisson's ratio ca.
PRCB	$\nu_{cb}$ , Poisson's ratio cb.
GAB	$G_{ab}$ , shear modulus ab.
GBC	$G_{bc}$ , shear modulus bc.
GCA	$G_{ca}$ , shear modulus ca.
AOPT	Material axes option, see <a href="#">Figure M2-1</a> :  EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a> . Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.  EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.  EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.  LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.

VARIABLE	DESCRIPTION
XP, YP, ZP	Define coordinates of point <b>p</b> for AOPT = 1 and 4.
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Define components of vector <b>v</b> for AOPT = 3 and 4.
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

### Remarks:

This material law is based on standard composite lay-up theory. The implementation, [Jones 1975], allows the calculation of the force,  $N$ , and moment,  $M$ , stress resultants from:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix}$$

where  $A_{ij}$  is the extensional stiffness,  $D_{ij}$  is the bending stiffness, and  $B_{ij}$  is the coupling stiffness which is a null matrix for symmetric lay-ups. The mid-surface strains and curvatures are denoted by  $\varepsilon_{ij}^0$  and  $\kappa_{ij}$  respectively. Since these stiffness matrices are symmetric, 18 terms are needed per shell element in addition to the shell resultants which are integrated in time. This is considerably less storage than would typically be required with through thickness integration which requires a minimum of eight history variables per integration point, e.g., if 100 layers are used 800 history variables would be stored. Not only is memory much less for this model, but the CPU time required is also considerably reduced.



Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
CIJ	$C_{ij}$ coefficients of stiffness matrix in the material coordinate system.
AOPT	Material axes option, see <a href="#">Figure M2-1</a> : <p>EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and</p>

VARIABLE	DESCRIPTION
	later.
XP, YP, ZP	Define coordinates of point <b>p</b> for AOPT = 1 and 4.
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Define components of vector <b>v</b> for AOPT = 3 and 4.
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

**Remarks:**

The calculation of the force,  $N_{ij}$ , and moment,  $M_{ij}$ , stress resultants is given in terms of the membrane strains,  $\varepsilon_i^0$ , and shell curvatures  $C_{ij}$ ,  $\kappa_i$ , as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

where  $C_{ij} = C_{ji}$ . In this model this symmetric matrix is transformed into the element local system and the coefficients are stored as element history variables. In model type \*MAT\_COMPOSITE\_DIRECT below, the resultants are already assumed to be given in the element local system which reduces the storage since the 21 coefficients are not stored as history variables as part of the element data.

The shell thickness is built into the coefficient matrix and, consequently, within the part ID, which references this material ID, the thickness must be uniform.

**\*MAT\_COMPOSITE\_DIRECT**

This is Material Type 118. This material is used for modeling the elastic responses of composites where a pre-integration is used to compute the extensional, bending, and coupling stiffness coefficients for use with the Belytschko-Tsay resultant shell formulation. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO						
Type	A8	F						

Card 2	1	2	3	4	5	6	7	8
Variable	C11	C12	C22	C13	C23	C33	C14	C24
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C34	C44	C15	C25	C35	C45	C55	C16
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	C26	C36	C46	C56	C66			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.

VARIABLE	DESCRIPTION
CIJ	$C_{ij}$ coefficients of the stiffness matrix.

**Remarks:**

The calculation of the force,  $N_{ij}$ , and moment,  $M_{ij}$ , stress resultants is given in terms of the membrane strains,  $\varepsilon_i^0$ , and shell curvatures,  $\kappa_i$ , as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

where  $C_{ij} = C_{ji}$ . In this model the stiffness coefficients are already assumed to be given in the element local system which reduces the storage. Great care in the element orientation and choice of the local element system, see \*CONTROL\_ACCURACY, must be observed if this model is used.

The shell thickness is built into the coefficient matrix and, consequently, within the part ID, which references this material ID, the thickness must be uniform.





**\*MAT\_119****\*MAT\_GENERAL\_NONLINEAR\_6DOF\_DISCRETE\_BEAM**

Card 5	1	2	3	4	5	6	7	8
Variable	LCIDTER	LCIDTES	LCIDTET	LCIDRER	LCIDRES	LCIDRET		
Type	I	I	I	I	I	I		

Card 6	1	2	3	4	5	6	7	8
Variable	UTFAILR	UTFAILS	UTFAILT	WTFAILR	WTFAILS	WTFAILT		
Type	F	F	F	F	F	F		

Card 7	1	2	3	4	5	6	7	8
Variable	UCFAILR	UCFAILS	UCFAILT	WCFAILR	WCFAILS	WCFAILT		
Type	F	F	F	F	F	F		

Card 8	1	2	3	4	5	6	7	8
Variable	IUR	IUS	IUT	IWR	IWS	IWT		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
KT	Translational stiffness for unloading option 2.0.
KR	Rotational stiffness for unloading option 2.0.

VARIABLE	DESCRIPTION
DAMPF	Damping factor for stability. Values in the neighborhood of unity are recommended. This damping factor is properly scaled to eliminate time step size dependency. Also, it is active if and only if the local stiffness is defined.
IFLAG	Flag for switching between the displacement (default IFLAG = 0) and linear strain (IFLAG = 1) formulations. The displacement formulation is the one used in all other models. For the linear strain formulation, the displacements and velocities are divided by the initial length of the beam.
IUNLD	<p>Unloading option (Also see <a href="#">Figure M119-1</a>):</p> <p>EQ.0.0: Loading and unloading follow loading curve</p> <p>EQ.1.0: Loading follows loading curve, unloading follows unloading curve. The unloading curve ID if undefined is taken as the loading curve.</p> <p>EQ.2.0: Loading follows loading curve, unloading follows unloading stiffness, KT or KR, to the unloading curve. The loading and unloading curves may only intersect at the origin of the axes.</p> <p>EQ.3.0: Quadratic unloading from peak displacement value to a permanent offset.</p>
OFFSET	Offset factor between 0 and 1.0 to determine permanent set upon unloading if the IUNLD = 3.0. The permanent sets in compression and tension are equal to the product of this offset value and the maximum compressive and tensile displacements, respectively.
LCIDTR	Load curve ID defining translational force resultant along local r-axis versus relative translational displacement. If zero, no stiffness related forces are generated for this degree of freedom. The loading curves must be defined from the most negative displacement to the most positive displacement. The force does not need to increase monotonically. The curves in this input are linearly extrapolated when the displacement range falls outside the curve definition.
LCIDTS	Load curve ID defining translational force resultant along local s-axis versus relative translational displacement.
LCIDTT	Load curve ID defining translational force resultant along local t-axis versus relative translational displacement.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDRR	Load curve ID defining rotational moment resultant about local r-axis versus relative rotational displacement.
LCIDRS	Load curve ID defining rotational moment resultant about local s-axis versus relative rotational displacement.
LCIDRT	Load curve ID defining rotational moment resultant about local t-axis versus relative rotational displacement.
LCIDTUR	Load curve ID defining translational force resultant along local r-axis versus relative translational displacement during unloading. The force values defined by this curve must increase monotonically from the most negative displacement to the most positive displacement. For IUNLD = 1.0, the slope of this curve must equal or exceed the loading curve for stability reasons. This is not the case for IUNLD = 2.0. For loading and unloading to follow the same path simply set LCIDTUR = LCIDTR. For options IUNLD = 0.0 or 3.0 the unloading curve is not required. For IUNLD = 2.0, if LCIDTUR is left blank or zero, the default is to use the same curve for unloading as for loading.
LCIDTUS	Load curve ID defining translational force resultant along local s-axis versus relative translational displacement during unloading.
LCIDTUT	Load curve ID defining translational force resultant along local t-axis versus relative translational displacement during unloading.
LCIDRUR	Load curve ID defining rotational moment resultant about local r-axis versus relative rotational displacement during unloading.
LCIDRUS	Load curve ID defining rotational moment resultant about local s-axis versus relative rotational displacement during unloading.
LCIDRUT	Load curve ID defining rotational moment resultant about local t-axis versus relative rotational displacement during unloading. If zero, no viscous forces are generated for this degree of freedom.
LCIDTDR	Load curve ID defining translational damping force resultant along local r-axis versus relative translational velocity.
LCIDTDS	Load curve ID defining translational damping force resultant along local s-axis versus relative translational velocity.
LCIDTDT	Load curve ID defining translational damping force resultant along local t-axis versus relative translational velocity.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDRDR	Load curve ID defining rotational damping moment resultant about local r-axis versus relative rotational velocity.
LCIDRDS	Load curve ID defining rotational damping moment resultant about local s-axis versus relative rotational velocity.
LCIDRDT	Load curve ID defining rotational damping moment resultant about local t-axis versus relative rotational velocity.
LCIDTER	Load curve ID defining translational damping force scale factor versus relative displacement in local r-direction.
LCIDTES	Load curve ID defining translational damping force scale factor versus relative displacement in local s-direction.
LCIDTET	Load curve ID defining translational damping force scale factor versus relative displacement in local t-direction.
LCIDRER	Load curve ID defining rotational damping moment resultant scale factor versus relative displacement in local r-rotation.
LCIDRES	Load curve ID defining rotational damping moment resultant scale factor versus relative displacement in local s-rotation.
LCIDRET	Load curve ID defining rotational damping moment resultant scale factor versus relative displacement in local t-rotation.
UTFAILR	Optional, translational displacement at failure in tension. If zero, the corresponding displacement, $u_r$ , is not considered in the failure calculation.
UTFAILS	Optional, translational displacement at failure in tension. If zero, the corresponding displacement, $u_s$ , is not considered in the failure calculation.
UTFAILT	Optional, translational displacement at failure in tension. If zero, the corresponding displacement, $u_t$ , is not considered in the failure calculation.
WTFAILR	Optional, rotational displacement at failure in tension. If zero, the corresponding rotation, $\theta_r$ , is not considered in the failure calculation.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
WTFAILS	Optional, rotational displacement at failure in tension. If zero, the corresponding rotation, $\theta_s$ , is not considered in the failure calculation.
WTFAILT	Optional rotational displacement at failure in tension. If zero, the corresponding rotation, $\theta_t$ , is not considered in the failure calculation.
UCFAILR	Optional, translational displacement at failure in compression. If zero, the corresponding displacement, $u_r$ , is not considered in the failure calculation. Define as a positive number.
UCFAILS	Optional, translational displacement at failure in compression. If zero, the corresponding displacement, $u_s$ , is not considered in the failure calculation. Define as a positive number.
UCFAILT	Optional, translational displacement at failure in compression. If zero, the corresponding displacement, $u_t$ , is not considered in the failure calculation. Define as a positive number.
WCFAILR	Optional, rotational displacement at failure in compression. If zero, the corresponding rotation, $\theta_r$ , is not considered in the failure calculation. Define as a positive number.
WCFAILS	Optional, rotational displacement at failure in compression. If zero, the corresponding rotation, $\theta_s$ , is not considered in the failure calculation. Define as a positive number.
WCFAILT	Optional, rotational displacement at failure in compression. If zero, the corresponding rotation, $\theta_t$ , is not considered in the failure calculation. Define as a positive number.
IUR	Initial translational displacement along local r-axis.
IUS	Initial translational displacement along local s-axis.
IUT	Initial translational displacement along local t-axis.
IWR	Initial rotational displacement about the local r-axis.
IWS	Initial rotational displacement about the local s-axis.
IWT	Initial rotational displacement about the local t-axis.

**Remarks:**

Catastrophic failure, which is based on displacement resultants, occurs if either of the following inequalities are satisfied:

$$\left(\frac{u_r}{u_r^{tfail}}\right)^2 + \left(\frac{u_s}{u_s^{tfail}}\right)^2 + \left(\frac{u_t}{u_t^{tfail}}\right)^2 + \left(\frac{\theta_r}{\theta_r^{tfail}}\right)^2 + \left(\frac{\theta_s}{\theta_s^{tfail}}\right)^2 + \left(\frac{\theta_t}{\theta_t^{tfail}}\right)^2 - 1 \geq 0$$

$$\left(\frac{u_r}{u_r^{cfail}}\right)^2 + \left(\frac{u_s}{u_s^{cfail}}\right)^2 + \left(\frac{u_t}{u_t^{cfail}}\right)^2 + \left(\frac{\theta_r}{\theta_r^{cfail}}\right)^2 + \left(\frac{\theta_s}{\theta_s^{cfail}}\right)^2 + \left(\frac{\theta_t}{\theta_t^{cfail}}\right)^2 - 1 \geq 0$$

After failure the discrete element is deleted. If failure is included either the tension failure or the compression failure or both may be used.

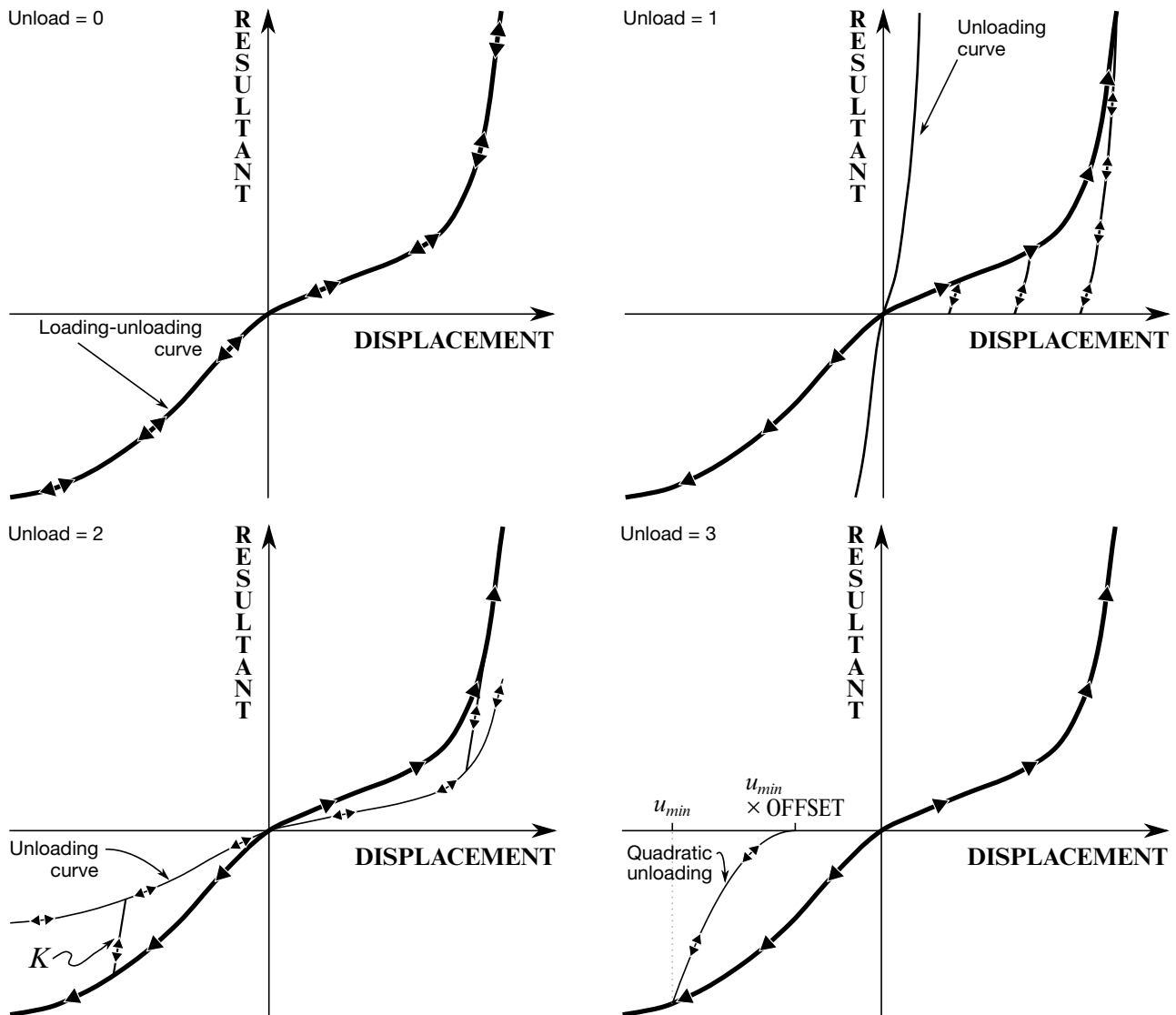


Figure M119-1. Load and unloading behavior.

There are two formulations for calculating the force. The first is the standard displacement formulation, where, for example, the force in a linear spring is

$$F = -K\Delta\ell$$

for a change in length of the beam of  $\Delta\ell$ . The second formulation is based on the linear strain, giving a force of

$$F = -K\frac{\Delta\ell}{\ell_0}$$

for a beam with an initial length of  $\ell_0$ . This option is useful when there are springs of different lengths but otherwise similar construction since it automatically reduces the stiffness of the spring as the length increases, allowing an entire family of springs to be modeled with a single material. Note that all the displacement and velocity components are divided by the initial length, and therefore the scaling applies to the damping and rotational stiffness.





Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 5	1	2	3	4	5	6	7	8
Variable	L1	L2	L3	L4	FF1	FF2	FF3	FF4
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 6	1	2	3	4	5	6	7	8
Variable	LCSS	LCLF	NUMINT	LCFO	LCFC	LCFN	VG Typ	DEXP
Type	F	F	F	F	F	F	F	F
Default	0	0	1.0	0	0	0	0	3.0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
N	Exponent for Power law. This value is only used if ATYP = 1 and LCSS = 0.

VARIABLE	DESCRIPTION
Q1	Gurson flow function parameter $q_1$ .
Q2	Gurson flow function parameter $q_2$ .
FC	Critical void volume fraction $f_c$ where voids begin to aggregate. This value is only used if LCFC = 0.
F0	Initial void volume fraction $f_0$ . This value is only used if LCF0 = 0.
EN	Mean nucleation strain $\varepsilon_N$ . GT.0.0: Constant value, LT.0.0: Load curve ID = (-EN) which defines mean nucleation strain $\varepsilon_N$ as a function of element length.
SN	Standard deviation $s_N$ of the normal distribution of $\varepsilon_N$ . GT.0.0: Constant value, LT.0.0: Load curve ID = (-SN) which defines standard deviation $s_N$ of the normal distribution of $\varepsilon_N$ as a function of element length.
FN	Void volume fraction of nucleating particles $f_N$ . This value is only used if LCFN = 0.
ETAN	Hardening modulus. This value is only used if ATYP = 2 and LCSS = 0.
ATYP	Type of hardening. EQ.0.0: Ideal plastic, $\sigma_Y = \text{SIGY}.$ EQ.1.0: Power law, $\sigma_Y = \text{SIGY} \times \left( \frac{\varepsilon^p + \text{SIGY}/E}{\text{SIGY}/E} \right)^{1/N}$ EQ.2.0: Linear hardening, $\sigma_Y = \text{SIGY} + \frac{E \times \text{ETAN}}{E - \text{ETAN}} \varepsilon^p.$ EQ.3.0: 8 points curve.
FF0	Failure void volume fraction $f_F$ . This value is only used if no curve is given by (L1, FF1) – (L4, FF4) and LCFF = 0.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EPS1 - EPS8	Effective plastic strain values. The first point must be zero corresponding to the initial yield stress. At least 2 points should be defined. These values are used if ATYP = 3 and LCSS = 0.
ES1 - ES8	Corresponding yield stress values to EPS1 – EPS8. These values are used if ATYP = 3 and LCSS = 0.
L1 - L4	Element length values. These values are only used if LCFF = 0
FF1 - FF4	Corresponding failure void volume fraction. These values are only used if LCFF = 0.
LCSS	Load curve ID or Table ID. ATYP is ignored with this option. Load curve ID defining effective stress versus effective plastic strain. Table ID defines for each strain rate value a load curve ID giving the effective stress versus effective plastic strain for that rate (see MAT_024). The stress-strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress-strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value.
	<u>NOTE:</u> The strain rate values defined in the table may be given as the natural logarithm of the strain rate. If the <i>first</i> stress-strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used. Since the tables are internally discretized to equally space the points, natural logarithms are necessary, for example, if the curves correspond to rates from $10^{-4}$ to $10^4$ .
LCFF	Load curve ID defining failure void volume fraction $f_F$ versus element length.
NUMINT	Number of integration points which must fail before the element is deleted. This option is available for shells and solids.  LT.0.0:  NUMINT  is percentage of integration points/layers which must fail before shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells.
LCF0	Load curve ID defining initial void volume fraction $f_0$ versus element length.

VARIABLE	DESCRIPTION
LCFC	Load curve ID defining critical void volume fraction $f_c$ versus element length.
LCFN	Load curve ID defining void volume fraction of nucleating particles $f_N$ versus element length.
VG Typ	Type of void growth behavior. EQ.0.0: Void growth in case of tension and void contraction in case of compression, but never below $f_0$ (default). EQ.1.0: Void growth only in case of tension. EQ.2.0: Void growth in case of tension and void contraction in case of compression, even below $f_0$ .
DEXP	Exponent value for damage history variable 16.

### Remarks:

The Gurson flow function is defined as:

$$\Phi = \frac{\sigma_M^2}{\sigma_Y^2} + 2q_1 f^* \cosh\left(\frac{3q_2 \sigma_H}{2\sigma_Y}\right) - 1 - (q_1 f^*)^2 = 0$$

where  $\sigma_M$  is the equivalent von Mises stress,  $\sigma_Y$  is the yield stress,  $\sigma_H$  is the mean hydrostatic stress. The effective void volume fraction is defined as

$$f^*(f) = \begin{cases} f & f \leq f_c \\ f_c + \frac{1/q_1 - f_c}{f_F - f_c} (f - f_c) & f > f_c \end{cases}$$

The growth of void volume fraction is defined as

$$\dot{f} = \dot{f}_G + \dot{f}_N$$

where the growth of existing voids is defined as

$$\dot{f}_G = (1 - f) \dot{\epsilon}_{kk}^p$$

and nucleation of new voids is defined as

$$\dot{f}_N = A \dot{\epsilon}_p$$

with function  $A$

$$A = \frac{f_N}{S_N \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\epsilon_p - \epsilon_N}{S_N}\right)^2\right]$$

Voids are nucleated only in tension.

**History variables:***Shell / Solid Description*

1 / 1	Void volume fraction
4 / 2	Triaxiality variable $\sigma_H/\sigma_M$
5 / 3	Effective strain rate
6 / 4	Growth of voids
7 / 5	Nucleation of voids
11 / 11	Dimensionless material damage value = $\begin{cases} \frac{(f-f_0)}{(f_c-f_0)} & f \leq f_c \\ 1 + \frac{(f-f_c)}{(f_F-f_c)} & f > f_c \end{cases}$
13 / 13	Deviatoric part of microscopic plastic strain
14 / 14	Volumetric part of macroscopic plastic strain
16 / 16	Dimensionless material damage value = $\left(\frac{f-f_0}{f_F-f_0}\right)^{1/DEXP}$



**\*MAT\_120\_JC****\*MAT\_GURSON\_JC**

Card 4	1	2	3	4	5	6	7	8
Variable	SIG1	SIG2	SIG3	SIG4	SIG5	SIG6	SIG7	SIG8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 5	1	2	3	4	5	6	7	8
Variable	LCDAM	L1	L2	D1	D2	D3	D4	LCJC
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 6	1	2	3	4	5	6	7	8
Variable	LCSS	LCFF	NUMINT	LCFO	LCFC	LCFN	VG Typ	DEXP
Type	F	F	F	F	F	F	F	F
Default	0	0	1	0	0	0	0	3.0

Optional Card (starting with version 971 release R4)

Card 7	1	2	3	4	5	6	7	8
Variable	KW	BETA	M					
Type	F	F	F					
Default	0	0	1.0					



<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
N	Exponent for Power law. This value is only used if ATYP = 1 and LCSS = 0.
Q1	Gurson flow function parameter $q_1$ .
Q2	Gurson flow function parameter $q_2$ .
FC	Critical void volume fraction $f_c$ where voids begin to aggregate.
F0	Initial void volume fraction $f_0$ . This value is only used if LCF0 = 0.
EN	Mean nucleation strain $\varepsilon_N$ . GT.0.0: Constant value, LT.0.0: Load curve ID = (-EN) which defines mean nucleation strain $\varepsilon_N$ as a function of element length.
SN	Standard deviation $s_N$ of the normal distribution of $\varepsilon_N$ . GT.0.0: Constant value, LT.0.0: Load curve ID = (-SN) which defines standard deviation $s_N$ of the normal distribution of $\varepsilon_N$ as a function of element length.
FN	Void volume fraction of nucleating particles $f_N$ . This value is only used if LCFN = 0.
ETAN	Hardening modulus. This value is only used if ATYP = 2 and LCSS = 0.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ATYP	Type of hardening. EQ.0.0: Ideal plastic, $\sigma_Y = \text{SIGY}.$ EQ.1.0: Power law, $\sigma_Y = \text{SIGY} \times \left( \frac{\varepsilon^p + \text{SIGY}/E}{\text{SIGY}/E} \right)^{1/N}$ EQ.2.0: Linear hardening, $\sigma_Y = \text{SIGY} + \frac{E \times \text{ETAN}}{E - \text{ETAN}} \varepsilon^p.$ EQ.3.0: 8 points curve.
FF0	Failure void volume fraction $f_F$ . This value is only used if LCFF = 0.
EPS1 - EPS8	Effective plastic strain values. The first point must be zero corresponding to the initial yield stress. At least 2 points should be defined. These values are used if ATYP = 3 and LCSS = 0.
ES1 - ES8	Corresponding yield stress values to EPS1 – EPS8. These values are used if ATYP = 3 and LCSS = 0.
LCDAM	Load curve defining scaling factor $\Lambda$ versus element length. Scales the Johnson-Cook failure strain (see remarks). If LCDAM = 0, no scaling is performed.
L1	Lower triaxiality factor defining failure evolution (Johnson-Cook).
L2	Upper triaxiality factor defining failure evolution (Johnson-Cook).
D1 - D4	Johnson-Cook damage parameters.
LCJC	Load curve defining scaling factor for Johnson-Cook failure versus triaxiality (see remarks). If LCJC > 0, parameters D1, D2 and D3 are ignored.

VARIABLE	DESCRIPTION
LCSS	<p>Load curve ID or Table ID. ATYP is ignored with this option. Load curve ID defining effective stress versus effective plastic strain. Table ID defines for each strain rate value a load curve ID giving the effective stress versus effective plastic strain for that rate (see MAT_024). The stress-strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress-strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value.</p> <p><u>NOTE:</u> The strain rate values defined in the table may be given as the natural logarithm of the strain rate. If the <i>first</i> stress-strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used. Since the tables are internally discretized to equally space the points, natural logarithms are necessary, for example, if the curves correspond to rates from <math>10^{-4}</math> to <math>10^4</math>.</p>
LCFF	<p>Load curve ID defining failure void volume fraction <math>f_F</math> versus element length.</p>
NUMINT	<p>Number of through thickness integration points which must fail before the element is deleted. This option is available for shells and solids.</p> <p>LT.0.0:  NUMINT  is percentage of integration points/layers which must fail before shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells.</p>
LCF0	<p>Load curve ID defining initial void volume fraction <math>f_0</math> versus element length.</p>
LCFC	<p>Load curve ID defining critical void volume fraction <math>f_c</math> versus element length.</p>
LCFN	<p>Load curve ID defining void volume fraction of nucleating particles <math>f_N</math> versus element length.</p>

VARIABLE	DESCRIPTION
VG Typ	Type of void growth behavior. EQ.0.0: Void growth in case of tension and void contraction in case of compression, but never below $f_0$ (default). EQ.1.0: Void growth only in case of tension. EQ.2.0: Void growth in case of tension and void contraction in case of compression, even below $f_0$ .
DEXP	Exponent value for damage history variable 16.
KW	Parameter $k_\omega$ for void growth in shear-dominated states. See remarks.
BETA	Parameter $\beta$ in Lode cosine function. See remarks.
M	Parameter for generalization of Johnson-Cook damage evolution. See remarks.

**Remarks:**

The Gurson flow function is defined as:

$$\Phi = \frac{\sigma_M^2}{\sigma_Y^2} + 2q_1 f^* \cosh\left(\frac{3q_2 \sigma_H}{2\sigma_Y}\right) - 1 - (q_1 f^*)^2 = 0$$

where  $\sigma_M$  is the equivalent von Mises stress,  $\sigma_Y$  is the yield stress,  $\sigma_H$  is the mean hydrostatic stress. The effective void volume fraction is defined as

$$f^*(f) = \begin{cases} f & f \leq f_c \\ f_c + \frac{1/q_1 - f_c}{f_F - f_c} (f - f_c) & f > f_c \end{cases}$$

The growth of void volume fraction is defined as

$$\dot{f} = \dot{f}_G + \dot{f}_N$$

where the growth of existing voids is defined as

$$\dot{f}_G = (1 - f) \dot{\epsilon}_{kk}^p + k_\omega \omega(\sigma) f (1 - f) \dot{\epsilon}_M^{pl} \frac{\sigma_Y}{\sigma_M}$$

The second term is an optional extension for shear failure proposed by Nahshon and Hutchinson [2008] with new parameter  $k_\omega$  (=0 by default), effective plastic strain rate in the matrix  $\dot{\epsilon}_M^{pl}$ , and Lode cosin function  $\omega(\sigma)$ :

$$\omega(\sigma) = 1 - \zeta^2 - \beta \cdot \zeta(1 - \zeta), \quad \zeta = \cos(3\theta) = \frac{27}{2} \frac{J_3}{\sigma_M^3}$$

with parameter  $\beta$ , Lode angle  $\theta$  and third deviatoric stress invariant  $J_3$ .

Nucleation of new voids is defined as

$$\dot{f}_N = A \dot{\epsilon}_M^{pl}$$

with function  $A$

$$A = \frac{f_N}{S_N \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\epsilon_M^{pl} - \epsilon_N}{S_N} \right)^2 \right]$$

Voids are nucleated only in tension.

The Johnson-Cook failure criterion is added in this material model. Based on the triaxiality ratio  $\sigma_H/\sigma_M$  failure is calculated as:

$$\sigma_H/\sigma_M > L_1 : \text{Gurson model}$$

$$L_1 \geq \sigma_H/\sigma_M \geq L_2 : \text{Gurson model and Johnson-Cook failure criteria}$$

$$L_2 < \sigma_H/\sigma_M : \text{Gurson model}$$

Johnson-Cook failure strain is defined as

$$\epsilon_f = \left[ D_1 + D_2 \exp \left( D_3 \frac{\sigma_H}{\sigma_M} \right) \right] (1 + D_4 \ln \dot{\epsilon}) \Lambda$$

where  $D_1, D_2, D_3$  and  $D_4$  are the Johnson-Cook failure parameters and  $\Lambda$  is a function for including mesh-size dependency. An alternative expression can be used, where the first term of the above equation (including  $D_1, D_2$  and  $D_3$ ) is replaced by a general function LCJC which depends on triaxiality

$$\epsilon_f = \text{LCJC} \times \left( \frac{\sigma_H}{\sigma_M} \right) (1 + D_4 \ln \dot{\epsilon}) \Lambda$$

The Johnson-Cook damage parameter  $D_f$  is calculated with the following evolution equation:

$$\dot{D}_f = \frac{\dot{\epsilon}^{pl}}{\epsilon_f} \Rightarrow D_f = \sum \frac{\Delta \epsilon^{pl}}{\epsilon_f}$$

where  $\Delta \epsilon^{pl}$  is the increment in effective plastic strain. The material fails when  $D_f$  reaches 1.0. A more general (non-linear) damage evolution is possible if  $M > 1$  is chosen:

$$\dot{D}_f = \frac{M}{\epsilon_f} D_f^{(1-\frac{1}{M})} \dot{\epsilon}^{pl}, \quad M \geq 1.0$$

**History variables:***Shell / Solid Description*

1 / 1	Void volume fraction
4 / 2	Triaxiality variable $\sigma_H/\sigma_M$
5 / 3	Effective strain rate
6 / 4	Growth of voids
7 / 5	Nucleation of voids
8 / 6	Johnson-Cook failure strain $\varepsilon_f$
9 / 7	Johnson-Cook damage parameter $D_f$
0 / 8	Domain variable: EQ.0 $\Rightarrow$ elastic stress update EQ.1 $\Rightarrow$ region (a) Gurson EQ.2 $\Rightarrow$ region (b) Gurson + Johnson-Cook EQ.3 $\Rightarrow$ region (c) Gurson
11 / 11	Dimensionless material damage value = $\begin{cases} \frac{(f-f_0)}{(f_c-f_0)} & f \leq f_c \\ 1 + \frac{(f-f_c)}{(f_F-f_c)} & f > f_c \end{cases}$
13 / 13	Deviatoric part of microscopic plastic strain
14 / 14	Volumetric part of macroscopic plastic strain
16 / 16	Dimensionless material damage value = $\left(\frac{f-f_0}{f_F-f_0}\right)^{1/DEXP}$







VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
N	Exponent for Power law. This value is only used if ATYP = 1 and LCSS = 0.
Q1	Parameter $q_1$ .
Q1	Parameter $q_2$ .
FC	Critical void volume fraction $f_c$
F0	Initial void volume fraction $f_0$ .
EN	Mean nucleation strain $\epsilon_N$ .
SN	Standard deviation $S_N$ of the normal distribution of $\epsilon_N$ .
FN	Void volume fraction of nucleating particles.
ETAN	Hardening modulus. This value is only used if ATYP = 2 and LCSS = 0.
ATYP	Type of hardening. EQ.0.0: Ideal plastic, $\sigma_Y = \text{SIGY}.$ EQ.1.0: Power law, $\sigma_Y = \text{SIGY} \times \left( \frac{\epsilon^p + \text{SIGY}/E}{\text{SIGY}/E} \right)^{1/N}$ EQ.2.0: Linear hardening, $\sigma_Y = \text{SIGY} + \frac{E \times \text{ETAN}}{E - \text{ETAN}} \epsilon^p.$ EQ.3.0: 8 points curve.
FF0	Failure void volume fraction. This value is used if no curve is given by the points (L1, FF1) – (L4, FF4) and LCLF = 0.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EPS1 - EPS8	Effective plastic strain values. The first point must be zero corresponding to the initial yield stress. This option is only used if ATYP equal to 3. At least 2 points should be defined. These values are used if ATYP = 3 and LCSS = 0.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8. These values are used if ATYP = 3 and LCSS = 0.
L1 - L4	Element length values. These values are only used if LCLF = 0.
FF1 - FF4	Corresponding failure void volume fraction. These values are only used if LCLF = 0.
LCSS	Load curve ID defining effective stress versus effective plastic strain. ATYP is ignored with this option.
LCLF	Load curve ID defining failure void volume fraction versus element length. The values L1 - L4 and FF1 - FF4 are ignored with this option.
NUMINT	Number of through thickness integration points which must fail before the element is deleted.
ALPHA	Parameter $\alpha$ . for the Rc-Dc model
BETA	Parameter $\beta$ . for the Rc-Dc model
GAMMA	Parameter $\gamma$ . for the Rc-Dc model
D0	Parameter $D_0$ . for the Rc-Dc model
B	Parameter $b$ . for the Rc-Dc model
LAMBDA	Parameter $\lambda$ . for the Rc-Dc model
DS	Parameter $D_s$ . for the Rc-Dc model
L	Characteristic element length for this material

**Remarks:**

The Gurson flow function is defined as:

$$\Phi = \frac{\sigma_M^2}{\sigma_Y^2} + 2q_1 f^* \cosh\left(\frac{3q_2 \sigma_H}{2\sigma_Y}\right) - 1 - (q_1 f^*)^2 = 0$$

where  $\sigma_M$  is the equivalent von Mises stress,  $\sigma_Y$  is the Yield stress,  $\sigma_H$  is the mean hydrostatic stress. The effective void volume fraction is defined as

$$f^*(f) = \begin{cases} f & f \leq f_c \\ f_c + \frac{1/q_1 - f_c}{f_F - f_c} (f - f_c) & f > f_c \end{cases}$$

The growth of the void volume fraction is defined as

$$\dot{f} = \dot{f}_G + \dot{f}_N$$

where the growth of existing voids is given as:

$$\dot{f}_G = (1 - f)\dot{\epsilon}_{kk}^p,$$

and nucleation of new voids as:

$$\dot{f}_N = A\dot{\epsilon}_p$$

in which  $A$  is defined as

$$A = \frac{f_N}{S_N \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{\epsilon_p - \epsilon_N}{S_N}\right)^2\right)$$

The Rc-Dc model is defined as the following:

The damage  $D$  is given by

$$D = \int \omega_1 \omega_2 d\epsilon^p$$

where  $\epsilon^p$  is the equivalent plastic strain,

$$\omega_1 = \left(\frac{1}{1 - \gamma \sigma_m}\right)^\alpha$$

is a triaxial stress weighting term and

$$\omega_2 = (2 - A_D)^\beta$$

is a asymmetric strain weighting term.

In the above  $\sigma_m$  is the mean stress and

$$A_D = \max\left(\frac{S_2}{S_3}, \frac{S_2}{S_1}\right)$$

Fracture is initiated when the accumulation of damage is

$$\frac{D}{D_c} > 1$$

where  $D_c$  is the a critical damage given by

$$D_c = D_0(1 + b|\nabla D|^\lambda)$$

A fracture fraction

$$F = \frac{D - D_c}{D_s}$$

defines the degradations of the material by the Rc-Dc model.

The characteristic element length is used in the calculation of  $\nabla D$ . Calculation of this factor is only done for element with smaller element length than this value.

**\*MAT\_GENERAL\_NONLINEAR\_1DOF\_DISCRETE\_BEAM**

This is Material Type 121. This is a very general spring and damper model. This beam is based on the MAT\_SPRING\_GENERAL\_NONLINEAR option and is a one-dimensional version of the 6DOF\_DISCRETE\_BEAM above. The forces generated by this model act along a line between the two connected nodal points. Additional unloading options have been included.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	IUNLD	OFFSET	DAMPF		
Type	A8	F	F	I	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	LCIDT	LCIDTU	LCIDTD	LCIDTE				
Type	I	I	I	I				

Card 3	1	2	3	4	5	6	7	8
Variable	UTFAIL	UCFAIL	IU					
Type	F	F	F					

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
K	Translational stiffness for unloading option 2.0.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
IUNLD	Unloading option (Also see <a href="#">Figure M119-1</a> ): EQ.0.0: Loading and unloading follow loading curve EQ.1.0: Loading follows loading curve, unloading follows unloading curve. The unloading curve ID if undefined is taken as the loading curve. EQ.2.0: Loading follows loading curve, unloading follows unloading stiffness, K, to the unloading curve. The loading and unloading curves may only intersect at the origin of the axes. EQ.3.0: Quadratic unloading from peak displacement value to a permanent offset.
OFFSET	Offset to determine permanent set upon unloading if the IUNLD = 3.0. The permanent sets in compression and tension are equal to the product of this offset value and the maximum compressive and tensile displacements, respectively.
DAMPF	Damping factor for stability. Values in the neighborhood of unity are recommended. This damping factor is properly scaled to eliminate time step size dependency. Also, it is active if and only if the local stiffness is defined.
LCIDT	Load curve ID defining translational force resultant along the axis versus relative translational displacement. If zero, no stiffness related forces are generated for this degree of freedom. The loading curves must be defined from the most negative displacement to the most positive displacement. The force does not need to increase monotonically for the loading curve. The curves are extrapolated when the displacement range falls outside the curve definition.
LCIDTU	Load curve ID defining translational force resultant along the axis versus relative translational displacement during unloading. The force values defined by this curve must increase monotonically from the most negative displacement to the most positive displacement. For IUNLD = 1.0, the slope of this curve must equal or exceed the loading curve for stability reasons. This is not the case for IUNLD = 2.0. For loading and unloading to follow the same path simply set LCIDTU = LCIDT.
LCIDTD	Load curve ID defining translational damping force resultant along the axis versus relative translational velocity.

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDTE	Load curve ID defining translational damping force scale factor versus relative displacement along the axis.
UTFAIL	Optional, translational displacement at failure in tension. If zero, failure in tension is not considered.
UCFAIL	Optional, translational displacement at failure in compression. If zero, failure in compression is not considered.
IU	Initial translational displacement along axis.





<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus, E
PR	Poisson's ratio, $\nu$
HR	Hardening rule: EQ.1.0: linear (default), EQ.2.0: exponential. EQ.3.0: load curve
P1	Material parameter: HR.EQ.1.0: Tangent modulus, HR.EQ.2.0: k, strength coefficient for exponential hardening
P2	Material parameter: HR.EQ.1.0: Yield stress HR.EQ.2.0: n, exponent
R00	R00, Lankford parameter determined from experiments
R45	R45, Lankford parameter determined from experiments
R90	R90, Lankford parameter determined from experiments
LCID	load curve ID for the load curve hardening rule
E0	$\epsilon_0$ for determining initial yield stress for exponential hardening. (Default = 0.0)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see <b>MAT_OPTION TROPIC_ELASTIC</b> for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <b>*DEFINE_COORDINATE_NODES</b>, and then rotated about the shell element normal by an angle <b>BETA</b>.</p> <p>EQ.2.0: globally orthotropic with material axes determined by the vector <b>a</b> defined below, as with <b>*DEFINE_COORDINATE_VECTOR</b>.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, <b>BETA</b>, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on <b>*DEFINE_COORDINATE_NODES</b>, <b>*DEFINE_COORDINATE_SYSTEM</b> or <b>*DEFINE_COORDINATE_VECTOR</b>). Available in R3 version of 971 and later.</p>
XP YP ZP	Coordinates of point <b>p</b> for AOPT = 1.
A1 A2 A3	Components of vector <b>a</b> for AOPT = 2.
V1 V2 V3	Components of vector <b>v</b> for AOPT = 3.
D1 D2 D3	Components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see <b>*ELEMENT_SHELL_BETA</b> .

**Remarks:**

The calculated effective stress is stored in history variable #4.

**\*MAT\_HILL\_3R\_3D**

This is Material Type 122\_3D. It combines orthotropic elastic behavior with Hill's 1948 anisotropic plasticity theory. Anisotropic plastic properties are given by 6 material parameters, *F*, *G*, *H*, *L*, *M*, *N* which are determined by experiments. This model is implemented for solid elements.

This keyword can be written either as \*MAT\_HILL\_3R\_3D, or \*MAT\_122\_3D.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EX	EY	EZ	PRXY	PRYZ	PRXZ
Type	I	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	GXY	GYZ	GXZ	F	G	H	L	M
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	N	HR	P1	P2				
Type	F	I	I/F	F				

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	I							

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification ID, must be a unique number.
RO	Mass density.
EX, EY, EZ	Young's modulus in <i>x</i> , <i>y</i> , and <i>z</i> directions, respectively. Negative values indicate (positive) curve numbers, where each curve is a function of temperature.
PRXY, PRYZ, PRXZ	Poisson's ratio, $\nu$ , in <i>xy</i> , <i>yz</i> and <i>xz</i> directions, respectively. Negative values indicate (positive) curve numbers, where each curve is a function of temperature.
GXY, GYZ, GXZ	Shear modulus in <i>xy</i> , <i>yz</i> and <i>xz</i> directions, respectively. Negative values indicate (positive) curve numbers, where each curve is a function of temperature.
F, G, H, L, M, N	Material constants in Hill's 1948 yield criterion (see Remarks). Negative values indicate (positive) curve numbers, where each curve is a function of temperature.
HR	Hardening rule: EQ.1: Stress-strain relationship is defined by load curve or 2D-table ID with parameter P1. P2 is ignored. EQ.2: Stress-strain relationship is defined by strength coefficient K (P1) and strain hardening coefficient n (P2), as in Swift's exponential hardening equation: $\sigma_{\text{yield}} = k(\epsilon + 0.01)^n.$

VARIABLE	DESCRIPTION
P1	<p>Material parameter:</p> <p>HR.EQ.1: Load curve or 2D-table ID defining stress-strain curve. If 2D-table ID, the table gives stress-strain curves for different temperatures.</p> <p>HR.EQ.2: <math>k</math>, strength coefficient in <math>\sigma_{yield} = k(\varepsilon + 0.01)^n</math>.</p>
P2	<p>Material parameter:</p> <p>HR.EQ.1: not used.</p> <p>HR.EQ.2.0: <math>n</math>, the exponent in <math>\sigma_{yield} = k(\varepsilon + 0.01)^n</math>.</p>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point <math>\mathbf{p}</math> in space and the global location of the element center; this is the a-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by the vectors <math>\mathbf{a}</math> and <math>\mathbf{d}</math>, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</p>
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.

VARIABLE	DESCRIPTION
BETA	Material angle in degrees for AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO.

**Hill's 1948 yield criterion:**

Hill's yield criterion is based on the assumptions that the material is orthotropic, that hydrostatic stress does not affect yielding, and that there is no Bauschinger effect. According to Hill, when the principal axes of anisotropy are the axes of reference, the yield surface has the form

$$f = \bar{\sigma}(\sigma) - \sigma_{\text{yield}}(\epsilon_p) = 0,$$

where the effective stress  $\bar{\sigma}$  (stored as history variable #2) is given by

$$(F + G)\bar{\sigma}^2 = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2,$$

and where  $F, G, H, L, M, N$  are material parameters of the current state of anisotropy, assuming three mutually orthogonal planes of symmetry at every point. The material  $z$ -direction is here the reference direction.

Let  $X, Y, Z$  be the tensile yield stresses in the principal directions of anisotropy, then

$$\frac{\sigma_{y0}^2}{X^2} = \frac{G + H}{F + G}, \quad \frac{\sigma_{y0}^2}{Y^2} = \frac{H + F}{F + G}, \quad \frac{\sigma_{y0}^2}{Z^2} = 1,$$

where  $\sigma_{y0} = \sigma_{\text{yield}}(0)$ .  $F, G, H$  are not uniquely determined, but the choice  $F+G = 1$  gives

$$F = \frac{Z^2}{2} \left( \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2} \right), \quad G = \frac{Z^2}{2} \left( \frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2} \right), \quad H = \frac{Z^2}{2} \left( \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} \right).$$

If  $R_{xy}, S_{zx}, T_{xy}$  are the yield stresses in shear with respect to the principal axes of

anisotropy, then

Material	<i>F</i>	<i>G</i>	<i>H</i>	<i>L</i>	<i>M</i>	<i>N</i>
AA5042	0.3341	0.5098	0.3569	1.5000	1.5000	1.8197

**Table M122-1.** AA5042 material constants (Hill's 1948 yield) - BM1 NUMISHEET 2011.

$$L = \frac{Z^2}{2R_{xy}^2}, \quad M = \frac{Z^2}{2S_{zx}^2}, \quad N = \frac{Z^2}{2T_{xy}^2}.$$

If  $F = G = H$ , and,  $L = M = N = 3F$ , the Hill criterion reduces to the Von-Mises criterion.

The strain hardening in this model can either defined by the load curve or by Swift's exponential hardening equation:  $\sigma_{\text{yield}} = k(\varepsilon + 0.01)^n$ .

### Validation and application:

1. This material model is suitable for metal forming application using solid elements to account for anisotropic plasticity. NUMISHEET conferences have provided material constants of Hill's 1948 yield for many commonly used materials. In this example, experimental results from benchmark 1 (BM1, AA5042) of the NUMISHEET 2011 are used to validate an equal-biaxial tension and two uniaxial tensile results in two different directions (rolling and 90°) on a single solid element.

As shown in [Figure M122-1](#)(top left), under the constraints imposed, a single element of solid type 2 is pulled in uniaxial tension in the  $x$ -direction. The resulting hardening curve is compared with experimental data provided (top right). Similarly, the element is pulled in uniaxial tension in the  $Y$ -direction under the constraints shown in [Figure M122-1](#)(middle left). The resulting hardening curve is compared with experimental data (middle right). In [Figure M122-1](#)(bottom left), the element is pulled in both  $x$ - and  $y$ -directions equally under the constraints shown, and the resulting hardening curve is compared with experimental data (bottom right). All computed results are satisfactory. The material constants used for the simulation are provided in [Table M122-1](#).

In addition, an element of type 1 is subjected to a shear test with a composite material, courtesy of CYBERNET SYSTEMS CO., LTD. Results compare well with the experiments, as shown in [Figure M122-2](#).

In real world application, the six material parameters required can be calibrated with nonlinear regression analysis (such as those available through LS-OPT) through a series of tensile tests in three orthogonal directions and three shear tests in three orthogonal planes.

2. This material model can also be applied in multi-scale simulation of fiberglass and laminated materials, according to CYBERNET SYSTEMS CO., LTD. The elastic coefficients can be calibrated analytically by a homogenization method with tensile tests in the three orthogonal directions and three pure shear tests in the three orthogonal planes.

**Revision information:**

The material model is available in explicit dynamics in both SMP and MPP starting in Revision 86100, and is available in implicit dynamics in both SMP and MPP starting in Revision 104178. It also supports temperature dependent Young's/shear modulus, Poisson ratios, and Hill parameters.



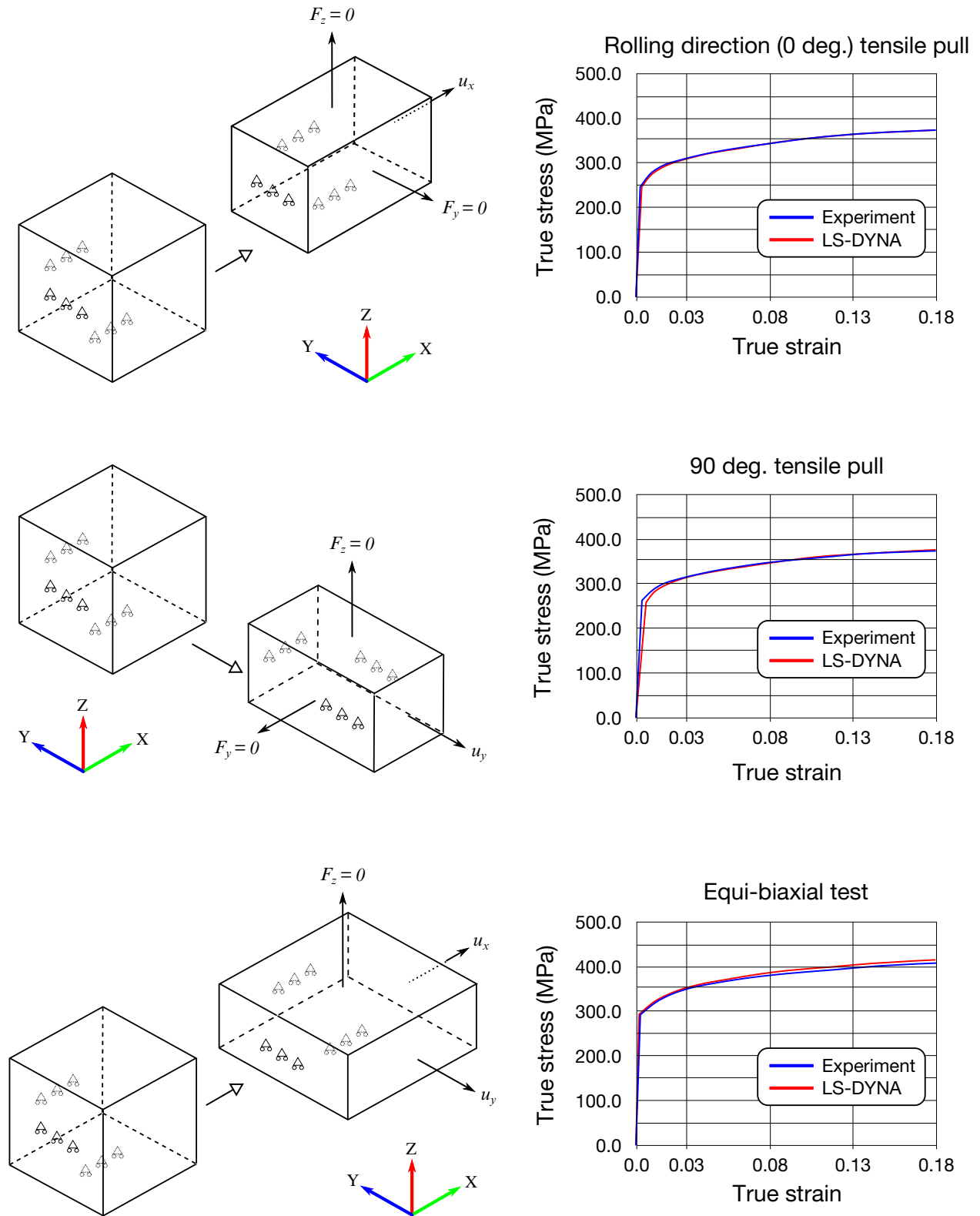
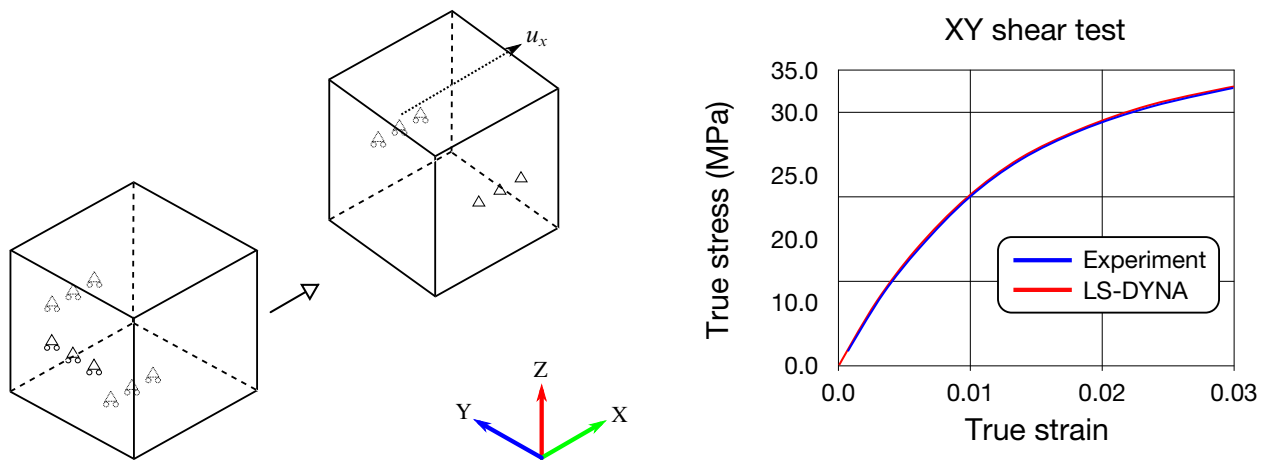


Figure M122-1. Validation with experiments - BM1 NUMISHEET 2011



**Figure M122-2.** Shear result validated with test results (Courtesy of CYBERNET SYSTEMS CO., LTD.)



<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus, E
PR	Poisson's ratio, $\nu$
HR	Hardening rule: EQ.1.0: n.a. EQ.2.0: n.a. EQ.3.0: load curve
R00	R00, Lankford parameter determined from experiments
R45	R45, Lankford parameter determined from experiments
R90	R90, Lankford parameter determined from experiments
LC00	Load curve ID for the yield curve in the 0 degree direction.
ICONV	Convexity option: EQ.0.0: convexity of the yield surface is not enforced, EQ.1.0: convexity of the yield surface is enforced.
LC90	Load curve ID for the yield curve in the 90 degree direction.
LC45	Load curve ID for the yield curve in the 45 degree direction.
LCEX	Absolute value is Load curve ID for the yield curve in shear or biaxial: LCEX.gt.0.0: shear yield is provided LCEX.lt.0.0: biaxial yield is provided

VARIABLE	DESCRIPTION
AOPT	<p>Material axes option (see <code>MAT_OPTION TROPIC_ELASTIC</code> for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <code>*DEFINE_COORDINATE_NODES</code>, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by the vector <b>a</b> defined below, as with <code>*DEFINE_COORDINATE_VECTOR</code>.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on <code>*DEFINE_COORDINATE_NODES</code>, <code>*DEFINE_COORDINATE_SYSTEM</code> or <code>*DEFINE_COORDINATE_VECTOR</code>). Available in R3 version of 971 and later.</p>
XP YP ZP	Coordinates of point <b>p</b> for AOPT = 1.
A1 A2 A3	Components of vector <b>a</b> for AOPT = 2.
V1 V2 V3	Components of vector <b>v</b> for AOPT = 3.
D1 D2 D3	Components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see <code>*ELEMENT_SHELL_BETA</code> .



Card 3	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 5 is required if and only if either the RATE or RTCL option is active.

Card 5	1	2	3	4	5	6	7	8
Variable	LCTSRF	EPS0	TRIAX					
Type	I	F	F					
Default	0	0	0					

**VARIABLE**

**DESCRIPTION**

- MID            Material identification. A unique number or label not exceeding 8 characters must be specified.
- RO            Mass density.
- E             Young's modulus.
- PR            Poisson's ratio.
- SIGY         Yield stress.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ETAN	Tangent modulus, ignored if (LCSS.GT.0) is defined.
FAIL	<p>Failure flag.</p> <p>LT.0.0: User defined failure subroutine, <code>matusr_24</code> in <code>dyn21.F</code>, is called to determine failure</p> <p>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.</p> <p>GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.</p>
TDEL	Minimum time step size for automatic element deletion.
C	Strain rate parameter, C, see formula below.
P	Strain rate parameter, P, see formula below.
LCSS	<p>Load curve ID or Table ID.</p> <p><b>Load Curve.</b> When LCSS is a Load curve ID, it is taken as defining effective stress versus effective plastic strain. If defined EPS1 - EPS8 and ES1 - ES8 are ignored.</p> <p><b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See <a href="#">Figure M24-1</a>. When the strain rate falls below the minimum value, the stress versus effective plastic strain curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value the stress versus effective plastic strain curve for the highest value of strain rate is used. The strain rate parameters: C and P, the curve ID, LCSR, EPS1 - EPS8, and ES1 - ES8 are ignored if a Table ID is defined.</p> <p><b>Logarithmically Defined Tables.</b> If the <i>first</i> stress-strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for <i>all</i> stress-strain curves. Since the tables are internally discretized to equally space the points, natural logarithms are necessary, for example, if the curves correspond to rates from <math>10^{-4}</math> to <math>10^4</math>. Computing natural logarithms can substantially increase the computational time on certain computer architectures.</p>
LCSR	Load curve ID defining strain rate scaling effect on yield stress.



VARIABLE	DESCRIPTION
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default), EQ.1.0: Viscoplastic formulation (recommended).
EPSTHIN	Thinning strain at failure. This number should be given as a positive number.
EPSMAJ	Major in plane strain at failure for shells (or) Major principal strain at failure for solids.  LT.0: EPSMAJ =  EPSMAJ  and filtering is activated. The last twelve values of the major strain is stored at each integration point and the average value is used to determine failure.
NUMINT	Number of integration points which must fail before the element is deleted. (If zero, all points must fail.) For fully integrated shell formulations, each of the $4 \times \text{NIP}$ integration points are counted individually in determining a total for failed integration points. NIP is the number of through-thickness integration points. As NUMINT approaches the total number of integration points (NIP for under integrated shells, $4 \times \text{NIP}$ for fully integrated shells), the chance of instability increases.  LT.0.0:  NUMINT  is percentage of integration points/layers which must fail before shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells.
EPS1 - EPS8	Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. WARNING: If the first point is nonzero the yield stress is extrapolated to determine the initial yield. If this option is used SIGY and ETAN are ignored and may be input as zero.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8.
LCTSRF	Load curve that defines the thinning strain at failure as a function of the plastic strain rate.

VARIABLE	DESCRIPTION
EPS0	EPS0 parameter for RTCL damage. EQ.0.0: (default) RTCL damage is inactive. GT.0.0: RTCL damage is active
TRIAx	RTCL damage triaxiality limit. EQ.0.0: (default) No limit. GT.0.0: Damage does not accumulate when triaxiality exceeds TRIAX.

**Remarks:**

Optional RTCL damage is used to fail elements when the damage function exceeds 1.0. During each solution cycle, if the plastic strain increment is greater than zero, an increment of RTCL damage is calculated by

$$\Delta f_{\text{damage}} = \frac{1}{\varepsilon_0} f\left(\frac{\sigma_H}{\bar{\sigma}}\right)_{\text{RTCL}} d\bar{\varepsilon}^p$$

where

$$f\left(\frac{\sigma_H}{\bar{\sigma}}\right)_{\text{RTCL}} = \begin{cases} 0 & \frac{\sigma_H}{\bar{\sigma}} \leq -\frac{1}{3} \\ 2 \frac{1 + \frac{\sigma_H}{\bar{\sigma}} \sqrt{12 - 27\left(\frac{\sigma_H}{\bar{\sigma}}\right)^2}}{3 \frac{\sigma_H}{\bar{\sigma}} + \sqrt{12 - 27\left(\frac{\sigma_H}{\bar{\sigma}}\right)^2}} & \frac{1}{3} < \frac{\sigma_H}{\bar{\sigma}} < \frac{1}{3} \\ \frac{1}{1.65} \exp\left(\frac{3\sigma_H}{2\bar{\sigma}}\right) & \frac{\sigma_H}{\bar{\sigma}} \geq \frac{1}{3} \end{cases}$$

and,

$\varepsilon_0$  = uniaxial fracture strain / critical damage value

$\sigma_H$  = hydrostatic stress

$\bar{\sigma}$  = effective stress

$d\bar{\varepsilon}^p$  = effective plastic strain increment

The increments are summed through time and the element is deleted when  $f_{\text{damage}} \geq 1.0$ . For  $0.0 < f_{\text{damage}} < 1.0$ , the element strength will not be degraded.

The value of  $f_{\text{damage}}$  is stored as history variable #9 and can be fringe plotted from d3plot files if the number of extra history variables requested is  $\geq 9$  on \*DATABASE\_EXTENT\_BINARY.

The optional TRIAX parameter can be used to prevent excessive RTCL damage growth and element erosion for badly shaped elements that might show unrealistically high values for the triaxiality. The triaxiality,  $\frac{\sigma_H}{\sigma}$ , is stored as history variable #11.

The EPSMAJ parameter is compared to the major principal strain in the following senses:

- For shells it is the maximum eigenvalue of the in-plane strain tensor that is incremented by the strain increments.
- For solid elements it is calculated as the maximum eigenvalue to the logarithmic strain tensor

$$\epsilon = \frac{1}{2} \ln(\mathbf{F}^T \mathbf{F}),$$

where  $\mathbf{F}$  is the global deformation gradient. In sum, both element types use a natural strain measure for determining failure, the major strain calculated in this way is output as history variable #7.

To get an idea about the probability of failure, an indicator  $D$  is computed internally:

$$D = \max \left( \frac{\bar{\epsilon}^p}{\text{FAIL}}, \frac{-\epsilon_3}{\text{EPSTHIN}}, \frac{\epsilon_I}{\text{EPSMAJ}} \right)$$

and stored as history variable #10.  $D$  ranges from 0 (intact) to 1 (failed).  $\bar{\epsilon}^p$ ,  $-\epsilon_3$ , and  $\epsilon_I$  are current values of effective plastic strain, thinning strain, and major in plane strain. This instability measure, including the RTCL damage, can also be retrieved from requesting material histories

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>				
<b>Label</b>	<b>Attributes</b>			<b>Description</b>
Instability	-	-	-	Failure indicator $\max(D, f_{\text{damage}})$
Plastic Strain Rate	-	-	-	Effective plastic strain rate $\dot{\epsilon}_{\text{eff}}^p$

For implicit calculations on this material involving severe nonlinear hardening the radial return method may result in inaccurate stress-strain response. Setting IACC = 1 on \*CONTROL\_ACCURACY activates a fully iterative plasticity algorithm, which will remedy this. This is not to be confused with the MITER flag on \*CONTROL\_SHELL, which governs the treatment of the plane stress assumption for shell elements. If any failure model is applied with this option, incident failure will initiate damage, and the stress will continuously degrade to zero before erosion for a deformation of 1% plastic strain. So for instance, if the failure strain is FAIL = 0.05, then the element is eroded when  $\bar{\epsilon}^p = 0.06$  and the material goes from intact to completely damaged between  $\bar{\epsilon}^p = 0.05$  and  $\bar{\epsilon}^p = 0.06$ . The reason is to enhance implicit performance by maintaining continuity in the internal forces.

# \*MAT\_124

# \*MAT\_PLASTICITY\_COMPRESSION\_TENSION

## \*MAT\_PLASTICITY\_COMPRESSION\_TENSION

This is Material Type 124. An isotropic elastic-plastic material where unique yield stress versus plastic strain curves can be defined for compression and tension. Also, failure can occur based on a plastic strain or a minimum time step size. Rate effects on the yield stress are modeled either by using the Cowper-Symonds strain rate model or by using two load curves that scale the yield stress values in compression and tension, respectively. Material rate effects, which are independent of the plasticity model, are based on a 6-term Prony series Maxwell mode that generates an additional stress tensor. The viscous stress tensor is superimposed on the stress tensor generated by the plasticity.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	C	P	FAIL	TDEL
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	0	0	10.E+20	0

Card 2	1	2	3	4	5	6	7	8
Variable	LCIDC	LCIDT	LCSRC	LCSRT	SRFLAG	LCFAIL	EC	RPCT
Type	I	I	I	I	F	I	F	F
Default	0	0	0	0	0	0	none	0

Card 3	1	2	3	4	5	6	7	8
Variable	PC	PT	PCUTC	PCUTT	PCUTF			
Type	F	F	F	F	F			
Default	0	0	0	0	0			

Card 4	1	2	3	4	5	6	7	8
Variable	K							
Type	F							

**Viscoelastic Constant Cards.** Up to 6 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 6 cards are used.

Card 5	1	2	3	4	5	6	7	8
Variable	Gi	BETAi						
Type	F	F						

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
C	Strain rate parameter, <i>C</i> , see formula below.
P	Strain rate parameter, <i>P</i> , see formula below.
FAIL	Failure flag. LT.0.0: User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic deletion of shell elements.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDC	Load curve ID defining effective stress versus effective plastic strain in compression. Enter positive yield stress and plastic strain values when defining this curve.
LCIDT	Load curve ID defining effective stress versus effective plastic strain in tension. Enter positive yield stress and plastic strain values when defining this curve.
LCSRC	Optional load curve ID defining strain rate scaling factor on yield stress versus strain rate when the material is in compression.
LCSRT	Optional load curve ID defining strain rate scaling factor on yield stress versus strain rate when the material is in tension.
SRFLAG	Formulation for rate effects: EQ.0.0: Total strain rate (based on the total strain tensor), EQ.1.0: Effective strain rate (based on deviatoric portion of the strain tensor). EQ.2.0: Effective plastic strain rate (viscoplastic).
LCFAIL	Optional load curve ID defining effective plastic strain at failure versus strain rate. If LCFAIL is specified, FAIL is ignored. See <a href="#">Remarks</a> for additional information.
EC	Optional Young's modulus for compression, $> 0$ .
RPCT	Fraction of PT and PC, used to define mean stress at which Young's modulus is E and EC, respectively. Young's modulus is E when mean stress $> RPCT \times PT$ , and EC when mean stress $< -RPCT \times PC$ . If the mean stress falls between $-RPCT \times PC$ and $RPCT \times PT$ , a linearly interpolated value is used.
PC	Compressive mean stress (pressure) at which the yield stress follows load curve ID, LCIDC. If the pressure falls between PC and PT a weighted average of the two load curves is used. Both PC and PT should be entered as positive values.
PT	Tensile mean stress at which the yield stress follows load curve ID, LCIDT.

VARIABLE	DESCRIPTION
PCUTC	Pressure cut-off in compression (PCUTC must be greater than or equal to zero). PCUTC (and PCUTT) apply only to element types that use a 3D stress update, e.g., solids, tshell formulations 3 and 5, and SPH. When the pressure cut-off is reached the deviatoric stress tensor is set to zero and the pressure remains at its compressive value. Like the yield stress, PCUTC is scaled to account for rate effects.
PCUTT	Pressure cut-off in tension (PCUTT must be less than or equal to zero). When the pressure cut-off is reached the deviatoric stress tensor and tensile pressure is set to zero. Like the yield stress, PCUTT is scaled to account for rate effects.
PCUTF	Pressure cut-off flag activation. EQ.0.0: Inactive, EQ.1.0: Active.
K	Optional bulk modulus for the viscoelastic material. If nonzero a Kelvin type behavior will be obtained. Generally, $K$ is set to zero.
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETA $i$	Optional shear decay constant for the $i^{\text{th}}$ term

**Remarks:**

The stress strain behavior follows a different curve in compression than it does in tension. Tension is determined by the sign of the mean stress where a positive mean stress (i.e., a negative pressure) is indicative of tension. Two curves must be defined giving the yield stress versus effective plastic strain for both the tension and compression regimes.

Mean stress is an invariant which can be expressed as  $(\sigma_x + \sigma_y + \sigma_z)/3$ . PC and PT define a range of mean stress values within which interpolation is done between the tensile yield surface and compressive yield surface. PC and PT are not true material properties but are just a numerical convenience so that the transition from one yield surface to the other is not abrupt as the sign of the mean stress changes. Both PC and PT are input as positive values as it is implied that PC is a compressive mean stress value and PT is tensile mean stress value.

Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor:

$$1 + \left[ \frac{\dot{\epsilon}}{C} \right]^{1/p}$$

Where, if SRFLAG = 0,  $\dot{\epsilon}$  is the total strain rate,

$$\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}.$$

The LCFAIL field is only applicable when at least one of the following four conditions are met:

1. SRFLAG = 2
2. LCSRC is nonzero
3. LCSRT is nonzero
4.  $G_i$ ,  $BETA_i$  values are provided.





**\*MAT\_125****\*MAT\_KINEMATIC\_HARDENING\_TRANSVERSELY\_ANISOTROPIC**

Card 2	1	2	3	4	5	6	7	8
Variable	CB	Y	SC1	K	RSAT	SB	H	SC2
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	0.0

Card 3	1	2	3	4	5	6	7	8
Variable	EA	COE	IOPT	C1	C2	IFLD		
Type	F	F	I	F	F	I		
Default	none	none	0	none	none	none		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's Modulus
PR	Poisson's ratio
R	Anisotropic hardening parameter
HLCID	Load curve ID in keyword *DEFINE_CURVE, where true strain and true stress relationship is characterized. This curve is used in conjunction with variable OPT, and not to be referenced or used in other keywords. <i>The use of this parameter is not recommended.</i>

VARIABLE	DESCRIPTION
OPT	Error calculation flag. When OPT = 2, LS-DYNA will perform error calculation based on the true stress-strain curve from uniaxial tension, specified by HLCID. The corrections will be made to the cyclic load curve, both in the loading and unloading portions. Since, in some cases where loading is more complex, the accumulated plastic strain could be large (say more than 30%), the input uniaxial stress-strain curve must have enough strain range to cover the maximum expected plastic strain. Note this variable must be set to a value of "2" if HLCID is specified and stress-strain curve is used. <i>The default value of "0" is recommended.</i>
CB	The uppercase $B$ defined in Yoshida & Uemori's equations.
Y	Hardening parameter appearing in Yoshida & Uemori's equations.
SC1	The lowercase $c$ defined in the following equations, and $C1$ as in the <a href="#">remarks</a> section.
K	Hardening parameter appearing in Yoshida & Uemori's equations.
RSAT	Hardening parameter appearing in Yoshida & Uemori's equations.
SB	The lowercase $b$ appearing in Yoshida & Uemori's equations.
H	Anisotropic parameter associated with work-hardening stagnation.
SC2	The lowercase $c$ defined in the following equations, and $C2$ as in the <a href="#">remarks</a> section. If SC2 = 0.0, left blank, or SC2 = SC1, then it turns into the basic model.
EA	Variable controlling the change of Young's modulus, $E^A$ in the following equations.
COE	Variable controlling the change of Young's modulus, $\zeta$ in the following equations.
IOPT	Modified kinematic hardening rule flag: EQ.0: Original Yoshida & Uemori formulation, EQ.1: Modified formulation. Define C1, C2 below.
C1, C2	Constants used to modify $R$ :

$$R = \text{RSAT} \times [(C_1 + \bar{\epsilon}^p)^{c_2} - C_1^{c_2}]$$

VARIABLE	DESCRIPTION
IFLD	ID of a load curve defining Forming Limit Diagram (FLD) under linear strain paths. In the load curve, abscissas represent minor strains while ordinates represent major strains. Define only when the option NLP is used. See the example in the <a href="#">remarks</a> section.

**The Yoshida & Uemori’s kinematic hardening model:**

According to F. Yoshida and T. Uemori’s paper titled “A model of large-strain cyclic plasticity describing the Bauschinger effect and work hardening stagnation” in 2002 International Journal of Plasticity 18, 661-686, and referring to [Figure M125-1](#),

$$\alpha_* = \alpha - \beta$$

$$\alpha_* = c \left[ \left( \frac{a}{Y} \right) (\sigma - \alpha) - \sqrt{\frac{a}{\alpha_*}} \alpha_* \right] \bar{\epsilon}^p$$

$$a = B + R - Y$$

The change of size and location for the bounding surface is defined as, referring to [Figure M125-2](#),

$$\dot{R} = k(R_{\text{sat}} - R)\dot{\bar{\epsilon}}^p,$$

$$\dot{\beta}' = k\left(\frac{2}{3}bD - \beta'\right)\dot{\bar{\epsilon}}^p$$

$$\sigma_{\text{bound}} = B + R + \beta$$

In Yoshida & Uemori’s model, there is work-hardening stagnation in the unloading process, and it is described as,

$$g_\sigma(\sigma', q', r') = \frac{3}{2}(\sigma' - q'):(\sigma' - q') - r^2$$

$$\dot{q}' = \mu(\beta' - q')$$

$$r = h\Gamma$$

$$\Gamma = \frac{3(\beta' - q'):\dot{\beta}'}{2r}$$

The change in Young’s modulus is defined as a function of effective strain,

$$E = E_0 - (E_0 - E_A)[1 - \exp(-\zeta\bar{\epsilon}^p)]$$

**Strain hardening saturation:**

Further improvements in the original Yoshida & Uemori’s model, as described in a paper “Determination of Nonlinear Isotropic/Kinematic Hardening Constitutive Parameter for AHSS

using Tension and Compression Tests”, by Ming F. Shi, Xinhai Zhu, Cedric Xia, Thomas Stoughton, in NUMISHEET 2008 proceedings, 137-142, 2008, included modifications to allow working hardening in large strain deformation region, avoiding the problem of earlier saturation, especially for Advanced High Strength Steel (AHSS). These types of steels exhibit continuous strain hardening behavior and a non-saturated isotropic hardening function. As described in the paper, the evolution equation for R (a part of the current radius of the bounding surface in deviatoric stress space), as is with the saturation type of isotropic hardening rule proposed in the original Yoshida & Uemori model,

$$\dot{R} = m(R_{sat} - R)\dot{\rho}$$

is modified as,

$$R = RSAT \times [(C_1 + \bar{\epsilon}^p)^{c_2} - C_1^{c_2}]$$

For saturation type of isotropic hardening rule, set IOPT = 0, applicable to most of Aluminum sheet materials. In addition, the paper provides detailed variables used for this material model for DDQ, HSLA, DP600, DP780 and DP980 materials. Since the symbols used in the paper are different from what are used here, the following table provides a reference between symbols used in the paper and variables here in this keyword:

B	Y	C	m	K	b	h	e <sup>0</sup>	N
CB	Y	SC1	K	Rsat	SB	H	C1	C2

Using the modified formulation and the material properties provided by the paper, the predicted and tested results compared very well both in a full cycle tension and compression test and in a pre-strained tension and compression test, according to the paper.

**A Failure Criterion for Nonlinear Strain Paths (NLP):**

The NLP failure criterion and corresponding post processing procedures are described in the entries for \*MAT\_036 and \*MAT\_037. The history variables for every element stored in d3plot files include:

1. Formability Index (F.I.): #1 (#24 after Revision 113708)
2. Strain ratio β (in-plane minor strain/major strain): #2 (#25 after Revision 113708)
3. Effective strain from the planar isotropic assumption: #3 (#26 after Revision 113708)

The entire time history can be plotted using Post/History menu in LS-PrePost v4.0. To enable the output of these history variables to the d3plot files, NEIPS on the \*DATABASE\_-EXTENT\_BINARY card must be set to at least 3. When plotting the formability index, first select the history var #1 from the Misc in the FriComp menu. The pull-down menu under

*FriComp* can be used to select minimum value 'Min' for necking failure determination (refer to *Tharrett and Stoughton's paper in 2003 SAE 2003-01-1157*). In *FriRang*, the option *None* is to be selected in the pull-down menu next to *Avg*. Lastly, set the simulation result to the last state in the *animation tool bar*. The index value ranges from 0.0 to 1.5. The non-linear forming limit is reached when the index reaches 1.0.

### Smoothing of the Strain Ratio $\beta$ :

Details related to the smoothing of  $\beta$  can be found in manual pages under *\*CONTROL\_FORMING\_TOLERANC*. This is available starting in Revision 113708.

### An Example of the NLP Option:

A partial keyword example is listed below when the option NLP is used. The traditional Forming Limit Diagram (FLD) which governs the linear strain paths only is defined by load curve ID 321.

```
*MAT_KINEMATIC_HARDENING_TRANSVERSELY_ANISOTROPIC_NLP
$#      mid      ro      e      pr      r      hclid      opt
        1      7.83E-9    2.07E+5    0.3      1.035
$#      cb      y      sc      k      rsat      sb      h
        422.8    304.2    398.5    28.0    702.7    136.9    0.91
$#      ea      coe      iopt     c1      c2      IFLD
        0.0      0.0      1      0.0065    0.545    321
*DEFINE_CURVE
$ traditional FLD data (major vs. minor)
321
-0.357,0.596
  :
-0.020,0.260
 0.000,0.239
 0.010,0.244
 0.020,0.249
  :
 0.239,0.354
 0.247,0.356
 0.262,0.361
 0.372,0.361
*END
```

### Application results:

Application of the modified Yoshida & Uemori's hardening rule in the metal forming industry has shown significant advantage in springback prediction accuracy, especially for AHSS type of sheet materials. As shown in [Figure M125-3](#) (left), predicted springback shape of an automotive shotgun (also called: upper load path/beam) using *\*MAT\_125* is compared with experimental measurements on a DP780 material. Prediction accuracy achieved over 92% with *\*MAT\_125* while about 61% correlation is found with *\*MAT\_037* ([Figure M125-3](#) right), a remarkable improvement.

In another example, NUMISHEET 2011 BM4 is used to demonstrate the application of the Young's modulus variations as a function of effective strain in prediction of springback. The sheet blank is a DP780 material with an initial thickness of 1.4mm. The simulation process is shown (Figure M125-4) as pre-straining (to 8%), springback, trimming, forming and springback. Young's modulus variations with effective strains are accounted for by curve fitting the provided experimental data to obtain the variables EA and COE, Figure M125-5. Referring to Figures M125-6 and M125-7, final springback shapes of the cross sectional view are compared with measurement provided, along with benchmark results from software X and Y. In addition, springback with no pre-straining is also conducted and correlated, shown in the same figures. Furthermore, hysteretic plasticity with a full cycle tension and compression simulation is done on one single shell element and the result is superimposed with experimental data, in Figure M125-8.

To improve convergence and for a faster simulation time, it is recommended that \*CONTROL\_IMPLICIT\_FORMING type '1' be used when conducting a springback simulation.

### About SC1 and SC2:

In F. Yoshida and T. Uemori's 2002 paper, the effect of variables SC1 and SC2 were discussed. According to the paper, variables SC1 and SC2 are used to describe the forward and reverse deformations of the cyclic plasticity curve, respectively. It allows for a more rapid change of work hardening rate in the vicinity of the initial yielding (~0.5% equivalent plastic strain), in the form of the following equations:

$$SC = \begin{cases} SC1 & \max(\bar{\alpha}_*) < B - Y \\ SC2 & \text{otherwise} \end{cases}$$

where  $\max(\bar{\alpha}_*)$  is the maximum value of  $\bar{\alpha}_*$ , and,

$$\bar{\alpha}_* = \sqrt{\frac{3}{2} \alpha_* : \alpha_*}$$

As shown in Figure M125-9 from Yoshida & Uemori's original paper, the effect of a curve fitting is shown for a high strength steel (SPFC) using both SC1 and SC2, which fits much better than the fitting using only SC1. In addition, in Figure M125-10, a much better fitting is demonstrated with SC1 and SC2 than with SC1 only for a DP980 material.

### Inclusion of shell normal stress:

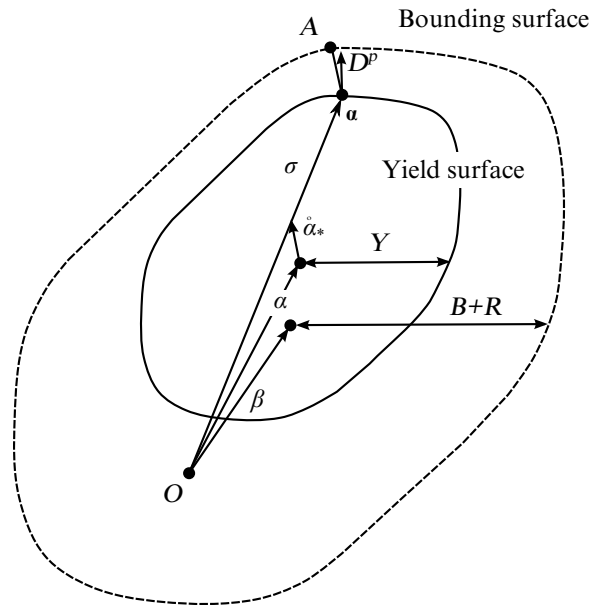
When \*LOAD\_SURFACE\_STRESS is used in the input deck together with \*MAT\_125, normal stresses (either from sliding contact or applied pressure) are accounted for during the simulation. The negative local z-stresses (select *z-stress* under *FCOMP* → *Stress* and select *local* under *FCOMP* in *LS-PrePost*) caused by the sliding contact or applied pressure

can be viewed from d3plot files after Revision 97158. It is found in some cases this inclusion can improve forming simulation accuracy.

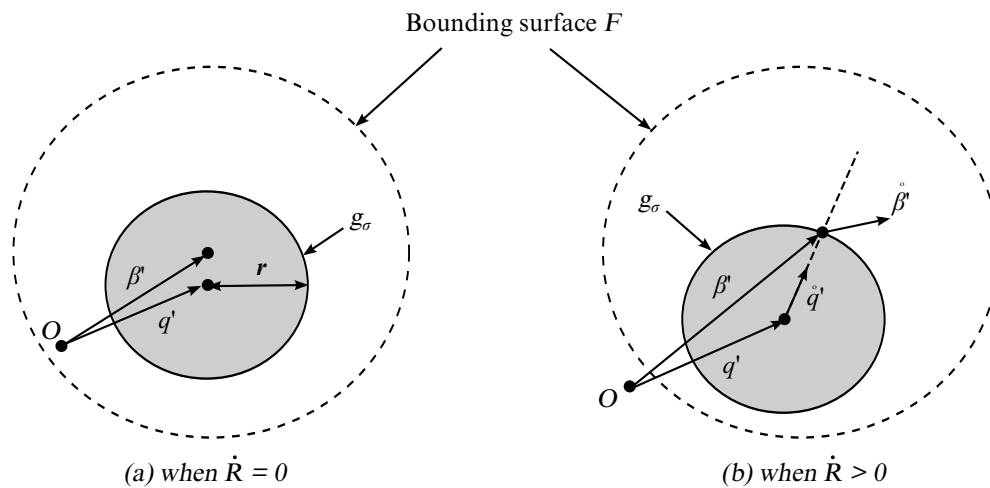
**Revision information:**

- 1) Revision 46217: the variables HLCID, OPT, IOPT, C1, and C2 are available.
- 2) Revision 74884: the variables SC1 and SC2 are available.
- 3) Revision 95594: the option NLP is available in explicit dynamic analysis, in both SMP and MPP.
- 4) Revision 97158: normal stresses inclusion is available. Later Revisions include various improvements and should be used.
- 5) Revision 113708: smoothing of  $\beta$  is available in both SMP and MPP; history variables change from 1, 2 and 3 to 24, 25 and 26, respectively.

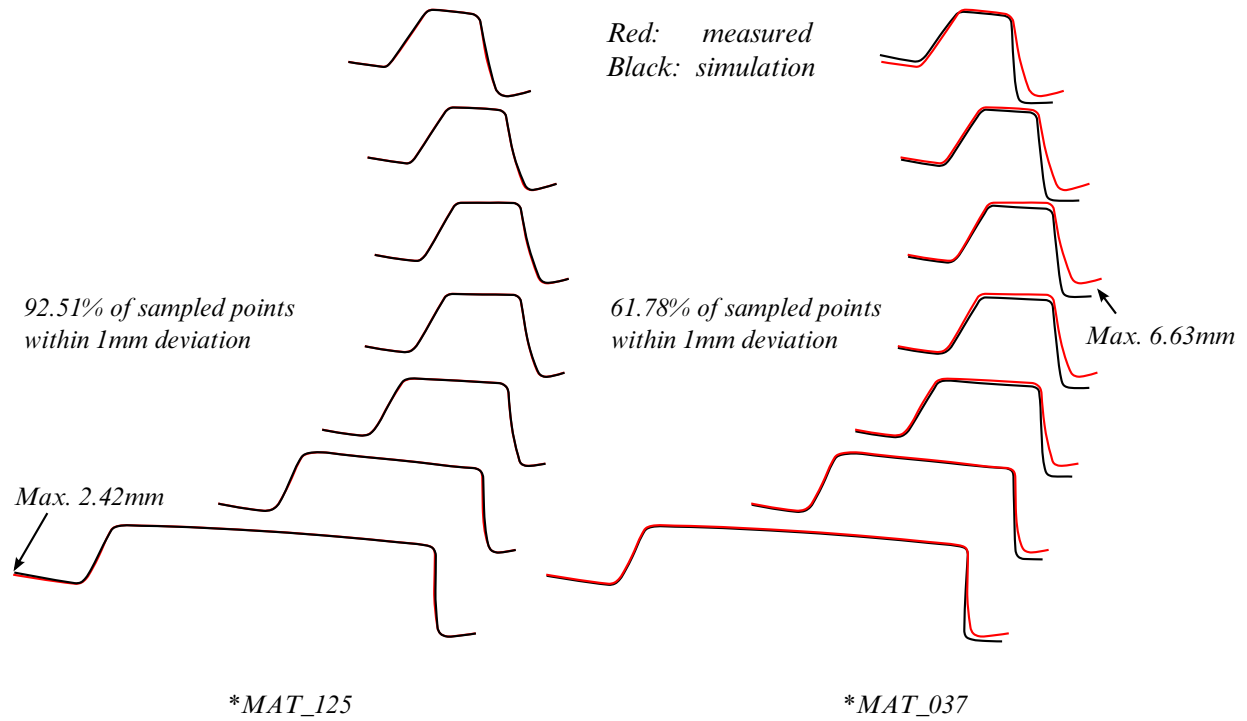




**Figure M125-1.** Schematic illustration of the two-surface model is the original center of the yield surface,  $\alpha_*$  is the current center for the yield surface;  $\alpha$  is the center of the bounding surface.  $\beta$  represents the relative position of the centers of the two surfaces.  $Y$  is the size of the yield surface and is constant throughout the deformation process.  $B+R$  represents the size of the bounding surface, with  $R$  being associated with isotropic hardening. *Reproduced from the original Yoshida and Uemori's paper.*



**Figure M125-2.** Change in bounding surface (*reproduced from the original Yoshida and Uemori's paper*).



**Figure M125-3.** Comparison of springback prediction on the A/S P load beam (reproduced from an original color contour map courtesy of Chrysler LLC and United States Steel Corporation).

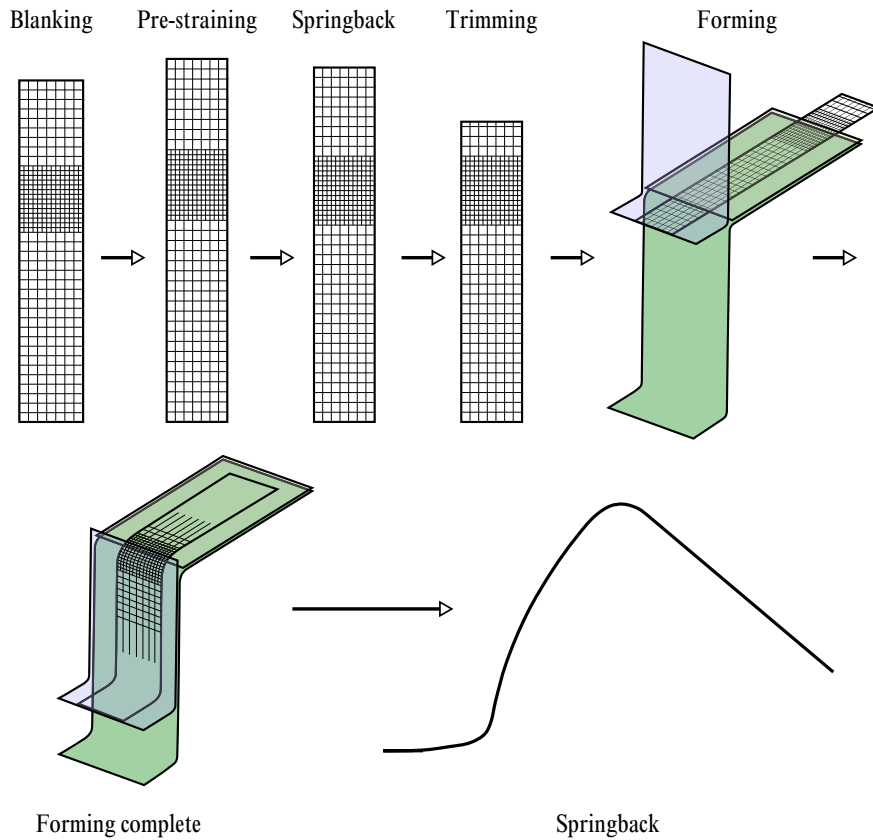


Figure M125-4. NUMISHEET 2011 Benchmark #4 simulation procedure.

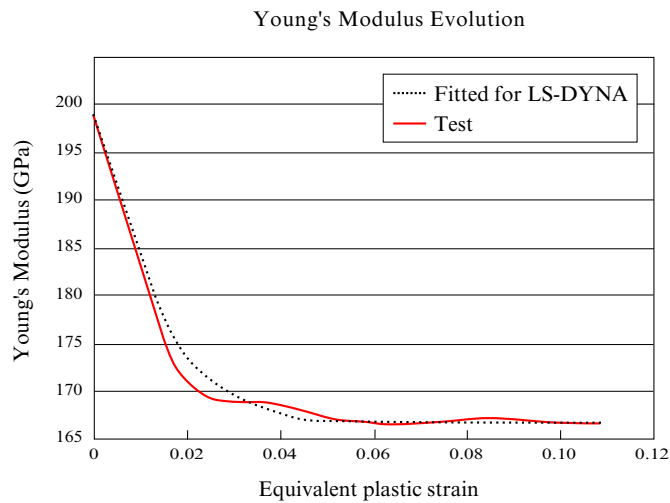
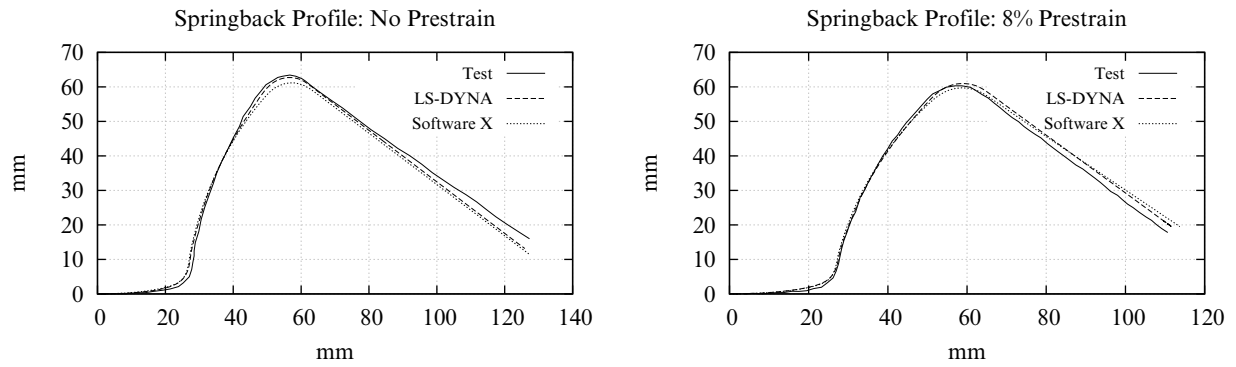
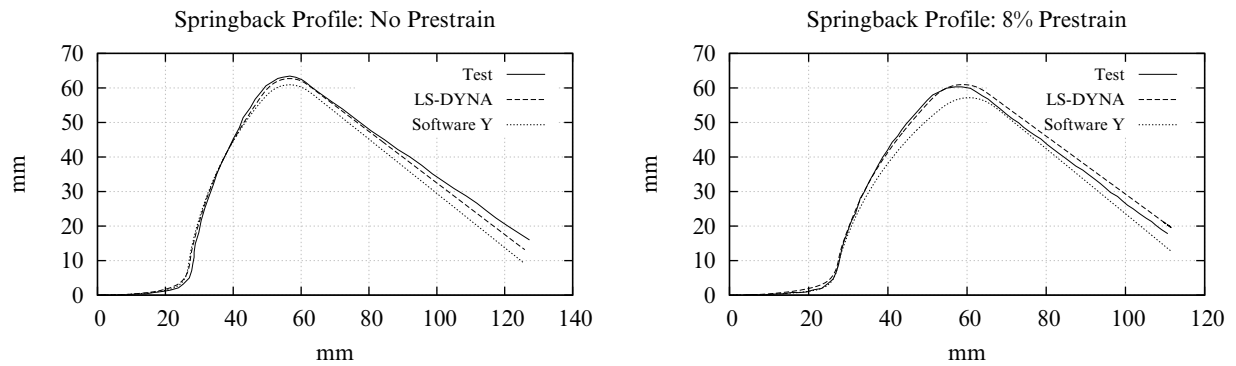


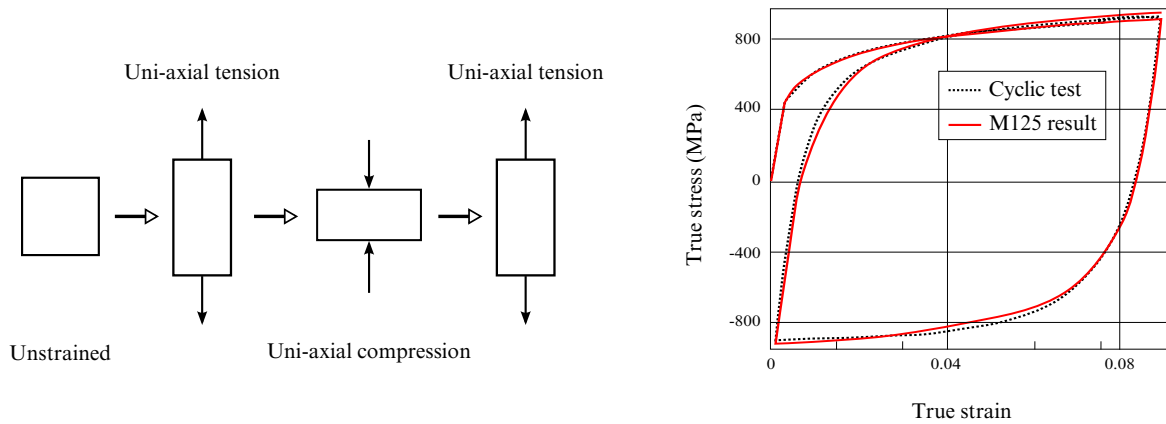
Figure M125-5. Curve fitting with coefficients: EA = 1.668E+05; COE = 95.0.



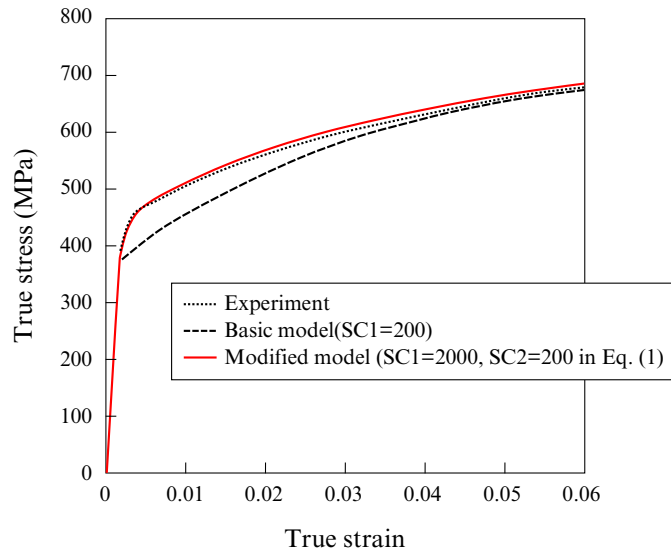
**Figure M125-6.** Comparison of springback profile with software X: 0% (left) and 8% prestrain (right)



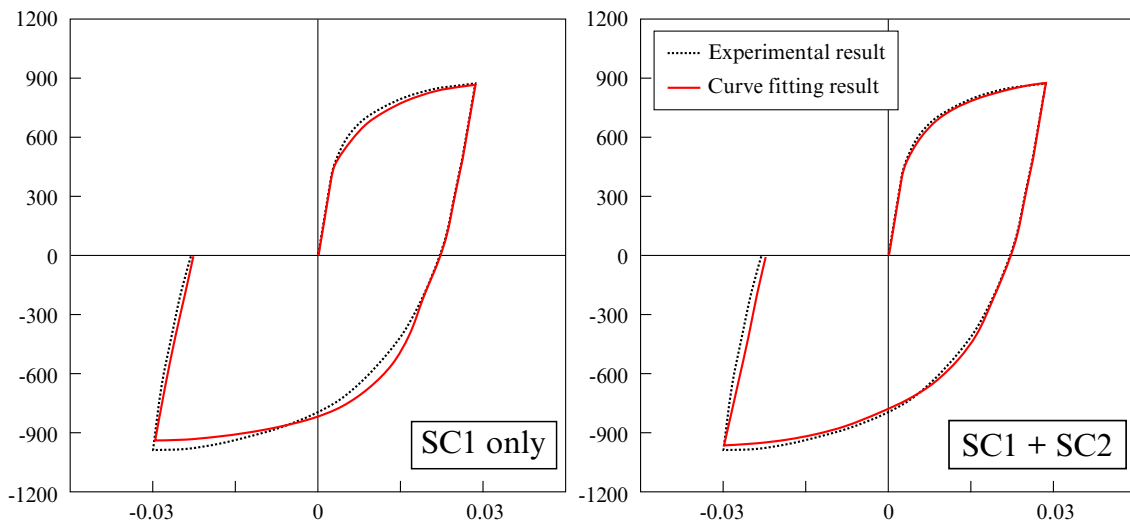
**Figure M125-7.** Comparison of springback profile with software Y: 0% (left) and 8% prestrain(right)



**Figure M125-8.** Cyclic plasticity verification on one element.



**Figure M125-9.** Effect of SC1 and SC2 (reproduced from the original Yoshida & Uemori's paper).



**Figure M125-10.** Material curve fitting comparison (reproduced from an original color slide courtesy of CYBERNET SYSTEMS CO., LTD.).

**\*MAT\_MODIFIED\_HONEYCOMB**

This is Material Type 126. The major use of this material model is for aluminum honeycomb crushable foam materials with anisotropic behavior. Three yield surfaces are available. In the first, nonlinear elastoplastic material behavior can be defined separately for all normal and shear stresses, which are considered to be fully uncoupled. In the second, a yield surface is defined that considers the effects of off-axis loading. The second yield surface is transversely isotropic. A drawback of this second yield surface is that the material can collapse in a shear mode due to low shear resistance. There was no obvious way of increasing the shear resistance without changing the behavior in purely uniaxial compression. Therefore, in the third option, the model has been modified so that the user can prescribe the shear and hydrostatic resistance in the material without affecting the uniaxial behavior. The choice of the second yield surface is flagged by the sign of the first load curve ID, LCA. The third yield surface is flagged by the sign of ECCU, which becomes the initial stress yield limit in simple shear. A description is given below.

The development of the second and third yield surfaces are based on experimental test results of aluminum honeycomb specimens at Toyota Motor Corporation.

The default element for this material is solid type 0, a nonlinear spring type brick element. The recommended hourglass control is the type 2 viscous formulation for one point integrated solid elements. The stiffness form of the hourglass control when used with this constitutive model can lead to nonphysical results since strain localization in the shear modes can be inhibited.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	VF	MU	BULK
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	.05	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	LCA	LCB	LCC	LCS	LCAB	LCBC	LCCA	LCSR
Type	F	F	F	F	F	F	F	F
Default	none	LCA	LCA	LCA	LCS	LCS	LCS	optional

Card 3	1	2	3	4	5	6	7	8
Variable	EAAU	EBBU	ECCU	GABU	GBCU	GCAU	AOPT	MACF
Type	F	F	F	F	F	F		I

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	D1	D2	D3	TSEF	SSEF	VREF	TREF	SHDFLG
Type	F	F	F	F	F	F	F	F

Additional card for AOPT = 3 or AOPT = 4.

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3					
Type	F	F	F					

Additional card for LCSR = -1.0

Card 7	1	2	3	4	5	6	7	8
Variable	LCSRA	LCSRB	LCSRC	LCSRAB	LCSRBC	LCSCA		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus for compacted honeycomb material.
PR	Poisson's ratio for compacted honeycomb material.
SIGY	Yield stress for fully compacted honeycomb.
VF	Relative volume at which the honeycomb is fully compacted. This parameter is ignored for corotational solid elements, types 0 and 9.
MU	$\mu$ , material viscosity coefficient. (default=.05) Recommended.
BULK	Bulk viscosity flag: EQ.0.0: bulk viscosity is not used. This is recommended. EQ.1.0: bulk viscosity is active and $\mu = 0$ . This will give results identical to previous versions of LS-DYNA.
LCA	Load curve ID, see *DEFINE_CURVE: LCA.LT.0: Yield stress as a function of the angle off the material axis in degrees. LCA.GT.0: sigma-aa versus normal strain component aa. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid element formulations a logarithmic strain is expected. See Remarks.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCB	<p>Load curve ID, see *DEFINE_CURVE:</p> <p>LCA.LT.0: strong axis hardening stress as a function of the volumetric strain.</p> <p>LCA.GT.0: sigma-bb versus normal strain component bb. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid element formulations a logarithmic strain is expected. Default LCB = LCA. See Remarks.</p>
LCC	<p>Load curve ID, see *DEFINE_CURVE:</p> <p>LCA.LT.0: weak axis hardening stress as a function of the volumetric strain.</p> <p>LCA.GT.0: sigma-cc versus normal strain component cc. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid element formulations a logarithmic strain is expected. Default LCC = LCA. See Remarks.</p>
LCS	<p>Load curve ID, see *DEFINE_CURVE:</p> <p>LCA.LT.0: damage curve giving shear stress multiplier as a function of the shear strain component. This curve definition is optional and may be used if damage is desired. IF SHDFLG = 0 (the default), the damage value multiplies the stress every time step and the stress is updated incrementally. The damage curve should be set to unity until failure begins. After failure the value should drop to 0.999 or 0.99 or any number between zero and one depending on how many steps are needed to zero the stress. Alternatively, if SHDFLG = 1, the damage value is treated as a factor that scales the shear stress compared to the undamaged value.</p> <p>LCA.GT.0: shear stress versus shear strain. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid element formulations a shear strain based on the deformed configuration is used. Default LCS = LCA. Each component of shear stress may have its own load curve. See Remarks.</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCAB	<p>Load curve ID, see *DEFINE_CURVE. Default LCAB = LCS:</p> <p>LCA.LT.0: damage curve giving shear ab-stress multiplier as a function of the ab-shear strain component. This curve definition is optional and may be used if damage is desired. See LCS above.</p> <p>LCA.GT.0: sigma-ab versus shear strain-ab. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid element formulations a shear strain based on the deformed configuration is used. See Remarks.</p>
LCBC	<p>Load curve ID, see *DEFINE_CURVE. Default LCBC = LCS:</p> <p>LCA.LT.0: damage curve giving bc-shear stress multiplier as a function of the ab-shear strain component. This curve definition is optional and may be used if damage is desired. See LCS above.</p> <p>LCA.GT.0: sigma-bc versus shear strain-bc. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid element formulations a shear strain based on the deformed configuration is used. See Remarks.</p>
LCCA	<p>Load curve ID, see *DEFINE_CURVE. Default LCCA = LCS:</p> <p>LCA.LT.0: damage curve giving ca-shear stress multiplier as a function of the ca-shear strain component. This curve definition is optional and may be used if damage is desired. See LCS above.</p> <p>LCA.GT.0: sigma-ca versus shear strain-ca. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid element formulations a shear strain based on the deformed configuration is used. See Remarks.</p>
LCSR	<p>Load curve ID, see *DEFINE_CURVE, for strain-rate effects defining the scale factor versus effective strain rate <math>\dot{\epsilon} = \sqrt{\frac{2}{3}(\dot{\epsilon}'_{ij}\dot{\epsilon}'_{ij})}</math>. This is optional. The curves defined above are scaled using this curve.</p>
EAAU	Elastic modulus $E_{\text{aaU}}$ in uncompressed configuration.
EBBU	Elastic modulus $E_{\text{bbU}}$ in uncompressed configuration.

VARIABLE	DESCRIPTION
ECCU	<p>Elastic modulus <math>E_{ccu}</math> in uncompressed configuration.</p> <p>LT.0.0: <math>\sigma_d^Y</math>,  ECCU  initial stress limit (yield) in simple shear. Also, LCA &lt; 0 to activate the transversely isotropic yield surface.</p>
GABU	<p>Shear modulus <math>G_{abu}</math> in uncompressed configuration.</p>
GBCU	<p>Shear modulus <math>G_{bcu}</math> in uncompressed configuration.</p>
GCAU	<p>Shear modulus <math>G_{cau}</math> in uncompressed configuration.</p> <p>ECCU.LT.0.0: <math>\sigma_p^Y</math>, GCAU initial stress limit (yield) in hydrostatic compression. Also, LCA &lt; 0 to activate the transversely isotropic yield surface.</p>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector v, and an originating point, p, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COOR-</p>

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
	DINATE_VECTOR).
MACF	Material axes change flag: EQ.1: No change, default, EQ.2: switch material axes a and b, EQ.3: switch material axes a and c, EQ.4: switch material axes b and c.
XP YP ZP	Coordinates of point <b>p</b> for AOPT = 1 and 4.
A1 A2 A3	Components of vector <b>a</b> for AOPT = 2.
D1 D2 D3	Components of vector <b>d</b> for AOPT = 2.
V1 V2 V3	Define components of vector <b>v</b> for AOPT = 3 and 4.
TSEF	Tensile strain at element failure (element will erode).
SSEF	Shear strain at element failure (element will erode).

VARIABLE	DESCRIPTION
VREF	This is an optional input parameter for solid elements types 1, 2, 3, 4, and 10. Relative volume at which the reference geometry is stored. At this time the element behaves like a nonlinear spring. The TREF, below, is reached first then VREF will have no effect.
TREF	This is an optional input parameter for solid elements types 1, 2, 3, 4, and 10. Element time step size at which the reference geometry is stored. When this time step size is reached the element behaves like a nonlinear spring. If VREF, above, is reached first then TREF will have no effect.
SHDFLG	Flag defining treatment of damage from curves LCS, LCAB, LCBC and LCCA (relevant only when LCA < 0): EQ.0.0: Damage reduces shear stress every time step, EQ.1.0: Damage = (shear stress)/(undamaged shear stress)
LCSRA	Optional load curve ID if LCSR = -1, see *DEFINE_CURVE, for strain rate effects defining the scale factor for the yield stress in the <i>a</i> -direction versus the <i>natural logarithm</i> of the absolute value of deviatoric strain rate in the <i>a</i> -direction. This curve is optional. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor.
LCSRB	Optional load curve ID if LCSR = -1, see *DEFINE_CURVE, for strain rate effects defining the scale factor for the yield stress in the <i>b</i> -direction versus the <i>natural logarithm</i> of the absolute value of deviatoric strain
LCSRC	Similar definition as for LCSA and LCSB above.
LCSRAB	Similar definition as for LCSA and LCSB above.
LCSRBC	Similar definition as for LCSA and LCSB above.
LCSRCA	Similar definition as for LCSA and LCSB above.

**Remarks:**

For efficiency it is strongly recommended that the load curve ID's: LCA, LCB, LCC, LCS, LCAB, LCBC, and LCCA, contain exactly the same number of points with corresponding strain values on the abscissa. If this recommendation is followed the cost of the table

lookup is insignificant. Conversely, the cost increases significantly if the abscissa strain values are not consistent between load curves.

For solid element formulations 1 and 2, the behavior before compaction is orthotropic where the components of the stress tensor are uncoupled, i.e., an a component of strain will generate resistance in the local *a*-direction with no coupling to the local *b* and *c* directions. The elastic moduli vary from their initial values to the fully compacted values linearly with the relative volume:

$$\begin{aligned} E_{aa} &= E_{aa0} + \beta(E - E_{aa0}) & G_{ab} &= G_{ab0} + \beta(G - G_{ab0}) \\ E_{bb} &= E_{bb0} + \beta(E - E_{bb0}) & G_{bc} &= G_{bc0} + \beta(G - G_{bc0}) \\ E_{cc} &= E_{cc0} + \beta(E - E_{cc0}) & G_{ca} &= G_{ca0} + \beta(G - G_{ca0}) \end{aligned}$$

where

$$\beta = \max \left[ \min \left( \frac{1 - V}{1 - V_f}, 1 \right), 0 \right]$$

and *G* is the elastic shear modulus for the fully compacted honeycomb material

$$G = \frac{E}{2(1 + \nu)}$$

The relative volume, *V*, is defined as the ratio of the current volume over the initial volume, and typically, *V* = 1 at the beginning of a calculation.

For corotational solid elements, types 0 and 9, the components of the stress tensor remain uncoupled and the uncompressed elastic moduli are used, that is, the fully compacted elastic moduli are ignored.

The load curves define the magnitude of the stress as the material undergoes deformation. The first value in the curve should be less than or equal to zero corresponding to tension and increase to full compaction. **Care should be taken when defining the curves so the extrapolated values do not lead to negative yield stresses.**

At the beginning of the stress update we transform each element's stresses and strain rates into the local element coordinate system. For the uncompacted material, the trial stress components are updated using the elastic interpolated moduli according to:

$$\begin{aligned} \sigma_{aa}^{n+1\text{trial}} &= \sigma_{aa}^n + E_{aa} \Delta \varepsilon_{aa} & \sigma_{ab}^{n+1\text{trial}} &= \sigma_{ab}^n + 2G_{ab} \Delta \varepsilon_{ab} \\ \sigma_{cc}^{n+1\text{trial}} &= \sigma_{cc}^n + E_{cc} \Delta \varepsilon_{cc} & \sigma_{bc}^{n+1\text{trial}} &= \sigma_{bc}^n + 2G_{bc} \Delta \varepsilon_{bc} \\ \sigma_{bb}^{n+1\text{trial}} &= \sigma_{bb}^n + E_{bb} \Delta \varepsilon_{bb} & \sigma_{ca}^{n+1\text{trial}} &= \sigma_{ca}^n + 2G_{ca} \Delta \varepsilon_{ca} \end{aligned}$$

If *LCA* > 0, each component of the updated stress tensor is checked to ensure that it does not exceed the permissible value determined from the load curves, e.g., if

$$|\sigma_{ij}^{n+1\text{trial}}| > \lambda \sigma_{ij}(\varepsilon_{ij})$$

then

$$\sigma_{ij}^{n+1} = \sigma_{ij}(\varepsilon_{ij}) \frac{\lambda \sigma_{ij}^{n+1\text{trial}}}{|\sigma_{ij}^{n+1\text{trial}}|}$$

On Card 3  $\sigma_{ij}(\varepsilon_{ij})$  is defined in the load curve specified in columns 31-40 for the aa stress component, 41-50 for the bb component, 51-60 for the cc component, and 61-70 for the ab, bc, cb shear stress components. The parameter  $\lambda$  is either unity or a value taken from the load curve number, LCSR, that defines  $\lambda$  as a function of strain-rate. Strain-rate is defined here as the Euclidean norm of the deviatoric strain-rate tensor.

If  $LCA < 0$ , a transversely isotropic yield surface is obtained where the uniaxial limit stress,  $\sigma^y(\varphi, \varepsilon^{\text{vol}})$ , can be defined as a function of angle  $\varphi$  with the strong axis and volumetric strain,  $\varepsilon^{\text{vol}}$ . In order to facilitate the input of data to such a limit stress surface, the limit stress is written as:

$$\sigma^y(\varphi, \varepsilon^{\text{vol}}) = \sigma^b(\varphi) + (\cos\varphi)^2 \sigma^s(\varepsilon^{\text{vol}}) + (\sin\varphi)^2 \sigma^w(\varepsilon^{\text{vol}})$$

where the functions  $\sigma^b$ ,  $\sigma^s$ , and  $\sigma^w$  are represented by load curves LCA, LCB, LCC, respectively. The latter two curves can be used to include the stiffening effects that are observed as the foam material crushes to the point where it begins to lock up. To ensure that the limit stress decreases with respect to the off-angle the curves should be defined such that following equations hold:

$$\frac{\partial \sigma^b(\varphi)}{\partial \varphi} \leq 0$$

and

$$\sigma^s(\varepsilon^{\text{vol}}) - \sigma^w(\varepsilon^{\text{vol}}) \geq 0.$$

A drawback of this implementation was that the material often collapsed in shear mode due to low shear resistance. There was no way of increasing the shear resistance without changing the behavior in pure uniaxial compression. We have therefore modified the model so that the user can optionally prescribe the shear and hydrostatic resistance in the material without affecting the uniaxial behavior. We introduce the parameters  $\sigma_p^Y(\varepsilon^{\text{vol}})$  and  $\sigma_d^Y(\varepsilon^{\text{vol}})$  as the *hydrostatic* and *shear limit stresses*, respectively. These are functions of the volumetric strain and are assumed given by

$$\begin{aligned} \sigma_p^Y(\varepsilon^{\text{vol}}) &= \sigma_p^Y + \sigma^s(\varepsilon^{\text{vol}}) \\ \sigma_d^Y(\varepsilon^{\text{vol}}) &= \sigma_d^Y + \sigma^s(\varepsilon^{\text{vol}})' \end{aligned}$$

where we have reused the densification function  $\sigma^s$ . The new parameters are the initial hydrostatic and shear limit stress values,  $\sigma_p^Y$  and  $\sigma_d^Y$ , and are provided by the user as

GCAU and  $|\text{ECCU}|$ , respectively. The negative sign of ECCU flags the third yield surface option whenever  $\text{LCA} < 0$ . The effect of the third formulation is that (i) for a uniaxial stress the stress limit is given by  $\sigma^Y(\phi, \varepsilon^{\text{vol}})$ , (ii) for a pressure the stress limit is given by  $\sigma_p^Y(\varepsilon^{\text{vol}})$  and (iii) for a simple shear the stress limit is given by  $\sigma_d^Y(\varepsilon^{\text{vol}})$ . Experiments have shown that the model may give noisy responses and inhomogeneous deformation modes if parameters are not chosen with care. We therefore recommend to (i) avoid large slopes in the function  $\sigma^P$ , (ii) let the functions  $\sigma^s$  and  $\sigma^w$  be slightly increasing and (iii) avoid large differences between the stress limit values  $\sigma^y(\phi, \varepsilon^{\text{vol}})$ ,  $\sigma_p^Y(\varepsilon^{\text{vol}})$  and  $\sigma_d^Y(\varepsilon^{\text{vol}})$ . These guidelines are likely to contradict how one would interpret test data and it is up to the user to find a reasonable trade-off between matching experimental results and avoiding the mentioned numerical side effects.

For fully compacted material (element formulations 1 and 2), we assume that the material behavior is elastic-perfectly plastic and updated the stress components according to:

$$s_{ij}^{\text{trial}} = s_{ij}^n + 2G\Delta\varepsilon_{ij}^{\text{dev}n+1/2}$$

where the deviatoric strain increment is defined as

$$\Delta\varepsilon_{ij}^{\text{dev}} = \Delta\varepsilon_{ij} - \frac{1}{3}\Delta\varepsilon_{kk}\delta_{ij}.$$

We now check to see if the yield stress for the fully compacted material is exceeded by comparing

$$s_{\text{eff}}^{\text{trial}} = \left( \frac{3}{2} s_{ij}^{\text{trial}} s_{ij}^{\text{trial}} \right)^{1/2}$$

the effective trial stress to the yield stress,  $\sigma_y$  (Card 1, field 41-50). If the effective trial stress exceeds the yield stress we simply scale back the stress components to the yield surface

$$s_{ij}^{n+1} = \frac{\sigma_y}{s_{\text{eff}}^{\text{trial}}} s_{ij}^{\text{trial}}.$$

We can now update the pressure using the elastic bulk modulus,  $K$

$$p^{n+1} = p^n - K\Delta\varepsilon_{kk}^{n+1/2}$$

$$K = \frac{E}{3(1-2\nu)}$$

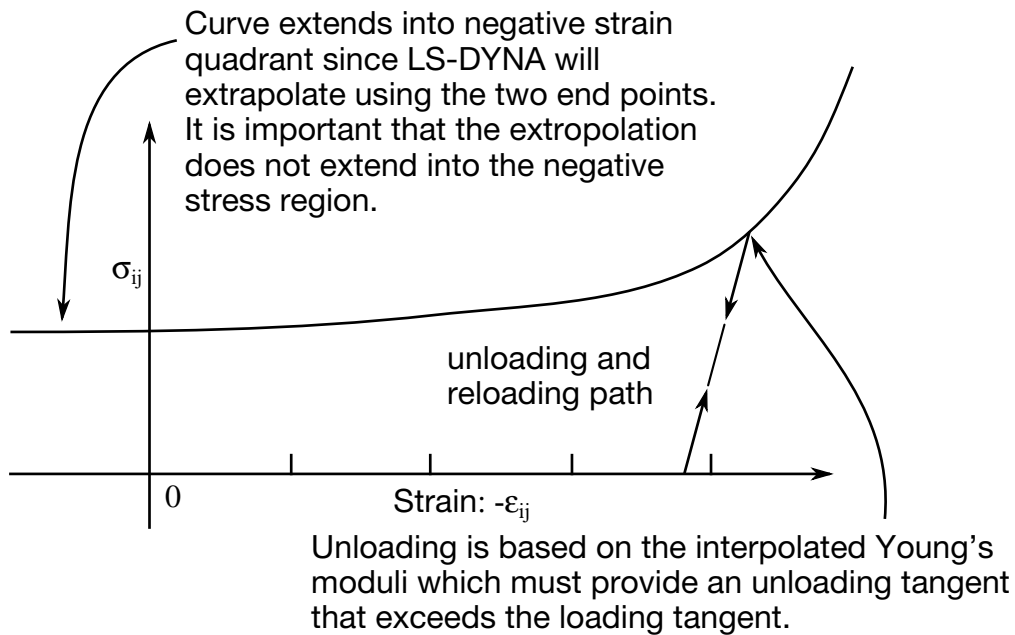
and obtain the final value for the Cauchy stress

$$\sigma_{ij}^{n+1} = s_{ij}^{n+1} - p^{n+1}\delta_{ij}$$

After completing the stress update we transform the stresses back to the global configuration.



For \*CONSTRAINED\_TIED\_NODES\_WITH\_FAILURE, the failure is based on the volume strain instead to the plastic strain.



**Figure M126-1.** Stress versus strain. Note that the “yield stress” at a strain of zero is nonzero. In the load curve definition the “time” value is the directional strain and the “function” value is the yield stress. Note that for element types 0 and 9 engineering strains are used, but for all other element types the rates are integrated in time.

**\*MAT\_ARRUDA\_BOYCE\_RUBBER**

This is Material Type 127. This material model provides a hyperelastic rubber model, see [Arruda and Boyce 1993] combined optionally with linear viscoelasticity as outlined by [Christensen 1980].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	G	N			
Type	A8	F	F	F	F			

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	TRAMP	NT					
Type	F	F	F					

**Viscoelastic Constant Cards.** Up to 6 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 6 cards are used.

Card 3	1	2	3	4	5	6	7	8
Variable	GI	BETA1						
Type	F	F						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
K	Bulk modulus
G	Shear modulus
N	Number of statistical links

VARIABLE	DESCRIPTION
LCID	Optional load curve ID of relaxation curve if constants $\beta_I$ are determined via a least squares fit. This relaxation curve is shown in <a href="#">Figure M76-1</a> . This model ignores the constant stress.
TRAMP	Optional ramp time for loading.
NT	Number of Prony series terms in optional fit. If zero, the default is 6. Currently, the maximum number is 6. Values less than 6, possibly 3-5 are recommended, since each term used adds significantly to the cost. Caution should be exercised when taking the results from the fit. Always check the results of the fit in the output file. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once a satisfactory fit has been achieved it is recommended that the coefficients which are written into the output file be input in future runs.
GI	Optional shear relaxation modulus for the $i$ th term.
BETAI	Optional decay constant if $i$ th term.

### Remarks:

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material a hydrostatic work term,  $W_H(J)$ , is included in the strain energy functional which is function of the relative volume,  $J$ , [Ogden 1984]:

$$W(J_1, J_2, J) = nk\theta \left[ \frac{1}{2}(J_1 - 3) + \frac{1}{20N}(J_1^2 - 9) + \frac{11}{1050N^2}(J_1^3 - 27) \right] \\ + nk\theta \left[ \frac{19}{7000N^3}(J_1^4 - 81) + \frac{519}{673750N^4}(J_1^5 - 243) \right] + W_H(J)$$

where the hydrostatic work term is in terms of the bulk modulus,  $K$ , and the third invariant,  $J$ , as:

$$W_H(J) = \frac{K}{2}(J - 1)^2$$

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl}(t - \tau) \frac{\partial E_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta t}$$

given by,

$$g(t) = \sum_{i=1}^n G_i e^{-\beta_i t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . The viscoelastic behavior is optional and an arbitrary number of terms may be used.



Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
C	Diastolic material coefficient.
B1	$b_1$ , diastolic material coefficient.
B2	$b_2$ , diastolic material coefficient.
B3	$b_3$ , diastolic material coefficient.
P	Pressure in the muscle tissue
B	Systolic material coefficient. Omit for the earlier model.
L0	$l_0$ , sarcomere length at which no active tension develops. Omit for the earlier model.
CA0MAX	$(Ca_0)_{\max}$ , maximum peak intracellular calcium concentrate. Omit for the earlier model.
LR	$l_R$ , Stress-free sarcomere length. Omit for the earlier model.
M	Systolic material coefficient. Omit for the earlier model.
BB	Systolic material coefficient. Omit for the earlier model.
CA0	$Ca_0$ , peak intracellular calcium concentration. Omit for the earlier model.
TMAX	$T_{\max}$ , maximum isometric tension achieved at the longest sarcomere length. Omit for the earlier model.
TACT	$t_{\text{act}}$ , time at which active contraction initiates. Omit for the earlier model

VARIABLE	DESCRIPTION
AOPT	<p>Material axes option (see MAT_OPTION TROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>P</math>, which define the centerline axis.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
MACF	<p>Material axes change flag for brick elements:</p> <p>EQ.1: No change, default,</p> <p>EQ.2: switch material axes a and b,</p> <p>EQ.3: switch material axes a and c,</p> <p>EQ.4: switch material axes b and c.</p>
XP, YP, ZP	$x_p$ $y_p$ $z_p$ , define coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	$a_1$ $a_2$ $a_3$ , define components of vector $\mathbf{a}$ for AOPT = 2.
D1, D2, D3	$d_1$ $d_2$ $d_3$ , define components of vector $\mathbf{d}$ for AOPT = 2.
V1, V2, V3	$v_1$ $v_2$ $v_3$ , define components of vector $\mathbf{v}$ for AOPT = 3 and 4.

VARIABLE	DESCRIPTION
BETA	Material angle in degrees for AOPT = 3, may be overridden on the element card, see *ELEMENT_SOLID_ORTHO.

**Remarks:**

1. The tissue model is described in terms of the energy functional that is transversely isotropic with respect to the local fiber direction,

$$W = \frac{C}{2}(e^Q - 1)$$

$$Q = b_f E_{11}^2 + b_t (E_{22}^2 + E_{33}^2 + E_{23}^2 + E_{32}^2) + b_{fs} (E_{12}^2 + E_{21}^2 + E_{13}^2 + E_{31}^2)$$

with  $C$ ,  $b_f$ ,  $b_t$ , and  $b_{fs}$  material parameters and  $E$  the Lagrange-Green strains.

The systolic contraction was modeled as the sum of the passive stress derived from the strain energy function and an active fiber directional component,  $T_0$ , which is a function of time,  $t$ ,

$$\underline{S} = \frac{\partial W}{\partial \underline{E}} - pJ\underline{C}^{-1} + T_0\{t, Ca_0, l\}$$

$$\underline{\sigma} = \frac{1}{J} \underline{E} \underline{S} \underline{E}^T$$

with  $\underline{S}$  the second Piola-Kirchoff stress tensor,  $\underline{C}$  the right Cauchy-Green deformation tensor,  $J$  the Jacobian of the deformation gradient tensor  $\underline{E}$ , and  $\underline{\sigma}$  the Cauchy stress tensor.

The active fiber directional stress component is defined by a time-varying elastance model, which at end-systole, is reduced to

$$T_0 = T_{\max} \frac{Ca_0^2}{Ca_0^2 + ECa_{50}^2} C_t$$

with  $T_{\max}$  the maximum isometric tension achieved at the longest sacromere length and maximum peak intracellular calcium concentration. The length-dependent calcium sensitivity and internal variable is given by,

$$ECa_{50} = \frac{(Ca_0)_{\max}}{\sqrt{\exp[B(l - l_0)] - 1}}$$

$$C_t = 1/2(1 - \cos w)$$

$$l = l_R \sqrt{2E_{11} + 1}$$



$$w = \pi \frac{0.25 + t_r}{t_r}$$

$$t_r = ml + bb$$

A cross-fiber, in-plane stress equivalent to 40% of that along the myocardial fiber direction is added.

2. The earlier tissue model is described in terms of the energy functional in terms of the Green strain components,  $E_{ij}$ ,

$$W(E) = \frac{C}{2}(e^Q - 1) + \frac{1}{2}P(I_3 - 1)$$

$$Q = b_1 E_{11}^2 + b_2 (E_{22}^2 + E_{33}^2 + E_{23}^2 + E_{32}^2) + b_3 (E_{12}^2 + E_{21}^2 + E_{13}^2 + E_{31}^2)$$

The Green components are modified to eliminate any effects of volumetric work following the procedures of Ogden. See the paper by Guccione *et al* [1991] for more detail.

**\*MAT\_LUNG\_TISSUE**

This is Material Type 129. This material model provides a hyperelastic model for heart tissue, see [Vawter 1980] combined optionally with linear viscoelasticity as outlined by [Christensen 1980].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	C	DELTA	ALPHA	BETA	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	C1	C2	LCID	TRAMP	NT			
Type	F	F	F	F	F			

**Viscoelastic Constant Cards.** Up to 6 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 6 cards are used.

Card 3	1	2	3	4	5	6	7	8
Variable	GI	BETA1						
Type	F	F						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
K	Bulk modulus
C	Material coefficient.
DELTA	$\Delta$ , material coefficient.
ALPHA	$\alpha$ , material coefficient.

VARIABLE	DESCRIPTION
BETA	$\beta$ , material coefficient.
C1	Material coefficient.
C2	Material coefficient.
LCID	Optional load curve ID of relaxation curve If constants $G_i$ and $\beta_i$ are determined via a least squares fit. This relaxation curve is shown in <a href="#">Figure M76-1</a> . This model ignores the constant stress.
TRAMP	Optional ramp time for loading.
NT	Number of Prony series terms in optional fit. If zero, the default is 6. Currently, the maximum number is 6. Values less than 6, possibly 3 - 5 are recommended, since each term used adds significantly to the cost. Caution should be exercised when taking the results from the fit. Always check the results of the fit in the output file. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once a satisfactory fit has been achieved it is recommended that the coefficients which are written into the output file be input in future runs.
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETA $i$	Optional decay constant if $i^{\text{th}}$ term

**Remarks:**

The material is described by a strain energy functional expressed in terms of the invariants of the Green Strain:

$$W(I_1, I_2) = \frac{C}{2\Delta} e^{(\alpha I_1^2 + \beta I_2)} + \frac{12C_1}{\Delta(1 + C_2)} [A^{(1+C_2)} - 1]$$

$$A^2 = \frac{4}{3} (I_1 + I_2) - 1$$

where the hydrostatic work term is in terms of the bulk modulus,  $K$ , and the third invariant,  $J$ , as:

$$W_H(J) = \frac{K}{2} (J - 1)^2$$

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl}(t - \tau) \frac{\partial E_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta t}$$

given by,

$$g(t) = \sum_{i=1}^n G_i e^{-\beta_i t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . The viscoelastic behavior is optional and an arbitrary number of terms may be used.

\*MAT\_SPECIAL\_ORTHOTROPIC

This is Material Type 130. This model is available the Belytschko-Tsay and the C0 triangular shell elements and is based on a resultant stress formulation. In-plane behavior is treated separately from bending in order to model perforated materials such as television shadow masks. If other shell formulations are specified, the formulation will be automatically switched to Belytschko-Tsay. As implemented, this material model cannot be used with user defined integration rules.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	YS	EP				
Type	A8	F	F	F				

Card 2	1	2	3	4	5	6	7	8
Variable	E11P	E22P	V12P	V21P	G12P	G23P	G31P	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	E11B	E22B	V12B	V21B	G12B	A0PT		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
YS	Yield stress. This parameter is optional and is approximates the yield condition. Set to zero if the behavior is elastic.
EP	Plastic hardening modulus.
E11P	$E_{11p}$ , for in plane behavior.
E22P	$E_{22p}$ , for in plane behavior.
V12P	$\nu_{12p}$ , for in plane behavior.
V11P	$\nu_{11p}$ , for in plane behavior.
G12P	$G_{12p}$ , for in plane behavior.
G23P	$G_{23p}$ , for in plane behavior.
G31P	$G_{31p}$ , for in plane behavior.
E11B	$E_{11p}$ , for bending behavior.
E22B	$E_{22p}$ , for bending behavior.
V12B	$\nu_{12b}$ , for bending behavior.
V21B	$\nu_{21b}$ , for bending behavior.
G12B	$G_{12b}$ , for bending behavior.

VARIABLE	DESCRIPTION
AOPT	<p>Material axes option (see MAT_{OPTION}TROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
A1, A2, A3	$(a_1, a_2, a_3)$ , define components of vector $\mathbf{a}$ for AOPT = 2.
D1, D2, D3	$(d_1, d_2, d_3)$ , define components of vector $\mathbf{d}$ for AOPT = 2.
V1, V2, V3	$(v_1, v_2, v_3)$ , define components of vector $\mathbf{v}$ for AOPT = 3.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

**Remarks:**

The in-plane elastic matrix for in-plane, plane stress behavior is given by:

$$\mathbf{C}_{\text{in plane}} = \begin{bmatrix} Q_{11p} & Q_{12p} & 0 & 0 & 0 \\ Q_{12p} & Q_{22p} & 0 & 0 & 0 \\ 0 & 0 & Q_{44p} & 0 & 0 \\ 0 & 0 & 0 & Q_{55p} & 0 \\ 0 & 0 & 0 & 0 & Q_{66p} \end{bmatrix}$$

The terms  $Q_{ijp}$  are defined as:

$$Q_{11p} = \frac{E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{22p} = \frac{E_{22p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{12p} = \frac{\nu_{12p}E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{44p} = G_{12p}$$

$$Q_{55p} = G_{23p}$$

$$Q_{66p} = G_{31p}$$

The elastic matrix for bending behavior is given by:

$$\mathbf{C}_{\text{bending}} = \begin{bmatrix} Q_{11b} & Q_{12b} & 0 \\ Q_{12b} & Q_{22b} & 0 \\ 0 & 0 & Q_{44b} \end{bmatrix}$$

The terms  $Q_{ijp}$  are similarly defined.

Because this is a resultant formulation, nothing is written to the six stress slots of d3plot. Resultant forces and moments may be written to elout and to dynain in place of the six stresses. The first two extra history variables may be used to complete output of the eight resultants to elout and dynain.



**\*MAT\_ISOTROPIC\_SMEARED\_CRACK**

This is Material Type 131. This model was developed by Lemmen and Meijer [2001] as a smeared crack model for isotropic materials. This model is available of solid elements only and is restricted to cracks in the x-y plane. Users should choose other models unless they have the report by Lemmen and Meijer [2001].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	ISPL	SIGF	GK	SR
Type	A8	F	F	F	I	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
ISPL	Failure option: EQ.0: Maximum principal stress criterion EQ.5: Smeared crack model EQ.6: Damage model based on modified von Mises strain
SIGF	Peak stress.
GK	Critical energy release rate.
SR	Strength ratio.

**Remarks:**

The following documentation is taken nearly verbatim from the documentation of Lemmen and Meijer [2001].

Three methods are offered to model progressive failure. The maximum principal stress criterion detects failure if the maximum (most tensile) principal stress exceeds  $\sigma_{\max}$ . Upon failure, the material can no longer carry stress.

The second failure model is the smeared crack model with linear softening stress-strain using equivalent uniaxial strains. Failure is assumed to be perpendicular to the principal strain directions. A rotational crack concept is employed in which the crack directions are related to the current directions of principal strain. Therefore crack directions may rotate in time. Principal stresses are expressed as

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} = \begin{bmatrix} \bar{E}_1 & 0 & 0 \\ 0 & \bar{E}_2 & 0 \\ 0 & 0 & \bar{E}_3 \end{bmatrix} \begin{pmatrix} \tilde{\epsilon}_1 \\ \tilde{\epsilon}_2 \\ \tilde{\epsilon}_3 \end{pmatrix} = \begin{pmatrix} \bar{E}_1 \tilde{\epsilon}_1 \\ \bar{E}_2 \tilde{\epsilon}_2 \\ \bar{E}_3 \tilde{\epsilon}_3 \end{pmatrix} \quad (131.1)$$

with  $\bar{E}_1$ ,  $\bar{E}_2$  and  $\bar{E}_3$  secant stiffness in the terms that depend on internal variables.

In the model developed for DYCOSS it has been assumed that there is no interaction between the three directions in which case stresses simply follow from

$$\sigma_j(\tilde{\epsilon}_j) = \begin{cases} E\tilde{\epsilon}_j & \text{if } 0 \leq \tilde{\epsilon}_j \leq \tilde{\epsilon}_{j,\text{ini}} \\ \bar{\sigma} \left( 1 - \frac{\tilde{\epsilon}_j - \tilde{\epsilon}_{j,\text{ini}}}{\tilde{\epsilon}_{j,\text{ult}} - \tilde{\epsilon}_{j,\text{ini}}} \right) & \text{if } \tilde{\epsilon}_{j,\text{ini}} < \tilde{\epsilon}_j \leq \tilde{\epsilon}_{j,\text{ult}} \\ 0 & \text{if } \tilde{\epsilon}_j > \tilde{\epsilon}_{j,\text{ult}} \end{cases} \quad (131.2)$$

with  $\bar{\sigma}$  the ultimate stress,  $\tilde{\epsilon}_{j,\text{ini}}$  the damage threshold, and  $\tilde{\epsilon}_{j,\text{ult}}$  the ultimate strain in  $j$ -direction. The damage threshold is defined as

$$\tilde{\epsilon}_{j,\text{ini}} = \frac{\bar{\sigma}}{E} \quad (131.3)$$

The ultimate strain is obtained by relating the crack growth energy and the dissipated energy

$$\int \int \bar{\sigma} d\tilde{\epsilon}_{j,\text{ult}} dV = GA \quad (131.4)$$

with  $G$  the energy release rate,  $V$  the element volume and  $A$  the area perpendicular to the principal strain direction. The one point elements LS-DYNA have a single integration point and the integral over the volume may be replaced by the volume. For linear softening it follows

$$\tilde{\epsilon}_{j,\text{ult}} = \frac{2GA}{V\bar{\sigma}} \quad (131.5)$$

The above formulation may be regarded as a damage equivalent to the maximum principle stress criterion.

The third model is a damage model represented by Brekelmans et. al [1991]. Here the Cauchy stress tensor  $\sigma$  is expressed as

$$\sigma = (1 - D)E\epsilon \quad (131.6)$$

where  $D$  represents the current damage and the factor  $(1-D)$  is the reduction factor caused by damage. The scalar damage variable is expressed as function of a so-called damage equivalent strain  $\epsilon d$

$$D = D(\varepsilon_d) = 1 - \frac{\varepsilon_{ini}(\varepsilon_{ult} - \varepsilon_d)}{\varepsilon_d(\varepsilon_{ult} - \varepsilon_{ini})} \quad (131.7)$$

and

$$\varepsilon_d = \frac{k-1}{2k(1-2\nu)} J_1 + \frac{1}{2k} \sqrt{\left(\frac{k-1}{1-2\nu} J_1\right)^2 + \frac{6k}{(1+\nu)^2} J_2} \quad (131.8)$$

where the constant  $k$  represents the ratio of the strength in tension over the strength in compression

$$k = \frac{\sigma_{ult, tension}}{\sigma_{ult, compression}} \quad (131.9)$$

$J_1$  resp.  $J_2$  are the first and second invariant of the strain tensor representing the volumetric and the deviatoric straining respectively

$$\begin{aligned} J_1 &= \text{tr}(\varepsilon) \\ J_2 &= \text{tr}(\varepsilon \cdot \varepsilon) - \frac{1}{3} [\text{tr}(\varepsilon)]^2 \end{aligned} \quad (131.10)$$

If the compression and tension strength are equal the dependency on the volumetric strain vanishes in (8) and failure is shear dominated. If the compressive strength is much larger than the strength in tension,  $k$  becomes small and the  $J_1$  terms in (131.8) dominate the behavior.



VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
EA	$E_a$ , Young's modulus in $a$ -direction.
EB	$E_b$ , Young's modulus in $b$ -direction.
EC	$E_c$ , Young's modulus in $c$ -direction
PRBA	$\nu_{ba}$ , Poisson's ratio $ba$ .
PRCA	$\nu_{ca}$ , Poisson's ratio $ca$ .
PRCB	$\nu_{cb}$ , Poisson's ratio $cb$ .
UINS	Ultimate interlaminar normal stress.
UISS	Ultimate interlaminar shear stress.
CERRMI	Critical energy release rate mode I
CERRMII	Critical energy release rate mode II
IND	Interlaminar normal direction : EQ.1.0: Along local $a$ axis EQ.2.0: Along local $b$ axis EQ.3.0: Along local $c$ axis
ISD	Interlaminar shear direction : EQ.4.0: Along local $ab$ axis EQ.5.0: Along local $bc$ axis EQ.6.0: Along local $ca$ axis
GAB	$G_{ab}$ , shear modulus $ab$ .
GBC	$G_{bc}$ , shear modulus $bc$ .
GCA	$G_{ca}$ , shear modulus $ca$ .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option, see Figure 2.1.</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in Figure 2.1. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>v</math> with the element normal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>v</math>, and an originating point, <math>P</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
XP YP ZP	Define coordinates of point $p$ for AOPT = 1 and 4.
A1 A2 A3	Define components of vector $a$ for AOPT = 2.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MACF	Material axes change flag for brick elements: EQ.1: No change, default, EQ.2: switch material axes a and b, EQ.3: switch material axes a and c, EQ.4: switch material axes b and c.
V1 V2 V3	Define components of vector <b>v</b> for AOPT = 3 and 4.
D1 D2 D3	Define components of vector <b>d</b> for AOPT = 2:
BETA	Material angle in degrees for AOPT = 3, may be overridden on the element card, see *ELEMENT_SOLID_ORTHO.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.

**Remarks:**

This is an orthotropic material with optional delamination failure for brittle composites. The elastic formulation is identical to the DYNA3D model that uses total strain formulation. The constitutive matrix **C** that relates to global components of stress to the global components of strain is defined as:

$$\mathbf{C} = \mathbf{T}^T \mathbf{C}_L \mathbf{T}$$

where **T** is the transformation matrix between the local material coordinate system and the global system and  $\mathbf{C}_L$  is the constitutive matrix defined in terms of the material constants of the local orthogonal material axes *a*, *b*, and *c* (see DYNA3D use manual).

Failure is described using linear softening stress strain curves for interlaminar normal and interlaminar shear direction. The current implementation for failure is essentially 2-D. Damage can occur in interlaminar normal direction and a single interlaminar shear direction. The orientation of these directions w.r.t. the principal material directions have to be specified by the user.

Based on specified values for the ultimate stress and the critical energy release rate bounding surfaces are defined

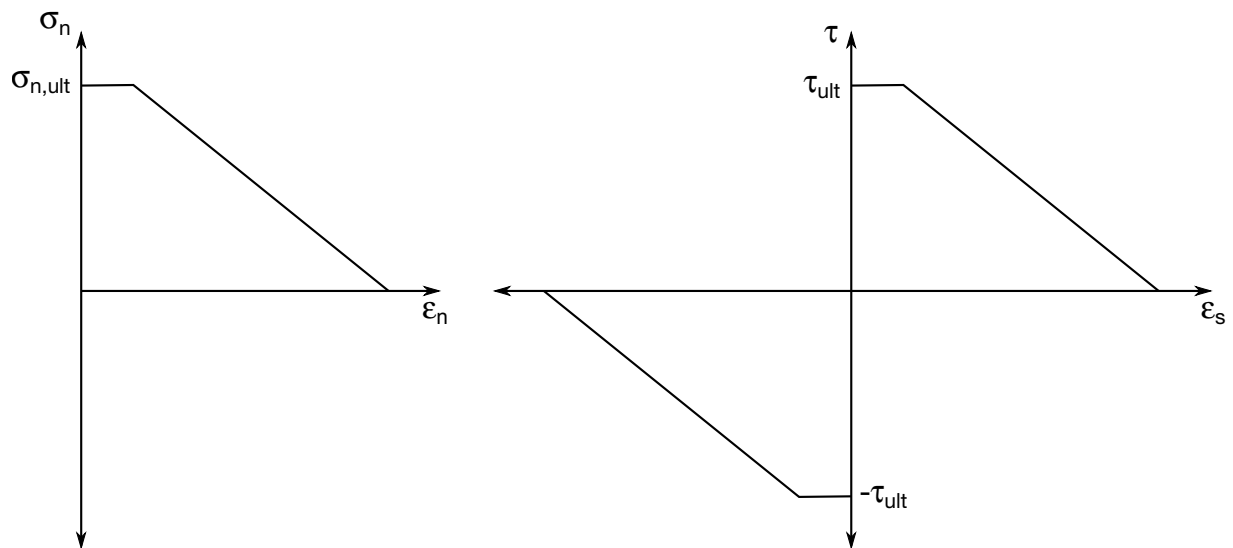
$$f_n = \sigma_n - \bar{\sigma}_n(\epsilon_n)$$

$$f_s = \sigma_s - \bar{\sigma}_s(\epsilon_s)$$

where the subscripts n and s refer to the normal and shear component. If stresses exceed the bounding surfaces inelastic straining occurs. The ultimate strain is obtained by relating the crack growth energy and the dissipated energy. For solid elements with a single integration point it can be derived

$$\epsilon_{i,ult} = \frac{2G_i A}{V\sigma_{i,ult}}$$

with  $G_i$  the critical energy release rate,  $V$  the element volume,  $A$  the area perpendicular to the active normal direction and  $\sigma_{i,ult}$  the ultimate stress. For the normal component failure can only occur under tensile loading. For shear component the behavior is symmetric around zero. The resulting stress bounds are depicted in Figure M132-1. Unloading is modeled with a Secant stiffness.



**Figure M132-1.** Shows stress bounds for the active normal component (left) and the archieve shear component (right).



\*MAT\_BARLAT\_YLD2000

This is Material Type 133. This model was developed by Barlat et al. [2003] to overcome some shortcomings of the six parameter Barlat model implemented as material 33 (MAT\_BARLAT\_YLD96) in LS-DYNA. This model is available for shell elements only.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	FIT	BETA	ITER	ISCALE
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	K	E0	N	C	P	HARD	A	
Type	F	F	F	F	F	F	F	

**Chaboche-Roussilier Card.** Additional Card for  $A < 0$ .

Card 3	1	2	3	4	5	6	7	8
Variable	CRC1	CRA1	CRC2	CRA2	CRC3	CRA3	CRC4	CRA4
Type	F	F	F	F	F	F	F	F

**Direct Material Parameter Card.** Additional card for  $FIT = 0$ .

Card 4	1	2	3	4	5	6	7	8
Variable	ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8
Type	F	F	F	F	F	F	F	F



Card 10	1	2	3	4	5	6	7	8
Variable	AOPT	OFFANG	P4	HTFLAG	HTA	HTB	HTC	HTD
Type	F	F	F	F	F	F	F	F

Card 11	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 12	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	USRFAIL	
Type	F	F	F	F	F	F	F	

**VARIABLE**

**DESCRIPTION**

- MID Material identification. A unique number or label not exceeding 8 characters must be specified.
- RO Mass density
- E Young’s modulus  
LE.0: -E is load curve ID for Young’s modulus vs. plastic strain
- PR Poisson’s ratio
- FIT Material parameter fit flag:  
EQ.0.0: Material parameters are used directly on card 3.  
EQ.1.0: Material parameters are determined from test data on cards 3 and 4
- BETA Hardening parameter. Any value ranging from 0 (isotropic hardening) to 1 (kinematic hardening) may be input.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ITER	<p>Plastic iteration flag:</p> <p>EQ.0.0: Plane stress algorithm for stress return</p> <p>EQ.1.0: Secant iteration algorithm for stress return</p> <p>ITER provides an option of using three secant iterations for determining the thickness strain increment as experiments have shown that this leads to a more accurate prediction of shell thickness changes for rapid processes. A significant increase in computation time is incurred with this option so it should be used only for applications associated with high rates of loading and/or for implicit analysis.</p>
ISCALE	<p>Yield locus scaling flag:</p> <p>EQ.0.0: Scaling on – reference direction = rolling direction (default)</p> <p>EQ.1.0: Scaling off – reference direction arbitrary</p>
K	<p>Material parameter:</p> <p>HARD.EQ.1.0: <math>k</math>, strength coefficient for exponential hardening</p> <p>HARD.EQ.2.0: <math>a</math> in Voce hardening law</p> <p>HARD.EQ.4.0: <math>k</math>, strength coefficient for Gosh hardening</p> <p>HARD.EQ.5.0: <math>a</math> in Hockett-Sherby hardening law</p>
E0	<p>Material parameter:</p> <p>HARD.EQ.1.0: <math>e_0</math>, strain at yield for exponential hardening</p> <p>HARD.EQ.2.0: <math>b</math> in Voce hardening law</p> <p>HARD.EQ.4.0: <math>\epsilon_0</math>, strain at yield for Gosh hardening</p> <p>HARD.EQ.5.0: <math>b</math> in Hockett-Sherby hardening law</p>
N	<p>Material parameter:</p> <p>HARD.EQ.1.0: <math>n</math>, exponent for exponential hardening</p> <p>HARD.EQ.2.0: <math>c</math> in Voce hardening law</p> <p>HARD.EQ.4.0: <math>n</math>, exponent for Gosh hardening</p> <p>HARD.EQ.5.0: <math>c</math> in Hockett-Sherby hardening law</p>
C	<p>Cowper-Symonds strain rate parameter, <math>C</math>, see formula below.</p>

VARIABLE	DESCRIPTION
P	Cowper-Symonds strain rate parameter, $p$ . $\sigma_y^v(\varepsilon_p, \dot{\varepsilon}_p) = \sigma_y(\varepsilon_p) \left( 1 + \left[ \frac{\dot{\varepsilon}_p}{C} \right]^{1/p} \right)$
HARD	Hardening law: EQ.1.0: Exponential hardening: $\sigma_y = k(\varepsilon_0 + \varepsilon_p)^n$ EQ.2.0: Voce hardening: $\sigma_y = a - be^{-c\varepsilon_p}$ EQ.3.0: Hansel hardening EQ.4.0: Gosh hardening: $\sigma_y = k(\varepsilon_0 + \varepsilon_p)^n - p$ EQ.5.0: Hockett-Sherby hardening: $\sigma_y = a - be^{-c\varepsilon_p^q}$ LT.0.0: Absolute value defines load curve ID or table ID with yield stress as functions of plastic strain and in the latter case also plastic strain rate.
A	Flow potential exponent. For face centered cubic (FCC) materials $A = 8$ is recommended and for body centered cubic (BCC) materials $A = 6$ may be used.
CRC $n$	Chaboche-Rousselier kinematic hardening parameters, see remarks.
CRA $n$	Chaboche-Rousselier kinematic hardening parameters, see remarks.
ALPHA1	$\alpha_1$ , see equations below
⋮	⋮
ALPHA8	$\alpha_8$ , see equations below
SIG00	Yield stress in 00 direction
SIG45	Yield stress in 45 direction
SIG90	Yield stress in 90 direction
R00	$R$ -value in 00 direction
R45	$R$ -value in 45 direction
R90	$R$ -value in 90 direction

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SIGXX	$xx$ -component of stress on yield surface (See Remark 2).
SIGYY	$yy$ -component of stress on yield surface (See Remark 2).
SIGXY	$xy$ -component of stress on yield surface (See Remark 2).
DXX	$xx$ -component of tangent to yield surface (See Remark 2).
DYY	$yy$ -component of tangent to yield surface (See Remark 2).
DXY	$xy$ -component of tangent to yield surface (See Remark 2).
CP	Adiabatic temperature calculation option: EQ.0.0: Adiabatic temperature calculation is disabled. GT.0.0: CP is the specific heat $C_p$ . Adiabatic temperature calculation is enabled.
T0	Initial temperature $T_0$ of the material if adiabatic temperature calculation is enabled.
TREF	Reference temperature for output of the yield stress as history variable.
TA0	Reference temperature $T_{A0}$ , the absolute zero for the used temperature scale, e.g. -273.15 if the Celsius scale is used and 0.0 if the Kelvin scale is used.
A	Martensite rate equation parameter $A$ , see equations below.
B	Martensite rate equation parameter $B$ , see equations below.
C	Martensite rate equation parameter $C$ , see equations below.
D	Martensite rate equation parameter $D$ , see equations below.
P	Martensite rate equation parameter $p$ , see equations below.
Q	Martensite rate equation parameter $Q$ , see equations below.
E0MART	Martensite rate equation parameter $E_{0(\text{mart})}$ , see equations below.

VARIABLE	DESCRIPTION
VM0	<p>The initial volume fraction of martensite <math>0.0 &lt; V_{m0} &lt; 1.0</math> may be initialised using two different methods:</p> <p>GT.0.0: <math>V_{m0}</math> is set to VM0.</p> <p>LT.0.0: Can be used only when there are initial plastic strains <math>\epsilon^p</math> present, e.g. when using *INITIAL_STRESS_SHELL. The absolute value of VM0 is then the load curve ID for a function <math>f</math> that sets <math>V_{m0} = f(\epsilon^p)</math>. The function <math>f</math> must be a monotonically nondecreasing function of <math>\epsilon^p</math>.</p>
AHS	Hardening law parameter $A_{HS}$ , see equations below.
BHS	Hardening law parameter $B_{HS}$ , see equations below.
M	Hardening law parameter $m$ , see equations below.
N	Hardening law parameter $n$ , see equations below.
EPS0	Hardening law parameter $\epsilon_0$ , see equations below.
HMART	Hardening law parameter $\Delta H_{\gamma \rightarrow \alpha'}$ , see equations below.
K1	Hardening law parameter $K_1$ , see equations below.
K2	Hardening law parameter $K_2$ , see equations below
AOPT	<p>Material axes option:</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in Figure M133-1. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR</p> <p>EQ.3.0: locally orthotropic material axes determined by offsetting the material axes by an angle, OFFANG, from a line determined by taking the cross product of the vector <math>v</math> with the normal to the plane of the element.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	later.
OFFANG	Offset angle for AOPT = 3
P4	Material parameter: HARD.EQ.4.0: $p$ in Gosh hardening law HARD.EQ.5.0: $q$ in Hocket-Sherby hardening law
HTFLAG	Heat treatment flag (see remarks): HTFLAG.EQ.0: Preforming stage HTFLAG.EQ.1: Heat treatment stage HTFLAG.EQ.2: Postforming stage
HTA	Load curve/Table ID for postforming parameter A
HTB	Load curve/Table ID for postforming parameter B
HTC	Load curve/Table ID for postforming parameter C
HTD	Load curve/Table ID for postforming parameter D
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2
USRFAIL	User defined failure flag EQ.0: no user subroutine is called EQ.1: user subroutine matusr_24 in dyn21.f is called

**Remarks:**

1. Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate. To ignore strain rate effects set both C and P to zero.

2. The yield condition for this material can be written



$$f(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \varepsilon_p) = \sigma_{\text{eff}}(\sigma_{xx} - 2\alpha_{xx} - \alpha_{yy}, \sigma_{yy} - 2\alpha_{yy} - \alpha_{xx}, \sigma_{xy} - \alpha_{xy}) - \sigma_Y^t(\varepsilon_p, \dot{\varepsilon}_p, \beta) \leq 0$$

where

$$\begin{aligned} \sigma_{\text{eff}}(s_{xx}, s_{yy}, s_{xy}) &= \left[ \frac{1}{2} (\varphi' + \varphi'') \right]^{1/a} \\ \varphi' &= |X'_1 - X'_2|^a \\ \varphi'' &= |2X''_1 + X''_2|^a + |X''_1 + 2X''_2|^a. \end{aligned}$$

The  $X'_i$  and  $X''_i$  are eigenvalues of  $X'_{ij}$  and  $X''_{ij}$  and are given by

$$\begin{aligned} X'_1 &= \frac{1}{2} \left( X'_{11} + X'_{22} + \sqrt{(X'_{11} - X'_{22})^2 + 4X'_{12}{}^2} \right) \\ X'_2 &= \frac{1}{2} \left( X'_{11} + X'_{22} - \sqrt{(X'_{11} - X'_{22})^2 + 4X'_{12}{}^2} \right) \end{aligned}$$

and

$$\begin{aligned} X''_1 &= \frac{1}{2} \left( X''_{11} + X''_{22} + \sqrt{(X''_{11} - X''_{22})^2 + 4X''_{12}{}^2} \right) \\ X''_2 &= \frac{1}{2} \left( X''_{11} + X''_{22} - \sqrt{(X''_{11} - X''_{22})^2 + 4X''_{12}{}^2} \right) \end{aligned}$$

respectively. The  $X'_{ij}$  and  $X''_{ij}$  are given by

$$\begin{aligned} \begin{pmatrix} X'_{11} \\ X'_{22} \\ X'_{12} \end{pmatrix} &= \begin{pmatrix} L'_{11} & L'_{12} & 0 \\ L'_{21} & L'_{22} & 0 \\ 0 & 0 & L'_{33} \end{pmatrix} \begin{pmatrix} s_{xx} \\ s_{yy} \\ s_{xy} \end{pmatrix} \\ \begin{pmatrix} X''_{11} \\ X''_{22} \\ X''_{12} \end{pmatrix} &= \begin{pmatrix} L''_{11} & L''_{12} & 0 \\ L''_{21} & L''_{22} & 0 \\ 0 & 0 & L''_{33} \end{pmatrix} \begin{pmatrix} s_{xx} \\ s_{yy} \\ s_{xy} \end{pmatrix} \end{aligned}$$

where,

$$\begin{aligned} \begin{pmatrix} L'_{11} \\ L'_{12} \\ L'_{21} \\ L'_{22} \\ L'_{33} \end{pmatrix} &= \frac{1}{3} \begin{pmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_7 \end{pmatrix} \\ \begin{pmatrix} L''_{11} \\ L''_{12} \\ L''_{21} \\ L''_{22} \\ L''_{33} \end{pmatrix} &= \frac{1}{9} \begin{pmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{pmatrix} \begin{pmatrix} \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_8 \end{pmatrix} \end{aligned}$$

The parameters  $\alpha_1$  to  $\alpha_8$  are the parameters that determines the shape of the yield surface.  $s_{xx}$ ,  $s_{yy}$ , and  $s_{xy}$  do not denote the deviatoric stress components but the arguments as used in the  $\sigma_{\text{eff}}$  function.

The material parameters can be determined from three uniaxial tests and a more general test. From the uniaxial tests the yield stress and R-values are used and from the general test an arbitrary point on the yield surface is used given by the stress components in the material system as

$$\sigma = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}$$

together with a tangent of the yield surface in that particular point. For the latter the tangential direction should be determined so that

$$d_{xx}\dot{\epsilon}_{xx}^p + d_{yy}\dot{\epsilon}_{yy}^p + 2d_{xy}\dot{\epsilon}_{xy}^p = 0$$

The biaxial data can be set to zero in the input deck for LS-DYNA to just fit the uniaxial data.

The effective stress (excluding back stress) can be output to the d3plot database through \*DEFINE\_MATERIAL\_HISTORIES.

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>		
<b>Label</b>	<b>Attributes</b>	<b>Description</b>
Effective Stress	- - - -	Effective stress $\sigma_{\text{eff}}(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ , see above

3. A kinematic hardening model is implemented following the works of Chaboche and Roussilier. A back stress  $\alpha$  is introduced such that the effective stress is computed as

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12})$$

The back stress is the sum of up to four terms according to

$$\alpha_{ij} = \sum_{k=1}^4 \alpha_{ij}^k$$

and the evolution of each back stress component is as follows

$$\delta\alpha_{ij}^k = C_k \left( a_k \frac{s_{ij}}{\sigma_{\text{eff}}} - \alpha_{ij}^k \right) \delta\epsilon_p$$

where  $C_k$  and  $a_k$  are material parameters,  $s_{ij}$  is the deviatoric stress tensor,  $\sigma_{\text{eff}}$  is the effective stress and  $\epsilon_p$  is the effective plastic strain. The yield condition is for this case modified according to

$$f(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \varepsilon_p) = \sigma_{\text{eff}}(\sigma_{xx} - 2\alpha_{xx} - \alpha_{yy}, \sigma_{yy} - 2\alpha_{yy} - \alpha_{xx}, \sigma_{xy} - \alpha_{xy}) \\ - \left\{ \sigma_Y^t(\varepsilon_p, \dot{\varepsilon}_p, 0) - \sum_{k=1}^4 a_k [1 - \exp(-C_k \varepsilon_p)] \right\} \leq 0$$

in order to get the expected stress strain response for uniaxial stress.

4. The Hansel hardening law is the same as in material 113 but is repeated here for the sake of convenience.

The hardening is temperature dependent and therefore this material model must be run either in a coupled thermo-mechanical solution, using prescribed temperatures or using the adiabatic temperature calculation option. Setting the parameter CP to the specific heat  $C_p$  of the material activates the adiabatic temperature calculation that calculates the temperature rate from the equation

$$\dot{T} = \sum_{i,j} \frac{\sigma_{ij} D_{ij}^p}{\rho C_p}$$

where  $\boldsymbol{\sigma} : \mathbf{D}^p$  (the numerator) is the plastically dissipated heat. Using the Kelvin scale is recommended, even though other scales may be used without problems.

The hardening behaviour is described by the following equations. The martensite rate equation is

$$\frac{\partial V_m}{\partial \bar{\varepsilon}^p} = \begin{cases} 0 & \varepsilon < E_{0(\text{mart})} \\ \frac{B}{A} V_m^p \left( \frac{1 - V_m}{V_m} \right)^{\frac{B+1}{B}} \frac{[1 - \tanh(C + D \times T)]}{2} \exp\left(\frac{Q}{T - T_{A0}}\right) & \bar{\varepsilon}^p \geq E_{0(\text{mart})} \end{cases}$$

Where

$\bar{\varepsilon}^p$  = effective plastic strain

$T$  = temperature

The martensite fraction is integrated from the above rate equation:

$$V_m = \int_0^{\varepsilon} \frac{\partial V_m}{\partial \bar{\varepsilon}^p} d\bar{\varepsilon}^p.$$

It always holds that  $0.0 < V_M < 1.0$ . The initial martensite content is  $V_{m0}$  and must be greater than zero and less than 1.0. Note that  $V_{M0}$  is not used during a restart or when initializing the  $V_m$  history variable using \*INITIAL\_STRESS\_SHELL.

The yield stress  $\sigma_y$  is

$$\sigma_y = \{B_{HS} - (B_{HS} - A_{HS})\exp(-m[\bar{\epsilon}^p + \epsilon_0]^n)\}(K_1 + K_2T) + \Delta H_{\gamma \rightarrow \alpha'} V_m.$$

The parameters p and B should fulfill the following condition

$$\frac{1 + B}{B} < p,$$

if not fulfilled then the martensite rate will approach infinity as  $V_m$  approaches zero. Setting the parameter  $\epsilon_0$  larger than zero, typical range 0.001-0.02 is recommended. A part from the effective true strain a few additional history variables are output, see below.

History variables that are output for post-processing:

*Variable Description*

- 24 Yield stress of material at temperature TREF. Useful to evaluate the strength of the material after e.g., a simulated forming operation.
  - 25 Volume fraction martensite,  $V_m$
  - 26 CP.EQ.0.0: Not used  
CP.GT.0.0: Temperature from adiabatic temperature calculation.
5. Heat treatment for increasing the formability of prestrained aluminum sheets can be simulated through the use of HTFLAG, where the intention is to run a forming simulation in steps involving preforming, springback, heat treatment and postforming. In each step the history is transferred to the next via the use of dynain (see \*INTERFACE\_SPRINGBACK). The first two steps are performed with HTFLAG = 0 according to standard procedures, resulting in a plastic strain field  $\epsilon_p^0$  corresponding to the prestrain. The heat treatment step is performed using HTFLAG = 1 in a coupled thermomechanical simulation, where the blank is heated. The coupling between thermal and mechanical is only that the maximum temperature  $T^0$  is stored as a history variable in the material model, this corresponding to the heat treatment temperature. Here it is important to export all history variables to the dynein file for the postforming step. In the final postforming step, HTFLAG = 2, the yield stress is then augmented by the Hockett-Sherby like term

$$\Delta\sigma = b - (b - a)\exp\left[-c(\epsilon_p - \epsilon_p^0)^d\right]$$

where  $a$ ,  $b$ ,  $c$  and  $d$  are given as tables as functions of the heat treatment temperature  $T^0$  and prestrain  $\epsilon_p^0$ . That is, in the table definitions each load curve corresponds to a given prestrain and the load curve value is with respect to the heat treatment temperature,

$$a = a(T^0, \epsilon_p^0), \quad b = b(T^0, \epsilon_p^0), \quad c = c(T^0, \epsilon_p^0), \quad d = d(T^0, \epsilon_p^0)$$

The effect of heat treatment is that the material strength decreases but hardening increases, thus typically,

$$a \leq 0, \quad b \geq a, \quad c > 0, \quad d > 0.$$

**\*MAT\_VISCOELASTIC\_FABRIC**

This is Material Type 134. The viscoelastic fabric model is a variation on the general viscoelastic model of material 76. This model is valid for 3 and 4 node membrane elements only and is strongly recommended for modeling isotropic viscoelastic fabrics where wrinkling may be a problem. For thin fabrics, buckling can result in an inability to support compressive stresses; thus, a flag is included for this option. If bending stresses are important use a shell formulation with model 76.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK				CSE	
Type	I	F	F				F	

If fitting is done from a relaxation curve, specify fitting parameters on card 2, *otherwise* if constants are set on Viscoelastic Constant Cards *LEAVE THIS CARD BLANK*.

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	NT	BSTART	TRAMP	LCIDK	NTK	BSTARTK	TRAMPK
Type	F	I	F	F	F	I	F	F

**Viscoelastic Constant Cards.** Up to 6 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 6 cards are used. These cards are not needed if relaxation data is defined. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included.

Card 3	1	2	3	4	5	6	7	8
Variable	GI	BETAI	KI	BETAKI				
Type	F	F	F	F				

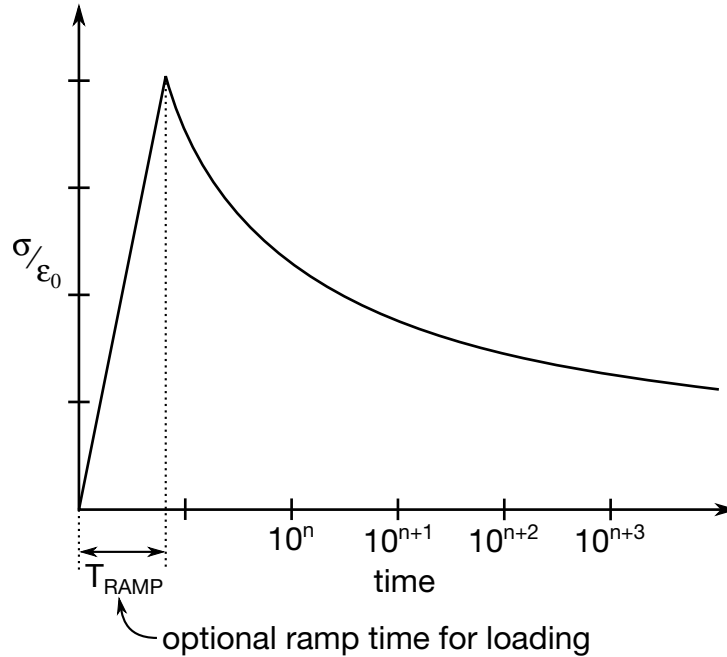
**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number must be specified.
RO	Mass density.

VARIABLE	DESCRIPTION
BULK	Elastic constant bulk modulus. If the bulk behavior is viscoelastic, then this modulus is used in determining the contact interface stiffness only.
CSE	Compressive stress flag (default = 0.0). EQ.0.0: don't eliminate compressive stresses EQ.1.0: eliminate compressive stresses
LCID	Load curve ID if constants, $G_i$ , and $\beta_i$ are determined via a least squares fit. This relaxation curve is shown below.
NT	Number of terms in shear fit. If zero the default is 6. Currently, the maximum number is set to 6.
BSTART	In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$ , $\beta_4$ is 10 times $\beta_3$ , and so on. If zero, BSTART = 0.01.
TRAMP	Optional ramp time for loading.
LCIDK	Load curve ID for bulk behavior if constants, $K_i$ , and $\beta_{\kappa_i}$ are determined via a least squares fit. This relaxation curve is shown below.
NTK	Number of terms desired in bulk fit. If zero the default is 6. Currently, the maximum number is set to 6.
BSTARTK	In the fit, $\beta_{\kappa_1}$ is set to zero, $\beta_{\kappa_2}$ is set to BSTARTK, $\beta_{\kappa_3}$ is 10 times $\beta_{\kappa_2}$ , $\beta_{\kappa_4}$ is 10 times $\beta_{\kappa_3}$ , and so on. If zero, BSTARTK = 0.01.
TRAMPK	Optional ramp time for bulk loading.
GI	Optional shear relaxation modulus for the ith term
BETAI	Optional shear decay constant for the ith term
KI	Optional bulk relaxation modulus for the ith term
BETAKI	Optional bulk decay constant for the ith term

**Remarks:**

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:



**Figure M134-1.** Stress Relaxation curve.

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  is the relaxation function. If we wish to include only simple rate effects for the deviatoric stresses, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^N G_m e^{-\beta_m t}$$

We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . An arbitrary number of terms, up to 6, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk moduli:

$$k(t) = \sum_{m=1}^N K_m e^{-\beta_{k_m} t}$$

For an example of a stress relaxation curve see [Figure M134-1](#). This curve defines stress versus time where time is defined on a logarithmic scale. For best results, the points defined in the load curve should be equally spaced on the logarithmic scale. Furthermore, the load curve should be smooth and defined in the positive quadrant. If nonphysical values are determined by least squares fit, LS-DYNA will terminate with an error message after the initialization phase is completed. If the ramp time for loading is included, then the relaxation which occurs during the loading phase is taken into account. This effect may or may not be important.



**\*MAT\_WTM\_STM**

This is material type 135. This anisotropic-viscoplastic material model adopts two yield criteria for metals with orthotropic anisotropy proposed by Barlat and Lian [1989] (Weak Texture Model) and Aretz [2004] (Strong Texture Model).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	NUMFI	EPSC	WC	TAUC
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	SIGMA0	QR1	CR1	QR2	CR2	K	LC	FLG
Type	F	F	F	F	F	F	F	F

**YLD2003 Card.** This card 3 format is used when FLG = 0.

Card 3	1	2	3	4	5	6	7	8
Variable	A1	A2	A3	A4	A5	A6	A7	A8
Type	F	F	F	F	F	F	F	F

**Yield Surface Card.** This card 3 format is used when FLG = 1.

Card 3	1	2	3	4	5	6	7	8
Variable	S00	S45	S90	SBB	R00	R45	R90	RBB
Type	F	F	F	F	F	F	F	F

**YLD89 Card.** This card 3 format used when FLG = 2.

Card 3	1	2	3	4	5	6	7	8
Variable	A	C	H	P				
Type	F	F	F	F				

Card 4	1	2	3	4	5	6	7	8
Variable	QX1	CX1	QX2	CX2	EDOT	M	EMIN	S100
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	AOPT	BETA						
Type	F	F						

Card 6	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label not exceeding 8 characters must be specified.

VARIABLE	DESCRIPTION
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
NUMFI	Number of through thickness integration points that must fail before the element is deleted (remember to change this number if switching between full and reduced integration type of elements).
EPSC	Critical value $\varepsilon_{tC}$ of the plastic thickness strain (used in the CTS fracture criterion).
WC	Critical value $W_c$ for the Cockcroft-Latham fracture criterion
TAUC	Critical value $\tau_c$ for the Bressan-Williams shear fracture criterion
SIGMA0	Initial mean value of yield stress $\sigma_0$ : GT.0.0: Constant value, LT.0.0: Load curve ID = -SIGMA0 which defines yield stress as a function of plastic strain. Hardening parameters QR1, CR1, QR2, and CR2 are ignored in that case.
QR1	Isotropic hardening parameter $Q_{R1}$
CR1	Isotropic hardening parameter $C_{R1}$
QR2	Isotropic hardening parameter $Q_{R2}$
CR2	Isotropic hardening parameter $C_{R2}$
K	$k$ equals half YLD2003 exponent $m$ . Recommended value for FCC materials is $m = 8$ , i.e. $k = 4$ .
LC	First load curve number for process effects, i.e. the load curve describing the relation between the pre-strain and the yield stress $\sigma_0$ . Similar curves for $Q_{R1}$ , $C_{R1}$ , $Q_{R2}$ , $C_{R2}$ , and $W_c$ must follow consecutively from this number.
A1	Yld2003 parameter $a_1$
A2	Yld2003 parameter $a_2$
A3	Yld2003 parameter $a_3$

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A4	Yld2003 parameter $a_4$
A5	Yld2003 parameter $a_5$
A6	Yld2003 parameter $a_6$
A7	Yld2003 parameter $a_7$
A8	Yld2003 parameter $a_8$
S00	Yield stress in $0^\circ$ direction
S45	Yield stress in $45^\circ$ direction
S90	Yield stress in $90^\circ$ direction
SBB	Balanced biaxial flow stress
R00	R-ratio in $0^\circ$ direction
R45	R-ratio in $45^\circ$ direction
R90	R-ratio in $90^\circ$ direction
RBB	Balance biaxial flow ratio
A	YLD89 parameter a
C	YLD89 parameter c
H	YLD89 parameter h
P	YLD89 parameter p
QX1	Kinematic hardening parameter $Q_{x1}$
CX1	Kinematic hardening parameter $C_{x1}$
QX2	Kinematic hardening parameter $Q_{x2}$
CX2	Kinematic hardening parameter $C_{x2}$
EDOT	Strain rate parameter $\dot{\epsilon}_0$
M	Strain rate parameter $m$

VARIABLE	DESCRIPTION
EMIN	<p>Lower limit of the isotropic hardening rate <math>\frac{dR}{d\bar{\epsilon}}</math>. This feature is included to model a non-zero and linear/exponential isotropic work hardening rate at large values of effective plastic strain. If the isotropic work hardening rate predicted by the utilized Voce-type work hardening rule falls below the specified value it is substituted by the prescribed value or switched to a power-law hardening if S100.NE.0. This option should be considered for problems involving extensive plastic deformations. If process dependent material characteristics are prescribed, i.e. if LC .GT. 0 the same minimum tangent modulus is assumed for all the prescribed work hardening curves. If instead EMIN.LT.0 then -EMIN defines the plastic strain value at which the linear or power-law hardening approximation commences.</p>
S100	<p>Yield stress at 100% strain for using a power-law approximation beyond the strain defined by EMIN.</p>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later..</p>
BETA	<p>Material angle in degrees for AOPT = 0 or 3, may be overwritten on the element card, see *ELEMENT_SHELL_BETA.</p>
XP YP ZP	<p>Coordinates of point p for AOPT = 1.</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A1 A2 A3	Components of vector <b>a</b> for AOPT = 2.
V1 V2 V3	Components of vector <b>v</b> for AOPT = 3
D1 D2 D3	Components of vector <b>d</b> for AOPT = 2.

**Remarks:**

If FLG = 1, i.e. if the yield surface parameters  $a_1$ - $a_8$  are identified on the basis of prescribed material data internally in the material routine, files with point data for plotting of the identified yield surface, along with the predicted directional variation of the yield stress and plastic flow are generated in the directory where the LS-DYNA analysis is run. Four different files are generated for each specified material.

These files are named according to the scheme:

1. Contour\_1#
2. Contour\_2#
3. Contour\_3#
4. R\_and\_S#

Where # is a value starting at 1.

The three first files contain contour data for plotting of the yield surface as shown in [Figure M135-2](#). To generate these plots a suitable plotting program should be adopted and for each file/plot, column A should be plotted vs. columns B. For a more detailed description of these plots it is referred to References. [Figure M135-3](#) further shows a plot generated from the final file named 'R\_and\_S#' showing the directional dependency of the normalized yield stress (column A vs. B) and plastic strain ratio (column B vs. C).

The yield condition for this material can be written

$$t(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \varepsilon^p, \dot{\varepsilon}^p) = \sigma_{\text{eff}}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) - \sigma_Y(\varepsilon^p, \dot{\varepsilon}^p)$$

where

$$\sigma_Y = [\sigma_0 + R(\varepsilon^p)] \left(1 + \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0}\right)^C$$

where the isotropic hardening reads

$$R(\varepsilon^p) = Q_{R1}[1 - \exp(-C_{R1}\varepsilon^p)] + Q_{R2}[1 - \exp(-C_{R2}\varepsilon^p)].$$

For the Weak Texture Model the yield function is defined as

$$\sigma_{\text{eff}} = \left[ \frac{1}{2} \{ a(k_1 + k_2)^m + a(k_1 - k_2)^m + C(2k_2)^m \} \right]^{\frac{1}{m}}$$

where

$$k_1 = \frac{\sigma_x + h \sigma_y}{2}$$

$$k_2 = \sqrt{\left( \frac{\sigma_x + h \sigma_y}{2} \right)^2 + (r \sigma_{xy})^2}.$$

For the Strong Texture Model the yield function is defined as

$$\sigma_{\text{eff}} = \left\{ \frac{1}{2} [(\sigma'_+)^m + (\sigma'_-)^m + (\sigma''_+ - \sigma''_-)^m] \right\}^{\frac{1}{m}}$$

where

$$\sigma'_{\pm} = \frac{a_8 \sigma_x + a_1 \sigma_y}{2} \pm \sqrt{\left( \frac{a_2 \sigma_x - a_3 \sigma_y}{2} \right)^2 + a_4^2 \sigma_{xy}^2}$$

$$\sigma''_{\pm} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left( \frac{a_5 \sigma_x - a_6 \sigma_y}{2} \right)^2 + a_7^2 \sigma_{xy}^2}.$$

Kinematic hardening can be included by

$$\alpha = \sum_{R=1}^2 \alpha_R$$

where each of the kinematic hardening variables  $\alpha_R$  is independent and obeys a nonlinear evolutionary equation in the form

$$\dot{\alpha}_R = C_{\alpha i} \left( Q_{\alpha i} \frac{\tau}{\sigma} - \alpha_R \right) \dot{\epsilon}^p$$

where the effective stress  $\bar{\sigma}$  is defined as

$$\bar{\sigma} = \sigma_{\text{eff}}(\tau)$$

where

$$\tau = \sigma - \alpha.$$

Critical thickness strain failure in a layer is assumed to occur when

$$\epsilon_t \leq \epsilon_{tc}$$

where  $\epsilon_{tc}$  is a material parameter. It should be noted that  $\epsilon_{tc}$  is a negative number (i.e. failure is assumed to occur only in the case of thinning).

Cockcraft and Latham fracture is assumed to occur when

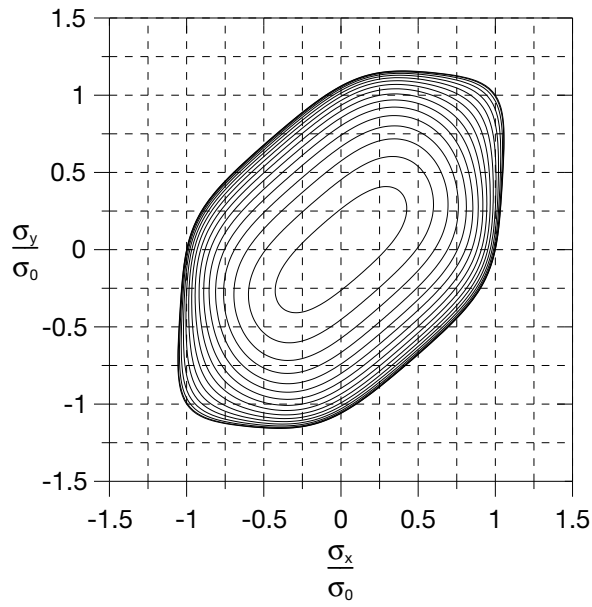
$$W = \int \max(\sigma_1, 0) d\epsilon^p \geq W_C$$

where  $\sigma_1$  is the maximum principal stress and  $W_C$  is a material parameter.

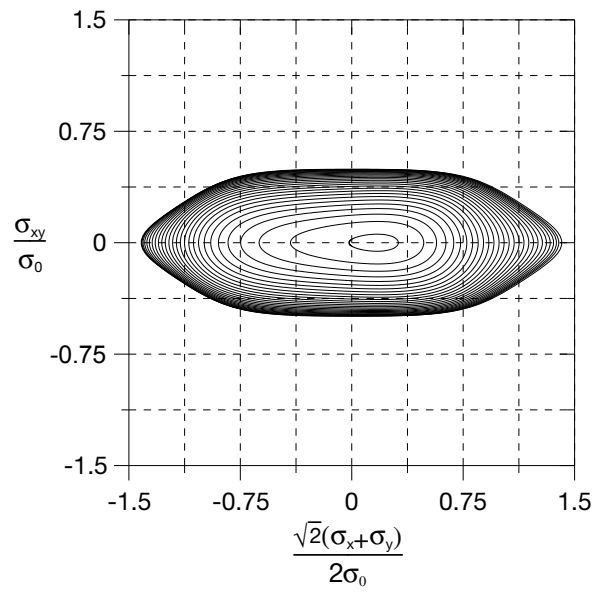
<i>History Variable</i>	<i>Description</i>
1	Isotropic hardening value $R_1$
2	Isotropic hardening value $R_2$
3	Increment in effective plastic strain $\Delta\bar{\epsilon}$
4	Not defined, for internal use in the material model
5	Not defined, for internal use in the material model
6	Not defined, for internal use in the material model
7	Failure in integration point EQ.0: No failure EQ.1: Failure due to EPSC, i.e. $\epsilon_t \geq \epsilon_{tc}$ . EQ.2: Failure due to WC, i.e. $W \geq W_C$ . EQ.3: Failure due to TAUC, i.e. $\tau \geq \tau_c$
8	Sum of incremental strain in local element x-direction: $\epsilon_{xx} = \sum \Delta\epsilon_{xx}$
9	Sum of incremental strain in local element y-direction: $\epsilon_{yy} = \sum \Delta\epsilon_{yy}$
10	Value of theh Cockcroft-Latham failure parameter $W = \sum \sigma_1 \Delta p$
11	Plastic strain component in thickness direction $\epsilon_t$
12	Mean value of increments in plastic strain through the thickness (For use with the non-local instability criterion. Note that constant lamella thickness is assumed and the instability criterion can give unrealistic results if used with a user-defined integration rule with varying lamella thickness.)
13	Not defined, for internal use in the material model
14	Nonlocal value $\rho = \frac{\Delta\epsilon_3}{\Delta\epsilon_3^{\Omega}}$

**Table M135-1.**

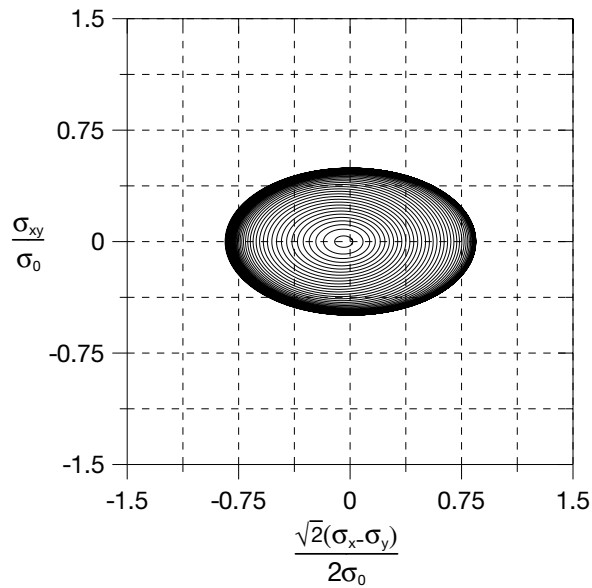




(A)

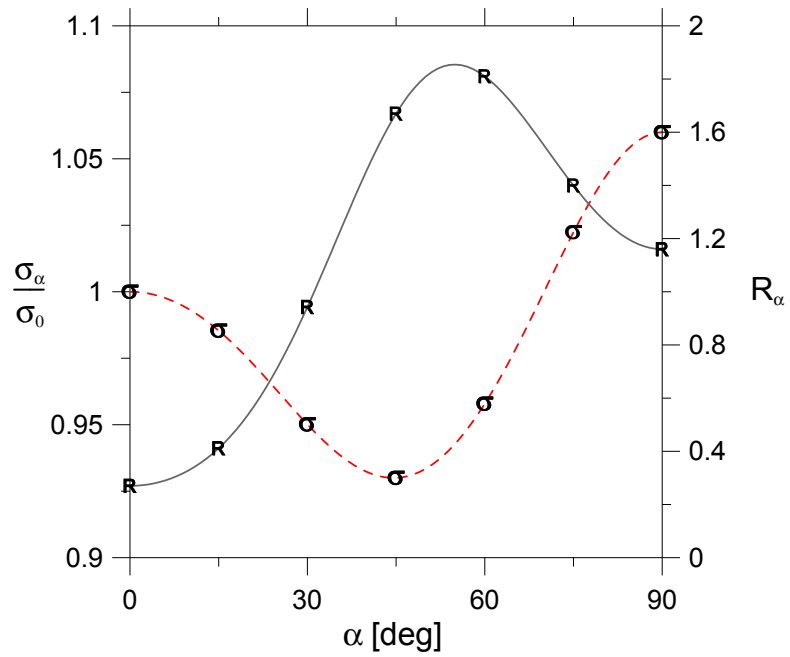


(B)



(C)

**Figure M135-2.** Contour plots of the yield surface generated from the files (a) 'Contour\_1<#>', (b) Contour\_2<#>', and (c) 'Contour\_3<#>'.



**Figure M135-3.** Predicted directional variation of the yield stress and plastic flow generated from the file 'R\_and\_S<#>'.

**\*MAT\_WTM\_STM\_PLC**

This is Material Type 135. This anisotropic material adopts the yield criteria proposed by Aretz [2004]. The material strength is defined by McCormick's constitutive relation for materials exhibiting negative steady-state Strain Rate Sensitivity (SRS). McCormick [1998] and Zhang, McCormick and Estrin [2001].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	NUMFI	EPSC	WC	TAUC
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	SIGMA0	QR1	CR1	QR2	CR2	K		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	A1	A2	A3	A4	A5	A6	A7	A8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	S	H	OMEGA	TD	ALPHA	EPS0		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	AOPT	BETA						
Type	F	F						

Card 6	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
NUMFI	Number of through thickness integration points that must fail before the element is deleted (remember to change this number if switching between full and reduced integration type of elements).
EPSC	Critical value $\epsilon_{tC}$ of the plastic thickness strain.
WC	Critical value $W_c$ for the Cockcroft-Latham fracture criterion.
TAUC	Critical value $\tau_c$ for the shear fracture criterion.
SIGMA0	Initial yield stress $\sigma_0$

VARIABLE	DESCRIPTION
QR1	Isotropic hardening parameter, $Q_{R1}$
CR1	Isotropic hardening parameter, $C_{R1}$
QR2	Isotropic hardening parameter, $Q_{R2}$
CR2	Isotropic hardening parameter, $C_{R2}$
K	$k$ equals half the exponent $m$ for the yield criterion
A1	Yld2003 parameter, $a_1$
A2	Yld2003 parameter, $a_2$
A3	Yld2003 parameter, $a_3$
A4	Yld2003 parameter, $a_4$
A5	Yld2003 parameter, $a_5$
A6	Yld2003 parameter, $a_6$
A7	Yld2003 parameter, $a_7$
A8	Yld2003 parameter, $a_8$
S	Dynamic strain aging parameter, $S$ .
H	Dynamic strain aging parameter, $H$ .
OMEGA	Dynamic strain aging parameter, $\Omega$ .
TD	Dynamic strain aging parameter, $t_d$ .
ALPHA	Dynamic strain aging parameter, $\alpha$ .
EPS0	Dynamic strain aging parameter, $\dot{\epsilon}_0$ .
AOPT	Material axes option (see Mat_OPTION TROPIC_ELASTIC for a more complete description) <p>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>, and then rotated about the shell element normal by the angle BETA. Nodes 1, 2 and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES.</p>

VARIABLE	DESCRIPTION
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: Locally orthotropic material axes determined by offsetting the material axes by an angle, BETA, from a line determined by taking the cross product of the vector $\mathbf{v}$ with the normal to the plane of the element.
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overwritten on the element card, see *ELEMENT_SHELL_BETA.
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.

**Remarks:**

The yield function is defined as

$$f = \bar{f}(\boldsymbol{\sigma}) - [\sigma_Y(t_a) + R(\varepsilon_p) + \sigma_v(\dot{\varepsilon}^p)]$$

where the equivalent stress  $\sigma_{eq}$  is defined as by an anisotropic yield criterion

$$\sigma_{eq} = \left[ \frac{1}{2} (|\sigma'_1|^m + |\sigma'_2|^m + |\sigma''_1 - \sigma''_2|) \right]^{\frac{1}{m}}$$

where

$$\begin{Bmatrix} \sigma'_1 \\ \sigma'_2 \end{Bmatrix} = \frac{a_8\sigma_{xx} + a_1\sigma_{yy}}{2} \pm \sqrt{\left(\frac{a_2\sigma_{xx} - a_3\sigma_{yy}}{2}\right)^2 + a_4^2\sigma_{xy}^2}$$

and

$$\begin{Bmatrix} \sigma''_1 \\ \sigma''_2 \end{Bmatrix} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{a_5\sigma_{xx} - a_6\sigma_{yy}}{2}\right)^2 + a_7^2\sigma_{xy}^2}$$

The strain hardening function  $R$  is defined by the extended Voce law

$$R(\varepsilon^p) = \sum_{i=1}^2 Q_{Ri} (1 - \exp(-C_{Ri} \varepsilon^p))$$

where  $\varepsilon^p$  is the effective (or accumulated) plastic strain, and  $Q_{Ri}$  and  $C_{Ri}$  are strain hardening parameters.

Viscous stress  $\sigma_v$  is given by

$$\sigma_v = (\dot{\varepsilon}^p) = s \ln \left( 1 + \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0} \right)$$

where  $S$  represents the instantaneous strain rate sensitivity (SRS) and  $\dot{\varepsilon}_0$  is a reference strain rate. In this model the yield strength, including the contribution from dynamic strain aging (DSA) is defined as

$$\sigma_Y(t_a) = \sigma_0 + SH \left[ 1 - \exp \left\{ - \left( \frac{t_a}{t_d} \right)^\alpha \right\} \right]$$

where  $\sigma_0$  is the yield strength for vanishing average waiting time,  $t_a$ , i.e. at high strain rates, and  $H$ ,  $\alpha$  and  $t_d$  are material constants linked to dynamic strain aging. It is noteworthy that  $\sigma_Y$  is an increasing function of  $t_a$ . The average waiting time is defined by the evolution equation

$$t_a = 1 - \frac{t_a}{t_{a,ss}}$$

where the quasi-steady waiting time  $t_{a,ss}$  is given as

$$t_{a,ss} = \frac{\Omega}{\dot{\varepsilon}^p}$$

where  $\Omega$  is the strain produced by all mobile dislocations moving to the next obstacle on their path.

**\*MAT\_CORUS\_VEGTER**

This is Material Type 136, a plane stress orthotropic material model for metal forming. Yield surface construction is based on the interpolation by second-order Bezier curves, and model parameters are determined directly from a set of mechanical tests conducted for a number of directions. For each direction, four mechanical tests are carried out: a uniaxial, an equi-biaxial, a plane strain tensile test and a shear test. These test results are used to determine the coefficients of the Fourier directional dependency field. For a more detailed description please see Vegter and Boogaard [2006].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	N	FBI	RB10	LCID
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	SYS	SIP	SHS	SHL	ESH	E0	ALPHA	LCID2
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT							
Type								

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		



Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

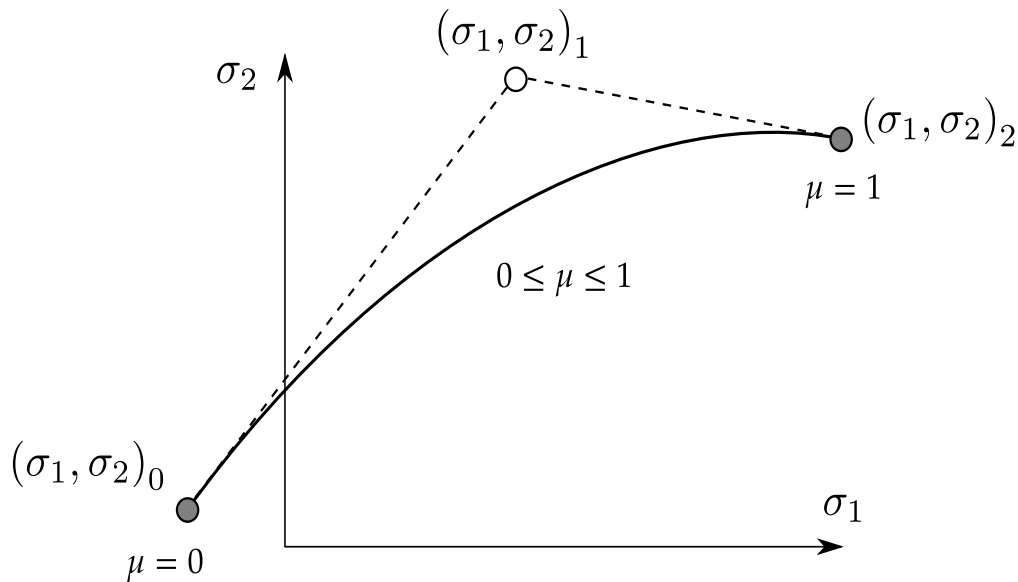
**Experimental Data Cards.** The next N cards (see N, card 1) contain experimental data obtained from four mechanical tests for a group of equidistantly placed directions  $\theta_i = \frac{i\pi}{2N}$ ,  $i = 0, 1, 2, \dots, N$ .

Card 6	1	2	3	4	5	6	7	8
Variable	FUN-I	RUN-I	FPS1-I	FPS2-I	FSH-I			
Type	F	F	F	F	F			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Material density
E	Elastic Young's modulus
PR	Poisson's ratio
N	N  is order of Fourier series (i.e., number of test groups minus one). The minimum number for  N  is 2, and the maximum is 12. GE.0.0: Explicit cutting-plane return mapping algorithm LT.0.0: Fully implicit return mapping algorithm (more robust)
FBI	Normalized yield stress $\sigma_{bi}$ for equi-biaxial test.
RBI0	Strain ratio $\rho_{bi}(0^\circ) = \dot{\epsilon}_2(0^\circ)/\dot{\epsilon}_1(0^\circ)$ for equi-biaxial test in the rolling direction.
LCID	Stress-strain curve ID. If defined, SYS, SIP, SHS, SHL, ESH, and E0 are ignored.
SYS	Static yield stress, $\sigma_0$ .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SIP	Stress increment parameter, $\Delta\sigma_m$ .
SHS	Strain hardening parameter for small strain, $\beta$ .
SHL	Strain hardening parameter for larger strain, $\Omega$ .
ESH	Exponent for strain hardening, $n$ .
E0	Initial plastic strain, $\epsilon_0$
ALPHA	$\alpha$ distribution of hardening used in the curve-fitting. $\alpha = 0$ pure kinematic hardening and $\alpha = 1$ provides pure isotropic hardening.
LCID2	Curve ID. The curve defines Young's modulus scaling factor with respect to the plastic strain. By default it is assumed that Young's modulus remains constant. Effective value is between 0 and 1.
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description): <ul style="list-style-type: none"> <li>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.</li> <li>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</li> <li>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</li> <li>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</li> </ul>
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.



**Figure M136-1.** Bézier interpolation curve.

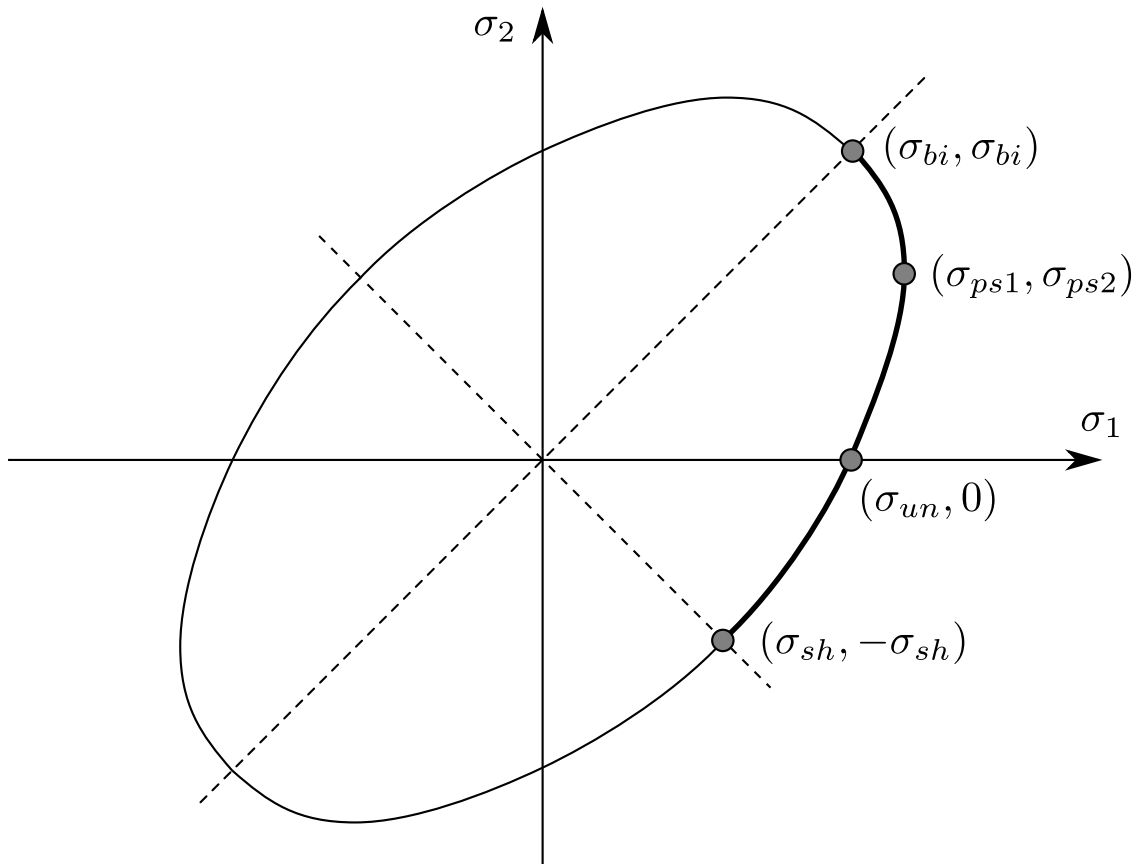
VARIABLE	DESCRIPTION
BETA	Material angle in degrees for AOPT = 0 and 3, may be overwritten on the element card, see *ELEMENT_SHELL_BETA.
FUN-I	Normalized yield stress $\sigma_{un}$ for uniaxial test for the ith direction.
RUN-I	Strain ratio (R-value) for uniaxial test for the ith direction.
FPS1-I	First normalized yield stress $\sigma_{ps1}$ for plain strain test for the ith direction.
FPS2-I	Second normalized yield stress $\sigma_{ps2}$ for plain strain test for the ith direction.
FSH-I	First normalized yield stress $\sigma_{sh}$ for pure shear test for the ith direction.

### Remarks:

The Vegter yield locus is section-wise defined by quadratic Bézier interpolation functions. Each individual curve uses 2 reference points and a hinge point in the principal plane stress space, see [Figure M136-1](#).

The mathematical description of the Bézier interpolation is given by:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_0 + 2\mu \left[ \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_1 - \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_0 \right] + \mu^2 \left[ \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_2 + \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_0 - 2 \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_1 \right]$$



**Figure M136-2.** Vegter yield surface.

where  $(\sigma_1, \sigma_2)_0$  is the first reference point,  $(\sigma_1, \sigma_2)_1$  is the hinge point, and  $(\sigma_1, \sigma_2)_2$  is the second reference point.  $\mu$  is a parameter which determines the location on the curve ( $0 \leq \mu \leq 1$ ).

Four characteristic stress states are selected as reference points: the equi-biaxial point  $(\sigma_{bi}, \sigma_{bi})$ , the plane strain point  $(\sigma_{ps1}, \sigma_{ps2})$ , the uniaxial point  $(\sigma_{un}, 0)$  and the pure shear point  $(\sigma_{sh}, -\sigma_{sh})$ , see Figure M136-2. Between the 4 stress points, 3 Bézier curves are used to interpolate the yield locus. Symmetry conditions are used to construct the complete surface. The yield locus in Figure M136-2 shows the reference points of experiments for one specific direction. The reference points can also be determined for other angles to the rolling direction (planar angle  $\theta$ ). E.g. if  $N = 2$  is chosen, normalized yield stresses for directions  $0^\circ$ ,  $45^\circ$ , and  $90^\circ$  should be defined. A Fourier series is used to interpolate intermediate angles between the measured points.

The Vegter yield function with isotropic hardening (ALPHA = 1) is given as:

$$\phi = \sigma_{eq}(\sigma_1, \sigma_2, \theta) - \sigma_y(\bar{\epsilon}^p)$$

with the equivalent stress  $\sigma_{eq}$  obtained from the appropriate Bézier function related to the current stress state. The uni-axial yield stress  $\sigma_y$  can be defined as stress-strain curve LCID or alternatively as a functional expression:

$$\sigma_y = \sigma_0 + \Delta\sigma_m \left[ \beta(\bar{\varepsilon}^p + \varepsilon_0) + \left(1 - e^{-\Omega(\bar{\varepsilon}^p + \varepsilon_0)}\right)^n \right]$$

In case of kinematic hardening ( $\text{ALPHA} < 1$ ), the standard stress tensor is replaced by a relative stress tensor, defined as the difference between the stress tensor and a back stress tensor.

To determine the yield stress or reference points of the Vegter yield locus, four mechanical tests have to be performed for different directions. A good description about the material characterization procedure can be found in Vegter et al. (2003).

**\*MAT\_COHESIVE\_MIXED\_MODE**

This is Material Type 138. This model is a simplification of \*MAT\_COHESIVE\_GENERAL restricted to linear softening. It includes a bilinear traction-separation law with quadratic mixed mode delamination criterion and a damage formulation. This material model can be used only with cohesive element formulations; see the variable ELFORM in \*SECTION\_SOLID and \*SECTION\_SHELL.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	EN	ET	GIC	GIIC
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	XMU	T	S	UND	UTD	GAMMA		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
ROFLG	Flag for whether density is specified per unit area or volume. ROFLG = 0 specified density per unit volume (default), and ROFLG = 1 specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero.
INTFAIL	The number of integration points required for the cohesive element to be deleted. If it is zero, the element will not be deleted even if it satisfies the failure criterion. The value of INTFAIL may range from 1 to 4, with 1 the recommended value.
EN	The stiffness (units of stress / length) normal to the plane of the cohesive element.
ET	The stiffness (units of stress / length) in the plane of the cohesive element.

VARIABLE	DESCRIPTION
GIC	Energy release rate for mode I (units of stress × length)
GIIC	Energy release rate for mode II (units of stress × length)
XMU	Exponent of the mixed mode criteria (see remarks below)
T	Peak traction (stress units) in normal direction LT.0.0: Load curve ID = (-T) which defines peak traction in normal direction as a function of element size. See remarks.
S	Peak traction (stress units) in tangential direction LT.0.0: Load curve ID = (-S) which defines peak traction in tangential direction as a function of element size. See remarks.
UND	Ultimate displacement in the normal direction
UTD	Ultimate displacement in the tangential direction
GAMMA	Additional exponent for Benzeggagh-Kenane law (default = 1.0)

**Remarks:**

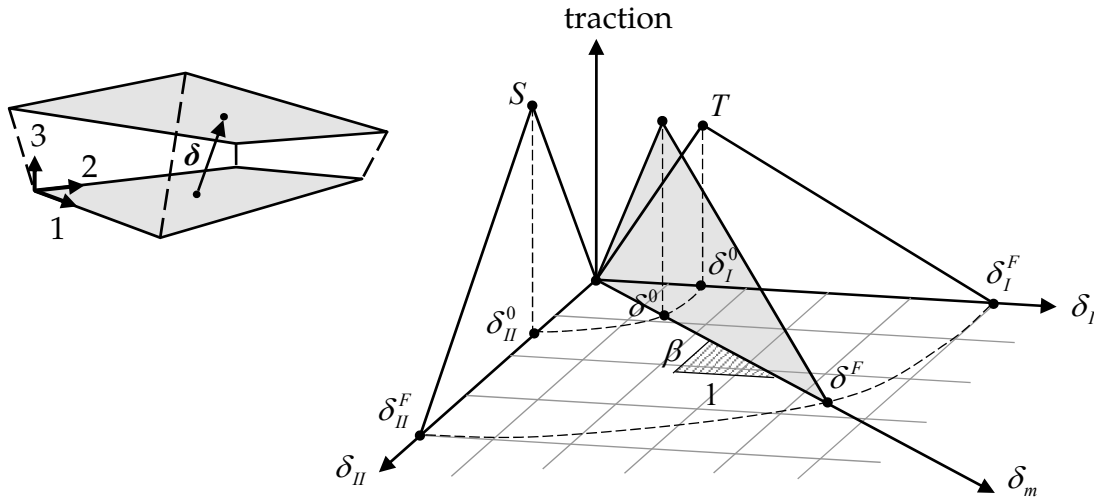
The ultimate displacements in the normal and tangential directions are the displacements at the time when the material has failed completely, i.e., the tractions are zero. The linear stiffness for loading followed by the linear softening during the damage provides an especially simple relationship between the energy release rates, the peak tractions, and the ultimate displacements:

$$GIC = T \times \frac{UND}{2}$$

$$GIIC = S \times \frac{UTD}{2}$$

If the peak tractions aren't specified, they are computed from the ultimate displacements. See Fiolka and Matzenmiller [2005] and Gerlach, Fiolka and Matzenmiller [2005].

In this cohesive material model, the total mixed-mode relative displacement  $\delta_m$  is defined as  $\delta_m = \sqrt{\delta_I^2 + \delta_{II}^2}$ , where  $\delta_I = \delta_3$  is the separation in normal direction (mode I) and  $\delta_{II} = \sqrt{\delta_1^2 + \delta_2^2}$  is the separation in tangential direction (mode II). The mixed-mode damage initiation displacement  $\delta^0$  (onset of softening) is given by



**Figure M138-1.** Mixed-mode traction-separation law

$$\delta^0 = \delta_I^0 \delta_{II}^0 \sqrt{\frac{1 + \beta^2}{(\delta_{II}^0)^2 + (\beta \delta_I^0)^2}}$$

where  $\delta_I^0 = T/EN$  and  $\delta_{II}^0 = S/ET$  are the single mode damage initiation separations and  $\beta = \delta_{II}/\delta_I$  is the “mode mixity” (see Figure M138-1). The ultimate mixed-mode displacement  $\delta^F$  (total failure) for the power law ( $XMU > 0$ ) is:

$$\delta^F = \frac{2(1 + \beta^2)}{\delta^0} \left[ \left( \frac{EN}{GIC} \right)^{XMU} + \left( \frac{ET \times \beta^2}{GIIC} \right)^{XMU} \right]^{-\frac{1}{XMU}}$$

and alternatively for the Benzeggagh-Kenane law [1996] ( $XMU < 0$ ):

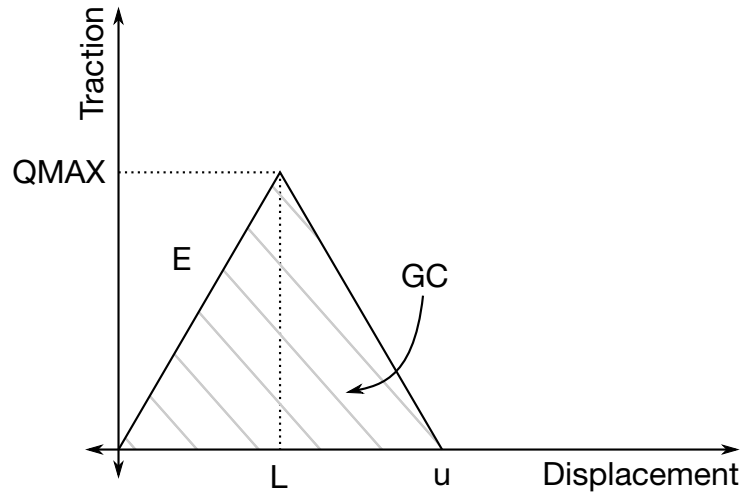
$$\delta^F = \frac{2}{\delta^0 \left( \frac{1}{1 + \beta^2} EN^\gamma + \frac{\beta^2}{1 + \beta^2} ET^\gamma \right)^{1/\gamma}} \left[ GIC + (GIIC - GIC) \left( \frac{\beta^2 \times ET}{EN + \beta^2 \times ET} \right)^{|XMU|} \right]$$

A reasonable choice for the exponent  $\gamma$  would be GAMMA = 1.0 (default) or GAMMA = 2.0.

In this model, damage of the interface is considered, i.e. irreversible conditions are enforced with loading/unloading paths coming from/pointing to the origin.

Peak tractions  $T$  and/or  $S$  can be defined as functions of characteristic element length (square root of midsurface area) via load curve. This option is useful to get nearly the same global responses (e.g. load-displacement curve) with coarse meshes when compared to a fine mesh solution. In general, lower peak traction values are needed for coarser meshes





**Figure M138-2.** Bilinear traction-separation

Two error checks have been implemented for this material model in order to ensure proper material data. Since the traction versus displacement curve is fairly simple (triangular shaped), equations can be developed to ensure that the displacement,  $L$ , at the peak load ( $Q_{MAX}$ ), is smaller than the ultimate distance for failure,  $u$ . See [Figure M138-2](#) for the used notation.

One has that

$$GC = \frac{1}{2} u \times Q_{MAX}$$

And,

$$L = \frac{Q_{MAX}}{E}.$$

To ensure that the peak is not past the failure point,  $\frac{u}{L}$  must be larger than 1.

$$u = \frac{2GC}{EL},$$

where  $GC$  is the energy release rate. This gives

$$\frac{u}{L} = \frac{2GC}{EL \times L} = \frac{2GC}{E \left( \frac{Q_{MAX}}{E} \right)^2} > 1.$$

The error checks are then done for tension and pure shear, respectively,

$$\frac{u}{L} = \frac{(2GIC)}{EN \left( \frac{T}{EN} \right)^2} > 1,$$

$$\frac{u}{L} = \frac{(2GIIC)}{ET \left( \frac{S}{ET} \right)^2} > 1.$$

**\*MAT\_MODIFIED\_FORCE\_LIMITED**

This is Material Type 139. This material for the Belytschko-Schwer resultant beam is an extension of material 29. In addition to the original plastic hinge and collapse mechanisms of material 29, yield moments may be defined as a function of axial force. After a hinge forms, the moment transmitted by the hinge is limited by a moment-plastic rotation relationship.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	DF	AOPT	YTFLAG	ASOFT
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	0.0	0.0	0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	M1	M2	M3	M4	M5	M6	M7	M8
Type	F	F	F	F	F	F	F	F
Default	none	0	0	0	0	0	0	0

Card 3	1	2	3	4	5	6	7	8
Variable	LC1	LC2	LC3	LC4	LC5	LC6	LC7	LC8
Type	F	F	F	F	F	F	F	F
Default	none	0	0	0	0	0	0	0

Card 4	1	2	3	4	5	6	7	8
Variable	LPS1	SFS1	LPS2	SFS2	YMS1	YMS2		
Type	F	F	F	F	F	F		
Default	0	1.0	LPS1	1.0	1.0E+20	YMS1		

Card 5	1	2	3	4	5	6	7	8
Variable	LPT1	SFT1	LPT2	SFT2	YMT1	YMT2		
Type	F	F	F	F	F	F		
Default	0	1.0	LPT1	1.0	1.0E+20	YMT1		

Card 6	1	2	3	4	5	6	7	8
Variable	LPR	SFR	YMR					
Type	F	F	F					
Default	0	1.0	1.0E+20					

Card 7	1	2	3	4	5	6	7	8
Variable	LYS1	SYS1	LYS2	SYS2	LYT1	SYT1	LYT2	SYT2
Type	F	F	F	F	F	F	F	F
Default	0	1.0	0	1.0	0	1.0	0	1.0





**\*MAT\_139****\*MAT\_MODIFIED\_FORCE\_LIMITED**

Card 16	1	2	3	4	5	6	7	8
Variable	LPMT2_1	LPMT2_2	LPMT2_3	LPMT2_4	LPMT2_5	LPMT2_6	LPMT2_7	LPMT2_8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 17	1	2	3	4	5	6	7	8
Variable	HMR_1	HMR_2	HMR_3	HMR_4	HMR_5	HMR_6	HMR_7	HMR_8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 18	1	2	3	4	5	6	7	8
Variable	LPMR_1	LPMR_2	LPMR_3	LPMR_4	LPMR_5	LPMR_6	LPMR_7	LPMR_8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
DF	Damping factor, see definition in notes below. A proper control for the timestep has to be maintained by the user!

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	Axial load curve option: EQ.0.0: axial load curves are force versus strain, EQ.1.0: axial load curves are force versus change in length. LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
YTFLAG	Flag to allow beam to yield in tension: EQ.0.0: beam does not yield in tension, EQ.1.0: beam can yield in tension.
ASOFT	Axial elastic softening factor applied once hinge has formed. When a hinge has formed the stiffness is reduced by this factor. If zero, this factor is ignored.
M1, M2, ..., M8	Applied end moment for force versus (strain/change in length) curve. At least one must be defined. A maximum of 8 moments can be defined. The values should be in ascending order.
LC1, LC2, ..., LC8	Load curve ID (see *DEFINE_CURVE) defining axial force versus strain/change in length (see AOPT) for the corresponding applied end moment. Define the same number as end moments. Each curve must contain the same number of points.
LPS1	Load curve ID for plastic moment versus rotation about s-axis at node 1. If zero, this load curve is ignored.
SFS1	Scale factor for plastic moment versus rotation curve about s-axis at node 1. Default = 1.0.
LPS2	Load curve ID for plastic moment versus rotation about s-axis at node 2. Default: is same as at node 1.
SFS2	Scale factor for plastic moment versus rotation curve about s-axis at node 2. Default: is same as at node 1.
YMS1	Yield moment about s-axis at node 1 for interaction calculations (default set to 1.0E+20 to prevent interaction).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
YMS2	Yield moment about s-axis at node 2 for interaction calculations (default set to YMS1).
LPT1	Load curve ID for plastic moment versus rotation about t-axis at node 1. If zero, this load curve is ignored.
SFT1	Scale factor for plastic moment versus rotation curve about t-axis at node 1. Default = 1.0.
LPT2	Load curve ID for plastic moment versus rotation about t-axis at node 2. Default: is the same as at node 1.
SFT2	Scale factor for plastic moment versus rotation curve about t-axis at node 2. Default: is the same as at node 1.
YMT1	Yield moment about t-axis at node 1 for interaction calculations (default set to 1.0E+20 to prevent interactions)
YMT2	Yield moment about t-axis at node 2 for interaction calculations (default set to YMT1)
LPR	Load curve ID for plastic torsional moment versus rotation. If zero, this load curve is ignored.
SFR	Scale factor for plastic torsional moment versus rotation (default = 1.0).
YMR	Torsional yield moment for interaction calculations (default set to 1.0E+20 to prevent interaction)
LYS1	ID of curve defining yield moment as a function of axial force for the s-axis at node 1.
SYS1	Scale factor applied to load curve LYS1.
LYS2	ID of curve defining yield moment as a function of axial force for the s-axis at node 2.
SYS2	Scale factor applied to load curve LYS2.
LYT1	ID of curve defining yield moment as a function of axial force for the t-axis at node 1.
SYT1	Scale factor applied to load curve LYT1.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
LYT2	ID of curve defining yield moment as a function of axial force for the t-axis at node 2.
SYT2	Scale factor applied to load curve LYT2.
LYR	ID of curve defining yield moment as a function of axial force for the torsional axis.
SYR	Scale factor applied to load curve LYR.
HMS1_n	Hinge moment for s-axis at node 1.
LPMS1_n	ID of curve defining plastic moment as a function of plastic rotation for the s-axis at node 1 for hinge moment HMS1_n
HMS2_n	Hinge moment for s-axis at node 2.
LPMS2_n	ID of curve defining plastic moment as a function of plastic rotation for the s-axis at node 2 for hinge moment HMS2_n
HMT1_n	Hinge moment for t-axis at node 1.
LPMT1_n	ID of curve defining plastic moment as a function of plastic rotation for the t-axis at node 1 for hinge moment HMT1_n
HMT2_n	Hinge moment for t-axis at node 2.
LPMT2_n	ID of curve defining plastic moment as a function of plastic rotation for the t-axis at node 2 for hinge moment HMT2_n
HMR_n	Hinge moment for the torsional axis.
LPMR_n	ID of curve defining plastic moment as a function of plastic rotation for the torsional axis for hinge moment HMR_n

**Remarks:**

This material model is available for the Belytschko resultant beam element only. Plastic hinges form at the ends of the beam when the moment reaches the plastic moment. The plastic moment versus rotation relationship is specified by the user in the form of a load curve and scale factor. The points of the load curve are (plastic rotation in radians, plastic moment). Both quantities should be positive for all points, with the first point being (zero, initial plastic moment). Within this constraint any form of characteristic may be used, including flat or falling curves. Different load curves and scale factors may be specified at each node and about each of the local s and t axes.

Axial collapse occurs when the compressive axial load reaches the collapse load. Collapse load versus collapse deflection is specified in the form of a load curve. The points of the load curve are either (true strain, collapse force) or (change in length, collapse force). Both quantities should be entered as positive for all points, and will be interpreted as compressive. The first point should be (zero, initial collapse load).

The collapse load may vary with end moment as well as with deflections. In this case several load-deflection curves are defined, each corresponding to a different end moment. Each load curve should have the same number of points and the same deflection values. The end moment is defined as the average of the absolute moments at each end of the beam and is always positive.

Stiffness-proportional damping may be added using the damping factor  $\lambda$ . This is defined as follows:

$$\lambda = \frac{2 \times \xi}{\omega}$$

where  $\xi$  is the damping factor at the reference frequency  $\omega$  (in radians per second). For example if 1% damping at 2Hz is required

$$\lambda = \frac{2 \times 0.01}{2\pi \times 2} = 0.001592$$

If damping is used, a small time step may be required. LS-DYNA does not check this so to avoid instability it may be necessary to control the time step via a load curve. As a guide, the time step required for any given element is multiplied by  $0.3L/c\lambda$  when damping is present (L = element length, c = sound speed).

### **Moment Interaction:**

Plastic hinges can form due to the combined action of moments about the three axes. This facility is activated only when yield moments are defined in the material input. A hinge forms when the following condition is first satisfied.

$$\left(\frac{M_r}{M_{r\text{yield}}}\right)^2 + \left(\frac{M_s}{M_{s\text{yield}}}\right)^2 + \left(\frac{M_t}{M_{t\text{yield}}}\right)^2 \geq 1$$

where,

$M_r, M_s, M_t$  = current moment

$M_{r\text{yield}}, M_{s\text{yield}}, M_{t\text{yield}}$  = yield moment

Note that scale factors for hinge behavior defined in the input will also be applied to the yield moments: for example,  $M_{s\text{yield}}$  in the above formula is given by the input yield moment about the local axis times the input scale factor for the local s axis. For strain-

softening characteristics, the yield moment should generally be set equal to the initial peak of the moment-rotation load curve.

On forming a hinge, upper limit moments are set. These are given by

$$M_{r_{upper}} = \max\left(M_{r'}, \frac{M_{r_{yield}}}{2}\right)$$

and similar conditions hold for  $M_{s_{upper}}$  and  $M_{t_{upper}}$ . Thereafter the plastic moments will be given by

$$M_{r_p} = \min(M_{r_{upper}}, M_{r_{curve}})$$

where,

$M_{r_p}$  = current plastic moment

$M_{r_{curve}}$  = moment from load curve at the current rotation scaled by the scale factor.

$M_{s_p}$  and  $M_{t_p}$  satisfy similar conditions.

The effect of this is to provide an upper limit to the moment that can be generated; it represents the softening effect of local buckling at a hinge site. Thus if a member is bent about its local s-axis it will then be weaker in torsion and about its local t-axis. For moments-softening curves, the effect is to trim off the initial peak (although if the curves subsequently harden, the final hardening will also be trimmed off).

It is not possible to make the plastic moment vary with the current axial load, but it is possible to make hinge formation a function of axial load and subsequent plastic moment a function of the moment at the time the hinge formed. This is discussed in the next section.

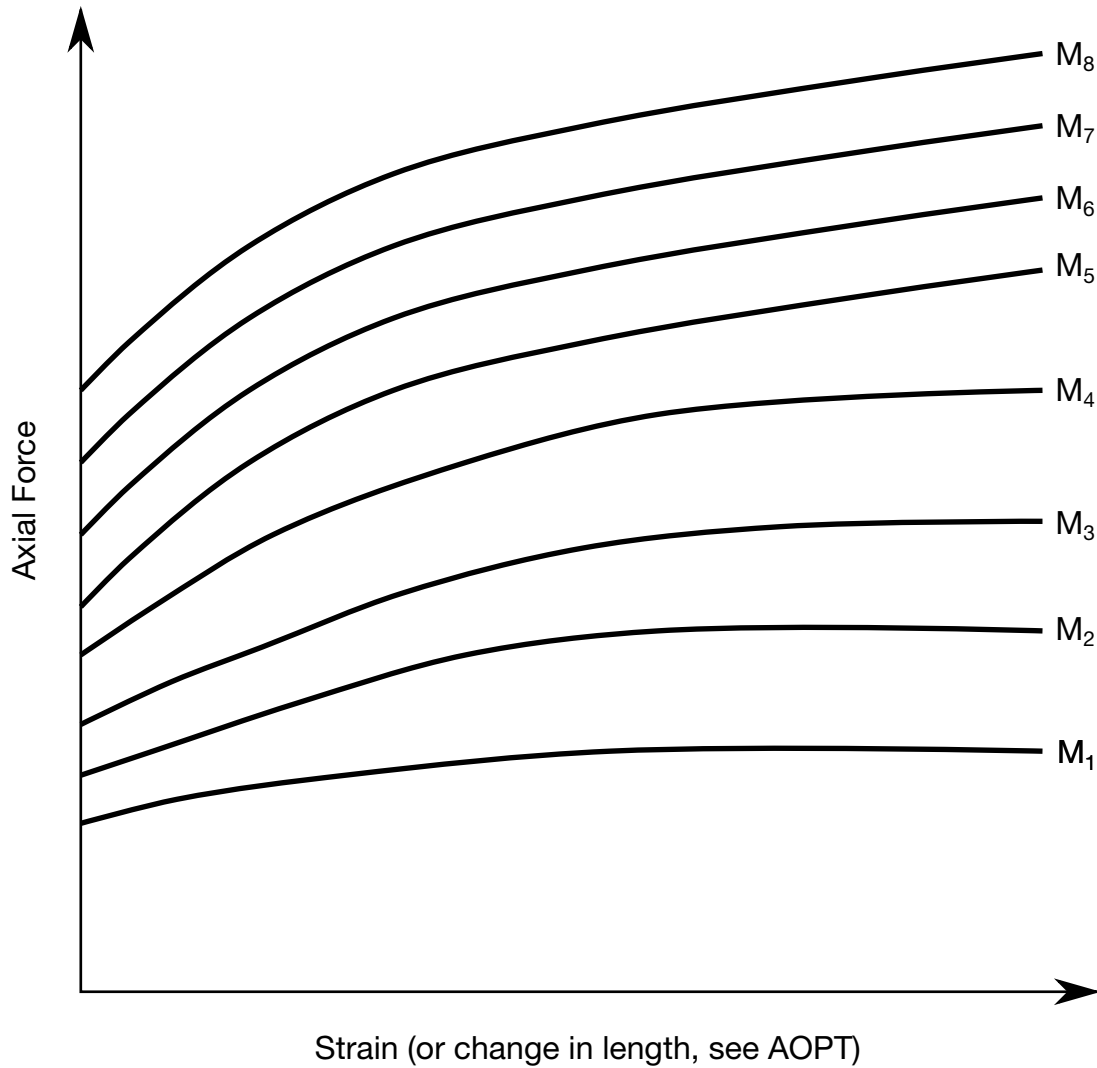
### **Independent plastic hinge formation:**

In addition to the moment interaction equation, Cards 7 through 18 allow plastic hinges to form independently for the s-axis and t-axis at each end of the beam and also for the torsional axis. A plastic hinge is assumed to form if any component of the current moment exceeds the yield moment as defined by the yield moment vs. axial force curves input on cards 7 and 8. If any of the 5 curves is omitted, a hinge will not form for that component. The curves can be defined for both compressive and tensile axial forces. If the axial force falls outside the range of the curve, the first or last point in the curve will be used. A hinge forming for one component of moment does not effect the other components.

Upon forming a hinge, the magnitude of that component of moment will not be permitted to exceed the current plastic moment.. The current plastic moment is obtained by interpolating between the plastic moment vs. plastic rotation curves input on cards 10, 12, 14, 16, or 18. Curves may be input for up to 8 hinge moments, where the hinge moment is defined as the yield moment at the time that the hinge formed. Curves must be input in

order of increasing hinge moment and each curve should have the same plastic rotation values. The first or last curve will be used if the hinge moment falls outside the range of the curves. If no curves are defined, the plastic moment is obtain from the curves on cards 4 through 6. The plastic moment is scaled by the scale factors on lines 4 to 6.

A hinge will form if either the independent yield moment is exceeded or if the moment interaction equation is satisfied. If both are true, the plastic moment will be set to the minimum of the interpolated value and  $M_{rp}$ .



**Figure M139-1.** The force magnitude is limited by the applied end moment. For an intermediate value of the end moment LS-DYNA interpolates between the curves to determine the allowable force value.

**\*MAT\_VACUUM**

This is Material Type 140. This model is a dummy material representing a vacuum in a multi-material Euler/ALE model. Instead of using ELFORM = 12 (under \*SECTION\_SOLID), it is better to use ELFORM = 11 with the void material defined as vacuum material instead.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RHO						
Type	A8	F						

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RHO	Estimated material density. This is used only as stability check.

**Remarks:**

1. The vacuum density is estimated. It should be small relative to air in the model (possibly at least  $10^3$  to  $10^6$  lighter than air).

**\*MAT\_RATE\_SENSITIVE\_POLYMER**

This is Material Type 141. This model, called the modified Ramaswamy-Stouffer model, is for the simulation of an isotropic ductile polymer with strain rate effects. See references; Stouffer and Dame [1996] and Goldberg and Stouffer [1999]. Uniaxial test data is used to fit the material parameters.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	Do	N	Zo	q
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	Omega							
Type	F							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Elastic modulus.
PR	Poisson's ratio
Do	Reference strain rate (= 1000 × max strain rate used in the test).
N	Exponent (see inelastic strain rate equation below)
Zo	Initial hardness of material
q	(see equations below).
Omega	Maximum internal stress.

**Remarks:**

The inelastic strain rate is defined as:

$$\dot{\epsilon}_{ij}^I = D_o \exp \left[ -0.5 \left( \frac{Z_o^2}{3K_2} \right)^N \right] \left( \frac{S_{ij} - \Omega_{ij}}{\sqrt{K_2}} \right)$$

where the  $K_2$  term is given as:

$$K_2 = 0.5(S_{ij} - \Omega_{ij})(S_{ij} - \Omega_{ij})$$

and represents the second invariant of the overstress tensor. The elastic components of the strain are added to the inelastic strain to obtain the total strain. The following relationship defines the back stress variable rate:

$$\dot{\Omega}_{ij} = \frac{2}{3}q\Omega_m\dot{\epsilon}_{ij}^I - q\Omega_{ij}\dot{\epsilon}_e^I$$

where  $q$  is a material constant,  $\Omega_m$  is a material constant that represents the maximum value of the internal stress, and  $\dot{\epsilon}_e^I$  is the effective inelastic strain rate.





Card 5	1	2	3	4	5	6	7	8
Variable	D1	D2	D3	V1	V2	V3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E11	Elastic modulus in axial direction.
E22	Elastic modulus in transverse direction (E22 = E33).
E12	Elastic shear modulus (E12 = E31).
E23	Elastic shear modulus in transverse plane.
G	Shear modulus.
K	Bulk modulus for contact stiffness.
I11	Load curve for nominal axial stress versus volumetric strain.
I22	Load curve ID for nominal transverse stresses versus volumetric strain (I22 = I33).
I12	Load curve ID for shear stress component 12 and 31 versus volumetric strain (I12 = I31).
I23	Load curve ID for shear stress component 23 versus volumetric strain.
IAA	Load curve ID (optional) for nominal stress versus volumetric strain for load at angle, ANG, relative to the material <i>a</i> -axis.
NSYM	Set to unity for a symmetric yield surface in volumetric compression and tension direction.
ANG	Angle corresponding to load curve ID, IAA.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MU	Damping coefficient for tensor viscosity which acts in both tension and compression. Recommended values vary between 0.05 to 0.10. If zero, tensor viscosity is not used, but bulk viscosity is used instead. Bulk viscosity creates a pressure as the element compresses that is added to the normal stresses, which can have the effect of creating transverse deformations when none are expected.
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>v</math> with the element normal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>v</math>, and an originating point, <math>P</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ISCL	Load curve ID for the strain rate scale factor versus the volumetric strain rate. The yield stress is scaled by the value specified by the load curve.
MACF	Material axes change flag: EQ.1: No change, default, EQ.2: switch material axes <b>a</b> and <b>b</b> , EQ.3: switch material axes <b>a</b> and <b>c</b> , EQ.4: switch material axes <b>b</b> and <b>c</b> .
XP YP ZP	Coordinates of point <b>p</b> for AOPT = 1 and 4.
A1 A2 A3	Components of vector <b>a</b> for AOPT = 2.
D1 D2 D3	Components of vector <b>d</b> for AOPT = 2.
V1 V2 V3	Define components of vector <b>v</b> for AOPT = 3 and 4.

**Remarks:**

This model behaves in a more physical way for off axis loading the material than, for example, \*MAT\_HONEYCOMB which can exhibit nonphysical stiffening for loading conditions that are off axis. The curves given for I11, I22, I12 and I23 are used to define a yield surface of Tsai-Wu-type that bounds the deviatoric stress tensor. Hence the elastic parameters E11, E12, E22 and E23 as well as G and K have to be defined in a consistent way.



**Figure M142-1.** Differences between options NSYM = 1 and NSYM = 0.

For the curve definitions volumetric strain  $\varepsilon_v = 1 - V/V_0$  is used as the abscissa parameter. If the symmetric option (NSYM = 1) is used, a curve for the first quadrant has to be given only. If NSYM = 0 is chosen, the curve definitions for I11, I22, I12 and I23 (and IAA) have to be in the first and second quadrant as shown in [Figure M142-1](#).

Tensor viscosity, which is activated by a nonzero value for MU, is generally more stable than bulk viscosity. A damping coefficient less than 0.01 has little effect, and a value greater than 0.10 may cause numerical instabilities.

\*MAT\_WOOD\_{OPTION}

This is Material Type 143. This is a transversely isotropic material and is available for solid elements. The user has the option of inputting his or her own material properties (<BLANK>), or requesting default material properties for Southern yellow pine (PINE) or Douglas fir (FIR). This model was developed by Murray [2002] under a contract from the FHWA.

Available options include:

<BLANK>

PINE

FIR

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	NPLOT	ITERS	IRATE	GHARD	IFAIL	IVOL
Type	A8	F	I	I	I	F	I	I

Card 2 for PINE and FIR keyword options.

Card 2	1	2	3	4	5	6	7	8
Variable	MOIS	TEMP	QUAL_T	QUAL_C	UNITS	IQUAL		
Type	F	F	F	F	I	I		

The following cards 2 through 6 are for option left blank.

Card 2	1	2	3	4	5	6	7	8
Variable	EL	ET	GLT	GTR	PR			
Type	F	F	F	F	F			

Card 3	1	2	3	4	5	6	7	8
Variable	XT	XC	YT	YC	SXY	SYZ		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable	GF1II	GF2II	BFIT	DMAXII	GF1 $\perp$	GF2 $\perp$	DFIT	DMAX $\perp$
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	FLPAR	FLPARC	POWPAR	FLPER	FLPERC	POWPER		
Type	F	F	F	F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	NPAR	CPAR	NPER	CPER				
Type	F	F	F	F				

The remaining cards *all* keyword options.

Card 7	1	2	3	4	5	6	7	8
Variable	AOPT	MACF	BETA					
Type	F	I	F					

Card 8	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 9	1	2	3	4	5	6	7	8
Variable	D1	D2	D3	V1	V2	V3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
NPLOT	Controls what is written as component 7 to the d3plot database. LS-PrePost always blindly labels this component as effective plastic strain.: EQ.1: Parallel damage (default). EQ.2: Perpendicular damage.
ITERS	Number of plasticity algorithm iterations. The default is one iteration.
IRATE	Rate effects option: EQ.0: Rate effects model turned off (default). EQ.1: Rate effects model turned on.
GHARD	Perfect plasticity override. Values greater than or equal to zero are allowed. Positive values model late time hardening in compression (an increase in strength with increasing strain). A zero value models perfect plasticity (no increase in strength with increasing strain). The default is zero.
IFAIL	Erosion perpendicular to the grain. EQ.0: No (default).

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	EQ.1: Yes (not recommended except for debugging).
IVOL	Flag to invoke erosion based on negative volume or strain increments greater than 0.01. EQ.0: No, do not apply erosion criteria. EQ.1: Yes, apply erosion criteria.
MOIS	Percent moisture content. If left blank, moisture content defaults to saturated at 30%.
TEMP	Temperature in °C. If left blank, temperature defaults to room temperature at 20 °C
QUAL_T	Quality factor options. These quality factors reduce the clear wood tension, shear, and compression strengths as a function of grade. EQ.0: Grade 1, 1D, 2, 2D. Predefined strength reduction factors are: Pine: QUAL_T = 0.47 in tension/shear. QUAL_C = 0.63 in compression. Fir: QUAL_T = 0.40 in tension/shear QUAL_C = 0.73 in compression. EQ.-1: DS-65 or SEI STR (pine and fir). Predefined strength reduction factors are: QUAL_T = 0.80 in tension/shear. QUAL_C = 0.93 in compression. EQ.-2: Clear wood. No strength reduction factors are applied: QUAL_T = 1.0. QUAL_C = 1.0. GT.0: User defined quality factor in tension. Values between 0 and 1 are expected. Values greater than one are allowed, but may not be realistic.
QUAL_C	User defined quality factor in compression. This input value is used if Qual_T > 0. Values between 0 and 1 are expected. Values greater than one are allowed, but may not be realistic. If left blank, a default value of Qual_C = Qual_T is used.



---

UNITS	Units options: EQ.0: GPa, mm, msec, Kg/mm <sup>3</sup> , kN. EQ.1: MPa, mm, msec, g/mm <sup>3</sup> , Nt. EQ.2: MPa, mm, sec, Mg/mm <sup>3</sup> , Nt. EQ.3: Psi, inch, sec, lb-s <sup>2</sup> /inch <sup>4</sup> , lb
IQUAL	Apply quality factors perpendicular to the grain: EQ.0: Yes (default). EQ.1: No.
EL	Parallel normal modulus
ET	Perpendicular normal modulus.
GLT	Parallel shear modulus (GLT = GLR).
GTR	Perpendicular shear modulus.
PR	Parallel major Poisson's ratio.
XT	Parallel tensile strength.
XC	Parallel compressive strength.
YT	Perpendicular tensile strength.
YC	Perpendicular compressive strength.
SXY	Parallel shear strength.
SYZ	Perpendicular shear strength.
GF1	Parallel fracture energy in tension.
GF2	Parallel fracture energy in shear.
BFIT	Parallel softening parameter.
DMAX	Parallel maximum damage.
GF1 <sup>⊥</sup>	Perpendicular fracture energy in tension.
GF2 <sup>⊥</sup>	Perpendicular fracture energy in shear.

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DFIT	Perpendicular softening parameter.
DMAX <sup>⊥</sup>	Perpendicular maximum damage.
FLPAR	Parallel fluidity parameter for tension and shear.
FLPARC	Parallel fluidity parameter for compression.
POWPAR	Parallel power.
FLPER	Perpendicular fluidity parameter for tension and shear.
FLPERC	Perpendicular fluidity parameter for compression.
POWPER	Perpendicular power.
NPAR	Parallel hardening initiation.
CPAR	Parallel hardening rate
NPER	Perpendicular hardening initiation.
CPER	Perpendicular hardening rate.
AOPT	Material axes option (see <i>MAT_OPTION TROPIC_ELASTIC</i> for a more complete description): <ul style="list-style-type: none"><li>EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by <i>*DEFINE_COORDINATE_NODES</i>.</li><li>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. This option is for solid elements only.</li><li>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with <i>*DEFINE_COORDINATE_VECTOR</i>.</li><li>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivi-</li></ul>

ty of the element, respectively.

EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector  $\mathbf{v}$ , and an originating point,  $P$ , which define the centerline axis. This option is for solid elements only.

LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on \*DEFINE\_COORDINATE\_NODES, \*DEFINE\_COORDINATE\_SYSTEM or \*DEFINE\_COORDINATE\_VECTOR). Available in R3 version of 971 and later.

MACF	Material axes change flag: EQ.1: No change, default, EQ.2: switch material axes a and b, EQ.3: switch material axes a and c, EQ.4: switch material axes b and c.
BETA	Material angle in degrees for AOPT = 3, may be overridden on the element card, see *ELEMENT_SOLID_ORTHO.
XP YP ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1 A2 A3	Components of vector $\mathbf{a}$ for AOPT = 2.
D1 D2 D3	Components of vector $\mathbf{d}$ for AOPT = 2.
V1 V2 V3	Define components of vector $\mathbf{v}$ for AOPT = 3 and 4.

**Remarks:**

Material property data is for clear wood (small samples without defects like knots), whereas real structures are composed of graded wood. Clear wood is stronger than graded wood. Quality factors (strength reduction factors) are applied to the clear wood strengths to account for reductions in strength as a function of grade. One quality factor (QUAL\_T) is applied to the tensile and shear strengths. A second quality factor (QUAL\_C) is applied to the compressive strengths. As an option, predefined quality factors are provided based on correlations between LS-DYNA calculations and test data for pine and fir posts impacted by bogie vehicles. By default, quality factors are applied to both the parallel and perpendicular to the grain strengths. An option is available (IQUAL) to eliminate application perpendicular to the grain.

**\*MAT\_PITZER\_CRUSHABLE\_FOAM**

This is Material Type 144. This model is for the simulation of isotropic crushable forms with strain rate effects. Uniaxial and triaxial test data have to be used. For the elastic response, the Poisson ratio is set to zero.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	G	PR	TY	SRTV	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	LCPY	LCUYS	LCSR	VC	DFLG			
Type	I	I	I	F	F			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
K	Bulk modulus.
G	Shear modulus
PR	Poisson's ratio
TY	Tension yield.
SRTV	Young's modulus (E)
LCPY	Load curve ID giving pressure versus volumetric strain, see <a href="#">Figure M75-1</a> .
LCUYS	Load curve ID giving uniaxial stress versus volumetric strain, see <a href="#">Figure M75-1</a> .
LCSR	Load curve ID giving strain rate scale factor versus volumetric strain rate.

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
VC	Viscous damping coefficient (.05 < recommended value < .50).
DFLG	Density flag: EQ.0.0: use initial density EQ.1.0: use current density (larger step size with less mass scaling).

**Remarks:**

The logarithmic volumetric strain is defined in terms of the relative volume,  $V$ , as:

$$\gamma = -\ln(V)$$

In defining the curves the stress and strain pairs should be positive values starting with a volumetric strain value of zero.



Card 5	1	2	3	4	5	6	7	8
Variable	FAILFL	DBETA	DDELTA	VPTAU				
Type	F	F	F	F				

Card 6	1	2	3	4	5	6	7	8
Variable	ALPHA1	THETA1	GAMMA1	BETA1	ALPHA2	THETA2	GAMMA2	BETA2
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
SHEAR	Shear modulus, G
BULK	Bulk modulus, K
GRUN	Gruneisen ratio (typically = 0), $\Gamma$
SHOCK	Shock velocity parameter (typically 0), $S_1$
PORE	Flag for pore collapse EQ.0.0: for Pore collapse EQ.1.0: for Constant bulk modulus (typical)
ALPHA	Shear failure parameter, $\alpha$
THETA	Shear failure parameter, $\theta$
GAMMA	Shear failure parameter, $\gamma$
BETA	Shear failure parameter, $\beta$
	$\sqrt{J'_2} = F_e(J_1) = \alpha - \gamma \exp(-\beta J_1) + \theta J_1$
EFIT	Dilatation damage mechanics parameter (no damage = 1)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FFIT	Dilatation damage mechanics parameter (no damage = 0)
ALPHAN	Kinematic strain hardening parameter, $N^\alpha$
CALPHAN	Kinematic strain hardening parameter, $c^\alpha$
R0	Initial cap surface ellipticity, R
X0	Initial cap surface $J_1$ (mean stress) axis intercept, $X(\kappa_0)$
IROCK	EQ.0: soils (cap can contract) EQ.1: rock/concrete
SECP	Shear enhanced compaction
AFIT	Ductile damage mechanics parameter (=1 no damage)
BFIT	Ductile damage mechanics parameter (=0 no damage)
RDAM0	Ductile damage mechanics parameter
W	Plastic Volume Strain parameter, W
D1	Plastic Volume Strain parameter, $D_1$
D2	Plastic Volume Strain parameter, $D_2$ $\varepsilon_V^P = W \left\{ 1 - \exp \left\{ -D_1 [X(\kappa) - X(\kappa_0)] - D_2 [(X(\kappa) - X(\kappa_0))]^2 \right\} \right\}$
NPLOT	History variable post-processed as effective plastic strain (See <a href="#">Table M145-1</a> for history variables available for plotting)
EPSMAX	Maximum permitted strain increment (default = 0) $\Delta \varepsilon_{\max} = 0.05(\alpha - N^\alpha - \gamma) \min \left( \frac{1}{G}, \frac{R}{9K} \right)$ (calculated default)
CFIT	Brittle damage mechanics parameter (=1 no damage)
DFIT	Brittle damage mechanics parameter (=0 no damage)
TFAIL	Tensile failure stress



VARIABLE	DESCRIPTION
FAILFL	Flag controlling element deletion and effect of damage on stress (see Remarks 1 and 2): EQ.1: $\sigma_{ij}$ reduces with increasing damage; element is deleted when fully damaged (default) EQ.-1: $\sigma_{ij}$ reduces with increasing damage; element is not deleted EQ.2: $S_{ij}$ reduces with increasing damage; element is deleted when fully damaged EQ.-2: $S_{ij}$ reduces with increasing damage; element is not deleted
DBETA	Rounded vertices parameter, $\Delta\beta_0$
DDELTA	Rounded vertices parameter, $\delta$
VPTAU	Viscoplasticity relaxation time parameter, $\tau$
ALPHA1	Torsion scaling parameter, $\alpha_1$ $\alpha_1 < 0 \rightarrow  \alpha_1  = \text{Friction Angle (degrees)}$
THETA1	Torsion scaling parameter, $\theta_1$
GAMMA1	Torsion scaling parameter, $\gamma_1$
BETA1	Torsion scaling parameter, $\beta_1$ $Q_1 = \alpha_1 - \gamma_1 \exp(-\beta_1 J_1) + \theta_1 J_1 \theta_2$
ALPHA2	Tri-axial extension scaling parameter, $\alpha_2$
THETA2	Tri-axial extension scaling parameter, $\theta_2$
GAMMA2	Tri-axial extension scaling parameter, $\gamma_2$
BETA2	Tri-axial extension scaling parameter, $\beta_2$ $Q_2 = \alpha_2 - \gamma_2 \exp(-\beta_2 J_1) + \theta_2 J_1$

**Remarks:**

1. FAILFL controls whether the damage accumulation applies to either the total stress tensor  $\sigma_{ij}$  or the deviatoric stress tensor  $S_{ij}$ . When FAILFL = 2, damage does not diminish the ability of the material to support hydrostatic stress.

2. FAILFL also serves as a flag to control element deletion. Fully damaged elements are deleted only if FAILFL is a positive value. When MAT\_145 is used with the ALE or EFG solvers, failed elements should not be eroded and so a negative value of FAILFL should be used.

### Output History Variables:

All the output parameters listed in [Table M145-1](#) is available for post-processing using LS-PrePost and its displayed list of History Variables. The LS-DYNA input parameter NEIPH should be set to 26; see for example the keyword input for \*DATABASE\_EXTENT\_BINARY.

<i>PLOT</i>	<i>Function</i>	<i>Description</i>
1	$X(\kappa)$	$J_1$ intercept of cap surface
2	$L(\kappa)$	$J_1$ value at cap-shear surface intercept
3	$R$	Cap surface ellipticity
4	$\tilde{R}$	Rubin function
5	$\epsilon_v^p$	Plastic volume strain
6		Yield Flag ( = 0 elastic)
7		Number of strain sub-increments
8	$G^\alpha$	Kinematic hardening parameter
9	$J_2^\alpha$	Kinematic hardening back stress
10		Effective strain rate
11		Ductile damage
12		Ductile damage threshold
13		Strain energy
14		Brittle damage
15		Brittle damage threshold
16		Brittle energy norm
17		$J_1$ (w/o visco-damage/plastic)
18		$J'_2$ (w/o visco-damage/plastic)
19		$J'_3$ (w/o visco-damage/plastic)
20		$\hat{J}_3$ (w/o visco-damage/plastic)
21	$\beta$	Lode Angle
22	d	Maximum damage parameter
23		future variable
24		future variable
25		future variable
26		future variable

**Table M145-1.** Output variables for post-processing using NPLOT parameter.

Sample Input for Concrete

Gran and Senseny [1996] report the axial stress versus strain response for twelve unconfined compression tests of concrete, used in scale-model reinforced-concrete wall tests. The Schwer & Murray Cap Model parameters provided below were used, see Schwer [2001], to model the unconfined compression test stress-strain response for the nominal 40 MPa strength concrete reported by Gran and Senseny. The basic units for the provided parameters are length in millimeters (mm), time in milliseconds (msec), and mass in grams (g). This base unit set yields units of force in Newtons (N) and pressure in Mega-Pascals (MPa).

Example MAT\_SCHWER\_MURRAY\_CAP\_MODEL deck

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	SHEAR	BULK	GRUN	SHOCK	PORE	
Value	A8	2.3E-3	1.048E4	1.168E4	0.0	0.0	1.	

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA	THETA	GAMMA	BETA	EFIT	FFIT	ALPHAN	CALPHA
Value	190.0	0.0	184.2	2.5E-3	0.999	0.7	2.5	2.5E3

Card 3	1	2	3	4	5	6	7	8
Variable	R0	X0	IROCK	SECP	AFIT	BFIT	RDAM0	
Value	5.0	100.0	1.0	0.0	0.999	0.3	0.94	

Card 4	1	2	3	4	5	6	7	8
Variable	W	D1	D2	NPLOT	EPSMAX	CFIT	DFIT	TFAIL
Value	5.0E-2	2.5E-4	3.5E-7	23.0	0.0	1.0	300.0	7.0

Card 5	1	2	3	4	5	6	7	8
Variable	FAILFG	DBETA	DDELTA	VPTAU				
Value	1.0	0.0	0.0	0.0				

Card 6	1	2	3	4	5	6	7	8
Variable	ALPHA1	THETA1	GAMMA1	BETA1	ALPHA2	THETA2	GAMMA2	BETA2
Value	0.747	3.3E-4	0.17	5.0E-2	0.66	4.0E-4	0.16	5.0E-2

### User Input Parameters and System of Units

Consider the following basic units:

Length:  $L$  (e.g. millimeters - mm )

Mass:  $M$  (e.g. grams - g )

Time:  $T$  (e.g. milliseconds - ms )

The following consistent unit systems can then be derived using Newton's Law, i.e.  $F = Ma$ .

Force:  $F = ML/T^2$  [ g-mm/ms<sup>2</sup> = Kg-m/s<sup>2</sup> = Newton - N ]

Stress:  $\sigma = F/L^2$  [ N/mm<sup>2</sup> = 10<sup>6</sup>N/m<sup>2</sup> = 10<sup>6</sup> Pascals = MPa ]

Density:  $\rho = M/L^3$  [ g/mm<sup>3</sup> = 10<sup>6</sup> Kg/m<sup>3</sup> ]

### User Inputs and Units

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	SHEAR	BULK	GRUN	SHOCK	PORE	
Units	I	Density M/L <sup>3</sup>	Stress: F/L <sup>2</sup>	Stress: F/L <sup>2</sup>				

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA	THETA	GAMMA	BETA	EFIT	FFIT	ALPHAN	CALPHA
Units	Stress: F/L <sup>2</sup>		Stress: F/L <sup>2</sup>	Stress <sup>-1</sup> : L <sup>2</sup> /F		Stress <sup>-1/2</sup> : L/F <sup>1/2</sup>	Stress: F/L <sup>2</sup>	Stress: F/L <sup>2</sup>

Card 3	1	2	3	4	5	6	7	8
Variable	R0	X0	IROCK	SECP	AFIT	BFIT	RDAM0	
Units		Stress: F/L <sup>2</sup>				Stress <sup>-1/2</sup> : L/F <sup>1/2</sup>	Stress <sup>1/2</sup> : F <sup>1/2</sup> /L	

Card 4	1	2	3	4	5	6	7	8
Variable	W	D1	D2	NPLOT	MAXEPS	CFIT	DFIT	TFAIL
Units		Stress <sup>-1</sup> : L <sup>2</sup> /F	Stress <sup>-2</sup> : L <sup>4</sup> /F <sup>2</sup>				Stress <sup>-1/2</sup> : L/F <sup>1/2</sup>	Stress: F/L <sup>2</sup>

Card 5	1	2	3	4	5	6	7	8
Variable	FAILFG	DBETA	DDELTA	VPTAU				
Units		Angle degrees		Time T				

Card 6	1	2	3	4	5	6	7	8
Variable	ALPHA1	THETA1	GAMMA1	BETA1	ALPHA2	THETA2	GAMMA2	BETA2
Units	Stress: F/L <sup>2</sup>		Stress: F/L <sup>2</sup>	Stress <sup>-1</sup> : L <sup>2</sup> /F	Stress: F/L <sup>2</sup>		Stress: F/L <sup>2</sup>	Stress <sup>-1</sup> : L <sup>2</sup> /F

## \*MAT\_146

## \*MAT\_1DOF\_GENERALIZED\_SPRING

### \*MAT\_1DOF\_GENERALIZED\_SPRING

This is Material Type 146. This is a linear spring or damper that allows different degrees-of-freedom at two nodes to be coupled.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	C	SCLN1	SCLN2	DOFN1	DOFN2
Type	A8	F	F	F	F	F	I	I

Card 2	1	2	3	4	5	6	7	8
Variable	CID1	CID2						
Type	I	I						

#### VARIABLE

#### DESCRIPTION

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
K	Spring stiffness.
C	Damping constant.
SCLN1	Scale factor on force at node 1. Default = 1.0.
SCLN2	Scale factor on force at node 2. Default = 1.0.
DOFN1	Active degree-of-freedom at node 1, a number between 1 to 6 where 1 is x-translation and 4 is x-rotation. If this parameter is defined in the SECTION_BEAM definition or on the ELEMENT_BEAM_SCALAR card, then the value here, if defined, is ignored.
DOFN2	Active degree-of-freedom at node 2, a number between 1 to 6. If this parameter is defined in the SECTION_BEAM definition or on the ELEMENT_BEAM_SCALAR card, then the value here, if defined, is ignored.

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
CID1	Local coordinate system at node 1. This coordinate system can be overwritten by a local system specified on the *ELEMENT_BEAM_-SCALAR or *SECTION_BEAM keyword input. If no coordinate system is specified, the global system is used.
CID2	Local coordinate system at node 2. If CID2 = 0, CID2 = CID1.

**\*MAT\_FHWA\_SOIL**

This is Material Type 147. This is an isotropic material with damage and is available for solid elements. The model has a modified Mohr-Coulomb surface to determine the pressure dependent peak shear strength. It was developed for applications involving roadbase soils by Lewis [1999] for the FHWA, who extended the work of Abbo and Sloan [1995] to include excess pore water effects.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	NPLOT	SPGRAV	RHOWAT	VN	GAMMAR	INTRMX
Type	A8	F	I	F	F	F	F	I
Default	none	none	1	none	1.0	0.0	0.0	1

Card 2	1	2	3	4	5	6	7	8
Variable	K	G	PHIMAX	AHYP	COH	ECCEN	AN	ET
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none	none	none

Card 3	1	2	3	4	5	6	7	8
Variable	MCONT	PWD1	PWKSK	PWD2	PHIRES	DINT	VDFM	DAMLEV
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	0.0	none	none	none



Card 4	1	2	3	4	5	6	7	8
Variable	EPSMAX							
Type	F							
Default	none							

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
NPLOT	Controls what is written as component 7 to the d3plot database. LS-PrePost always blindly labels this component as effective plastic strain. EQ.1: Effective Strain EQ.2: Damage Criterion Threshold EQ.3: Damage (diso) EQ.4: Current Damage Criterion EQ.5: Pore Water Pressure EQ.6: Current Friction Angle (phi)
SPGRAV	Specific Gravity of Soil used to get porosity.
RHOWATt	Density of water in model units - used to determine air void strain (saturation)
VN	Viscoplasticity parameter (strain-rate enhanced strength)
GAMMAr	Viscoplasticity parameter (strain-rate enhanced strength)
ITERMAXx	Maximum number of plasticity iterations (default 1)
K	Bulk Modulus (non-zero)
G	Shear modulus (non-zero)
PHIMAX	Peak Shear Strength Angle (friction angle) (radians)

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AHYP	Coefficient A for modified Drucker-Prager Surface
COH	Cohesion ñ Shear Strength at zero confinement (overburden)
ECCEN	Eccentricity parameter for third invariant effects
AN	Strain hardening percent of phi max where non-linear effects start
ET	Strain Hardening Amount of non-linear effects
MCONT	Moisture Content of Soil (Determines amount of air voids) (0.0 - 1.00)
PWD1	Parameter for pore water effects on bulk modulus
PWKSK	Skeleton bulk modulus- Pore water parameter ñ set to zero to eliminate effects
PWD2	Parameter for pore water effects on the effective pressure (confinement)
PHIRES	The minimum internal friction angle, radians (residual shear strength)
DINT	Volumetric Strain at Initial damage threshold, EMBED Equation.3
VDFM	Void formation energy (like fracture energy)
DAMLEV	Level of damage that will cause element deletion (0.0 - 1.00)
EPSMAX	Maximum principle failure strain

**\*MAT\_FHWA\_SOIL\_NEBRASKA**

This is an option to use the default properties determined for soils used at the University of Nebraska (Lincoln). The default units used for this material are millimeter, millisecond, and kilograms. If different units are desired, the conversion factors must be input.

This is Material Type 147. This is an isotropic material with damage and is available for solid elements. The model has a modified Mohr-Coulomb surface to determine the pressure dependent peak shear strength. It was developed for applications involving road base soils.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	FCTIM	FCTMAS	FCTLEN				
Type	A8	F	I	F	F	F	F	I
Default	none	none	1	none	1.0	0.0	0.0	1

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
FCTIM	Factor to multiply milliseconds by to get desired time units
FCTMAS	Factor to multiply kilograms by to get desired mass units
FCTLEN	Factor to multiply millimeters by to get desired length units

**Remarks:**

1. As an example, if time units of seconds are desired, then  $FCTIM = 0.001$



**Card 3 for Per mass Calculation.** Method (A) RUNIV = blank or 0.0.

Card 3	1	2	3	4	5	6	7	8
Variable	CPmass1	CPmass 2	CPmass 3	CPmass 4	CPmass 5	CPmass6	CPmass 7	CPmass 8
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

**Card 2 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

Card 2	1	2	3	4	5	6	7	8
Variable	MOLWT1	MOLWT2	MOLWT3	MOLWT4	MOLWT5	MOLWT6	MOLWT7	MOLWT8
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none
Remark	2							

**Card 3 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

Card 3	1	2	3	4	5	6	7	8
Variable	CPmole1	CPmole2	CPmole3	CPmole4	CPmole5	CPmole6	CPmole7	CPmole8
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none
Remark	2							

**Card 4 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

Card 4	1	2	3	4	5	6	7	8
Variable	B1	B2	B3	B4	B5	B6	B7	B8
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none
Remark	2							

**Card 5 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

Card 5	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none
Remark	2							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
IADIAB	This flag (default = 0) is used to turn ON/OFF adiabatic compression logs for an ideal gas (remark 5). EQ.0: OFF (default) EQ.1: ON
RUNIV	Universal gas constant in per-mole unit (8.31447 J/(mole*K)).
CVmass1 - CVmass8	If RUNIV is BLANK or zero (method A): Heat capacity at constant volume for up to eight different gases in per-mass unit.

$C_p(T)$	CPmole	$B$	$C$
$\frac{\text{J}}{\text{kg K}}$	$\frac{\text{J}}{\text{mole K}}$	$\frac{\text{J}}{\text{mole K}^2}$	$\frac{\text{J}}{\text{mole K}^3}$

Figure M148-1. Standard SI units.

VARIABLE	DESCRIPTION
CPmass1 - CPmass8	If RUNIV is BLANK or zero (method A): Heat capacity at constant pressure for up to eight different gases in per-mass unit.
MOLWT1 - MOLWT8	If RUNIV is nonzero (method B): Molecular weight of each ideal gas in the mixture (mass-unit/mole).
CPmole1 - CPmole8	If RUNIV is nonzero (method B): Heat capacity at constant pressure for up to eight different gases in per-mole unit. These are nominal heat capacity values typically at STP. These are denoted by the variable "A" in the equation in remark 2.
B1 - B8	If RUNIV is nonzero (method B): First order coefficient for a temperature dependent heat capacity at constant pressure for up to eight different gases. These are denoted by the variable "B" in the equation in remark 2.
C1 - C8	If RUNIV is nonzero (method B): Second order coefficient for a temperature dependent heat capacity at constant pressure for up to eight different gases. These are denoted by the variable "C" in the equation in remark 2.

**Remarks:**

- There are 2 methods of defining the gas properties for the mixture. If RUNIV is BLANK or ZERO → Method (A) is used to define constant heat capacities where per-mass unit values of  $C_v$  and  $C_p$  are input. Only cards 2 and 3 are required for this method. Method (B) is used to define constant or temperature dependent heat capacities where per-mole unit values of  $C_p$  are input. Cards 2 - 5 are required for this method.
- The per-mass-unit, temperature-dependent, constant-pressure heat capacity is

$$C_p(T) = \frac{[\text{CPmole} + B \times T + C \times T^2]}{\text{MOLWT}}$$

See [table M148-1](#).

3. The initial temperature and the density of the gas species present in a mesh or part at time zero is specified by the keyword `*INITIAL_GAS_MIXTURE`.
4. The ideal gas mixture is assumed to be thermal equilibrium, that is, all species are at the same temperature (T). The gases in the mixture are also assumed to follow Dalton's Partial Pressure Law,  $P = \sum_i^{\text{ngas}} P_i$ . The partial pressure of each gas is then  $P_i = \rho_i R_{\text{gas}_i} T$  where  $R_{\text{gas}_i} = \frac{R_{\text{univ}}}{MW}$ . The individual gas species temperature equals the mixture temperature. The temperature is computed from the internal energy where the *mixture internal energy per unit volume* is used,

$$e_V = \sum_i^{\text{ngas}} \rho_i C_{V_i} T_i = \sum_i^{\text{ngas}} \rho_i C_{V_i} T$$

$$T = T_i = \frac{e_V}{\sum_i^{\text{ngas}} \rho_i C_{V_i}}$$

In general, the advection step conserves momentum and internal energy, but not kinetic energy. This can result in energy lost in the system and lead to a pressure drop. In `*MAT_GAS_MIXTURE` the dissipated kinetic energy is automatically converted into heat (internal energy). Thus in effect the total energy is conserved instead of conserving just the internal energy. This numerical scheme has been shown to improve accuracy in some cases. However, the user should always be vigilant and check the physics of the problem closely.

5. As an example consider an airbag surrounded by ambient air. As the inflator gas flows into the bag, the ALE elements cut by the airbag fabric shell elements will contain some inflator gas inside and some ambient air outside. The multi-material element treatment is not perfect. Consequently the temperature of the outside air may be made artificially high after the multi-material element treatment. To prevent the outside ambient air from getting artificially high T, set `IDIAB = 1` for the ambient air outside. Simple adiabatic compression equation is then assumed for the outside air. The use of this flag may be needed, but only when that air is modeled by the `*MAT_GAS_MIXTURE` card.

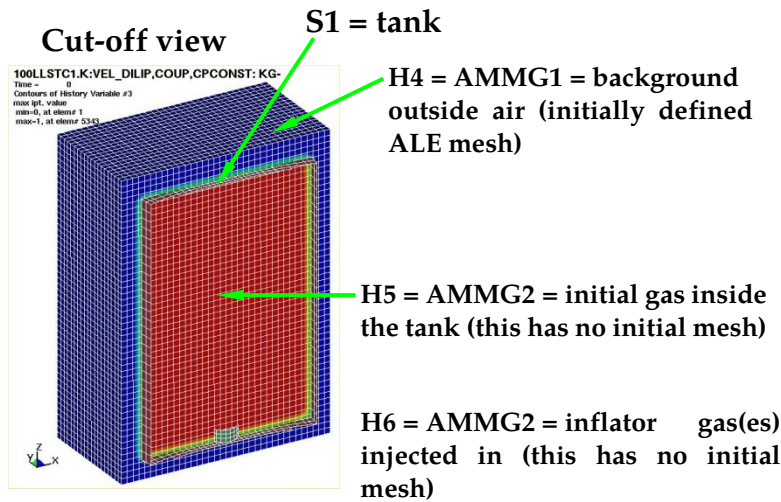
### Example:

Consider a tank test model where the Lagrangian tank (Part S1) is surrounded by an ALE air mesh (Part H4 = AMMGID 1). There are 2 ALE parts which are defined but initially have no corresponding mesh: part 5 (H5 = AMMGID 2) is the resident gas inside the tank at  $t = 0$ , and part 6 (H6 = AMMGID 2) is the inflator gas(es) which is injected into the tank when  $t > 0$ . AMMGID stands for ALE Multi-Material Group ID. Please see figure and input below. The `*MAT_GAS_MIXTURE` (MGM) card defines the gas properties of ALE parts H5 & H6. The MGM card input for both method (A) and (B) are shown.



The \*INITIAL\_GAS\_MIXTURE card is also shown. It basically specifies that "AMMGID 2 may be present in part or mesh H4 at t = 0, and the initial density of this gas is defined in the rho1 position which corresponds to the 1<sup>st</sup> material in the mixture (or H5, the resident gas)."

Example configuration:



Sample input:

```

$-----
*PART
H5 = initial gas inside the tank
$      PID      SECID      MID      EOSID      HGID      GRAV      ADPOPT      TMID
$      5         5         5         0         5         0         0
*SECTION_SOLID
$      5         11         0
$-----
$ Example 1: Constant heat capacities using per-mass unit.
$*MAT_GAS_MIXTURE
$      MID      IADIAB      R_univ
$      5         0         0
$      Cv1_mas  Cv2_mas  Cv3_mas  Cv4_mas  Cv5_mas  Cv6_mas  Cv7_mas  Cv8_mas
$718.7828911237.56228
$      Cp1_mas  Cp2_mas  Cp3_mas  Cp4_mas  Cp5_mas  Cp6_mas  Cp7_mas  Cp8_mas
$1007.00058 1606.1117
$-----
$ Example 2: Variable heat capacities using per-mole unit.
$*MAT_GAS_MIXTURE
$      MID      IADIAB      R_univ
$      5         0      8.314470
$      MW1      MW2      MW3      MW4      MW5      MW6      MW7      MW8
$      0.0288479 0.02256
$      Cp1_mol  Cp2_mol  Cp3_mol  Cp4_mol  Cp5_mol  Cp6_mol  Cp7_mol  Cp8_mol
$      29.049852 36.23388
$      B1         B2         B3         B4         B5         B6         B7         B8
$      7.056E-3 0.132E-1
$      C1         C2         C3         C4         C5         C6         C7         C8
$      -1.225E-6 -0.190E-5
$-----
$ One card is defined for each AMMG that will occupy some elements of a mesh set

```

# \*MAT\_148

# \*MAT\_GAS\_MIXTURE

\*INITIAL\_GAS\_MIXTURE

\$	SID	STYPE	MMGID	T0				
	4	1	1	298.15				
\$	RHO1	RHO2	RHO3	RHO4	RHO5	RHO6	RHO7	RHO8

1.17913E-9

\*INITIAL\_GAS\_MIXTURE

\$	SID	STYPE	MMGID	T0				
	4	1	2	298.15				
\$	RHO1	RHO2	RHO3	RHO4	RHO5	RHO6	RHO7	RHO8

1.17913E-9

-----



Card 6	1	2	3	4	5	6	7	8
Variable	C2ND	C3	C4	C5	C6	C7ND	C8ND	C9ND
Type	F	F	F	F	F	F	F	F

Card 7	1	2	3	4	5	6	7	8
Variable	C10	A1	A2	A3	A4	A_XX	A_YY	A_ZZ
Type	F	F	F	F	F	F	F	F

Card 8	1	2	3	4	5	6	7	8
Variable	A_XY	A_YZ	A_XZ	ALPHXX	ALPHYY	ALPHZZ	ALPHXY	ALPHYZ
Type	F	F	F	F	F	F	F	F

Card 9	1	2	3	4	5	6	7	8
Variable	ALPHXZ	DKAPPA	PHIO	PHICR	DLBDAG	FACTOR	RSWTCH	DMGOPT
Type	F	F	F	F	F	F	F	F

Card 10	1	2	3	4	5	6	7	8
Variable	DELASO	DIMPLO	ATOL	RTOL	DINTER			
Type	F	F	F	F	F			

Leave this card blank (but include it!).

Card 11	1	2	3	4	5	6	7	8
Variable								
Type								

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RHO	Material density.
E	Young's modulus
PR	Poisson's ratio
RGAS	universal gas constant.
BVECT	Burger's vector
D0	pre-exponential diffusivity coefficient
QD	activation energy
CV	specific heat at constant volume
ADRAG	drag intercept
BDRAG	drag coefficient
DMTHTA	shear modulus temperature coefficient
DMPHI	shear modulus damage coefficient
DNTHTA	bulk modulus temperature coefficient
DNPFI	bulk modulus damage coefficient
THETA0	reference temperature
THETAM	melt temperature
BETA0	coefficient of thermal expansion at reference temperature

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BTHETA	thermal expansion temperature coefficient
DMR	damage rate sensitivity parameter
DNUC1	nucleation coefficient 1
DNUC2	nucleation coefficient 2
DNUC3	nucleation coefficient 3
DNUC4	nucleation coefficient 4
DM1	coefficient of yield temperature dependence
DM2	coefficient of yield temperature dependence
DM3	coefficient of yield temperature dependence
DM4	coefficient of yield temperature dependence
DM5	coefficient of yield temperature dependence
Q1ND	dimensionless activation energy for f
Q2ND	dimensionless activation energy for rd
Q3ND	dimensionless activation energy for Rd
Q4ND	dimensionless activation energy Rs
CALPHA	coefficient for backstress alpha
CKAPPA	coefficient for internal stress kappa
C1	parameter for flow rule exponent n
C2ND	parameter for transition rate f
C3	parameter for alpha dynamic recovery rd
C4	parameter for alpha hardening h
C5	parameter for kappa dynamic recovery Rd
C6	parameter for kappa hardening H
C7ND	parameter kappa static recovery Rs

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
C8ND	parameter for yield
C9ND	parameter for temperature dependence of flow rule exponent n
C10	parameter for static recovery (set = 1)
A1	plastic anisotropy parameter
A2	plastic anisotropy parameter
A3	plastic anisotropy parameter
A4	plastic anisotropy parameter
A_XX	initial structure tensor component
A_YY	initial structure tensor component
A_ZZ	initial structure tensor component
A_XY	initial structure tensor component
A_YZ	initial structure tensor component
A_XZ	initial structure tensor component
ALPHXX	initial backstress component
ALPHYY	initial backstress component
ALPHZZ	initial backstress component
ALPHXY	initial backstress component
ALPHYZ	initial backstress component
ALPHXZ	initial backstress component
DKAPPA	initial isotropic internal stress
PHI0	initial isotropic porosity
PHICR	critical cutoff porosity
DLBDAG	slip system geometry parameter
FACTOR	fraction of plastic work converted to heat, adiabatic

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RSWTCH	rate sensitivity switch
DMGOPT	Damage model option parameter EQ.1.0: pressure independent Cocks/Ashby 1980 EQ.2.0: pressure dependent Cocks/Ashby 1980 EQ.3.0: pressure dependent Cocks 1989
DELASO	Temperature option EQ.0.0: driven externally EQ.1.0: adiabatic
DIMPLO	Implementation option flag EQ.1.0: combined viscous drag and thermally activated dislocation motion EQ.2.0: separate viscous drag and thermally activated dislocation motion
ATOL	absolute error tolerance for local Newton iteration
RTOL	relative error tolerance for local Newton iteration
DNITER	maximum number of iterations for local Newton iteration

**Remarks:**

$$\dot{\bar{\alpha}} = h \mathbf{d}^p - r_d \dot{\bar{\epsilon}}^p \bar{\alpha} \alpha$$

$$\dot{\kappa} = (H - R_d \kappa) \dot{\bar{\epsilon}}^p - R_s \kappa \sinh(Q_s \kappa)$$

$$\mathbf{d}^p = \sqrt{\frac{3}{2}} \dot{\bar{\epsilon}}^p \mathbf{n}, \dot{\bar{\epsilon}}^p = f \sinh^n \left[ \left\langle \frac{\bar{\sigma}}{\kappa + Y} - 1 \right\rangle \right]$$



$\dot{\varepsilon}^p$ – equation	$\alpha$ – equation	$\kappa$ – equation
$f = c_2 \exp\left(\frac{Q_1}{\theta}\right)$	$r_d = c_3 \exp\left(\frac{-Q_2}{\theta}\right)$	$R_d = c_5 \exp\left(\frac{-Q_3}{\theta}\right)$
$n = \frac{c_9}{\theta} - c_1$	$h = c_4 \hat{\mu}(\theta)$	$H = c_6 \hat{\mu}(\theta)$
$Y = c_8 \hat{Y}(\theta)$		$R_s = c_7 \exp\left(\frac{-Q_4}{\theta}\right)$
		$Q_s = c_{10} \exp\left(\frac{-Q_5}{\theta}\right)$

**Table M151-1.** Plasticity Material Functions of EMMI Model.

Void growth:

$$\dot{\varphi} = \frac{3}{\sqrt{2}} (1 - \varphi) \hat{G}(\bar{\sigma}_{eq}, \bar{p}, \varphi) \dot{\varepsilon}^p$$

$$\hat{G}(\bar{\sigma}_{eq}, \bar{p}, \varphi) = \frac{3}{\sqrt{3}} \left[ \frac{1}{(1 - \varphi)m + 1} - 1 \right] \sinh \left[ \frac{2(2m - 1) \langle \bar{p} \rangle}{2m + 1} \frac{1}{\bar{\sigma}_{eq}} \right]$$

**\*MAT\_DAMAGE\_3**

This is Material Type 153. This model has two back stress terms for kinematic hardening combined with isotropic hardening and a damage model for modeling low cycle fatigue and failure. Huang [2006] programmed this model and provided it as a user subroutine with the documentation that follows. It is available for beam, shell and solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	HARDI	BETA	LCSS
Type	A8	F	F	F	F	F	F	I

Card 2	1	2	3	4	5	6	7	8
Variable	HARDK1	GAMMA1	HARDK2	GAMMA2	SRC	SRP	HARDK3	GAMMA3
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	IDAM	IDS	IDEP	EPSD	S	T	DC	
Type	I	I	I	F	F	F	F	

## Optional Card 4

Card 4	1	2	3	4	5	6	7	8
Variable	HARDK4	GAMMA4						
Type	F	F						

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label not exceeding 8 characters must be specified.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RO	Mass density, $\rho$
E	Young's modulus, $E$
PR	Poisson's ratio, $\nu$
SIGY	Initial yield stress, $\sigma_{y0}$ (ignored if LCSS.GT.0)
HARDI	Isotropic hardening modulus, $H$ (ignored if LCSS.GT.0)
BETA	Isotropic hardening parameter, $\beta$ . Set $\beta = 0$ for linear isotropic hardening. (Ignored if LCSS.GT.0 or if HARDI.EQ.0.)
LCSS	Load curve ID defining effective stress vs. effective plastic strain for isotropic hardening. The first abscissa value must be zero corresponding to the initial yield stress. The first ordinate value is the initial yield stress.
HARDK1	Kinematic hardening modulus $C_1$
GAMMA1	Kinematic hardening parameter $\gamma_1$ . Set $\gamma_1 = 0$ for linear kinematic hardening. Ignored if (HARDK1.EQ.0) is defined.
HARDK2	Kinematic hardening modulus $C_2$
GAMMA2	Kinematic hardening parameter $\gamma_2$ . Set $\gamma_2 = 0$ for linear kinematic hardening. Ignored if (HARDK2.EQ.0) is defined.
SRC	Strain rate parameter, $C$ , for Cowper Symonds strain rate model, see below. If zero, rate effects are not considered.
SRP	Strain rate parameter, $P$ , for Cowper Symonds strain rate model, see below. If zero, rate effects are not considered.
HARDK3	Kinematic hardening modulus $C_3$
GAMMA3	Kinematic hardening parameter $\gamma_3$ . Set $\gamma_3 = 0$ for linear kinematic hardening. Ignored if (HARDK3.EQ.0) is defined.
IDAM	Isotropic damage flag EQ.0: damage is inactivated. IDS, IDEP, EPSD, S, T, DC are ignored. EQ.1: damage is activated

VARIABLE	DESCRIPTION
IDS	Output stress flag EQ.0: undamaged stress is $\tilde{\sigma}$ output EQ.1: damaged stress is $\tilde{\sigma}(1 - D)$ output
IDEP	Damaged plastic strain EQ.0: plastic strain is accumulated $r = \int \dot{\epsilon}^{pl}$ EQ.1: damaged plastic strain is accumulated $r = \int (1 - D)\dot{\epsilon}^{pl}$
EPSD	Damage threshold $r_d$ . Damage accumulation begins when $r > r_d$
S	Damage material constant $S$ . Default = $\sigma_{y0}/200$
T	Damage material constant $t$ . Default = 1
DC	Critical damage value $D_c$ . When damage value reaches critical, the element is deleted from calculation. Default = 0.5
HARDK4	Kinematic hardening modulus $C_4$
GAMMA4	Kinematic hardening parameter $\gamma_4$ . Set $\gamma_4 = 0$ for linear kinematic hardening. Ignored if (HARDK4.EQ.0) is defined.

**Remarks:**

This model is based on the work of Lemaitre [1992], and Dufailly and Lemaitre [1995]. It is a pressure-independent plasticity model with the yield surface defined by the function

$$F = \bar{\sigma} - \sigma_y = 0$$

where  $\sigma_v$  is uniaxial yield stress

$$\sigma_y = \sigma_{y0} + \frac{H}{\beta} [1 - \exp(-\beta r)]$$

By setting  $\beta = 0$ , a linear isotropic hardening is obtained

$$\sigma_y = \sigma_{y0} + Hr$$

where  $\sigma_{v0}$  is the initial yield stress. And  $\bar{\sigma}$  is the equivalent von Mises stress, with respect to the deviatoric effective stress

$$s_e = dev[\tilde{\sigma}] - \alpha = \mathbf{s} - \alpha$$

where  $\mathbf{s}$  is deviatoric stress and  $\alpha$  is the back stress, which is decomposed into several components

$$\alpha = \sum_j \alpha_j$$

and  $\tilde{\sigma}$  is effective stress (undamaged stress), based on Continuum Damage Mechanics model [Lemaitre 1992]

$$\tilde{\sigma} = \frac{\sigma}{1 - D}$$

where  $D$  is the isotropic damage scalar, which is bounded by 0 and 1

$$0 \leq D \leq 1$$

$D = 0$  represents a damage-free material RVE (representative volume element), while  $D = 1$  represents a fully broken material RVE in two parts. In fact, fracture occurs when  $D = D_c < 1$ , modeled as element removal. The evolution of the isotropic damage value related to ductile damage and fracture (the case where the plastic strain or dissipation is much larger than the elastic one, [Lemaitre 1992]) is defined as

$$\dot{D} = \begin{cases} \left(\frac{Y}{S}\right)^t \dot{\epsilon}^{pl} & r > r_d \& \frac{\sigma_m}{\sigma_{eq}} > -\frac{1}{3} \\ 0 & \text{otherwise} \end{cases}$$

where  $\frac{\sigma_m}{\sigma_{eq}}$  is the stress triaxiality,  $r_d$  is damage threshold,  $S$  is a material constant, and  $Y$  is strain energy density release rate.

$$Y = \frac{1}{2} \boldsymbol{\epsilon}^{el} : \mathbf{D}^{el} : \boldsymbol{\epsilon}^{el}$$

Where  $\mathbf{D}^{el}$  represents the fourth-order elasticity tensor,  $\boldsymbol{\epsilon}^{el}$  is elastic strain. And  $t$  is a material constant, introduced by Dufailly and Lemaitre [1995], to provide additional degree of freedom for modeling low-cycle fatigue ( $t = 1$  in Lemaitre [1992]). Dufailly and Lemaitre [1995] also proposed a simplified method to fit experimental results and get  $S$  and  $t$ .

The equivalent Mises stress is defined as

$$\bar{\sigma}(\mathbf{s}_e) = \sqrt{\frac{3}{2} \mathbf{s}_e : \mathbf{s}_e} = \sqrt{\frac{3}{2}} \|\mathbf{s}_e\|$$

The model assumes associated plastic flow

$$\dot{\boldsymbol{\epsilon}}^{pl} = \frac{\partial F}{\partial \boldsymbol{\sigma}} d\lambda = \frac{3}{2} \frac{\mathbf{s}_e}{\bar{\sigma}} d\lambda$$

Where  $d\lambda$  is the plastic consistency parameter. The evolution of the kinematic component of the model is defined as [Armstrong and Frederick 1966]:

$$\begin{cases} \dot{\alpha}_j = \frac{2}{3} C_j \dot{\boldsymbol{\epsilon}}^{pl} - \gamma_j \alpha_j \dot{\boldsymbol{\epsilon}}^{pl} & \text{IDEP} = 0 \\ \dot{\alpha}_j = (1 - D) \left( \frac{2}{3} C_j \dot{\boldsymbol{\epsilon}}^{pl} - \gamma_j \alpha_j \dot{\boldsymbol{\epsilon}}^{pl} \right) & \text{IDEP} = 1 \end{cases}$$

The damaged plastic strain is accumulated as

$$\begin{cases} r = \int \dot{\epsilon}^{pl} & IDEP = 0 \\ r = \int (1 - D) \dot{\epsilon}^{pl} & IDEP = 1 \end{cases}$$

where  $\dot{\epsilon}^{pl}$  is the equivalent plastic strain rate

$$\dot{\epsilon}^{pl} = \sqrt{\frac{2}{3} \dot{\epsilon}^{pl} : \dot{\epsilon}^{pl}}$$

where  $\dot{\epsilon}^{pl}$  represents the rate of plastic flow.

Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate.

Table 153.1 shows the difference between MAT 153 and MAT 104/105. MAT 153 is less computationally expensive than MAT 104/105. Kinematic hardening, which already exists in MAT 103, is included in MAT 153, but not in MAT 104/105.

	MAT 153	MAT 104	MAT 105
Computational cost	1.0	3.0	3.0
Isotropic hardening	One component	Two components	One component
Kinematic hardening	Four components	N/A	N/A
Output stress	IDS = 0 $\Rightarrow \tilde{\sigma}$ IDS = 1 $\Rightarrow \tilde{\sigma}(1 - D)$	$\tilde{\sigma}(1 - D)$	$\tilde{\sigma}(1 - D)$
Damaged plastic strain	IDEP = 0 $\Rightarrow$ $r = \int \dot{\tilde{\epsilon}}^{pl}$ IDEP = 1 $\Rightarrow$ $r = \int (1 - D) \dot{\tilde{\epsilon}}^{pl}$	$r = \int (1 - D) \dot{\tilde{\epsilon}}^{pl}$	$r = \int (1 - D) \dot{\tilde{\epsilon}}^{pl}$
Accumulation when	$\frac{\sigma_m}{\sigma_{eq}} > -\frac{1}{3}$	$\sigma_1 > 0$	$\sigma_1 > 0$
Isotropic plasticity	Yes	Yes	Yes
Anisotropic plasticity	No	Yes	No
Isotropic damage	Yes	Yes	Yes
Anisotropic damage	No	Yes	No

**Table M153-1.** Differences between MAT 153 and MAT 104/105

**\*MAT\_DESHPANDE\_FLECK\_FOAM**

This is material type 154 for solid elements. This material is for modeling aluminum foam used as a filler material in aluminum extrusions to enhance the energy absorbing capability of the extrusion. Such energy absorbers are used in vehicles to dissipate energy during impact. This model was developed by Reyes, Hopperstad, Berstad, and Langseth [2002] and is based on the foam model by Deshpande and Fleck [2000].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RHO	E	PR	ALPHA	GAMMA		
Type	A8	F	F	F	F	F		
Default	none	none	none	none	none	none		

Card 2	1	2	3	4	5	6	7	8
Variable	EPSD	ALPHA2	BETA	SIGP	DERFI	CFAIL	PFAIL	NUM
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RHO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
ALPHA	Controls shape of yield surface.
GAMMA	See remarks.
EPSD	Densification strain.



VARIABLE	DESCRIPTION
ALPHA2	See remarks.
BETA	See remarks.
SIGP	See remarks.
DERFI	Type of derivation used in material subroutine EQ.0: Numerical derivation EQ.1: Analytical derivation
CFAIL	Failure volumetric strain.
PFAIL	Failure principal stress. Must be sustained NUM (>0) timesteps to fail element.
NUM	Number of timesteps at or above PFAIL to trigger element failure.

**Remarks:**

The yield stress function  $\Phi$  is defined by:

$$\Phi = \hat{\sigma} - \sigma_y$$

The equivalent stress  $\hat{\sigma}$  is given by:

$$\hat{\sigma}^2 = \frac{\sigma_{VM}^2 + \alpha^2 \sigma_m^2}{1 + \left(\frac{\alpha}{3}\right)^2}$$

where,  $\sigma_{VM}$ , is the von Mises effective stress:

$$\sigma_{VM} = \sqrt{\frac{2}{3} \boldsymbol{\sigma}^{dev} : \boldsymbol{\sigma}^{dev}}$$

In this equation  $\sigma_m$  and  $\boldsymbol{\sigma}^{dev}$  are the mean and deviatoric stress:

$$\boldsymbol{\sigma}^{dev} = \boldsymbol{\sigma} - \sigma_m \mathbf{I}$$

The yield stress  $\sigma_y$  can be expressed as:

$$\sigma_y = \sigma_p + \gamma \frac{\hat{\epsilon}}{\epsilon_D} + \alpha_2 \ln \left[ \frac{1}{1 - \left(\frac{\hat{\epsilon}}{\epsilon_D}\right)^\beta} \right]$$

Here,  $\sigma_p$ ,  $\alpha_2$ ,  $\gamma$  and  $\beta$  are material parameters. The densification strain  $\epsilon_D$  is defined as:

$$\epsilon_D = -\ln \left( \frac{\rho_f}{\rho_{f0}} \right)$$

where  $\rho_f$  is the foam density and  $\rho_{f0}$  is the density of the virgin material.



**\*MAT\_155****\*MAT\_PLASTICITY\_COMPRESSION\_TENSION\_EOS**

Card 4	1	2	3	4	5	6	7	8
Variable	K							
Type	F							

**Viscoelastic Constant Cards.** Card Format for viscoelastic constants. Up to 6 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 6 cards are used.

Optional	1	2	3	4	5	6	7	8
Variable	GI	BETA1						
Type	F	F						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
C	Strain rate parameter, C, see formula below.
P	Strain rate parameter, P, see formula below.
FAIL	Failure flag. LT.0.0: User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic element deletion.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDC	Load curve ID defining yield stress versus effective plastic strain in compression.
LCIDT	Load curve ID defining yield stress versus effective plastic strain in tension.
LCSRC	Optional load curve ID defining strain rate scaling effect on yield stress when the material is in compression.
LCSRT	Optional load curve ID defining strain rate scaling effect on yield stress when the material is in tension.
SRFLAG	Formulation for rate effects: EQ.0.0: Total strain rate, EQ.1.0: Deviatoric strain rate.
PC	Compressive mean stress (pressure) at which the yield stress follows load curve ID, LCIDC. If the pressure falls between PC and PT a weighted average of the two load curves is used.
PT	Tensile mean stress at which the yield stress follows load curve ID, LCIDT.
PCUTC	Pressure cut-off in compression. When the pressure cut-off is reached the deviatoric stress tensor is set to zero. The compressive pressure is not, however, limited to PCUTC. Like the yield stress, PCUTC is scaled to account for rate effects.
PCUTT	Pressure cut-off in tension. When the pressure cut-off is reached the deviatoric stress tensor and tensile pressure is set to zero. Like the yield stress, PCUTT is scaled to account for rate effects.
PCUTF	Pressure cut-off flag. EQ.0.0: Inactive, EQ.1.0: Active.
SCALEP	Scale factor applied to the yield stress after the pressure cut-off is reached in either compression or tension. If SCALEP = 0 (default), the deviatoric stress is set to zero after the cut-off is reached.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SCALEE	Scale factor applied to the yield stress after the strain exceeds the failure strain set by FAIL. If SCALEE = 0 (default), the deviatoric strain is set to zero if the failure strain is exceeded. IF both SCALEP > 0 and SCALEE > 0 and both failure conditions are met, then the minimum scale factor is used.
K	Optional bulk modulus for the viscoelastic material. If nonzero a Kelvin type behavior will be obtained. Generally, K is set to zero.
GI	Optional shear relaxation modulus for the ith term
BETAI	Optional shear decay constant for the ith term

**Remarks:**

The stress strain behavior follows a different curve in compression than it does in tension. Tension is determined by the sign of the mean stress where a positive mean stress (i.e., a negative pressure) is indicative of tension. Two curves must be defined giving the yield stress versus effective plastic strain for both the tension and compression regimes.

Mean stress is an invariant which can be expressed as  $(\sigma_x + \sigma_y + \sigma_z)/3$ . PC and PT define a range of mean stress values within which interpolation is done between the tensile yield surface and compressive yield surface. PC and PT are not true material properties but are just a numerical convenience so that the transition from one yield surface to the other is not abrupt as the sign of the mean stress changes. Both PC and PT are input as positive values as it is implied that PC is a compressive mean stress value and PT is tensile mean stress value.

Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate,

$$\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}.$$

History Variables:

History Variable	Description
4	Tensile pressure cutoff (set to zero if tensile or compressive failure occurs)
5	The cutoff flag, initially equals 1, set to 0 if tensile or compressive failure occurs
6	The failure mode flag EQ.0: if no failure EQ.1: if compressive failure EQ.2: if tensile failure EQ.3: if failure by plastic strain
7	The current flow stress

**\*MAT\_MUSCLE**

This is material type 156 for truss elements. This material is a Hill-type muscle model with activation and a parallel damper. Also, see \*MAT\_SPRING\_MUSCLE where a description of the theory is available.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	SNO	SRM	PIS	SSM	CER	DMP
Type	A8	F	F	F	F	F	F	F
Default								

Card 2	1	2	3	4	5	6	7	8
Variable	ALM	SFR	SVS	SVR	SSP			
Type	F	F	F	F	F			
Default	0.0	1.0	1.0	1.0	0.0			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Material density in the initial undeformed configuration.
SNO	Initial stretch ratio, $\frac{l_0}{l_{orig}}$ , i.e., the length as defined by the nodal points at $t = 0$ divided by the original initial length. The density for the nodal mass calculation is $RO/SNO$ , or $\frac{l_{orig}}{l_0} \rho$ .
SRM	Maximum strain rate.
PIS	Peak isometric stress corresponding to the dimensionless value of unity in the dimensionless stress versus strain function, see SSP below.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
SSM	Strain when the dimensionless stress versus strain function, SSP below, reaches its maximum stress value.
CER	Constant, governing the exponential rise of SSP. Required if SSP = 0.
DMP	Damping constant (stress × time units).
ALM	Activation level vs. time. LT.0: absolute value gives load curve ID GE.0: constant value of ALM is used
SFR	Scale factor for strain rate maximum vs. activation level, $a(t)$ . LT.0: absolute value gives load curve ID GE.0: constant value of 1.0 is used
SVS	Active dimensionless tensile stress vs. the stretch ratio, $\frac{l}{l_{orig}}$ . LT.0: absolute value gives load curve ID GE.0: constant value of 1.0 is used
SVR	Active dimensionless tensile stress vs. the normalized strain rate, $\dot{\epsilon}$ . LT.0: absolute value gives load curve ID GE.0: constant value of 1.0 is used
SSP	Isometric dimensionless stress vs. the stretch ratio, $\frac{l}{l_{orig}}$ for the parallel elastic element. LT.0: absolute value gives load curve ID or table ID (see below) EQ.0: exponential function is used (see below) GT.0: constant value of 0.0 is used

**Remarks:**

The material behavior of the muscle model is adapted from \*MAT\_S15, the spring muscle model and treated here as a standard material. The initial length of muscle is calculated automatically. The force, relative length and shortening velocity are replaced by stress, strain and strain rate. A new parallel damping element is added.

The strain  $\varepsilon$  and normalized strain rate  $\dot{\varepsilon}$  are defined respectively as

$$\begin{aligned}\varepsilon &= \frac{l}{l_{\text{orig}}} - 1 \\ &= \text{SNO} \times \frac{l}{l_0} - 1\end{aligned}$$

and,

$$\begin{aligned}\dot{\varepsilon} &= \frac{l}{l_{\text{orig}}} \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{\text{max}}} \\ &= \text{SNO} \times \frac{l}{l_0} \times \frac{\dot{\varepsilon}}{\text{SFR} \times \text{SRM}}\end{aligned}$$

where  $\dot{\varepsilon} = \Delta\varepsilon/\Delta t$  (current strain increment divided by current time step),  $l$  = current muscle length, and  $l_{\text{orig}}$  = original muscle length.

From the relation above, it is known:

$$l_{\text{orig}} = \frac{l_0}{1 + \varepsilon_0}$$

where  $\varepsilon_0 = \text{SNO} - 1$  and  $l_0$  = muscle length at time 0.

Stress of Contractile Element is:

$$\sigma_1 = \sigma_{\text{max}} a(t) f\left(\frac{l}{l_{\text{orig}}}\right) g(\dot{\varepsilon})$$

where  $\sigma_{\text{max}} = \text{PIS}$ ,  $a(t) = \text{ALM}$ ,  $f(l/l_{\text{orig}}) = \text{SVS}$ , and  $g(\dot{\varepsilon}) = \text{SVR}$ .

Stress of Passive Element is:

$$\sigma_2 = \begin{cases} \sigma_{\text{max}} h\left(\frac{l}{l_{\text{orig}}}\right) & \text{for curve} \\ \sigma_{\text{max}} h\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{\text{max}}}, \frac{l}{l_{\text{orig}}}\right) & \text{for table} \end{cases}$$

where  $h = \text{SSP}$ . For  $\text{SSP} < 0$ , the absolute value gives a load curve ID or table ID. The load curve defines isometric dimensionless stress  $h$  versus stretch ratio  $l/l_{\text{orig}}$ . The table defines for each normalized strain rate  $\dot{\varepsilon}$  a load curve giving the isometric dimensionless stress  $h$  versus stretch ratio  $l/l_{\text{orig}}$  for that rate.

For exponential relationship (SSP = 0):

$$h\left(\frac{1}{l_{\text{orig}}}\right) = \begin{cases} 0 & \frac{1}{l_{\text{orig}}} < 1 \\ \frac{1}{\exp(\text{CER}) - 1} \left[ \exp\left(\frac{\text{CER}}{\text{SSM}} \varepsilon\right) - 1 \right] & \frac{1}{l_{\text{orig}}} \geq 1 \quad \text{CER} \neq 0 \\ \frac{\varepsilon}{\text{SSM}} & \frac{1}{l_{\text{orig}}} \geq 1 \quad \text{CER} = 0 \end{cases}$$

Stress of Damping Element is:

$$\sigma_3 = \text{DMP} \times \frac{l}{l_{\text{orig}}} \dot{\varepsilon}$$

Total Stress is:

$$\sigma = \sigma_1 + \sigma_2 + \sigma_3$$



Card 6	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		EXTRA
Type	F	F	F	F	F	F		F

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	IHIS
Type	F	F	F	F	F	F	F	F

Two additional cards for EXTRA = 1 or 2.

Card 8	1	2	3	4	5	6	7	8
Variable	XT	XC	YT	YC	SXY	FF12		NCFAIL
Type	F	F	F	F	F	F		F

Card 9	1	2	3	4	5	6	7	8
Variable	ZT	ZC	SYZ	SZX	FF23	FF31		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID Material identification. A unique number or label not exceeding 8 characters must be specified.

RO Mass density.

SIGY Initial yield stress

LCSS Load curve ID or Table ID.

**Load Curve.** When LCSS is a Load curve ID, it is taken as defining effective stress versus effective plastic strain. If defined QR1, CR1,

VARIABLE	DESCRIPTION
	<p>QR2, and CR2 are ignored.</p> <p><b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See <a href="#">Figure M24-1</a>. When the strain rate falls below the minimum value, the stress versus effective plastic strain curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value the stress versus effective plastic strain curve for the highest value of strain rate is used.</p> <p><b>Logarithmically Defined Tables.</b> If the <i>first</i> stress-strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for <i>all</i> stress-strain curves. Since the tables are internally discretized to equally space the points, natural logarithms are necessary, for example, if the curves correspond to rates from <math>10^{-4}</math> to <math>10^4</math>. Computing natural logarithms can substantially increase the computational time on certain computer architectures.</p>
QR1	Isotropic hardening parameter
CR1	Isotropic hardening parameter
QR2	Isotropic hardening parameter
CR2	Isotropic hardening parameter
$C_{ij}$	The $i, j$ term in the $6 \times 6$ anisotropic constitutive matrix. Note that 1 corresponds to the $a$ material direction, 2 to the $b$ material direction, and 3 to the $c$ material direction.
R00	$R_{00}$ for shell (Default = 1.0)
R45	$R_{45}$ for shell (Default = 1.0)
R90	$R_{90}$ for shell (Default = 1.0)
F	$F$ for brick (Default = 1/2)
G	$G$ for brick (Default = 1/2)
H	$H$ for brick (Default = 1/2)
L	$L$ for brick (Default = 3/2)
M	$M$ for brick (Default = 3/2)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
N	N for brick (Default = 3/2)
S11	Yield stress in local-x direction (shells only). This input is ignored when $R00, R45, R90 > 0.$
S22	Yield stress in local-y direction (shells only). This input is ignored when $R00, R45, R90 > 0.$
S33	Yield stress in local-z direction (shells only). This input is ignored when $R00, R45, R90 > 0.$
S12	Yield stress in local-xy direction (shells only). This input is ignored when $R00, R45, R90 > 0.$
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description.  EQ.0.0: locally orthotropic with material axes by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA.  EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.  EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector $\mathbf{v}$ with the element normal.  LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
VP	Formulation for rate effects: EQ.0.0: scale yield stress (default), EQ.1.0: viscoplastic formulation.
MACF	Material axes change flag for brick elements: EQ.1: No change, default, EQ.2: switch material axes $a$ and $b$ , EQ.3: switch material axes $a$ and $c$ , EQ.4: switch material axes $b$ and $c$ .
XP, YP, ZP	coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	components of vector $\mathbf{a}$ for AOPT = 2.
D1, D2, D3	components of vector $\mathbf{d}$ for AOPT = 2.
V1, V2, V3	components of vector $\mathbf{v}$ for AOPT = 3 and 4.
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overridden on the element card, see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO.
IHIS	Flag for material properties initialization. EQ.0: material properties defined in Cards 1-5 are used GE.1: Use *INITIAL_STRESS_SOLID/SHELL to initialize material properties on an element-by-element basis for solid or shell elements, respectively (see Remarks below).
EXTRA	Flag to input further data (see Remarks below): EQ.1.0: Tsai-Wu failure criterion parameters (cards 8 and 9) EQ.2.0: Tsai-Hill failure criterion parameters (cards 8 and 9)
XT	Longitudinal tensile strength, $a$ -axis. GT.0.0: constant value LT.0.0: Load curve ID = (-XT) which defines the longitudinal tensile strength vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.



VARIABLE	DESCRIPTION
XC	Longitudinal compressive strength, <i>a</i> -axis (positive value). GT.0.0: constant value LT.0.0: Load curve ID = (-XC) which defines the longitudinal compressive strength vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
YT	Transverse tensile strength, <i>b</i> -axis. GT.0.0: constant value LT.0.0: Load curve ID = (-YT) which defines the transverse tensile strength vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
YC	Transverse compressive strength, <i>b</i> -axis (positive value). GT.0.0: constant value LT.0.0: Load curve ID = (-YC) which defines the transverse compressive strength vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
SXY	Shear strength, <i>ab</i> -plane. GT.0.0: constant value LT.0.0: Load curve ID = (-SXY) which defines the shear strength vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.
FF12	Scale factor between -1 and +1 for interaction term F12, see Remarks.
NCFAIL	Number of timesteps to reduce stresses until element deletion. The default is NCFAIL = 10.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ZT	<p>Transverse tensile strength, <i>c</i>-axis (<i>solid elements only</i>).</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-ZT) which defines the transverse tensile strength vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.</p>
ZC	<p>Transverse compressive strength, <i>c</i>-axis (positive value) (<i>solid elements only</i>).</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-ZC) which defines the transverse compressive strength vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.</p>
SYZ	<p>Shear strength, <i>bc</i>-plane (<i>solid elements only</i>).</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-SYZ) which defines the shear strength vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.</p>
SZX	<p>Shear strength, <i>ca</i>-plane (<i>solid elements only</i>).</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-SZX) which defines the shear strength vs. strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as natural logarithm of the strain rate.</p>
FF23	<p>Scale factor between -1 and +1 for interaction term F23, see Remarks (<i>solid elements only</i>).</p>
FF31	<p>Scale factor between -1 and +1 for interaction term F31, see Remarks (<i>solid elements only</i>).</p>

**Description of IHIS (Solid Elements):**

Several of this material's parameters may be overwritten on an element-by-element basis through history variables using the \*INITIAL\_STRESS\_SOLID keyword. Bitwise (binary) expansion of IHIS determines which material properties are to be read:

$$\text{IHIS} = a_3 \times 8 + a_2 \times 4 + a_1 \times 2 + a_0,$$

where each  $a_i$  is a binary flag set to either 1 or 0. The  $a_i$  are interpreted according to the following table.

Flag	Description	Variables	#
$a_0$	Material directions	$q_{11}, q_{12}, q_{13}, q_{31}, q_{32}, q_{33}$	6
$a_1$	Anisotropic stiffness	$C_{ij}$	21
$a_2$	Anisotropic constants	F, G, H, L, M, N	6
$a_3$	Stress-strain Curve	LCSS	1

The NHISV field on \*INITIAL\_STRESS\_SOLID must be set equal to the sum of the number of variables to be read in, which depends on IHIS (and the  $a_i$ ):

$$\text{NHISV} = 6a_0 + 21a_1 + 6a_2 + a_3.$$

Then, in the following order, \*INITIAL\_STRESS\_SOLID processes the history variables, HISV $_i$ , as:

1. 6 material direction parameters when  $a_0 = 1$
2. 21 anisotropic stiffness parameters when  $a_1 = 1$
3. 6 anisotropic constants when  $a_2 = 1$
4. 1 parameters when  $a_3 = 1$

The  $q_{ij}$  terms are the first and third rows of a rotation matrix for the rotation from a co-rotational element's system and the  $a$ - $b$ - $c$  material directions. The  $c_{ij}$  terms are the upper triangular terms of the symmetric stiffness matrix,  $c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{22}, c_{23}, c_{24}, c_{25}, c_{26}, c_{33}, c_{34}, c_{35}, c_{36}, c_{44}, c_{45}, c_{46}, c_{55}, c_{56},$  and  $c_{66}$ .

**Description of IHIS (Shell Elements):**

Several of this material's parameters may be overwritten on an element-by-element basis through history variables using the \*INITIAL\_STRESS\_SHELL keyword. Bitwise (binary) expansion of IHIS determines which material properties are to be read:

$$\text{IHIS} = a_4 \times 16 + a_3 \times 8 + a_2 \times 4 + a_1 \times 2 + a_0,$$

where each  $a_i$  is a binary flag set to either 1 or 0. The  $a_i$  are interpreted according to the following table.

Flag	Description	Variables	#
$a_0$	Material directions	$q_1, q_2$	2
$a_1$	Anisotropic stiffness	$C_{ij}$	21
$a_2$	Anisotropic constants	$r_{00}, r_{45}, r_{90}$	3
$a_3$	Stress-strain Curve	LCSS	1
$a_4$	Strength limits	$XT, XC, YT, YC, SXY$	5

The NHISV field on \*INITIAL\_STRESS\_SHELL must be set equal to the sum of the number of variables to be read in, which depends on IHIS (and the  $a_i$ ):

$$NHISV = 2a_0 + 21a_1 + 3a_2 + a_3 + 5a_4.$$

Then, in the following order, \*INITIAL\_STRESS\_SHELL processes the history variables, HISV $_i$ , as:

5. 2 material direction parameters when  $a_0 = 1$
6. 21 anisotropic stiffness parameters when  $a_1 = 1$
7. 3 anisotropic constants when  $a_2 = 1$
8. 1 parameters when  $a_3 = 1$
9. 5 strength parameters when  $a_4 = 1$

The  $q_i$  terms are the material direction cosine and sinus for the rotation from a co-rotational element's system to the  $a$ - $b$ - $c$  material directions. The  $c_{ij}$  terms are the upper triangular terms of the symmetric stiffness matrix,  $c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{22}, c_{23}, c_{24}, c_{25}, c_{26}, c_{33}, c_{34}, c_{35}, c_{36}, c_{44}, c_{45}, c_{46}, c_{55}, c_{56}$ , and  $c_{66}$ .

### **Tsai-Wu failure criterion (EXTRA = 1):**

Brittle failure with different strengths in tension and compression in all main material directions can be invoked with EXTRA = 1 and the definition of corresponding parameters on Cards 8 and 9. The model used is the phenomenological Tsai-Wu failure criterion which requires that

$$\begin{aligned} & \left( \frac{1}{XT} - \frac{1}{XC} \right) \sigma_{aa} + \left( \frac{1}{YT} - \frac{1}{YC} \right) \sigma_{bb} + \left( \frac{1}{ZT} - \frac{1}{ZC} \right) \sigma_{cc} \\ & + \frac{1}{XT \cdot XC} \sigma_{aa}^2 + \frac{1}{YT \cdot YC} \sigma_{bb}^2 + \frac{1}{ZT \cdot ZC} \sigma_{cc}^2 + \frac{1}{SXY^2} \sigma_{ab}^2 + \frac{1}{SYZ^2} \sigma_{bc}^2 + \frac{1}{SZX^2} \sigma_{ca}^2 \\ & + 2 \cdot F_{12} \cdot \sigma_{aa} \sigma_{bb} + 2 \cdot F_{23} \cdot \sigma_{bb} \sigma_{cc} + 2 \cdot F_{31} \cdot \sigma_{cc} \sigma_{aa} < 1 \end{aligned}$$

for the 3-dimensional case (solid elements) with three planes of symmetry with respect to the material coordinate system. The interaction terms  $F_{12}, F_{23}$ , and  $F_{31}$  are given by

$$F_{12} = FF12 \cdot \sqrt{\frac{1}{X_T \cdot X_C \cdot Y_T \cdot Y_C}}, \quad F_{23} = FF23 \cdot \sqrt{\frac{1}{Y_T \cdot Y_C \cdot Z_T \cdot Z_C}}, \quad F_{31} = FF31 \cdot \sqrt{\frac{1}{Z_T \cdot Z_C \cdot X_T \cdot X_C}}$$

For the 2-dimensional case of plane stress (shell elements) this expression reduces to:

$$\left(\frac{1}{X_T} - \frac{1}{X_C}\right) \sigma_{aa} + \left(\frac{1}{Y_T} - \frac{1}{Y_C}\right) \sigma_{bb} + \frac{1}{X_T \cdot X_C} \sigma_{aa}^2 + \frac{1}{Y_T \cdot Y_C} \sigma_{bb}^2 \\ + \frac{1}{S_{XY}^2} \sigma_{ab}^2 + 2 \cdot F_{12} \cdot \sigma_{aa} \sigma_{bb} < 1$$

If these conditions are violated, then the stress tensor will be reduced to zero over NCFAIL time steps and then the element gets eroded. A small value for NCFAIL (< 50) is recommended to avoid unphysical behavior, the default is 10. The default values for the strengths  $X_T, X_C, Y_T, Y_C, Z_T, Z_C, S_{XY}, S_{YZ}$ , and  $S_{ZX}$  are  $1e20$ , i.e. basically no limits. The scale factors FF12, FF23, and FF31 for the interaction terms are zero by default.

### Tsai-Hill failure criterion (EXTRA = 2):

Brittle failure with different strengths in tension and compression in all main material directions can be invoked with EXTRA = 2 and the definition of corresponding parameters on Cards 8 and 9 (FF12, FF23 and FF31 are not used in this model). The model based on the HILL criterion which can be written as

$$(G + H)\sigma_{aa}^2 + (F + H)\sigma_{bb}^2 + (F + G)\sigma_{cc}^2 - 2H\sigma_{aa}\sigma_{bb} - 2F\sigma_{bb}\sigma_{cc} - 2G\sigma_{cc}\sigma_{aa} \\ + 2N\sigma_{ab}^2 + 2L\sigma_{bc}^2 + 2M\sigma_{ca}^2 < 1$$

for the 3-dimensional case. The constants H,F,G,N,L,M can be expressed in terms of the

strength limits (which then becomes the TSAI-HILL criterion), where the current stress

state defines whether the compressive or the tensile strength limit will enter into the

equation:

$$G + H = \frac{1}{X_i^2} ; F + H = \frac{1}{Y_i^2} ; F + G = \frac{1}{Z_i^2} ; 2N = \frac{1}{SXY^2} ; 2L = \frac{1}{SYZ^2} ; 2N = \frac{1}{SZX^2}$$

$$H = 0.5 \cdot \left( \frac{1}{X_i^2} + \frac{1}{Y_i^2} - \frac{1}{Z_i^2} \right) ; F = 0.5 \cdot \left( \frac{1}{Y_i^2} + \frac{1}{Z_i^2} - \frac{1}{X_i^2} \right) ; G = 0.5 \cdot \left( \frac{1}{X_i^2} + \frac{1}{Z_i^2} - \frac{1}{Y_i^2} \right)$$

$$X_i = \begin{cases} XT & \text{if } \sigma_{aa} > 0 \\ XC & \text{if } \sigma_{aa} < 0 \end{cases} ; Y_i = \begin{cases} YT & \text{if } \sigma_{bb} > 0 \\ YC & \text{if } \sigma_{bb} < 0 \end{cases} ; Z_i = \begin{cases} ZT & \text{if } \sigma_{cc} > 0 \\ ZC & \text{if } \sigma_{cc} < 0 \end{cases}$$

For the 2-dimensional case of plane stress (shell elements) the TSAI-HILL criterion reduces to:

$$(G + H)\sigma_{aa}^2 + (F + H)\sigma_{bb}^2 - 2H\sigma_{aa}\sigma_{bb} + 2N\sigma_{ab}^2 < 1$$

with

$$G + H = \frac{1}{X_i^2} ; F + H = \frac{1}{Y_i^2} ; H = 0.5 \cdot \left( \frac{1}{X_i^2} \right) ; 2N = \frac{1}{SXY^2}$$

If these conditions are violated, then the stress tensor will be reduced to zero over NCFAIL time steps and then the element gets eroded. A small value for NCFAIL (< 50) is recommended to avoid unphysical behavior, the default is 10. The default values for the strengths XT, XC, YT, YC, ZT, ZC, SXY, SYZ, and SZX are 1e20, i.e. basically no limits.



**\*MAT\_158****\*MAT\_RATE\_SENSITIVE\_COMPOSITE\_FABRIC**

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

Card 6	1	2	3	4	5	6	7	8
Variable	E11C	E11T	E22C	E22T	GMS			
Type	F	F	F	F	F			

Card 7	1	2	3	4	5	6	7	8
Variable	XC	XT	YC	YT	SC			
Type	F	F	F	F	F			

Card 8	1	2	3	4	5	6	7	8
Variable	K							
Type	F							

**Viscoelastic Cards.** Up to 6 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 6 cards are used.

Optional	1	2	3	4	5	6	7	8
Variable	GI	BETA1						
Type	F	F						



VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
EA	$E_a$ , Young's modulus - longitudinal direction
EB	$E_b$ , Young's modulus - transverse direction
(EC)	$E_c$ , Young's modulus - normal direction (not used)
PRBA	$\nu_{ba}$ , Poisson's ratio ba
PRCA	$\nu_{ca}$ , Poisson's ratio ca, can be defined in card 4, col. 7, default = PRBA
PRCB	$\nu_{cb}$ , Poisson's ratio cb, can be defined in card 4, col. 8, default = PRBA
TAU1	$\tau_1$ , stress limit of the first slightly nonlinear part of the shear stress versus shear strain curve. The values $\tau_1$ and $\gamma_1$ are used to define a curve of shear stress versus shear strain. These values are input if FS, defined below, is set to a value of -1.
GAMMA1	$\gamma_1$ , strain limit of the first slightly nonlinear part of the shear stress versus shear strain curve.
GAB	$G_{ab}$ , shear modulus ab
GBC	$G_{bc}$ , shear modulus bc
GCA	$G_{ca}$ , shear modulus ca
SLIMT1	Factor to determine the minimum stress limit after stress maximum (fiber tension).
SLIMC1	Factor to determine the minimum stress limit after stress maximum (fiber compression).
SLIMT2	Factor to determine the minimum stress limit after stress maximum (matrix tension).
SLIMC2	Factor to determine the minimum stress limit after stress maximum (matrix compression).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SLIMS	Factor to determine the minimum stress limit after stress maximum (shear).
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):  EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.  EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.  EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle (BETA) from a line in the plane of the element defined by the cross product of the vector v with the element normal.  LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
TSIZE	Time step for automatic element deletion.
ERODS	Maximum effective strain for element layer failure. A value of unity would equal 100% strain.
SOFT	Softening reduction factor for strength in the crashfront.
FS	Failure surface type:  EQ.1.0: smooth failure surface with a quadratic criterion for both the fiber (a) and transverse (b) directions. This option can be used with complete laminates and fabrics.  EQ.0.0: smooth failure surface in the transverse (b) direction with a limiting value in the fiber (a) direction. This model is appropriate for unidirectional (UD) layered composites only.  EQ.-1: faceted failure surface. When the strength values are reached then damage evolves in tension and compression for both the fiber and transverse direction. Shear

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	behavior is also considered. This option can be used with complete laminates and fabrics.
XP, YP, ZP	Define coordinates of point <b>p</b> for AOPT = 1.
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Define components of vector <b>v</b> for AOPT = 3.
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.
E11C	Strain at longitudinal compressive strength, a-axis.
E11T	Strain at longitudinal tensile strength, a-axis.
E22C	Strain at transverse compressive strength, b-axis.
E22T	Strain at transverse tensile strength, b-axis.
GMS	Strain at shear strength, ab plane.
XC	Longitudinal compressive strength
XT	Longitudinal tensile strength, see below.
YC	Transverse compressive strength, b-axis, see below.
YT	Transverse tensile strength, b-axis, see below.
SC	Shear strength, ab plane.
K	Optional bulk modulus for the viscoelastic material. If nonzero a Kelvin type behavior will be obtained. Generally, K is set to zero.
GI	Optional shear relaxation modulus for the ith term
BETAI	Optional shear decay constant for the ith term

**Remarks:**

See the remark for material type 58, \*MAT\_LAMINATED\_COMPOSITE\_FABRIC, for the treatment of the composite material.

Rate effects are taken into account through a Maxwell model using linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t-\tau)$  is the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional. Since we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^N G_m e^{-\beta_m t}$$

We characterize this in the input by the shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . An arbitrary number of terms, not exceeding 6, may be used when applying the viscoelastic model. The composite failure is not directly affected by the presence of the viscous stress tensor.

**\*MAT\_CSCM\_{OPTION}**

This is material type 159. This is a smooth or continuous surface cap model and is available for solid elements in LS-DYNA. The user has the option of inputting his own material properties (<BLANK> option), or requesting default material properties for normal strength concrete (CONCRETE).

Available options include:

<BLANK>

CONCRETE

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	NPLOT	INCRE	IRATE	ERODE	RECOV	ITRETRC
Type	A8	F	I	F	I	F	F	I

Card 2	1	2	3	4	5	6	7	8
Variable	PRED							
Type	F							

Card 3 for CONCRETE keyword option.

Card 3	1	2	3	4	5	6	7	8
Variable	FPC	DAGG	UNITS					
Type	F	F	I					



<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
NPLOT	Controls what is written as component 7 to the d3plot database. LS-Prepost always blindly labels this component as effective plastic strain: EQ.1: Maximum of brittle and ductile damage (default). EQ.2: Maximum of brittle and ductile damage, with recovery of brittle damage. EQ.3: Brittle damage. EQ.4: Ductile damage. EQ.5: $\kappa$ (intersection of cap with shear surface). EQ.6: X0 (intersection of cap with pressure axis). EQ.7: $\varepsilon_v^p$ (plastic volume strain).
INCRE	Maximum strain increment for subincrementation. If left blank, a default value is set during initialization based upon the shear strength and stiffness.
IRATE	Rate effects options: EQ.0: Rate effects model turned off (default). EQ.1: Rate effects model turned on.
ERODE	Elements erode when damage exceeds 0.99 and the maximum principal strain exceeds ERODE-1.0. For erosion that is independent of strain, set ERODE equal to 1.0. Erosion does not occur if ERODE is less than 1.0.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RECOV	<p>The modulus is recovered in compression when RECOV is equal to 0 (default). The modulus remains at the brittle damage level when RECOV is equal to 1. Partial recovery is modeled for values of RECOV between 0 and 1. Two options are available:</p> <p>EQ.1: Input a value between 0 and 1. Recovery is based upon the sign of the pressure invariant only.</p> <p>EQ.2: Input a value between 10 and 11. Recovery is based upon the sign of both the pressure and volumetric strain. In this case, RECOV = RECOV-10, and a flag is set to request the volumetric strain check.</p>
IRETRC	<p>Cap retraction option:</p> <p>EQ.0: Cap does not retract (default).</p> <p>EQ.1: Cap retracts.</p>
PRED	<p>Pre-existing damage (<math>0 \leq \text{PreD} &lt; 1</math>). If left blank, the default is zero (no pre-existing damage).</p>

**Define for the CONCRETE option only:**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FPC	<p>Unconfined compression strength, <math>f'_c</math>. Material parameters are internally fit to data for unconfined compression strengths between about 20 and 58 Mpa (2,901 to 8,412 psi), with emphasis on the midrange between 28 and 48 MPa (4,061 and 6,962 psi). If left blank, default for FPC is 30 MPa.</p>
DAGG	<p>Maximum aggregate size, Dagg. Softening is fit to data for aggregate sizes between 8 and 32 mm (0.3 and 1.3 inches). If left blank, default for DAGG is 19 mm (3/4 inch).</p>
UNITS	<p>Units options:</p> <p>EQ.0: GPa, mm, msec, Kg/mm<sup>3</sup>, kN</p> <p>EQ.1: MPa, mm, msec, g/mm<sup>3</sup>, N</p> <p>EQ.2: MPa, mm, sec, Mg/mm<sup>3</sup>, N</p> <p>EQ.3: Psi, inch, sec, lbf-s<sup>2</sup>/inch<sup>4</sup>, lbf</p> <p>EQ.4: Pa, m, sec, kg/m<sup>3</sup>, N</p>



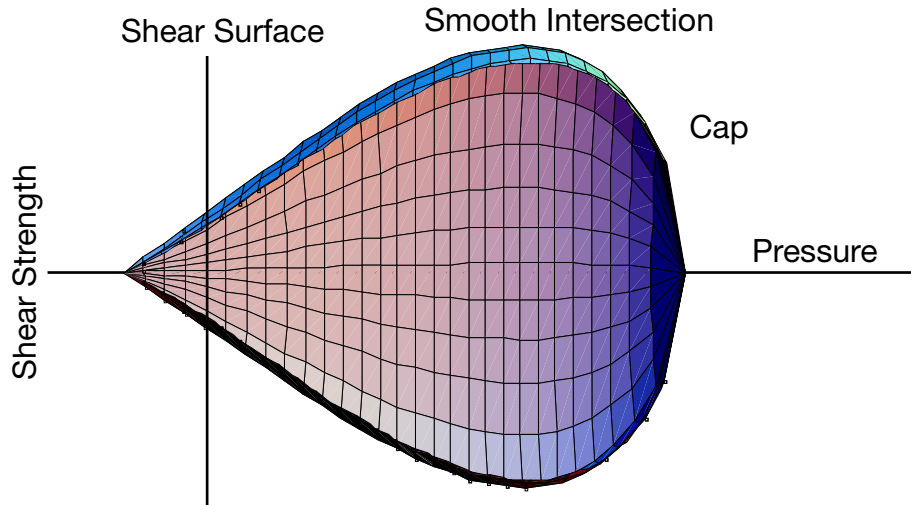
Define for <BLANK> option only:

<b>VARIABLE</b>	<b>DESCRIPTION</b>
G	Shear modulus.
K	Bulk modulus.
ALPHA	Tri-axial compression surface constant term, $\alpha$ .
THETA	Tri-axial compression surface linear term, $\theta$ .
LAMDA	Tri-axial compression surface nonlinear term, $\lambda$ .
BETA	Tri-axial compression surface exponent, $\beta$ .
ALPHA1	Torsion surface constant term, $\alpha_1$ .
THETA1	Torsion surface linear term, $\theta_1$ .
LAMDA1	Torsion surface nonlinear term, $\lambda_1$ .
BETA1	Torsion surface exponent, $\beta_1$ .
ALPHA2	Tri-axial extension surface constant term, $\alpha_2$ .
THETA2	Tri-axial extension surface linear term, $\theta_2$ .
LAMDA2	Tri-axial extension surface nonlinear term, $\lambda_2$ .
BETA2	Tri-axial extension surface exponent, $\beta_2$ .
NH	Hardening initiation, $N_H$ .
CH	Hardening rate, $C_H$ .
R	Cap aspect ratio, $R$ .
X0	Cap initial location, $X_0$ .
W	Maximum plastic volume compaction, $W$ .
D1	Linear shape parameter, $D_1$ .
D2	Quadratic shape parameter, $D_2$ .
B	Ductile shape softening parameter, $B$ .

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
GFC	Fracture energy in uniaxial stress $G_{fc}$ .
D	Brittle shape softening parameter, D.
GFT	Fracture energy in uniaxial tension, $G_{ft}$ .
GFS	Fracture energy in pure shear stress, $G_{fs}$ .
PWRC	Shear-to-compression transition parameter.
PWRT	Shear-to-tension transition parameter.
PMOD	Modify moderate pressure softening parameter.
ETA0C	Rate effects parameter for uniaxial compressive stress, $\eta_{0c}$ .
NC	Rate effects power for uniaxial compressive stress, $N_c$ .
ETA0T	Rate effects parameter for uniaxial tensile stress, $\eta_{0t}$ .
NT	Rate effects power for uniaxial tensile stress, $N_t$ .
OVERC	Maximum overstress allowed in compression.
OVERT	Maximum overstress allowed in tension.
SRATE	Ratio of effective shear stress to tensile stress fluidity parameters.
REPOW	Power which increases fracture energy with rate effects.

**Model Formulation and Input Parameters:**



**Figure M159-1.** General shape of concrete model yield surface in two dimensions.

This is a cap model with a smooth intersection between the shear yield surface and hardening cap, as shown in [Figure M159-1](#). The initial damage surface coincides with the yield surface. Rate effects are modeled with viscoplasticity. For a complete theoretical description, with references and example problems see [Murray 2007] and [Murray, Abu-Odeh and Bligh 2007].

*Stress Invariants.* The yield surface is formulated in terms of three stress invariants:  $J_1$  is the first invariant of the stress tensor,  $J'_2$  is the second invariant of the deviatoric stress tensor, and  $J'_3$  is the third invariant of the deviatoric stress tensor. The invariants are defined in terms of the deviatoric stress tensor,  $S_{ij}$  and pressure,  $P$ , as follows:

$$J_1 = 3P$$

$$J'_2 = \frac{1}{2} S_{ij} S_{ij}$$

$$J'_3 = \frac{1}{3} S_{ij} S_{jk} S_{ki}$$

*Plasticity Surface.* The three invariant yield function is based on these three invariants, and the cap hardening parameter,  $\kappa$ , as follows:

$$f(J_1, J'_2, J'_3, \kappa) = J'_2 - \mathfrak{R}^2 F_f^2 F_c$$

Here  $F_f$  is the shear failure surface,  $F_c$  is the hardening cap, and  $\mathfrak{R}$  is the Rubin three-invariant reduction factor. The cap hardening parameter  $\kappa$  is the value of the pressure invariant at the intersection of the cap and shear surfaces.

Trial elastic stress invariants are temporarily updated via the trial elastic stress tensor,  $\sigma^T$ . These are denoted  $J_1^T$ ,  $J_2^T$ , and  $J_3^T$ . Elastic stress states are modeled when  $f(J_1^T, J_2^T, J_3^T, \kappa^T) \leq 0$ . Elastic-plastic stress states are modeled when  $f(J_1^T, J_2^T, J_3^T, \kappa^T) > 0$ . In this case, the

plasticity algorithm returns the stress state to the yield surface such that  $f(J_1^P, J_2^P, J_3^P, \kappa^P) = 0$ . This is accomplished by enforcing the plastic consistency condition with associated flow.

Shear Failure Surface. The strength of concrete is modeled by the shear surface in the tensile and low confining pressure regimes:

$$F_f(J_1) = \alpha - \lambda \exp^{-\beta J_1} + \theta J_1$$

Here the values of  $\alpha, \beta, \lambda$ , and  $\theta$  are selected by fitting the model surface to strength measurements from triaxial compression (TXC) tests conducted on plain concrete cylinders.

Rubin Scaling Function. Concrete fails at lower values of  $\sqrt{3J_2}$  (principal stress difference) for triaxial extension (TXE) and torsion (TOR) tests than it does for TXC tests conducted at the same pressure. The Rubin scaling function  $\mathfrak{R}$  determines the strength of concrete for any state of stress relative to the strength for TXC, via  $\mathfrak{R}F_f$ . Strength in torsion is modeled as  $Q_1F_f$ . Strength in TXE is modeled as  $Q_2F_f$ , where:

$$Q_1 = \alpha_1 - \lambda_1 \exp^{-\beta_1 J_1} + \theta_1 J_1$$

$$Q_2 = \alpha_2 - \lambda_2 \exp^{-\beta_2 J_1} + \theta_2 J_1$$

Cap Hardening Surface. The strength of concrete is modeled by a combination of the cap and shear surfaces in the low to high confining pressure regimes. The cap is used to model plastic volume change related to pore collapse (although the pores are not explicitly modeled). The isotropic hardening cap is a two-part function that is either unity or an ellipse:

$$F_c(J_1, \kappa) = 1 - \frac{[J_1 - L(\kappa)][|J_1 - L(\kappa)| + J_1 - L(\kappa)]}{2 [X(\kappa) - L(\kappa)]^2}$$

where  $L(\kappa)$  is defined as:

$$L(\kappa) = \begin{cases} \kappa & \text{if } \kappa > \kappa_0 \\ \kappa_0 & \text{otherwise} \end{cases}$$

The equation for  $F_c$  is equal to unity for  $J_1 \leq L(\kappa)$ . It describes the ellipse for  $J_1 > L(\kappa)$ . The intersection of the shear surface and the cap is at  $J_1 = \kappa$ .  $\kappa_0$  is the value of  $J_1$  at the *initial* intersection of the cap and shear surfaces before hardening is engaged (before the cap moves). The equation for  $L(\kappa)$  restrains the cap from retracting past its initial location at  $\kappa_0$ .

The intersection of the cap with the  $J_1$  axis is at  $J_1 = X(\kappa)$ . This intersection depends upon the cap ellipticity ratio  $R$ , where  $R$  is the ratio of its major to minor axes:

$$X(\kappa) = L(\kappa) + RF_f[L(\kappa)]$$

The cap moves to simulate plastic volume change. The cap expands ( $X(\kappa)$  and  $\kappa$  increase) to simulate plastic volume compaction. The cap contracts ( $X(\kappa)$  and  $\kappa$  decrease) to simulate plastic volume expansion, called dilation. The motion (expansion and contraction) of the cap is based upon the hardening rule:

$$\varepsilon_v^p = W \left[ 1 - e^{-D_1(X-X_0) - D_2(X-X_0)^2} \right]$$

Here  $\varepsilon_v^p$  the plastic volume strain,  $W$  is the maximum plastic volume strain, and  $D_1$  and  $D_2$  are model input parameters.  $X_0$  is the initial location of the cap when  $\kappa = \kappa_0$ .

The five input parameters ( $X_0$ ,  $W$ ,  $D_1$ ,  $D_2$ , and  $R$ ) are obtained from fits to the pressure-volumetric strain curves in isotropic compression and uniaxial strain.  $X_0$  determines the pressure at which compaction initiates in isotropic compression.  $R$ , combined with  $X_0$ , determines the pressure at which compaction initiates in uniaxial strain.  $D_1$ , and  $D_2$  determine the shape of the pressure-volumetric strain curves.  $W$  determines the maximum plastic volume compaction.

Shear Hardening Surface. In unconfined compression, the stress-strain behavior of concrete exhibits nonlinearity and dilation prior to the peak. Such behavior is modeled with an initial shear yield surface,  $N_H F_f$ , which hardens until it coincides with the ultimate shear yield surface,  $F_f$ . Two input parameters are required. One parameter,  $N_H$ , initiates hardening by setting the location of the initial yield surface. A second parameter,  $C_H$ , determines the rate of hardening (amount of nonlinearity).

Damage. Concrete exhibits softening in the tensile and low to moderate compressive regimes.

$$\sigma_{ij}^d = (1 - d) \sigma_{ij}^{vp}$$

A scalar damage parameter,  $d$ , transforms the viscoplastic stress tensor without damage, denoted  $\sigma^{vp}$ , into the stress tensor with damage, denoted  $\sigma^d$ . Damage accumulation is based upon two distinct formulations, which we call brittle damage and ductile damage. The initial damage threshold is coincident with the shear plasticity surface, so the threshold does not have to be specified by the user.

Ductile Damage. Ductile damage accumulates when the pressure ( $P$ ) is compressive and an energy-type term,  $\tau_c$ , exceeds the damage threshold,  $\tau_{0c}$ . Ductile damage accumulation depends upon the total strain components,  $\varepsilon_{ij}$ , as follows:

$$\tau_c = \sqrt{\frac{1}{2} \sigma_{ij} \varepsilon_{ij}}$$

The stress components  $\sigma_{ij}$  are the elasto-plastic stresses (with kinematic hardening) calculated before application of damage and rate effects.

Brittle Damage. Brittle damage accumulates when the pressure is tensile and an energy-type term,  $\tau_t$ , exceeds the damage threshold,  $\tau_{0t}$ . Brittle damage accumulation depends upon the maximum principal strain,  $\varepsilon_{\max}$ , as follows:

$$\tau_t = \sqrt{E \varepsilon_{\max}^2}$$

Softening Function. As damage accumulates, the damage parameter  $d$  increases from an initial value of zero, towards a maximum value of one, via the following formulations:

$$\begin{aligned} \text{Brittle Damage: } d(\tau_t) &= \frac{0.999}{D} \left[ \frac{1 + D}{1 + D e^{-C(\tau_t - \tau_{0t})}} - 1 \right] \\ \text{Ductile Damage: } d(\tau_c) &= \frac{d_{\max}}{B} \left[ \frac{1 + B}{1 + B e^{-A(\tau_c - \tau_{0c})}} - 1 \right] \end{aligned}$$

The damage parameter that is applied to the six stresses is equal to the current maximum of the brittle or ductile damage parameter. The parameters A and B or C and D set the shape of the softening curve plotted as stress-displacement or stress-strain. The parameter  $d_{\max}$  is the maximum damage level that can be attained. It is calculated internally and is less than one at moderate confining pressures. The compressive softening parameter, A, may also be reduced with confinement, using the input parameter  $p_{\text{mod}}$ , as follows:

$$A = A(d_{\max} + 0.001)^{p_{\text{mod}}}$$

Regulating Mesh Size Sensitivity. The concrete model maintains constant fracture energy, regardless of element size. The fracture energy is defined here as the area under the stress-displacement curve from peak strength to zero strength. This is done by internally formulating the softening parameters A and C in terms of the element length,  $l$  (cube root of the element volume), the fracture energy,  $G_f$ , the initial damage threshold,  $\tau_{0t}$  or  $\tau_{0c}$ , and the softening shape parameters, D or B.

The fracture energy is calculated from up to five user-specified input parameters: GFC, GFS, GFT, PWRC, and PWRT. The user specifies three distinct fracture energy values. These are the fracture energy in uniaxial tensile stress, GFT, pure shear stress, GFS, and uniaxial compressive stress, GFC. The model internally selects the fracture energy from equations which interpolate between the three fracture energy values as a function of the stress state (expressed via two stress invariants). The interpolation equations depend upon the user-specified input powers PWRC and PWRT, as follows.

$$\begin{aligned} \text{Tensile Pressure: } G_f &= \text{GFS} + \left( \frac{-J_1}{\sqrt{3J_2'}} \right)^{\overbrace{\text{PWRT}}^{k_t}} [\text{GFT} - \text{GFS}] \\ \text{Compressive Pressure: } G_f &= \text{GFS} + \left( \frac{J_1}{\sqrt{3J_2'}} \right)^{\overbrace{\text{PWRC}}^{k_c}} [\text{GFC} - \text{GFS}] \end{aligned}$$

The internal parameters  $k_c$  and  $k_t$  are restricted to the interval [0,1].

Element Erosion. An element loses all strength and stiffness as  $d \rightarrow 1$ . To prevent computational difficulties with very low stiffness, element erosion is available as a user

option. An element erodes when  $d > 0.99$  and the maximum principal strain is greater than a user supplied input value, ERODE-1.0.

*Viscoplastic Rate Effects.* At each time step, the viscoplastic algorithm interpolates between the elastic trial stress,  $\sigma_{ij}^T$ , and the inviscid stress (without rate effects),  $\sigma_{ij}^P$ , to set the viscoplastic stress (with rate effects),  $\sigma_{ij}^{VP}$ :

$$\sigma_{ij}^{VP} = (1 - \gamma)\sigma_{ij}^T + \gamma\sigma_{ij}^P$$

where,

$$\gamma = \frac{\Delta t / \eta}{1 + \Delta t / \eta}$$

This interpolation depends upon the effective fluidity coefficient,  $\eta$ , and the time step,  $\Delta t$ . The effective fluidity coefficient is internally calculated from five user-supplied input parameters and interpolation equations:

$$\begin{aligned} \text{Tensile Pressure: } \eta &= \eta_s + \left( \frac{-J_1}{\sqrt{3J_2'}} \right)^{\text{PWRT}} [\eta_t - \eta_s] \\ \text{Compressive Pressure: } \eta &= \eta_s + \left( \frac{J_1}{\sqrt{3J_2'}} \right)^{\text{PWRC}} [\eta_c - \eta_s] \end{aligned}$$

where,

$$\eta_s = \text{SRATE} \times \eta_t$$

$$\eta_t = \frac{\text{ETA0T}}{\dot{\epsilon}^{\text{NT}}}$$

$$\eta_c = \frac{\text{ETA0C}}{\dot{\epsilon}^{\text{NC}}}$$

The input parameters are ETA0T and NT for fitting uniaxial tensile stress data, ETA0X and NC for fitting the uniaxial compressive stress data, and SRATE for fitting shear stress data. The effective strain rate is  $\dot{\epsilon}$ .

This viscoplastic model may predict substantial rate effects at high strain rates ( $\dot{\epsilon} > 100$ ). To limit rate effects at high strain rates, the user may input overstress limits in tension OVERT and compression OVERC. These input parameters limit calculation of the fluidity parameter, as follows:

$$\text{if } E\dot{\epsilon}\eta > \text{OVER}, \text{ then } \eta = \frac{m}{E\dot{\epsilon}}$$

where  $m = \text{OVERT}$  when the pressure is tensile, and  $m = \text{OVERC}$  when the pressure is compressive.

The user has the option of increasing the fracture energy as a function of effective strain rate via the REPOW input parameter, as follows:

$$G_f^{\text{rate}} = G_f \left( 1 + \frac{E\dot{\epsilon}\eta}{f'} \right)^{\text{REPOW}}$$

Here  $G_f^{\text{rate}}$  is the fracture energy enhanced by rate effects, and  $f'$  is the yield strength before application of rate effects (which is calculated internally by the model). The term in brackets is greater than, or equal to one, and is the approximate ratio of the dynamic to static strength.



**\*MAT\_ALE\_INCOMPRESSIBLE**

This is Material Type 160. This card allows to solve incompressible flows with the ALE solver. It should be used with the element formulation 6 and 12 in \*SECTION\_SOLID (elform = 6 or 12). A projection method enforces the incompressibility condition.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PC	MU				
Type	I	F	F	F				
Default	none	none	0.0	0.0				

Card 2	1	2	3	4	5	6	7	8
Variable	TOL	DTOUT	NCG	METH				
Type	F	F	I	I				
Default	1e-8	1e10	50	-7				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material ID. A unique number or label not exceeding 8 characters must be specified. Material ID is referenced in the *PART card and must be unique
RO	Material density
PC	Pressure cutoff (< or = 0.0)
MU	Dynamic viscosity coefficient
TOL	Tolerance for the convergence of the conjugate gradient
DTOUT	Time interval between screen outputs
NCG	Maximum number of loops in the conjugate gradient

<b>VARIABLE</b>	<b>DESCRIPTION</b>
METH	Conjugate gradient methods: EQ.-6: solves the poisson equation for the pressure EQ.-7: solves the poisson equation for the pressure increment



**\*MAT\_161, MAT\_162****\*MAT\_COMPOSITE\_MSC**

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

Card 5	1	2	3	4	5	6	7	8
Variable	SAT	SAC	SBT	SBC	SCT	SFC	SFS	SAB
Type	F	F	F	F	F	F	F	F

Card 6	1	2	3	4	5	6	7	8
Variable	SBC	SCA	SFFC	AMODEL	PHIC	E_LIMT	S_DELM	
Type	F	F	F	F	F	F	F	

Card 7	1	2	3	4	5	6	7	8
Variable	OMGMX	ECRSH	EEXPN	CERATE1	AM1			
Type	F	F	F	F	F			

**Failure Card.** Additional card for DMG keyword option.

Card 8	1	2	3	4	5	6	7	8
Variable	AM2	AM3	AM4	CERATE2	CERATE3	CERATE4		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label not exceeding 8 characters must be specified.

RO	Mass density
EA	$E_a$ , Young's modulus - longitudinal direction
EB	$E_b$ , Young's modulus - transverse direction
EC	$E_c$ , Young's modulus - through thickness direction
PRBA	$\nu_{ba}$ , Poisson's ratio ba
PRCA	$\nu_{ca}$ , Poisson's ratio ca
PRCB	$\nu_{cb}$ , Poisson's ratio cb
GAB	$G_{ab}$ , shear modulus ab
GBC	$G_{bc}$ , shear modulus bc
GCA	$G_{ca}$ , shear modulus ca
AOPT	Material axes option, see Figure 2.1:  EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in Figure 2.1. Nodes 1, 2, and 4 of an element are identical to the Nodes used for the definition of a coordinate system by *DEFINE_COORDINATE_NODES.  EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center, to define the a-direction.  EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.  EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector $v$ with the element normal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.  EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector $v$ , and an originating point, $p$ , which define the centerline axis. This op-

tion is for solid elements only.

LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on \*DEFINE\_COORDINATE\_NODES, \*DEFINE\_COORDINATE\_SYSTEM or \*DEFINE\_COORDINATE\_VECTOR). Available in R3 version of 971 and later.

MACF	Material axes change flag: EQ.1: No change, default, EQ.2: switch material axes a and b, EQ.3: switch material axes a and c, EQ.4: switch material axes b and c.
XP, YP, ZP	Define coordinates of point <b>p</b> for AOPT = 1 and 4.
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Define components of vector <b>v</b> for AOPT = 3 and 4.
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2.
BETA	Layer in-plane rotational angle in degrees.
SAT	Longitudinal tensile strength
SAC	Longitudinal compressive strength
SBT	Transverse tensile strength
SBC	Transverse compressive strength
SCT	Through thickness tensile strength
SFC	Crush strength
SFS	Fiber mode shear strength
SAB	Matrix mode shear strength, ab plane, see below.
SBC	Matrix mode shear strength, bc plane, see below.
SCA	Matrix mode shear strength, ca plane, see below.
SFFC	Scale factor for residual compressive strength

AMODEL	Material models: EQ.1.0: Unidirectional layer model EQ.2.0: Fabric layer model
PHIC	Coulomb friction angle for matrix and delamination failure, < 90
E_LIMT	Element eroding axial strain
S_DELM	Scale factor for delamination criterion
OMGMX	Limit damage parameter for elastic modulus reduction
ECRSH	Limit compressive volume strain for element eroding
EEXPN	Limit tensile volume strain for element eroding
CERATE1	Coefficient for strain rate dependent strength properties
AM1	Coefficient for strain rate softening property for fiber damage in a direction.
AM2	Coefficient for strain rate softening property for fiber damage in b direction.
AM3	Coefficient for strain rate softening property for fiber crush and punch shear damage.
AM4	Coefficient for strain rate softening property for matrix and delamination damage.
CERATE2	Coefficient for strain rate dependent axial moduli.
CERATE3	Coefficient for strain rate dependent shear moduli.
CERATE4	Coefficient for strain rate dependent transverse moduli.

**Material Models:**

The unidirectional and fabric layer failure criteria and the associated property degradation models for material 161 are described as follows. All the failure criteria are expressed in terms of stress components based on ply level stresses ( $\sigma_a, \sigma_b, \sigma_c, \tau_{ab}, \tau_{bc}, \tau_{ca}$ ) and the associated elastic moduli are ( $E_a, E_b, E_c, G_{ab}, G_{bc}, G_{ca}$ ). Note that for the unidirectional model, a, b and c denote the fiber, in-plane transverse and out-of-plane directions, respectively, while for the fabric model, a, b and c denote the in-plane fill, in-plane warp and out-of-plane directions, respectively.

**Unidirectional lamina model:**

Three criteria are used for fiber failure, one in tension/shear, one in compression and another one in crush under pressure. They are chosen in terms of quadratic stress forms as follows:

Tensile/shear fiber mode:

$$f_1 = \left( \frac{\langle \sigma_a \rangle}{S_{aT}} \right)^2 + \left( \frac{\tau_{ab}^2 + \tau_{ca}^2}{S_{FS}^2} \right) - 1 = 0$$

Compression fiber mode:

$$f_2 = \left( \frac{\langle \sigma'_a \rangle}{S_{aC}} \right)^2 - 1 = 0, \quad \sigma'_a = -\sigma_a + \left\langle -\frac{\sigma_b + \sigma_c}{2} \right\rangle$$

Crush mode:

$$f_3 = \left( \frac{\langle p \rangle}{S_{FC}} \right)^2 - 1 = 0, \quad p = -\frac{\sigma_a + \sigma_b + \sigma_c}{3}$$

where  $\langle \ \rangle$  are Macaulay brackets,  $S_{aT}$  and  $S_{aC}$  are the tensile and compressive strengths in the fiber direction, and  $S_{FS}$  and  $S_{FC}$  are the layer strengths associated with the fiber shear and crush failure, respectively.

Matrix mode failures must occur without fiber failure, and hence they will be on planes parallel to fibers. For simplicity, only two failure planes are considered: one is perpendicular to the planes of layering and the other one is parallel to them. The matrix failure criteria for the failure plane perpendicular and parallel to the layering planes, respectively, have the forms:

Perpendicular matrix mode:

$$f_4 = \left( \frac{\langle \sigma_b \rangle}{S_{bT}} \right)^2 + \left( \frac{\tau_{bc}}{S'_{bc}} \right)^2 + \left( \frac{\tau_{ab}}{S_{ab}} \right)^2 - 1 = 0$$

Parallel matrix mode (Delamination):

$$f_5 = S^2 \left\{ \left( \frac{\langle \sigma_c \rangle}{S_{bT}} \right)^2 + \left( \frac{\tau_{bc}}{S''_{bc}} \right)^2 + \left( \frac{\tau_{ca}}{S_{ca}} \right)^2 \right\} - 1 = 0$$

where  $S_{bT}$  is the transverse tensile strength. Based on the Coulomb-Mohr theory, the shear strengths for the transverse shear failure and the two axial shear failure modes are assumed to be the forms,

$$S_{ab} = S_{ab}^{(0)} + \tan(\varphi) \langle -\sigma_b \rangle$$

$$S'_{bc} = S_{bc}^{(0)} + \tan(\varphi) \langle -\sigma_b \rangle$$

$$S_{ca} = S_{ca}^{(0)} + \tan(\varphi) \langle -\sigma_c \rangle$$



$$S_{bc}'' = S_{bc}^{(0)} + \tan(\varphi)\langle -\sigma_c \rangle$$

where  $\varphi$  is a material constant as  $\tan(\varphi)$  is similar to the coefficient of friction, and  $S_{ab}^{(0)}$ ,  $S_{ca}^{(0)}$  and  $S_{bc}^{(0)}$  are the shear strength values of the corresponding tensile modes.

Failure predicted by the criterion of  $f_4$  can be referred to as transverse matrix failure, while the matrix failure predicted by  $f_5$ , which is parallel to the layer, can be referred as the delamination mode when it occurs within the elements that are adjacent to the ply interface. Note that a scale factor  $S$  is introduced to provide better correlation of delamination area with experiments. The scale factor  $S$  can be determined by fitting the analytical prediction to experimental data for the delamination area.

When fiber failure in tension/shear mode is predicted in a layer by  $f_1$ , the load carrying capacity of that layer is completely eliminated. All the stress components are reduced to zero instantaneously (100 time steps to avoid numerical instability). For compressive fiber failure, the layer is assumed to carry a residual axial load, while the transverse load carrying capacity is reduced to zero. When the fiber compressive failure mode is reached due to  $f_2$ , the axial layer compressive strength stress is assumed to reduce to a residual value  $S_{RC}$  ( $=SFFC \times S_{AC}$ ). The axial stress is then assumed to remain constant, i.e.,  $\sigma_a = -S_{RC}$ , for continuous compressive loading, while the subsequent unloading curve follows a reduced axial modulus to zero axial stress and strain state. When the fiber crush failure occurs, the material is assumed to behave elastically for compressive pressure,  $p > 0$ , and to carry no load for tensile pressure,  $p < 0$ .

When a matrix failure (delamination) in the a-b plane is predicted, the strength values for  $S_{ca}^{(0)}$  and  $S_{bc}^{(0)}$  are set to zero. This results in reducing the stress components  $\sigma_c$ ,  $\tau_{bc}$  and  $\tau_{ca}$  to the fractured material strength surface. For tensile mode,  $\sigma_c > 0$ , these stress components are reduced to zero. For compressive mode,  $\sigma_c < 0$ , the normal stress  $\sigma_c$  is assumed to deform elastically for the closed matrix crack. Loading on the failure envelop, the shear stresses are assumed to 'slide' on the fractured strength surface (frictional shear stresses) like in an ideal plastic material, while the subsequent unloading shear stress-strain path follows reduced shear moduli to the zero shear stress and strain state for both  $\tau_{bc}$  and  $\tau_{ca}$  components.

The post failure behavior for the matrix crack in the a-c plane due to  $f_4$  is modeled in the same fashion as that in the a-b plane as described above. In this case, when failure occurs,  $S_{ab}^{(0)}$  and  $S_{bc}^{(0)}$  are reduced to zero instantaneously. The post fracture response is then governed by failure criterion of  $f_5$  with  $S_{ab}^{(0)} = 0$  and  $S_{bc}^{(0)} = 0$ . For tensile mode,  $\sigma_b > 0$ ,  $\sigma_b$ ,  $\tau_{ab}$  and  $\tau_{bc}$  are zero. For compressive mode,  $\sigma_b < 0$ ,  $\sigma_b$  is assumed to be elastic, while  $\tau_{ab}$  and  $\tau_{bc}$  'slide' on the fracture strength surface as in an ideal plastic material, and the unloading path follows reduced shear moduli to the zero shear stress and strain state. It should be noted that  $\tau_{bc}$  is governed by both the failure functions and should lie within or on each of these two strength surfaces.

**Fabric lamina model:**

The fiber failure criteria of Hashin for a unidirectional layer are generalized to characterize the fiber damage in terms of strain components for a plain weave layer. The fill and warp fiber tensile/shear failure are given by the quadratic interaction between the associated axial and shear stresses, i.e.

$$f_6 = \left( \frac{\langle \sigma_a \rangle}{S_{aT}} \right)^2 + \frac{(\tau_{ab}^2 + \tau_{ca}^2)}{S_{aFS}^2} - 1 = 0$$

$$f_7 = \left( \frac{\langle \sigma_b \rangle}{S_{bT}} \right)^2 + \frac{(\tau_{ab}^2 + \tau_{bc}^2)}{S_{bFS}^2} - 1 = 0$$

where  $S_{aT}$  and  $S_{bT}$  are the axial tensile strengths in the fill and warp directions, respectively, and  $S_{aFS}$  and  $S_{bFS}$  are the layer shear strengths due to fiber shear failure in the fill and warp directions. These failure criteria are applicable when the associated  $\sigma_a$  or  $\sigma_b$  is positive. It is assumed  $S_{aFS} = SFS$ , and

$$S_{bFS} = SFS \times \frac{S_{bT}}{S_{aT}}$$

When  $\sigma_a$  or  $\sigma_b$  is compressive, it is assumed that the in-plane compressive failure in both the fill and warp directions are given by the maximum stress criterion, i.e.

$$f_8 = \left[ \frac{\langle \sigma'_a \rangle}{S_{aC}} \right]^2 - 1 = 0, \quad \sigma'_a = -\sigma_a + \langle -\sigma_c \rangle$$

$$f_9 = \left[ \frac{\langle \sigma'_b \rangle}{S_{bC}} \right]^2 - 1 = 0, \quad \sigma'_b = -\sigma_b + \langle -\sigma_c \rangle$$

where  $S_{aC}$  and  $S_{bC}$  are the axial compressive strengths in the fill and warp directions, respectively. The crush failure under compressive pressure is

$$f_{10} = \left( \frac{\langle p \rangle}{S_{FC}} \right)^2 - 1 = 0, \quad p = -\frac{\sigma_a + \sigma_b + \sigma_c}{3}$$

A plain weave layer can fail under in-plane shear stress without the occurrence of fiber breakage. This in-plane matrix failure mode is given by

$$f_{11} = \left( \frac{\tau_{ab}}{S_{ab}} \right)^2 - 1 = 0$$

where  $S_{ab}$  is the layer shear strength due to matrix shear failure.

Another failure mode, which is due to the quadratic interaction between the thickness stresses, is expected to be mainly a matrix failure. This through the thickness matrix failure criterion is

$$f_{12} = S^2 \left\{ \left( \frac{\langle \sigma_c \rangle}{S_{cT}} \right)^2 + \left( \frac{\tau_{bc}}{S_{bc}} \right)^2 + \left( \frac{\tau_{ca}}{S_{ca}} \right)^2 \right\} - 1 = 0$$

where  $S_{cT}$  is the through the thickness tensile strength, and  $S_{bc}$ , and  $S_{ca}$  are the shear strengths assumed to depend on the compressive normal stress  $\sigma_c$ , i.e.,

$$\begin{Bmatrix} S_{ca} \\ S_{bc} \end{Bmatrix} = \begin{Bmatrix} S_{ca}^{(0)} \\ S_{bc}^{(0)} \end{Bmatrix} + \tan(\varphi) \langle -\sigma_c \rangle$$

When failure predicted by this criterion occurs within elements that are adjacent to the ply interface, the failure plane is expected to be parallel to the layering planes, and, thus, can be referred to as the delamination mode. Note that a scale factor S is introduced to provide better correlation of delamination area with experiments. The scale factor S can be determined by fitting the analytical prediction to experimental data for the delamination area.

Similar to the unidirectional model, when fiber tensile/shear failure is predicted in a layer by f6 or f7, the load carrying capacity of that layer in the associated direction is completely eliminated. For compressive fiber failure due to by f8 or f9, the layer is assumed to carry a residual axial load in the failed direction, while the load carrying capacity transverse to the failed direction is assumed unchanged. When the compressive axial stress in a layer reaches the compressive axial strength  $S_{aC}$  or  $S_{bC}$ , the axial layer stress is assumed to be reduced to the residual strength  $S_{aRC}$  or  $S_{bRC}$  where  $S_{aRC} = \text{SFFC} \times S_{aC}$  and  $S_{bRC} = \text{SFFC} \times S_{bC}$ . The axial stress is assumed to remain constant, i.e.,  $\sigma_a = -S_{aCR}$  or  $\sigma_b = -S_{bCR}$ , for continuous compressive loading, while the subsequent unloading curve follows a reduced axial modulus. When the fiber crush failure is occurred, the material is assumed to behave elastically for compressive pressure,  $p > 0$ , and to carry no load for tensile pressure,  $p < 0$ .

When the in-plane matrix shear failure is predicted by f11 the axial load carrying capacity within a failed element is assumed unchanged, while the in-plane shear stress is assumed to be reduced to zero.

For through the thickness matrix (delamination) failure given by equations f12, the in-plane load carrying capacity within the element is assumed to be elastic, while the strength values for the tensile mode,  $S_{ca}^{(0)}$  and  $S_{bc}^{(0)}$ , are set to zero. For tensile mode,  $\sigma_c > 0$ , the through the thickness stress components are reduced to zero. For compressive mode,  $\sigma_c < 0$ ,  $\sigma_c$  is assumed to be elastic, while  $\tau_{bc}$  and  $\tau_{ca}$  'slide' on the fracture strength surface as in an ideal plastic material, and the unloading path follows reduced shear moduli to the zero shear stress and strain state.

The effect of strain-rate on the layer strength values of the fiber failure modes is modeled by the strain-rate dependent functions for the strength values  $\{S_{RT}\}$  as

$$\{S_{RT}\} = \{S_0\} \left( 1 + C_{\text{rate1}} \ln \frac{\{\dot{\epsilon}\}}{\dot{\epsilon}_0} \right)$$

$$\{S_{RT}\} = \begin{Bmatrix} S_{aT} \\ S_{aC} \\ S_{bT} \\ S_{bC} \\ S_{FC} \\ S_{FS} \end{Bmatrix}, \quad \{\dot{\epsilon}\} = \begin{Bmatrix} |\dot{\epsilon}_a| \\ |\dot{\epsilon}_a| \\ |\dot{\epsilon}_b| \\ |\dot{\epsilon}_b| \\ |\dot{\epsilon}_c| \\ (\dot{\epsilon}_{ca}^2 + \dot{\epsilon}_{bc}^2)^{1/2} \end{Bmatrix}$$

where  $C_{rate}$  is the strain-rate constants, and  $\{S_0\}$  are the strength values of  $\{S_{RT}\}$  at the reference strain-rate  $\dot{\epsilon}_0$ .

**Damage model:**

The damage model is a generalization of the layer failure model of Material 161 by adopting the MLT damage mechanics approach, Matzenmiller et al. [1995], for characterizing the softening behavior after damage initiation. Complete model description is given in Yen [2002]. The damage functions, which are expressed in terms of ply level engineering strains, are converted from the above failure criteria of fiber and matrix failure modes by neglecting the Poisson’s effect. Elastic moduli reduction is expressed in terms of the associated damage parameters  $\omega_i$ :

$$E'_i = (1 - \omega_i)E_i$$

$$\omega_i = 1 - \exp\left(-\frac{r_i^{m_i}}{m_i}\right), \quad r_i \geq 0, \quad i = 1, \dots, 6,$$

where  $E_i$  are the initial elastic moduli,  $E'_i$  are the reduced elastic moduli,  $r_i$  are the damage thresholds computed from the associated damage functions for fiber damage, matrix damage and delamination, and  $m_i$  are material damage parameters, which are currently assumed to be independent of strain-rate. The damage function is formulated to account for the overall nonlinear elastic response of a lamina including the initial ‘hardening’ and the subsequent softening beyond the ultimate strengths.

In the damage model (material 162), the effect of strain-rate on the nonlinear stress-strain response of a composite layer is modeled by the strain-rate dependent functions for the elastic moduli  $\{E_{RT}\}$  as

$$\{E_{RT}\} = \{E_0\} \left( 1 + \{C_{rate}\} \ln \frac{\{\dot{\epsilon}\}}{\dot{\epsilon}_0} \right)$$

$$\{E_{RT}\} = \begin{Bmatrix} E_a \\ E_b \\ E_c \\ G_{ab} \\ G_{bc} \\ G_{ca} \end{Bmatrix}, \quad \{\dot{\epsilon}\} = \begin{Bmatrix} |\dot{\epsilon}_a| \\ |\dot{\epsilon}_b| \\ |\dot{\epsilon}_c| \\ |\dot{\epsilon}_{ab}| \\ |\dot{\epsilon}_{bc}| \\ |\dot{\epsilon}_{ca}| \end{Bmatrix}, \quad \{C_{rate}\} = \begin{Bmatrix} C_{rate2} \\ C_{rate2} \\ C_{rate4} \\ C_{rate3} \\ C_{rate3} \\ C_{rate3} \end{Bmatrix}$$

where  $\{C_{rate}\}$  are the strain-rate constants.  $\{E_0\}$  are the modulus values of  $\{E_{RT}\}$  at the reference strain-rate  $\dot{\epsilon}_0$ .

### Element Erosion:

A failed element is eroded in any of three different ways:

1. If fiber tensile failure in a unidirectional layer is predicted in the element and the axial tensile strain is greater than E\_LIMT. For a fabric layer, both in-plane directions are failed and exceed E\_LIMT.
2. If compressive relative volume in a failed element is smaller than ECRSH.
3. If tensile relative volume in a failed element is greater than EEXPV.

### Damage History Parameters:

Information about the damage history variables for the associated failure modes can be plotted in LS-PrePost. These additional history variables are tabulated below:

History Variable	Description	Value	LS-PrePost History Variable
1. efa(I)	Fiber mode in a		7
2. efb(I)	Fiber mode in b	0-elastic	8
3. efp(I)	Fiber crush mode		9
4. em(I)	Perpendicular matrix mode	$\geq 1$ -failed	10
5. ed(I)	Parallel matrix/ delamination mode		11
6. delm(I)	delamination mode		12

**\*MAT\_MODIFIED\_CRUSHABLE\_FOAM**

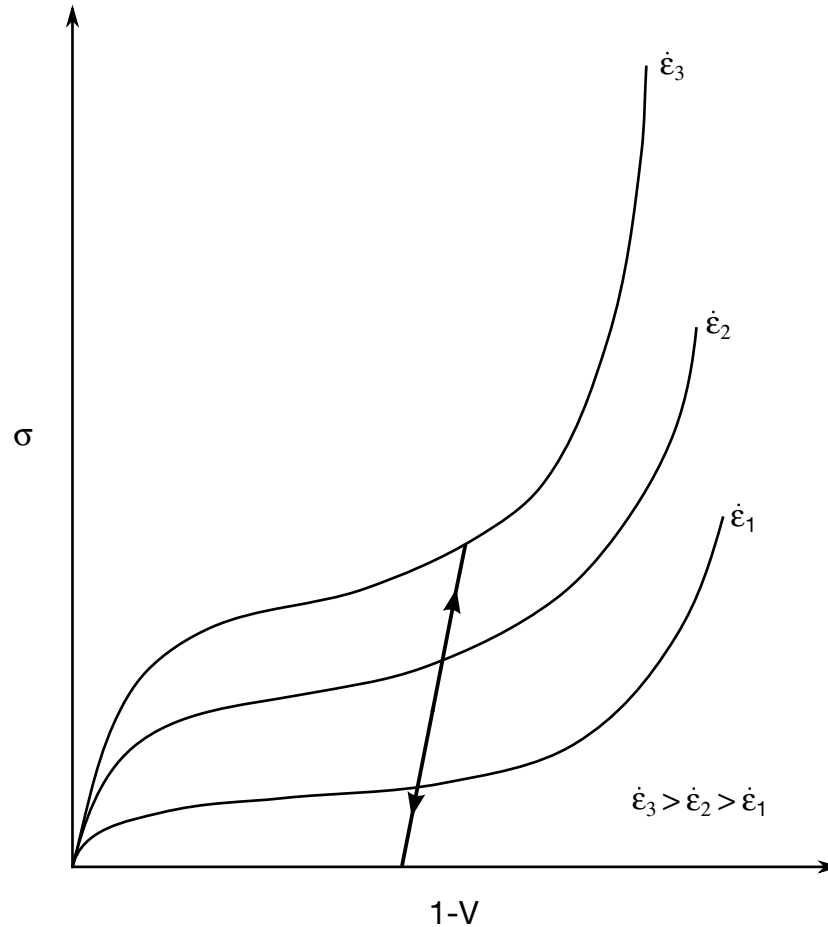
This is Material Type 163 which is dedicated to modeling crushable foam with optional damping, tension cutoff, and strain rate effects. Unloading is fully elastic. Tension is treated as elastic-perfectly-plastic at the tension cut-off value.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	TID	TSC	DAMP	NCYCLE
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.10	12.

Card 2	1	2	3	4	5	6	7	8
Variable	SRCLMT	SFLAG						
Type	F	I						
Default	1.E+20	0						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
TID	Table ID defining yield stress versus volumetric strain, $\gamma$ , at different strain rates.
TSC	Tensile stress cutoff. A nonzero, positive value is strongly recommended for realistic behavior.
DAMP	Rate sensitivity via damping coefficient ( $.05 <$ recommended value $<.50$ ).



**Figure M163-1.** Rate effects are defined by a family of curves giving yield stress versus volumetric strain where  $V$  is the relative volume.

VARIABLE	DESCRIPTION
NCYCLE	Number of cycles to determine the average volumetric strain rate.
SRCLMT	Strain rate change limit.
SFLAG	The strain rate in the table may be the true strain rate (SFLAG = 0) or the engineering strain rate (SFLAG = 1).

#### Remarks:

The volumetric strain is defined in terms of the relative volume,  $V$ , as:

$$\gamma = 1 - V$$

The relative volume is defined as the ratio of the current to the initial volume. In place of the effective plastic strain in the D3PLOT database, the integrated volumetric strain is output.

This material is an extension of material 63, \*MAT\_CRUSHABLE\_FOAM. It allows the yield stress to be a function of both volumetric strain rate and volumetric strain. Rate effects are accounted for by defining a table of curves using \*DEFINE\_TABLE. Each curve defines the yield stress versus volumetric strain for a different strain rate. The yield stress is obtained by interpolating between the two curves that bound the strain rate.

To prevent high frequency oscillations in the strain rate from causing similar high frequency oscillations in the yield stress, a modified volumetric strain rate is used when interpolating to obtain the yield stress. The modified strain rate is obtained as follows. If NYCLE is  $> 1$ , then the modified strain rate is obtained by a time average of the actual strain rate over NCYCLE solution cycles. For SRCLMT  $> 0$ , the modified strain rate is capped so that during each cycle, the modified strain rate is not permitted to change more than SRCLMT multiplied by the solution time step.



**\*MAT\_BRAIN\_LINEAR\_VISCOELASTIC**

This is Material Type 164. This material is a Kelvin-Maxwell model for modeling brain tissue, which is valid for solid elements only. See Remarks below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	G0	GI	DC	FO	S0
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
BULK	Bulk modulus (elastic)
G0	Short-time shear modulus, $G_0$
GI	Long-time (infinite) shear modulus, $G_\infty$
DC	Maxwell decay constant, $\beta$ [FO = 0.0] or Kelvin relaxation constant, $\tau$ [FO = 1.0]
FO	Formulation option: EQ.0.0: Maxwell, EQ.1.0: Kelvin.

VARIABLE	DESCRIPTION
SO	<p>Strain (logarithmic) output option to control what is written as component 7 to the d3plot database. (LS-PrePost always blindly labels this component as effective plastic strain.) The maximum values are updated for each element each time step:</p> <p>EQ.0.0: maximum principal strain that occurs during the calculation,</p> <p>EQ.1.0: maximum magnitude of the principal strain values that occurs during the calculation,</p> <p>EQ.2.0: maximum effective strain that occurs during the calculation.</p>

**Remarks:**

The shear relaxation behavior is described for the Maxwell model by:

$$G(t) = G + (G_0 - G_\infty)e^{-\beta t}$$

A Jaumann rate formulation is used

$$\overset{\nabla}{\sigma}_{ij} = 2 \int_0^t G(t - \tau) D'_{ij}(\tau) dt$$

where the prime denotes the deviatoric part of the stress rate,  $\overset{\nabla}{\sigma}_{ij}$ , and the strain rate  $D_{ij}$ . For the Kelvin model the stress evolution equation is defined as:

$$\dot{s}_{ij} + \frac{1}{\tau} s_{ij} = (1 + \delta_{ij}) G_0 \dot{e}_{ij} + (1 + \delta_{ij}) \frac{G_\infty}{\tau} \dot{e}_{ij}$$

The strain data as written to the d3plot database may be used to predict damage, see [Bandak 1991].

**\*MAT\_PLASTIC\_NONLINEAR\_KINEMATIC**

This is Material Type 165. This relatively simple model, based on a material model by Lemaitre and Chaboche [1990], is suited to model nonlinear kinematic hardening plasticity. The model accounts for the nonlinear Bauschinger effect, cyclic hardening, and ratcheting. Huang [2006] programmed this model and provided it as a user subroutine. It is a very cost effective model and is available shell and solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	H	C	GAMMA
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	FS							
Type	F							
Default	1.E+16							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Initial yield stress, $\sigma_{y0}$ .
H	Isotropic plastic hardening modulus
C	Kinematic hardening modulus

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GAMMA	Kinematic hardening parameter, $\gamma$ .
FS	Failure strain for eroding elements.

---

**Remarks:**

If the isotropic hardening modulus,  $H$ , is nonzero, the size of the surface increases as function of the equivalent plastic strain,  $\epsilon^p$ :

$$\sigma_y = \sigma_{y0} + H\epsilon^p$$

The rate of evolution of the kinematic component is a function of the plastic strain rate:

$$\dot{\alpha} = [Cn - \gamma\alpha]\dot{\epsilon}^p$$

where,  $n$ , is the flow direction. The term,  $\gamma\alpha\dot{\epsilon}^p$ , introduces the nonlinearity into the evolution law, which becomes linear if the parameter,  $\gamma$ , is set to zero.

**\*MAT\_PLASTIC\_NONLINEAR\_KINEMATIC\_B**

This is Material Type 165B. This material model is implemented to model the cyclic fatigue behavior. This model applies to both shell and solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	RE	B	Q	C1
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	GAMMA1	C2	GAMMA2	C3	GAMMA3			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's Modulus
PR	Poisson's ratio
RE	Yield stress, see Remarks.
B	Material parameter, see Remarks.
Q	Material parameter, see Remarks.

VARIABLE	DESCRIPTION
C1, GAMMA1, C2, GAMMA2, C3, GAMMA3	Material parameters, see Remarks.

### A model of elastoplastic cyclic hardening:

This material model is based on a 2013 paper by S. Plessis which modeled a double-notched specimen with Herbland (2009) material model. The elastoplastic stress tensor is given by:

$$\underline{\sigma} = \underline{\sigma}_M - \underline{L}; \underline{\varepsilon}^p$$

where,  $\underline{\sigma}_M$  is elastic stress,  $\underline{\varepsilon}^p$  is the plastic strain tensor. In a one-dimensional problem, the above equation becomes:

$$\sigma = \sigma_M - L' \varepsilon^p$$

where,  $L'$  is a parameter identified with FEM on a monotonic loading.

In the elasticity domain:

$$f = J_2(\underline{\sigma} - X_T) - R_e - R \leq 0$$

where  $J_2$  is the second stress invariant,  $\underline{\sigma}$  is the stress tensor,  $R$  is the isotropic hardening variable,  $R_e$  (variable RE) is the yield stress.

Evolution law of the isotropic hardening variable  $R$ :

$$\dot{R} = b X(Q - R) \dot{p}$$

where  $b$  (variable B) and  $Q$  (variable Q) are two material parameters,  $\dot{p}$  is the plastic strain rate defined by:

$$\dot{p} = \sqrt{\frac{2}{3} \underline{\dot{\varepsilon}}^p : \underline{\dot{\varepsilon}}^p}$$

with  $\underline{\dot{\varepsilon}}^p$  the plastic strain tensor.

Evolution law of the variable of kinematic hardening:

$$\underline{\dot{X}}_T = \sum_i \underline{\dot{X}}_i$$

with,

$$\underline{\dot{X}}_i = \frac{2}{3} C_i \underline{\dot{\varepsilon}}^p - \gamma_i \underline{X}_i \dot{p}$$

where  $\underline{\dot{X}}_T$  is the kinematic hardening tensor, and is the sum of three tensors  $\dot{X}_i$  ( $i = 1\sim 3$ ), each dependent on the one set of material coefficients  $C_i$  (variables C1, C2, C3) and  $\gamma_i$  (variables GAMMA1, GAMMA2, GAMMA3).

**Revision information:**

This material model is available starting in Revision 102594.

**\*MAT\_166****\*MAT\_MOMENT\_CURVATURE\_BEAM****\*MAT\_MOMENT\_CURVATURE\_BEAM**

This is Material Type 166. This material is for performing nonlinear elastic or multi-linear plastic analysis of Belytschko-Schwer beams with user-defined axial force-strain, moment curvature and torque-twist rate curves. If strain, curvature or twist rate is located outside the curves, use extrapolation to determine the corresponding rigidity. For multi-linear plastic analysis, the user-defined curves are used as yield surfaces.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	ELAF	EPFLG	CTA	CTB	CTT
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	0.0	0.0	0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	N1	N2	N3	N4	N5	N6	N7	N8
Type	F	F	F	F	F	F	F	F
Default	none	none	0.0 / none	0.0	0.0	0.0	0.0	0.0

Card 3	1	2	3	4	5	6	7	8
Variable	LCMS1	LCMS2	LCMS3	LCMS4	LCMS5	LCMS6	LCMS7	LCMS8
Type	F	F	F	F	F	F	F	F
Default	none	none	0.0 / none	0.0	0.0	0.0	0.0	0.0



Card 4	1	2	3	4	5	6	7	8
Variable	LCMT1	LCMT2	LCMT3	LCMT4	LCMT5	LCMT6	LCMT7	LCMT8
Type	F	F	F	F	F	F	F	F
Default	none	none	0.0 / none	0.0	0.0	0.0	0.0	0.0

Card 5	1	2	3	4	5	6	7	8
Variable	LCT1	LCT2	LCT3	LCT4	LCT5	LCT6	LCT7	LCT8
Type	F	F	F	F	F	F	F	F
Default	none	none	0.0 / none	0.0	0.0	0.0	0.0	0.0

**Multilinear Plastic Analysis Card.** Additional card for EPFLG = 1.

Card 6	1	2	3	4	5	6	7	8
Variable	CFA	CFB	CFT	HRULE	REPS	RBETA	RCAPAY	RCAPAZ
Type	F	F	F	F	F	F	F	F
Default	1.0	1.0	1.0	0.0	1.0E+20	1.0E+20	1.0E+20	1.0E+20

**VARIABLE**

**DESCRIPTION**

- MID Material identification. A unique number or label not exceeding 8 characters must be specified.
- RO Mass density
- E Young's modulus. This variable controls the time step size and must be chosen carefully. Increasing the value of E will decrease the time step size.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ELAF	Load curve ID for the axial force-strain curve
EPFLG	Function flag EQ.0.0: nonlinear elastic analysis EQ.1.0: multi-linear plastic analysis
CTA, CTB, CTT	Type of axial force-strain, moment-curvature, and torque-twist rate curves EQ.0.0: curve is symmetric EQ.1.0: curve is asymmetric  For symmetric curves, all data point must be in the first quadrant and at least three data points need to be given, starting from the origin, ensued by the yield point.  For asymmetric curves, at least five data points are needed and exactly one point must be at the origin. The two points on both sides of the origin record the positive and negative yield points.  The last data point(s) has no physical meaning: it serves only as a control point for inter or extrapolation.  The curves are input by the user and treated in LS-DYNA as a linearly piecewise function. The curves must be monotonically increasing, while the slopes must be monotonically decreasing
N1 - N8	Axial forces at which moment-curvature curves are given. The axial forces must be ordered monotonically increasing. At least two axial forces must be defined if the curves are symmetric. At least three axial forces must be defined if the curves are asymmetric.
LCMS1 - LCMS8	Load curve IDs for the moment-curvature curves about axis S under corresponding axial forces.
LCMT1 - LCMT8	Load curve IDs for the moment-curvature curves about axis T under corresponding axial forces.
LCT1 - LCT8	Load curve IDs for the torque-twist rate curves under corresponding axial forces.
CFA, CFB, CFT	For multi-linear plastic analysis only. Ratio of axial, bending and torsional elastic rigidities to their initial values, no less than 1.0 in value.

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
HRULE	Hardening rule, for multi-linear plastic analysis only. EQ.0.0: isotropic hardening GT.0.0.AND.LT.1.0: mixed hardening EQ.1.0: kinematic hardening
REPS	Rupture effective plastic axial strain
RBETA	Rupture effective plastic twist rate
RCAPAY	Rupture effective plastic curvature about axis S
RCAPAZ	Rupture effective plastic curvature about axis T

**\*MAT\_MCCORMICK**

This is Material Type 167. This is a constitutive model for finite plastic deformities in which the material's strength is defined by McCormick's constitutive relation for materials exhibiting negative steady-state Strain Rate Sensitivity (SRS). McCormick [1988] and Zhang, McCormick and Estrin [2001].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY			
Type	A8	F	F	F	F			

Card 2	1	2	3	4	5	6	7	8
Variable	Q1	C1	Q2	C2				
Type	F	F	F	F				

Card 3	1	2	3	4	5	6	7	8
Variable	S	H	OMEGA	TD	ALPHA	EPS0		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Initial yield stress
Q1	Isotropic hardening parameter, $Q_1$
C1	Isotropic hardening parameter, $C_1$

VARIABLE	DESCRIPTION
Q2	Isotropic hardening parameter, $Q_2$
C2	Isotropic hardening parameter, $C_2$
S	Dynamic strain aging parameter, $S$
H	Dynamic strain aging parameter, $H$
OMEGA	Dynamic strain aging parameter, $\Omega$
TD	Dynamic strain aging parameter, $t_d$
ALPHA	Dynamic strain aging parameter, $\alpha$
EPS0	Reference strain rate, $\dot{\epsilon}_0$

### Remarks:

The uniaxial stress-strain curve is given in the following form:

$$\sigma(\epsilon^p, \dot{\epsilon}^p) = \sigma_Y(t_a) + R(\epsilon^p) + \sigma_v(\dot{\epsilon}^p)$$

Viscous stress  $\sigma_v$  is given by

$$\sigma_v(\dot{\epsilon}^p) = S \times \ln \left( 1 + \frac{\dot{\epsilon}^p}{\dot{\epsilon}_0} \right)$$

where  $S$  represents the instantaneous strain rate sensitivity and  $\dot{\epsilon}_0$  is a reference strain rate.

In the McCormick model the yield strength including the contribution from dynamic strain aging (DSA) is defined as

$$\sigma_Y(t_a) = \sigma_o + S \times H \times \left[ 1 - \exp \left\{ - \left( \frac{t_a}{t_d} \right)^\alpha \right\} \right]$$

where  $\sigma_o$  is the yield strength for vanishing average waiting time  $t_a$ , and  $H$ ,  $\alpha$ , and  $t_d$  are material constants linked to dynamic strain aging.

The average waiting time is defined by the evolution equation

$$\dot{t}_a = 1 - \frac{t_a}{t_{a,ss}}$$

where the quasi-steady state waiting time  $t_{a,ss}$  is given as

$$t_{a,ss} = \frac{\Omega}{\dot{\epsilon}^p}$$

The strain hardening function  $R$  is defined by the extended Voce Law

$$R(\varepsilon^p) = Q_1[1 - \exp(-C_1\varepsilon^p)] + Q_2[1 - \exp(-C_2\varepsilon^p)].$$

## \*MAT\_POLYMER

This is material type 168. This model is implemented for brick elements.

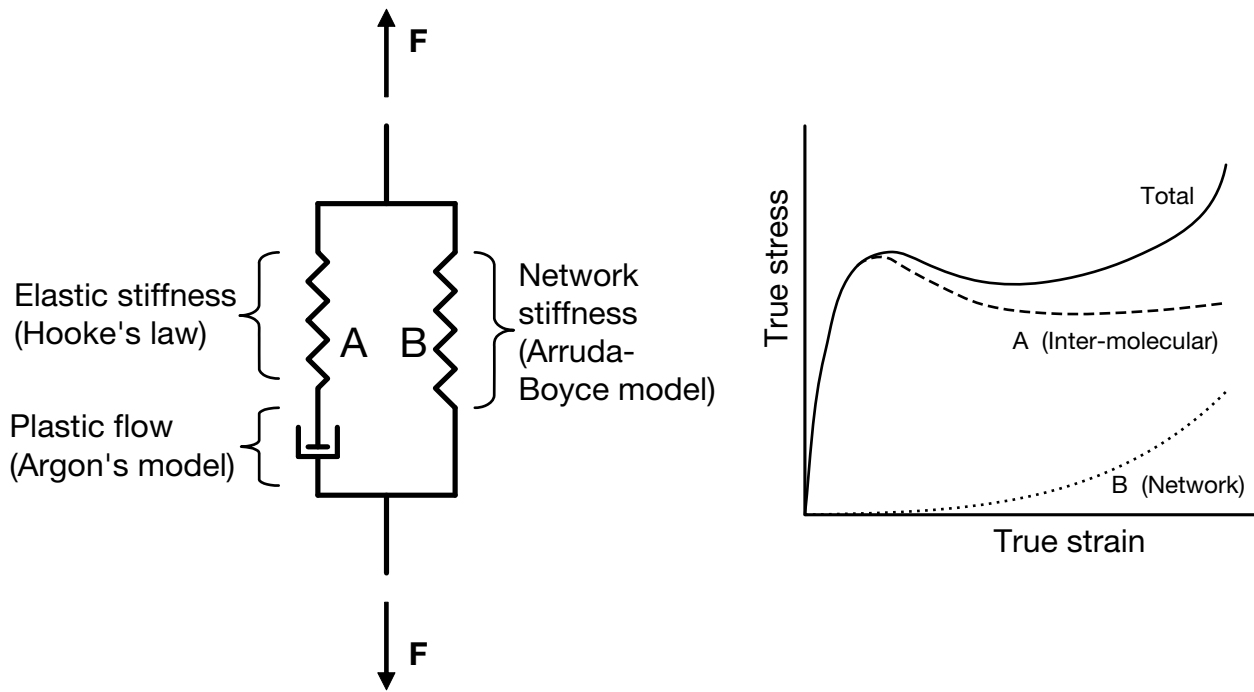
Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	GAMMA0	DG	SC	ST
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	TEMP	K	CR	N	C			
Type	F	F	F	F	F			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass Density.
E	Young's modulus, $E$ .
PR	Poisson's ratio, $\nu$ .
GAMMA0	Pre-exponential factor, $\dot{\gamma}_{0A}$ .
DG	Energy barrier to flow, $\Delta G$ .
SC	Shear resistance in compression, $S_c$ .
ST	Shear resistance in tension, $S_t$ .
TEMP	Absolute temperature, $\theta$ .
K	Boltzmann constant, $k$ .
CR	Product, $C_r = nk\theta$ .
N	Number of 'rigid links' between entanglements, $N$ .
C	Relaxation factor, $C$ .

Remarks:



**Figure M168-1.** Stress decomposition in inter-molecular and network contributions.

The polymer is assumed to have two basic resistances to deformation:

1. An inter-molecular barrier to deformation related to relative movement between molecules.
2. An evolving anisotropic resistance related to straightening of the molecule chains.

The model which is implemented and presented in this paper is mainly based on the framework suggested by Boyce et al. [2000]. Going back to the original work by Haward and Thackray [1968], they considered the uniaxial case only. The extension to a full 3D formulation was proposed by Boyce et al. [1988]. Moreover, Boyce and co-workers have during a period of 20 years changed or further developed the parts of the original model. Haward and Thackray [1968] used an Eyring model to represent the dashpot in Fig. M168-1, while Boyce et al. [2000] employed the double-kink model of Argon [1973] instead. Part B of the model, describing the resistance associated with straightening of the molecules, contained originally a one-dimensional Langevin spring [Haward and Thackray, 1968], which was generalized to 3D with the eight-chain model by Arruda and Boyce [1993].

The main structure of the model presented by Boyce et al. [2000] is kept for this model. Recognizing the large elastic deformations occurring for polymers, a formulation based on



a Neo-Hookean material is here selected for describing the spring in resistance A in [Figure M168-1](#).

Referring to [Figure M168-1](#), it is assumed that the deformation gradient tensor is the same for the two resistances (Part A and B)

$$\mathbf{F} = \mathbf{F}_A = \mathbf{F}_B$$

while the Cauchy stress tensor for the system is assumed to be the sum of the Cauchy stress tensors for the two parts

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_A + \boldsymbol{\sigma}_B.$$

### Part A: Inter-molecular resistance:

The deformation is decomposed into elastic and plastic parts,  $\mathbf{F}_A = \mathbf{F}_A^e \cdot \mathbf{F}_A^p$ , where it is assumed that the intermediate configuration  $\bar{\Omega}_A$  defined by  $\mathbf{F}_A^p$  is invariant to rigid body rotations of the current configuration. The velocity gradient in the current configuration  $\Omega$  is defined by

$$\mathbf{L}_A = \dot{\mathbf{F}}_A \cdot \mathbf{F}_A^{-1} = \mathbf{L}_A^e + \mathbf{L}_A^p$$

Owing to the decomposition,  $\mathbf{F}_A = \mathbf{F}_A^e \cdot \mathbf{F}_A^p$ , the elastic and plastic rate-of-deformation and spin tensors are defined by

$$\begin{aligned} \mathbf{L}_A^e &= \mathbf{D}_A^e + \mathbf{W}_A^e = \dot{\mathbf{F}}_A^e \cdot (\mathbf{F}_A^e)^{-1} \\ \mathbf{L}_A^p &= \mathbf{D}_A^p + \mathbf{W}_A^p = \mathbf{F}_A^e \cdot \dot{\mathbf{F}}_A^p \cdot (\mathbf{F}_A^p)^{-1} \cdot (\mathbf{F}_A^e)^{-1} = \mathbf{F}_A^e \cdot \bar{\mathbf{L}}_A^p \cdot (\mathbf{F}_A^e)^{-1} \end{aligned}$$

where  $\bar{\mathbf{L}}_A^p = \dot{\mathbf{F}}_A^p \cdot (\mathbf{F}_A^p)^{-1}$ . The Neo-Hookean material represents an extension of Hooke's law to large elastic deformations and may be chosen for the elastic part of the deformation when the elastic behavior is assumed to be isotropic.

$$\boldsymbol{\tau}_A = \lambda_0 \ln J_A^e \mathbf{I} + \mu_0 (\mathbf{B}_A^e - \mathbf{I})$$

where  $\boldsymbol{\tau}_A = J_A \boldsymbol{\sigma}_A$  is the Kirchhoff stress tensor of Part A and  $J_A^e = \sqrt{\det \mathbf{B}_A^e} = J_A$  is the Jacobian determinant. The elastic left Cauchy-Green deformation tensor is given by  $\mathbf{B}_A^e = \mathbf{F}_A^e \cdot \mathbf{F}_A^{eT}$ .

The flow rule is defined by

$$\mathbf{L}_A^p = \dot{\gamma}_A^p \mathbf{N}_A$$

where

$$\mathbf{N}_A = \frac{1}{\sqrt{2} \tau_A} \boldsymbol{\tau}_A^{\text{dev}}, \quad \tau_A = \sqrt{\frac{1}{2} \text{tr}(\boldsymbol{\tau}_A^{\text{dev}})^2}$$

and  $\boldsymbol{\tau}_A^{\text{dev}}$  is the stress deviator. The rate of flow is taken to be a thermally activated process

$$\dot{\gamma}_A^p = \dot{\gamma}_{0A} \exp \left[ -\frac{\Delta G(1 - \tau_A/s)}{k\theta} \right]$$

where  $\dot{\gamma}_{0A}$  is a pre-exponential factor,  $\Delta G$  is the energy barrier to flow,  $s$  is the shear resistance,  $k$  is the Boltzmann constant and  $\theta$  is the absolute temperature. The shear resistance  $s$  is assumed to depend on the stress triaxiality  $\sigma^*$ ,

$$s = s(\sigma^*), \quad \sigma^* = \frac{\text{tr } \boldsymbol{\sigma}_A}{3\sqrt{3}\tau_A}$$

The exact dependence is given by a user-defined load curve, which is linear between the shear resistances in compression and tension. These resistances are denoted  $s_c$  and  $s_t$ , respectively.

### Part B: Network resistance:

The network resistance is assumed to be nonlinear elastic with deformation gradient  $\mathbf{F}_B = \mathbf{F}_B^N$ , i.e. any viscoplastic deformation of the network is neglected. The stress-stretch relation is defined by

$$\boldsymbol{\tau}_B = \frac{nk\theta}{3} \frac{\sqrt{N}}{\bar{\lambda}_N} \mathcal{L}^{-1} \left( \frac{\bar{\lambda}_N}{\sqrt{N}} \right) (\bar{\mathbf{B}}_B^N - \bar{\lambda}_N^2 \mathbf{I})$$

where  $\tau_B = J_B \sigma_B$  is the Kirchhoff stress for Part B,  $n$  is the chain density and  $N$  the number of 'rigid links' between entanglements. In accordance with Boyce et. al [2000], the product,  $nk\theta$  is denoted  $C_R$  herein. Moreover,  $\mathcal{L}^{-1}$  is the inverse Langevin function,  $\mathcal{L}(\beta) = \coth\beta - 1/\beta$ , and further

$$\bar{\mathbf{B}}_B^N = \bar{\mathbf{F}}_B^N \cdot \bar{\mathbf{F}}_B^{N^T}, \quad \bar{\mathbf{F}}_B^N = J_B^{-1/3} \mathbf{F}_B^N, \quad J_B = \det \mathbf{F}_B^N, \quad \bar{\lambda}_N = \left[ \frac{1}{3} \text{tr } \bar{\mathbf{B}}_B^N \right]^{\frac{1}{2}}$$

The flow rule defining the rate of molecular relaxation reads

$$\mathbf{L}_B^F = \dot{\gamma}_B^F \mathbf{N}_B$$

where

$$\mathbf{N}_B = \frac{1}{\sqrt{2}} \frac{\boldsymbol{\tau}_B^{\text{dev}}}{\tau_B}, \quad \tau_B = \sqrt{\frac{1}{2} \boldsymbol{\tau}_B^{\text{dev}} : \boldsymbol{\tau}_B^{\text{dev}}}$$

The rate of relaxation is taken equal to

$$\dot{\gamma}_B^F = C \left( \frac{1}{\bar{\lambda}_F - 1} \right) \tau_B$$

where

$$\bar{\lambda}_F = \left[ \frac{1}{3} \text{tr} \left( \mathbf{F}_B^F \{ \mathbf{F}_B^F \}^T \right) \right]^{\frac{1}{2}}$$

The model has been implemented into LS-DYNA using a semi-implicit stress-update scheme [Moran et. al 1990], and is available for the explicit solver only.

**\*MAT\_ARUP\_ADHESIVE**

This is Material Type 169. This material model was written for adhesive bonding in aluminum structures. The plasticity model is not volume-conserving, and hence avoids the spuriously high tensile stresses that can develop if adhesive is modeled using traditional elasto-plastic material models. It is available **only** for solid elements of formulations 1, 2 and 15. The smallest dimension of the element is assumed to be the through-thickness dimension of the bond, unless THKDIR = 1.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	TENMAX	GCTEN	SHRMAX	GCSHR
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	10 <sup>20</sup>	10 <sup>20</sup>	10 <sup>20</sup>	10 <sup>20</sup>

Card 2	1	2	3	4	5	6	7	8
Variable	PWRT	PWRS	SHRP	SHT_SL	EDOT0	EDOT2	THKDIR	EXTRA
Type	F	F	F	F	F	F	F	F
Default	2.0	2.0	0.0	0.0	1.0	0.0	0.0	0.0

**Additional card for Extra = 1 or 3.**

Card 3	1	2	3	4	5	6	7	8
Variable	TMAXE	GCTE	SMAXE	GCSE	PWRTE	PWRSE		
Type	F	F	F	F	F	F		
Default	10 <sup>20</sup>	10 <sup>20</sup>	10 <sup>20</sup>	10 <sup>20</sup>	2.0	2.0		

**Additional card for Extra = 1 or 3.**

Card 4	1	2	3	4	5	6	7	8
Variable	FACET	FACCT	FACES	FACCS	SOFTT	SOFTS		
Type	F	F	F	F	F	F		
Default	1.0	1.0	1.0	1.0	1.0	1.0		

**Dynamic Strain Rate Card.** Additional card for EDOT2 ≠ 0.

Card 5	1	2	3	4	5	6	7	8
Variable	SDFAC	SGFAC	SDEFAC	SGEFAC				
Type	F	F	F	F				
Default	1.0	1.0	1.0	1.0				

**Bond Thickness Card.** Additional card for Extra = 2 or 3.

Card 6	1	2	3	4	5	6	7	8
Variable	BTHK	OUTFAIL	FSIP					
Type	F	F	F					
Default	0.0	0.0	0.0					

**VARIABLE**

**DESCRIPTION**

- MID Material identification. A unique number or label not exceeding 8 characters must be specified.
- RO Mass density.
- E Young's modulus.
- PR Poisson's ratio.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TENMAX	Maximum through-thickness tensile stress GT.0.0: constant value LT.0.0:  TENMAX  is ID of a *DEFINE_FUNCTION (see Remarks)
GCTEN	Energy per unit area to fail the bond in tension GT.0.0: constant value LT.0.0:  GCTEN  is ID of a *DEFINE_FUNCTION (see Remarks)
SHRMAX	Maximum through-thickness shear stress GT.0.0: constant value LT.0.0:  SHRMAX  is ID of a *DEFINE_FUNCTION (see Remarks)
GCSHR	Energy per unit area to fail the bond in shear GT.0.0: constant value LT.0.0:  GCSHR  is ID of a *DEFINE_FUNCTION (see Remarks)
PWRT	Power law term for tension
PWRS	Power law term for shear
SHRP	Shear plateau ratio (Optional) GT.0.0: constant value LT.0.0:  SHRP  is ID of a *DEFINE_FUNCTION (see Remarks)
SHT_SL	Slope (non-dimensional) of yield surface at zero tension (See Remarks)
EDOT0	Strain rate at which the "static" properties apply
EDOT2	Strain rate at which the "dynamic" properties apply (Card 5)
THKDIR	Through-thickness direction flag (See remarks) EQ.0.0: smallest element dimension (default) EQ.1.0: direction from nodes 1-2-3-4 to nodes 5-6-7-8

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EXTRA	Flag to input further data: EQ.1.0: interfacial failure properties (cards 3 and 4) EQ.2.0: bond thickness and more (card 6) EQ.3.0: both of the above
TMAXE	Maximum tensile force per unit length on edges of joint
GCTE	Energy per unit length to fail the edge of the bond in tension
SMAXE	Maximum shear force per unit length on edges of joint
GCSE	Energy per unit length to fail the edge of the bond in shear
PWRTE	Power law term for tension
PWRSE	Power law term for shear
FACET	Stiffness scaling factor for edge elements – tension
FACCT	Stiffness scaling factor for interior elements – tension
FACES	Stiffness scaling factor for edge elements – shear
FACCS	Stiffness scaling factor for interior elements – shear
SOFTT	Factor by which the tensile strength is reduced when a neighbor fails
SOFTS	Factor by which the shear strength is reduced when a neighbor fails
SDFAC	Factor on TENMAX and SHRMAX at strain rate EDOT2 GT.0.0: constant value LT.0.0:  SDFAC  is ID of a *DEFINE_FUNCTION (see Remarks)
SGFAC	Factor on GCTEN and GCSHR at strain rate EDOT2 GT.0.0: constant value LT.0.0:  SGFAC  is ID of a *DEFINE_FUNCTION (see Remarks)
SDEFAC	Factor on TMAXE and SMAXE at strain rate EDOT2
SGEFAC	Factor on GCTE and GCSE at strain rate EDOT2

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BTHK	Bond thickness (overrides thickness from element dimensions) LT.0.0:   BTHK   is bond thickness, but critical time step remains unaffected. Helps to avoid very small time steps, but it can affect stability.
OUTFAIL	Flag for additional output to messag file: Information about damage initiation time, failure function terms and forces. EQ.0.0: off EQ.1.0: on
FSIP	Effective in-plane strain at failure.

**Remarks:**

The through-thickness direction is identified from the smallest dimension of each element by default (THKDIR = 0.0). It is expected that this dimension will be smaller than in-plane dimensions (typically 1-2mm compared with 5-10mm). If this is not the case, one can set the through-thickness direction via element numbering (THKDIR = 1.0). Then the thickness direction is expected to point from lower face (nodes 1-2-3-4) to upper face (nodes 5-6-7-8). For wedge elements these faces are the two triangular faces (nodes 1-2-5) and (nodes 3-4-6).

The bond thickness is assumed to be the element size in the thickness direction. This may be overridden using BTHK. In this case the behavior becomes independent of the element thickness. The elastic stiffness is affected by BTHK, so it is necessary to set the characteristic element length to a smaller value

$$l_e^{\text{new}} = \sqrt{\text{BTHK} \times l_e^{\text{old}}}.$$

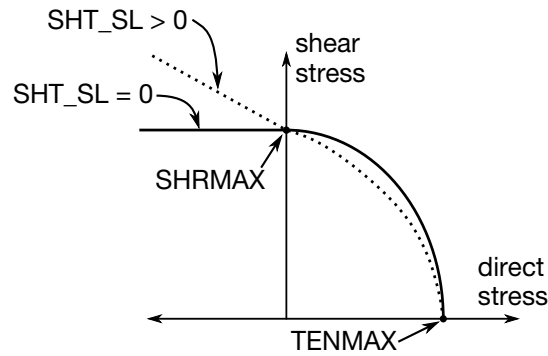
This again affects the critical time step of the element, i.e. a small BTHK can decrease the element time step significantly.

In-plane stresses are set to zero: it is assumed that the stiffness and strength of the substrate is large compared with that of the adhesive, given the relative thicknesses.

If the substrate is modeled with shell elements, it is expected that these will lie at the mid-surface of the substrate geometry. Therefore the solid elements representing the adhesive will be thicker than the actual bond. If the elastic compliance of the bond is significant, this can be corrected by increasing the elastic stiffness property E.

The yield and failure surfaces are treated as a power-law combination of direct tension and shear across the bond:





**Figure M169-1.** Figure illustrating the yield surface.

$$\left(\frac{\sigma}{\sigma_{\max}}\right)^{\text{PWRT}} + \left(\frac{\tau}{\tau_{\max} - \text{SHT\_SL} \times \sigma}\right)^{\text{PWRS}} = 1.0$$

At yield SHT\_SL is the slope of the yield surface at  $\sigma = 0$ . See [Figure M169-1](#)

The stress-displacement curves for tension and shear are shown in [Figure M169-2](#). In both cases, GC is the area under the curve. The displacement to failure in tension is given by

$$d_{ft} = 2 \left(\frac{\text{GCTEN}}{\text{TENMAX}}\right),$$

subject to a lower limit

$$d_{ft, \min} = \left(\frac{2L_0}{E'}\right) \text{TENMAX}$$

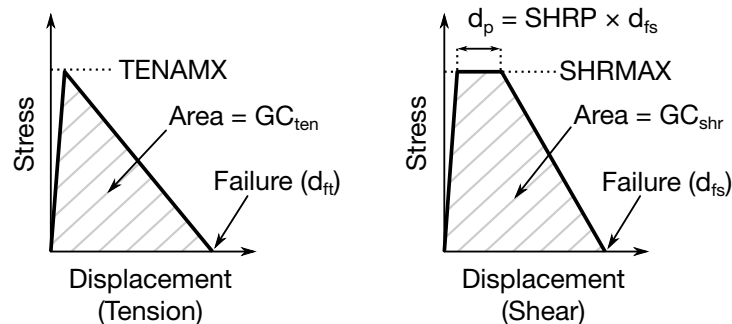
where  $L_0$  is the initial element thickness (or BTHK if used) and

$$E' = \frac{E(1 - \nu)}{(1 - 2\nu)(1 + \nu)}.$$

If GCTEN is input such that  $d_{ft} < d_{ft, \min}$ , LS-DYNA will automatically increase GCTEN to make  $d_{ft} = d_{ft, \min}$ . Therefore, GCTEN has a minimum value of

$$\text{GCTEN} \geq \frac{L_0}{E'} (\text{TENMAX})^2$$

Similarly, the minimum value for GCSHR is



**Figure M169-2.** Stress-Displacement Curves for Tension and Shear.

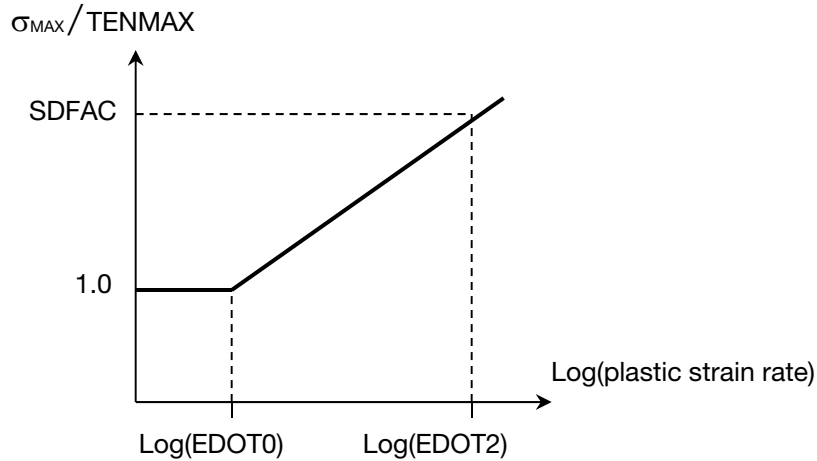


Figure M169-3. Figure illustrating rate effects.

$$GCSHR \geq \frac{L_0}{G} (SHRMAX)^2$$

where  $G$  is the elastic shear modulus.

Because of the algorithm used, yielding in tension across the bond does not require strains in the plane of the bond – unlike the plasticity models, plastic flow is not treated as volume-conserving.

The Plastic Strain output variable has a special meaning:

- 0 < PS < 1: PS is the maximum value of the yield function experienced since time zero
- 1 < PS < 2: the element has yielded and the strength is reducing towards failure – yields at PS = 1, fails at PS = 2.

The damage cause by cohesive deformation (0 at first yield to 1 at failure) and by interfacial deformation are stored in the first two extra history variables. These can be plotted if NEIPH on \*DATABASE\_EXTENT\_BINARY is 2 or more. By this means, the reasons for failure may be assessed.

When the plastic strain rate rises above EDOT0, rate effects are assumed to scale with the logarithm of the plastic strain rate, as in the example shown in Figure M169-3 for cohesive tensile strength with dynamic factor SDFAC. The same form of relationship is applied for the other dynamic factors. If EDOT0 is zero or blank, no rate effects are applied. Rate effects are applied using the viscoplastic method.

Interfacial failure is assumed to arise from stress concentrations at the edges of the bond – typically the strength of the bond becomes almost independent of bond length. This type of failure is usually more brittle than cohesive failure. To simulate this, LS-DYNA identifies the free edges of the bond (made up of element faces that are not shared by other

elements of material type \*MAT\_ARUP\_ADHESIVE, excluding the faces that bond to the substrate). Only these elements can fail initially. The neighbors of failed elements can then develop free edges and fail in turn.

In real adhesive bonds, the stresses at the edges can be concentrated over very small areas; in typical finite element models the elements are much too large to capture this. Therefore the concentration of loads onto the edges of the bond is accomplished artificially, by stiffening elements containing free edges (e.g. FACET, FACES > 1) and reducing the stiffness of interior elements (e.g. FACCT, FACCS < 1). Interior elements are allowed to yield at reduced loads (equivalent to TMAXE × FACET/FACCT and SMAXE × FACES/FACCS) to prevent excessive stresses developing before the edge elements have failed - but cannot be damaged until they become edge elements after the failure of their neighbors.

Parameters TENMAX, GCTEN, SHRMAX, GCSHR, SHRP, SDFAC, and SGFAC can be defined as negative values. In that case, the absolute values refer to \*DEFINE\_FUNCTION ID's. The arguments of those functions include several properties of both connection partners if corresponding solid elements are in a tied contact with shell elements.

These functions depend on:

(t1, t2) = thicknesses of both bond partners

(sy1, sy2) = initial yield stresses at plastic strain of 0.002

(sm1, sm2) = maximum engineering yield stresses (necking points)

r = strain rate

a = element area

For TENMAX = -100 such a function could look like:

```
*DEFINE_FUNCTION
100
func (t1, t2, sy1, sy2, sm1, sm2, r, a) = 0.5 * (sy1 + sy2)
```

Since material parameters have to be identified from both bond partners during initialization, this feature is only available for a subset of material models at the moment, namely no. 24, 120, 123, and 124.



Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E11P	$E_{11p}$ , for in plane behavior.
E22P	$E_{22p}$ , for in plane behavior.
V12P	$\nu_{12p}$ , for in plane behavior.
V11P	$\nu_{11p}$ , for in plane behavior.
G12P	$G_{12p}$ , for in plane behavior.
G23P	$G_{23p}$ , for in plane behavior.
G31P	$G_{31p}$ , for in plane behavior.
E11B	$E_{11b}$ , for bending behavior.
E22B	$E_{22b}$ , for bending behavior.
V12B	$\nu_{12b}$ , for bending behavior.
V21B	$\nu_{21b}$ , for bending behavior.
G12B	$G_{12b}$ , for bending behavior.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
LN11	Yield curve ID for $N_{11}$ , the in-plane force resultant.
LN22	Yield curve ID for $N_{22}$ , the in-plane force resultant.
LN12	Yield curve ID for $N_{12}$ , the in-plane force resultant.
LQ1	Yield curve ID for $Q_1$ , the transverse shear resultant.
LQ2	Yield curve ID for $Q_2$ , the transverse shear resultant.
LM11	Yield curve ID for $M_{11}$ , the moment.
LM22	Yield curve ID for $M_{22}$ , the moment.
LM12	Yield curve ID for $M_{12}$ , the moment.
A1, A2, A3	$(a_1, a_2, a_3)$ , define components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	$(v_1, v_2, v_3)$ , define components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	$(d_1, d_2, d_3)$ , define components of vector $\mathbf{d}$ for AOPT = 2.

VARIABLE	DESCRIPTION
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

**Remarks:**

The in-plane elastic matrix for in-plane, plane stress behavior is given by:

$$\mathbf{C}_{\text{in plane}} = \begin{bmatrix} Q_{11p} & Q_{12p} & 0 & 0 & 0 \\ Q_{12p} & Q_{22p} & 0 & 0 & 0 \\ 0 & 0 & Q_{44p} & 0 & 0 \\ 0 & 0 & 0 & Q_{55p} & 0 \\ 0 & 0 & 0 & 0 & Q_{66p} \end{bmatrix}$$

The terms  $Q_{ijp}$  are defined as:

$$Q_{11p} = \frac{E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{22p} = \frac{E_{22p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{12p} = \frac{\nu_{12p}E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{44p} = G_{12p}$$

$$Q_{55p} = G_{23p}$$

$$Q_{66p} = G_{31p}$$

The elastic matrix for bending behavior is given by:

$$\mathbf{C}_{\text{bending}} = \begin{bmatrix} Q_{11b} & Q_{12b} & 0 \\ Q_{12b} & Q_{22b} & 0 \\ 0 & 0 & Q_{44b} \end{bmatrix}$$

The terms  $Q_{ijp}$  are similarly defined.

Because this is a resultant formulation, no stresses are output to d3plot, and forces and moments are reported to elout in place of stresses.

**\*MAT\_171****\*MAT\_STEEL\_CONCENTRIC\_BRACE****\*MAT\_STEEL\_CONCENTRIC\_BRACE**

This is Material Type 171. It represents the cyclic buckling and tensile yielding behavior of steel braces and is intended primarily for seismic analysis. Use only for beam elements with ELFORM = 2 (Belytschko-Schwer beam).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	YM	PR	SIGY	LAMDA	FBUCK	FBUCK2
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	See Remarks	See Remarks	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	CCBRF	BCUR						
Type	F	F						
Default	See Remarks							

Card 3	1	2	3	4	5	6	7	8
Variable	TS1	TS2	TS3	TS4	CS1	CS2	CS3	CS4
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	= TS1	= TS2	= TS3	= TS4

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density



<b>VARIABLE</b>	<b>DESCRIPTION</b>
YM	Young's Modulus
PR	Poisson's Ratio
SIGY	Yield stress
LAMDA	Slenderness ratio (optional – see note)
FBUCK	Initial buckling load (optional – see note. If used, should be positive)
FBUCK2	Optional extra term in initial buckling load – see note
CCBRF	Reduction factor on initial buckling load for cyclic behavior
BCUR	Optional load curve giving compressive buckling load (y-axis) versus compressive strain (x-axis - both positive)
TS1 - TS4	Tensile axial strain thresholds 1 to 4
CS1 - CS4	Compressive axial strain thresholds 1 to 4

**Remarks:**

The brace element is intended to represent the buckling, yielding and cyclic behavior of steel elements such as tubes or I-sections that carry only axial loads. Empirical relationships are used to determine the buckling and cyclic load-deflection behavior. A single beam element should be used to represent each structural element.

The cyclic behavior is shown in the graph (compression shown as negative force and displacement).

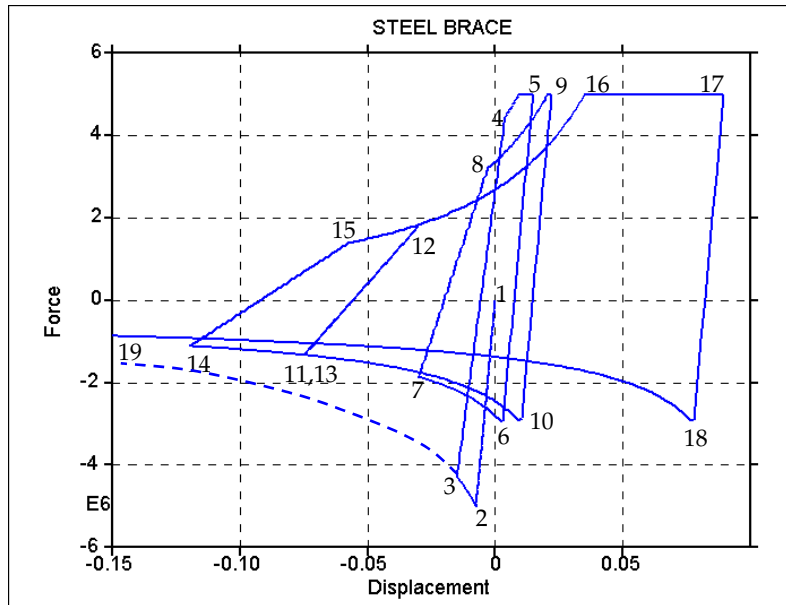


Figure M171-1.

The initial buckling load (point 2) is:

$$F_b \text{ initial} = \text{FBUCK} + \frac{\text{FBUCK2}}{L^2}$$

where FBUCK, FBUCK2 are input parameters and L is the length of the beam element. If neither FBUCK nor FBUCK2 are defined, the default is that the initial buckling load is

$$\text{SIGY} \times A,$$

where A is the cross sectional area. The buckling curve (shown dashed) has the form:

$$F(d) = \frac{F_b \text{ initial}}{\sqrt{A\delta + B}}$$

where  $\delta$  is  $\text{abs}(\text{strain}/\text{yield strain})$ , and A and B are internally-calculated functions of slenderness ratio ( $\lambda$ ) and loading history.

The member slenderness ratio  $\lambda$  is defined as  $\frac{kL}{r}$ , where k depends on end conditions, L is the element length, and r is the radius of gyration such that  $Ar^2 = I$  (and  $I = \min(I_{yy}, I_{zz})$ );  $\lambda$  will by default be calculated from the section properties and element length using  $k = 1$ . Optionally, this may be overridden by input parameter LAMDA to allow for different end conditions.

Optionally, the user may provide a buckling curve BCUR. The points of the curve give compressive displacement (x-axis) versus force (y-axis); the first point should have zero displacement and the initial buckling force. Displacement and force should both be positive. The initial buckling force must not be greater than the yield force.

The tensile yield force (point 5 and section 16-17) is defined by

$$F_y = \text{SIGY} \times A,$$

where yield stress SIGY is an input parameter and A is the cross-sectional area.

Following initial buckling and subsequent yield in tension, the member is assumed to be damaged. The initial buckling curve is then scaled by input parameter CCBRF, leading to reduced strength curves such as segments 6-7, 10-14 and 18-19. This reduction factor is typically in the range 0.6 to 1.0 (smaller values for more slender members). By default, CCBRF is calculated using SEAOC 1990:

$$\text{CCBRF} = \frac{1}{\left(1 + \frac{0.5\lambda}{\pi \sqrt{\frac{E}{0.5\sigma_y}}}\right)}$$

When tensile loading is applied after buckling, the member must first be straightened before the full tensile yield force can be developed. This is represented by a reduced unloading stiffness (e.g. segment 14-15) and the tensile reloading curve (segments 8-9 and 15-16). Further details can be found in Bruneau, Uang, and Whittaker [1998] and Structural Engineers Association of California [1974, 1990, 1996].

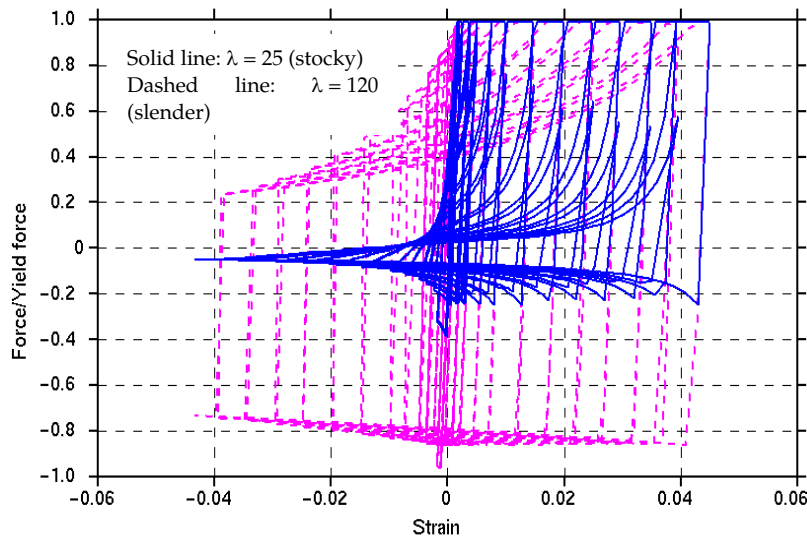


Figure M171-2.

The response of stocky (low  $\lambda$ ) and slender (high  $\lambda$ ) braces are compared in the graph. These differences are achieved by altering the input value LAMDA (or the section properties of the beam) and FBUCK.

**Output:**

Axial Strain and Internal Energy may be plotted from the INTEGRATED beam results menus in Oasys Ltd. Post processors: D3PLOT and T/HIS.

FEMA thresholds are the total axial strains (defined by change of length/initial length) at which the element is deemed to have passed from one category to the next, e.g. "Elastic", "Immediate Occupancy", "Life Safe", etc. During the analysis, the maximum tensile and compressive strains ("high tide strains") are recorded. These are checked against the user-defined limits TS1 to TS4 and CS1 to CS4. The output flag is then set to 0, 1, 2, 3, or 4 according to which limits have been passed. The value in the output files is the highest such flag from tensile or compressive strains. To plot this data, select INTEGRATED beam results, Integration point 4, Axial Strain.

Maximum plastic strains in tension and compression are also output. These are defined as maximum total strain to date minus the yield or first buckling strain for tensile and compressive plastic strains respectively. To plot these, select INTEGRATED beam results, Integration point 4, "shear stress XY" and "shear stress XZ" for tensile and compressive plastic strains, respectively.

## \*MAT\_CONCRETE\_EC2

This is Material Type 172, for shell and Hughes-Liu beam elements only. The material model can represent plain concrete only, reinforcing steel only, or a smeared combination of concrete and reinforcement. The model includes concrete cracking in tension and crushing in compression, and reinforcement yield, hardening and failure. Properties are thermally sensitive; the material model can be used for fire analysis. Material data and equations governing the behavior (including thermal properties) are taken from Eurocode 2 (EC2). See notes below for more details of how the standard is applied in the material model. Although the material model offers many options, a reasonable response may be obtained by entering only RO, FC and FT for plain concrete. If reinforcement is present, YMREINF, SUREINF, FRACRX, FRACRY must be defined, or for an alternative way to model the reinforcement, see \*MAT\_203/\*MAT\_HYSTERETIC\_REINFORCEMENT. Note that, from release R10 onwards, the number of possible cracks has been increased from 2 (0 and 90 degrees) to 4 – see notes below.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	FC	FT	TYPEC	UNITC	ECUTEN	FCC
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	0.0	1.0	1.0	0.0025	FC

Card 2	1	2	3	4	5	6	7	8
Variable	ESOFT	LCHAR	MU	TAUMXF	TAUMXC	ECRAGG	AGGSZ	UNITL
Type	F	F	F	F	F	F	F	F
Default	See notes	0.0	0.4	$10^{20}$	$1.161 \times FT$	0.001	0.0	1.0



Omit if ISHCHK = 0

Card 7	1	2	3	4	5	6	7	8
Variable	TYPSEC	P_OR_F	EFFD	GAMSC	ERODET	ERODEC	ERODER	TMPOFF
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	2.0	0.01	0.05	0.0

Additional card for TYPEC = 6 or 9.

Card 8	1	2	3	4	5	6	7	8
Variable	ECL_6	ECSP69	GAMCE9	PHIEF9				
Type	F	F	F	F				
Default	see notes	see notes	0.0	0.0				

Define this card only if FT is negative.

Card 9	1	2	3	4	5	6	7	8
Variable	FT2	FTSHR	LCFTT	WRO_G	ZSURF			
Type	F	F	F	F	F			
Default	abs(FT)	abs(FT2)	0.0	0.0	0.0			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
FC	Compressive strength of concrete (stress units). Meaning depends on TYPEC. TYPEC = 1,2,3,4,5,7,8:FC is the actual compressive strength TYPEC = 6: FC is the unconfined compressive strength used in Mander equations. TYPEC = 9: FC is the characteristic compressive strength (fck in EC2 1-1). See also FCC and the notes below.
FT	Tensile stress to cause cracking. Negative value to read card 9.
TYPEC	Concrete aggregate type for stress-strain-temperature relationships EQ.1.0: Siliceous (default), Draft EC2 Annex (fire engineering) EQ.2.0: Calcareous, Draft EC2 Annex (fire engineering) EQ.3.0: Non-thermally-sensitive using ET3, ECU3 EQ.4.0: Lightweight EQ.5.0: Fiber-reinforced EQ.6.0: Non-thermally-sensitive, Mander algorithm EQ.7.0: Siliceous, EC2 1-2:2004 (fire engineering) EQ.8.0: Calcareous, EC2 1-2:2004 (fire engineering) EQ.9.0: EC2 1-1:2004 (general and buildings) To obtain the pre-R9 behaviour, i.e. maximum of 2 cracks, add 100 to TYPEC. For example, 109 means 2 cracks, EC2 1-1:2004 (general and buildings).
UNITC	Factor to convert stress units to MPa (used in shear capacity checks and for application of EC2 formulae when TYPEC = 9) e.g. if model units are Newtons and metres, UNITC=10 <sup>-6</sup> .



<b>VARIABLE</b>	<b>DESCRIPTION</b>
ECUTEN	Strain to fully open a crack.
FCC	Relevant only if TYPEC = 6 or 9. TYPEC = 6: FCC is the compressive strength of confined concrete used in Mander equations. Default: unconfined properties are assumed. TYPEC = 9: FCC is the actual compressive strength. If blank, this will be set equal to the mean compressive strength (fcm in EC2 1-1) as required for serviceability calculations (8MPa greater than FC). For ultimate load calculations the user may set FCC to a factored characteristic compressive strength. See notes below.
ESOFT	Tension stiffening (Slope of stress-strain curve post-cracking in tension)
MU	Friction on crack planes (max shear = $\mu \times$ compressive stress)
TAUMXF	Maximum friction shear stress on crack planes (ignored if AGGSZ > 0 - see notes).
TAUMXC	Maximum through-thickness shear stress after cracking (see notes).
ECRAGG	Strain parameter for aggregate interlock (ignored if AGGSZ > 0 - see notes).
AGGSZ	Aggregate size (length units - used in NS3473 aggregate interlock formula - see notes).
UNITL	Factor to convert length units to millimeters (used only if AGGSZ > 0 - see notes) e.g. if model unit is meters, UNITL = 1000.
LCHAR	Characteristic length at which ESOFT applies, also used as crack spacing in aggregate-interlock calculation
YMREINF	Young's Modulus of reinforcement
PRREINF	Poisson's Ratio of reinforcement
SUREINF	Ultimate stress of reinforcement

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TYPER	Type of reinforcement for stress-strain-temperature relationships EQ.1.0: Hot rolled reinforcing steel, Draft EC2 Annex (fire) EQ.2.0: Cold worked reinforcing steel (default), Draft EC2 Annex (fire) EQ.3.0: Quenched/tempered prestressing steel, Draft EC2 Annex (fire) EQ.4.0: Cold worked prestressing steel, Draft EC2 Annex (fire) EQ.5.0: Non-thermally sensitive using load curve LCRSU. EQ.7.0: Hot rolled reinforcing steel, EC2 1-2:2004 (fire) EQ.8.0: Cold worked reinforcing steel, EC2 1-2:2004 (fire)
FRACRX	Fraction of reinforcement ( <i>x</i> -axis). For example, to obtain 1% reinforcement set FRACR = 0.01.
FRACRY	Fraction of reinforcement ( <i>y</i> -axis). For example, to obtain 1% reinforcement set FRACR = 0.01.
LCRSU	Load curve for TYPER = 5, giving non-dimensional factor on SUREINF versus plastic strain (overrides stress-strain relationships from EC2).
LCALPS	Optional load curve giving thermal expansion coefficient of reinforcement vs temperature – overrides relationship from EC2.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>LT.0.0: This option has not yet been implemented for this material model.</p>
ET36	Young's Modulus of concrete (TYPEC = 3 and 6). For other values of TYPEC, the Young's Modulus is calculated internally: see notes.
PRT36	Poisson's Ratio of concrete (TYPEC = all).
ECUT36	Strain to failure of concrete in compression (TYPEC = 3 and 6).
LCALPC	Optional load curve giving thermal expansion coefficient of concrete vs temperature – overrides relationship from EC2.
DEGRAD	If non-zero, the compressive strength of concrete parallel to an open crack will be reduced (see notes).
ISHCHK	Set this flag to 1 to input Card 7 (shear capacity check and other optional input data).
UNLFAC	Stiffness degradation factor after crushing (0.0 to 1.0 – see notes).
XP, YP, ZP	Not used.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	Material angle in degrees for AOPT = 0 and AOPT = 3. BETA may be overridden on the element card, see *ELEMENT_SHELL_BETA
TYPESC	Type of shear capacity check EQ.1.0: BS 8110 EQ.2.0: ACI
P_OR_F	If BS8110 shear check, percent reinforcement – e.g. if 0.5%, input 0.5. If ACI shear check, ratio (cylinder strength/FC) - defaults to 1.
EFFD	Effective section depth (length units), used in shear capacity check. This is usually the section depth excluding the cover concrete.
GAMSC	Load factor used in BS8110 shear capacity check.
ERODET	Crack-opening strain at which element is deleted
ERODEC	Compressive strain used in erosion criteria, see notes
ERODER	Reinforcement plastic strain used in erosion criteria, see notes
TMPOFF	Constant to be added to the model's temperature unit to convert into degrees Celsius, e.g., if the model's temperature unit is degrees Kelvin, set TMPOFF to -273. Degrees Celsius temperatures are then used throughout the material model, e.g., for LCALPC as well as for the default thermally-sensitive properties.
EC1_6	Strain at maximum compressive stress for Type 6 concrete.
ECSP69	Spalling strain in compression for TYPEC = 6 and 9.
GAMCE9	Material factor that divides the Youngs Modulus (TYPEC = 9).
PHIEF9	Effective creep ratio (TYPEC = 9).
FT2	Tensile strength used for calculating tensile response.
FTSHR	Tensile strength used for calculating post-crack shear response.
LCFTT	Load curve defining factor on tensile strength versus time.
WRO_G	Density times gravity for water pressure in cracks
ZSURF	Z-coordinate of water surface (for water pressure in cracks)

**Remarks:**

This material model can be used to represent unreinforced concrete ( $FRACR = 0$ ), steel ( $FRACR = 1$ ), or reinforced concrete with evenly distributed reinforcement ( $0 < FRACR < 1$ ). Concrete is modelled as an initially-isotropic material with a non-rotating smeared crack approach in tension, together with a plasticity model for compressive loading. Reinforcement is treated as separate sets of bars in the local element  $x$  and  $y$  axes. The reinforcement is assumed not to carry through-thickness shear or in-plane shear. Therefore, this material model should not be used to model steel-only sections, i.e. do not create a section in which all the integration points are of \*MAT\_172 with  $FRACRX$ ,  $FRACRY = 1$ .

**Creating Reinforced Concrete Sections:**

Reinforced concrete sections for shell or beam elements may be created using \*PART\_COMPOSITE (for shells) or \*INTEGRATION\_BEAM (for beams) to define the section. Create one Material definition representing the concrete using MAT\_CONCRETE\_EC2 with  $FRACR = 0$ . Create another Material definition representing the reinforcement using MAT\_CONCRETE\_EC2 with  $FRACRX$  and/or  $FRACRY = 1$ . The Material ID of each integration point is then set to represent either concrete or steel. The position of each integration point within the cross-section and its cross-sectional area are chosen to represent the actual distribution of reinforcement.

**Options for TYPEC and TYPER**

Eurocode 2 (EC2) contains different sections applicable to general structural engineering versus fire engineering. The latter contains different data for different types of concrete and steel, and has been revised during its history. TYPEC and TYPER control the version and section of the EC2 document from which the material data is taken, and the types of concrete and steel being represented. In the descriptions of TYPEC and TYPER above, "Draft EC2 Annex (fire engineering)" means data taken from the 1995 draft Eurocode 2 Part 1-2 (for fire engineering), ENV 1992-1-2:1995. These are the defaults, and are suitable for general use where elevated temperatures are not considered.

EC2 was then issued in 2004 (described above as EC2 1-2:2004 (fire)) with revised stress-strain data at elevated temperatures (TYPEC and TYPER = 7 or 8). These settings are recommended for analyses with elevated temperatures.

Meanwhile Eurocode 2 Part 1-1 (for general structural engineering), EC2 1-1:2004, contains material data and formulae that differ from Part 1-2; these are obtained by setting TYPEC = 9. This setting is recommended where compatibility is required with the structural engineering data and assumptions of Part 1-1 of the Eurocode.

A further option for modelling concrete, TYPEC = 6, is provided for applications such as seismic engineering in which the different stress-strain behaviors of confined versus

unconfined concrete needs to be captured. This option uses equations by Mander et al, and does not relate directly to Eurocode 2.

## **Material Behavior: Concrete**

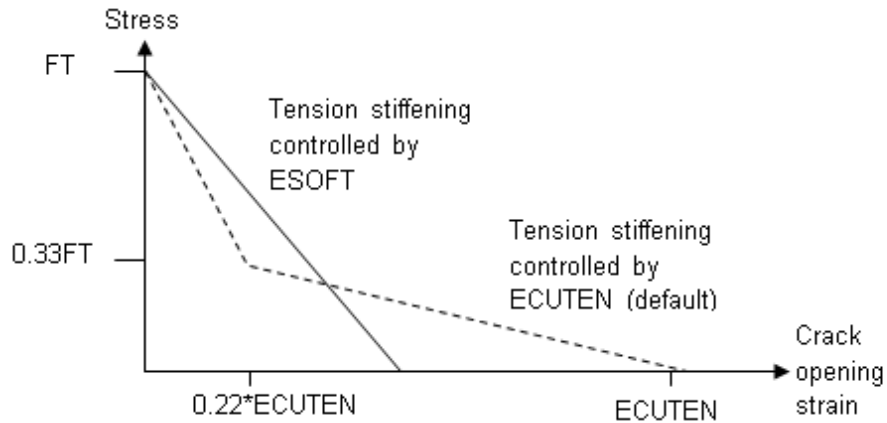
### Thermal sensitivity

For TYPEC = 1,2,4,5,7,8, the material properties are thermally-sensitive. If no temperatures are defined in the model, it behaves as if at 20degC. Pre-programmed relationships between temperature and concrete properties are taken from the EC2 document. The thermal expansion coefficient is as defined in EC2, is non-zero by default, and is a function of temperature. This may be overridden by inputting the curve LCALPC. TYPEC = 3, 6 and 9 are not thermally sensitive and have no thermal expansion coefficient by default.

### Tensile response

The concrete is assumed to crack in tension when the maximum in-plane principal stress reaches FT. A non-rotating smeared crack approach is used. Cracks can open and close repeatedly under hysteretic loading. When a crack is closed it can carry compression according to the normal compressive stress-strain relationships. The direction of the crack relative to the element coordinate system is stored when the crack first forms. The material can carry compression parallel to the crack even when the crack is open. Further cracks may then form at pre-determined angles to the first crack, if the tensile stress in that direction reaches FT. In versions up to R9.0, the number of further cracks is limited to one, at 90 degrees to the first crack. In versions starting from R10, up to three further cracks can form, at 45, 90 and 135 degrees to the first crack. The tensile stress is limited to FT only in the available crack directions. The tensile stress in other directions is unlimited, and could exceed FT. This is a limitation of the non-rotating crack approach and may lead to models being non-conservative, i.e. the response is stronger than implied by the input. The increase of the possible number of cracks from two to four significantly reduces this error, and may therefore cause models to seem "weaker" in R10 than in R9 under some loading conditions. An option to revert to the previous 2-crack behavior is available in R10 – add 100 to TYPEC.

After initial cracking, the tensile stress reduces with increasing tensile strain. A finite amount of energy must be absorbed to create a fully open crack - in practice the reinforcement holds the concrete together, allowing it to continue to take some tension (this effect is known as tension-stiffening). The options available for the stress-strain relationship are shown below. The bilinear relationship is used by default. The simple linear relationship applies only if ESOFT > 0 and ECUTEN = 0.



**Figure M172-1.** Tensile Behaviour of Concrete

LCHAR can optionally be used to maintain constant energy per unit area of crack irrespective of mesh size, i.e. the crack opening displacement is fixed rather than the crack opening strain.  $LCHAR \times ECUTEN$  is then the displacement to fully open a crack. For the actual elements, crack opening displacement is estimated by  $strain \times \sqrt{area}$ . Note that if LCHAR is defined, it is also used as the crack spacing in the NS 3473 aggregate interlock calculation.

For the thermally-sensitive values of TYPEC, the relationship of FT with temperature is taken from EC2 – there is no input option to change this. FT is assumed to remain at its input value at temperatures up to 100°C, then to reduce linearly with temperature to zero at 600°C. Up to 500°C, the crack opening strain ECUTEN increases with temperature such that the fracture energy to open the crack remains constant. Above 500 deg C the crack opening strain does not increase further.

In some concrete design codes and standards, it is stipulated that the tensile strength of concrete should be assumed to be zero. However, for MAT\_CONCRETE\_EC2 it is not recommended to set FT to zero, because:

- Cracks will form at random orientations caused by small dynamic tensile stresses, leading to unexpected behavior when the loading increases because the crack orientations are fixed when the cracks first form;
- The shear strength of cracked concrete may then also become zero in the analysis (according to the aggregate interlock formula, the post-crack shear strength is assumed proportional to FT).

These problems may be tackled by using the inputs on Card 9. Firstly, separate tensile strengths may be input for the tensile response and for calculating the shear strength of cracked concrete. Secondly, by using the load curve LCFTT, the tensile strength may be ramped gradually down to zero after the static loads have been applied, ensuring that the cracks will form in the correct orientation

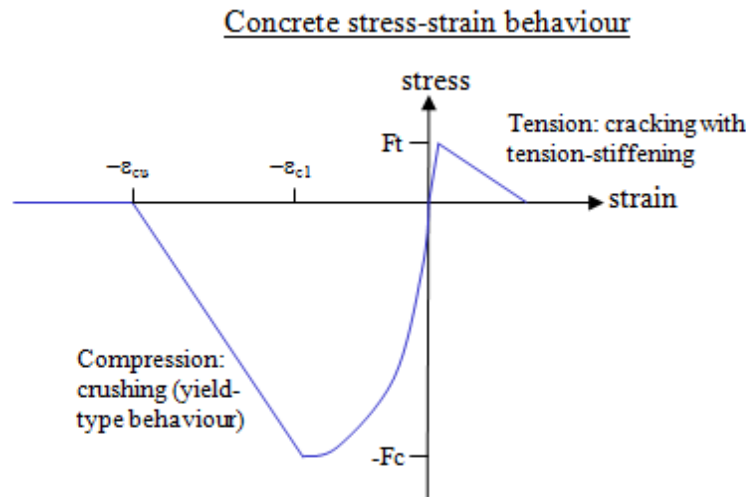
Compressive response: TYPEC = 1,2,4,5,7,8

For TYPEC = 1,2,4,5,7,8, the compressive behavior of the concrete initially follows a stress-strain curve defined in EC2 as:

$$\text{Stress} = FC_{\max} \times \left[ \left( \frac{\epsilon}{\epsilon_{cl}} \right) \times \frac{3}{2 + \left( \frac{\epsilon}{\epsilon_{cl}} \right)^3} \right]$$

where  $\epsilon_{cl}$  is the strain at which the ultimate compressive strength  $FC_{\max}$  is reached, and  $\epsilon$  is the current equivalent uniaxial compressive strain.

The initial elastic modulus is given by  $E = 3 \times FC_{\max} / 2\epsilon_{cl}$ . On reaching  $FC_{\max}$ , the stress decreases linearly with increasing strain, reaching zero at a strain  $\epsilon_{cu}$ . Strains  $\epsilon_{cl}$  and  $\epsilon_{cu}$  are by default taken from EC2 and are functions of temperature. At 20°C they take values 0.0025 and 0.02 respectively.  $FC_{\max}$  is also a function of temperature, given by the input parameter FC (which applies at 20°C) times a temperature-dependent softening factor taken from EC2. The differences between TYPEC = 1,2,4,5,7,8 are limited to (a) different reductions of FC at elevated temperatures, and (b) different values of  $\epsilon_{cl}$  at elevated temperatures.



**Figure M172-2.** Concrete stress strain behavior

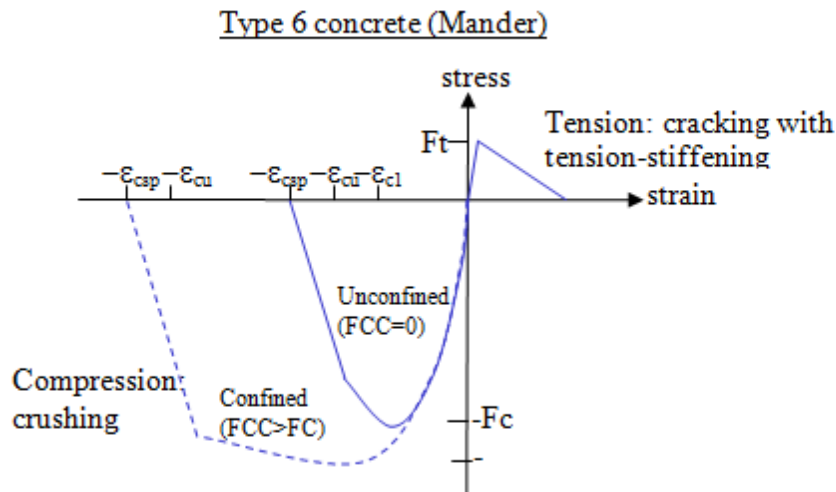
Compressive response: TYPEC = 3

For TYPEC = 3, the user over-rides the default values of Young's Modulus and  $\epsilon_{cu}$  using ET36 and ECUT36 respectively. In this case, the strain  $\epsilon_{cl}$  is calculated from the elastic stiffness, and there is no thermal sensitivity. The stress-strain behaviour follows the same form as described above.

Compressive response: TYPEC = 6



For TYPEC = 6, the above compressive crushing behaviour is replaced with the equations proposed by Mander. This algorithm can model unconfined or confined concrete; for unconfined, leave FCC blank. For confined concrete, input the confined compressive strength as FCC.



**Figure M172-3.** Type 6 concrete

Default values for type 6 are calculated as follows:

$$\epsilon_{cl} = 0.002 \times \left[ 1 + 5 \left( \frac{FCC6}{FC} - 1 \right) \right]$$

$$\epsilon_{cu} = 1.1 \times \epsilon_c$$

$$\epsilon_{csp} = \epsilon_{cu} + 2 \frac{FCC}{E}$$

Note that for unconfined concrete, FCC6 = FC causing  $\epsilon_{cl}$  to default to 0.002.

Compressive response: TYPEC = 9

For TYPEC = 9, the input parameter FC is the characteristic cylinder strength in the stress units of the model. FC x UNITC is assumed to be  $f_{ck}$ , the strength class in MPa units. The mean tensile strength  $f_{ctm}$ , mean Young's Modulus  $E_{cm}$ , and the strains used to construct the stress-strain curve such as  $\epsilon_{cl}$  are by default evaluated automatically from tabulated functions of  $f_{ck}$  given in Table 3.1 of EC2. The compressive strength of the material is given by the input parameter FCC, which defaults to the mean compressive strength  $f_{cm}$  defined

in EC2 as  $f_{ck} + 8\text{MPa}$ ). The user may override the default compressive strength by inputting FCC explicitly. The stress-strain curve follows this form:

$$\frac{\text{Stress}}{\text{FCC}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta}$$

Where FCC is the input parameter FCC (default:  $= (f_{ck} + 8\text{MPa})/\text{UNITC}$ ),

$$\eta = \text{strain}/\varepsilon_{cl},$$

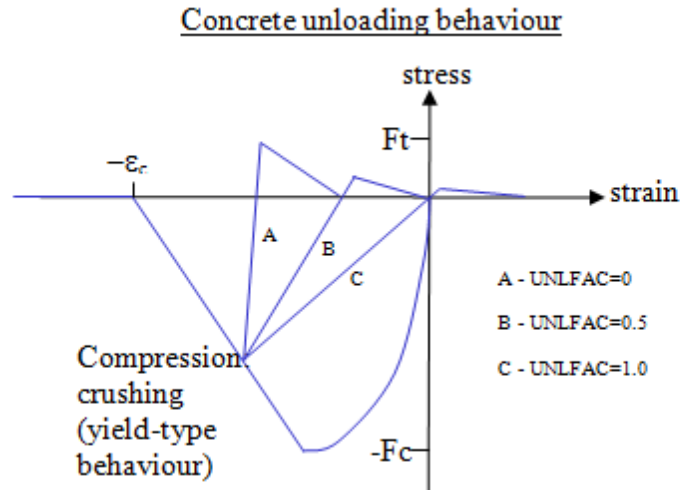
$$k = 1.05E \times \varepsilon_{c1}/\text{FCC}$$

E is the Young's Modulus.

The default parameters are intended to be appropriate for a serviceability analysis (mean properties), so default FT =  $f_{ctm}$  and default E =  $E_{cm}$ . For an ultimate load analysis, FCC should be the "design compressive strength" (normally the factored characteristic strength, including any appropriate material factors); FT should be input as the factored characteristic tensile strength; GAMCE9 may be input (a material factor that divides the Young's Modulus so  $E = E_{cm}/\text{GAMCE9}$ ); and a creep factor PHIEF9 may be input: this scales  $\varepsilon_{cl}$  by  $(1+\text{PHIEF9})$ .

#### **Unload/reload stiffness (all concrete types):**

During compressive loading, the elastic modulus will be reduced according to the parameter UNLFAC (default = 0.5). UNLFAC = 0.0 means no reduction, i.e. the initial elastic modulus will apply during unloading and reloading. UNLFAC = 1.0 means that unloading results in no permanent strain. Intermediate values imply a permanent strain linearly interpolated between these extremes.



**Figure M172-4.** Concrete unloading behavior

Tensile strength is reduced by the same factor as the elastic modulus as described in the paragraph above.

#### Optional compressive strength degradation due to cracking:

By default, the compressive strength of cracked and uncracked elements is the same. If DEGRAD is non-zero, the formula from BS8110 is used to reduce compressive strength parallel to the crack while the crack is open:

$$\text{Reduction factor} = \min \left( 1.0, \frac{1.0}{0.8 + 100\varepsilon_t} \right),$$

where  $\varepsilon_t$  is the tensile strain normal to the crack.

#### Shear strength on cracking planes:

Before cracking, the through-thickness shear stress in the concrete is unlimited. For cracked elements, shear stress on the crack plane (magnitude of shear stress including element-plane and through-thickness terms) is treated in one of two ways:

1. If  $AGGSZ > 0$ , the relationship from Norwegian standard NS3473 is used to model the aggregate-interlock that allows cracked concrete to carry shear loading. In this case, UNITL must be defined. This is the factor that converts model length units to millimetres, i.e. the aggregate size in millimetres =  $AGGSZ \times UNITL$ . The formula in NS3473 also requires the crack width in millimetres: this is estimated from  $UNITL \times \varepsilon_{cro} \times L_e$ , here  $\varepsilon_{cro}$  is the crack opening strain and  $L_e$  is the crack spacing, taken as LCHAR if non-zero, or equal to element size if LCHAR is zero. Optionally, TAUMXC may be used to set the maximum shear stress when the crack is closed and the normal stress is zero – by default this is equal to 1.161FT from the

formulae in NS3473. If TAUMXC is defined, the shear stress from the NS3473 formula is scaled by TAUMXC / 1.161FT.

2. If AGGSZ = 0, the aggregate interlock is modeled by this formula:

$$\tau_{\max} = \frac{\text{TAUMXC}}{1.0 + \frac{\varepsilon_{\text{cro}}}{\text{ECRAGG}}} + \min(\text{MU} \times \sigma_{\text{comp}}, \text{TAUMXF})$$

Where  $\tau_{\max}$  is the maximum shear stress carried across a crack;  $\sigma_{\text{comp}}$  is the compressive stress across the crack (this is zero if the crack is open); ECRAGG is the crack opening strain at which the input shear strength TAUMXC is halved. Again, TAUMXC defaults to 1.161FT.

Note that if a shear capacity check is specified, the above applies only to in-plane shear, while the through-thickness shear is unlimited.

### Reinforcement

The reinforcement is treated as separate bars providing resistance only in the local  $x$  and  $y$  directions – it does not carry shear in-plane or out of plane.

For TYPER = 1,2,3,4,7,8, the behaviour is thermally sensitive and follows stress-strain relationships of a form defined in EC2. At 20°C (or if no thermal input is defined) the behaviour is elastic-perfectly-plastic with Young's Modulus EREINF and ultimate stress SUREINF, up to the onset of failure, after which the stress reduces linearly with increasing strain until final failure. At elevated temperatures there is a nonlinear transition between the elastic phase and the perfectly plastic phase, and EREINF and SUREINF are scaled down by temperature-dependent factors defined in EC2. The strain at which failure occurs depends on the reinforcement type (TYPER) and the temperature. For example, for hot-rolled reinforcing steel at 20°C failure begins at 15% strain and is complete at 20% strain. The thermal expansion coefficient is as defined in EC2 and is a function of temperature. This may be overridden by inputting the curve LCAPLS. The differences between TYPER = 1,2,4,7,8 are limited to (a) different reductions of EREINF and SUREINF at elevated temperatures, (b) different nonlinear transitions between elastic and plastic phases and (c) the strains at which softening begins and is complete.

The default stress-strain curve for reinforcement may be overridden using TYPER = 5 and LCRSU. In this case, the reinforcement properties are not temperature-sensitive and the yield stress is given by SUREINF  $\times f(\varepsilon_p)$ , where  $f(\varepsilon_p)$  is the load curve value at the current plastic strain. To include failure of the reinforcement, the curve should reduce to zero at the desired failure strain and remain zero for higher strains. Note that by default LS-DYNA re-interpolates the input curve to have 100 equally-spaced points; if the last point on the curve is at very high strain, then the initial part of the curve may become poorly defined.

**Local directions:**

AOPT and associated data are used to define the directions of the reinforcement bars. If the reinforcement directions are not consistent across neighbouring elements, the response may be less stiff than intended – this is equivalent to the bars being bent at the element boundaries. See material type 2 for description of the different AOPT settings.

**Shear capacity check:**

Shear reinforcement is not included explicitly in this material model. However, a shear capacity check can be made, to show regions that require shear reinforcement. The assumption is that the structure will not yield or fail in through-thickness shear, because sufficient shear reinforcement will be added. Set ISHCHK and TYPESC to 1. Give the percentage reinforcement (P\_OR\_F), effective depth of section EFFD (this typically excludes the cover concrete), and load factor GAMSC. These are used in Table 3.8 of BS 8110-1:1997 to determine the design shear stress. The “shear capacity” is this design shear stress times the total section thickness (i.e. force per unit width), modified according to Equation 6b of BS 8110 to allow for axial load. The “shear demand” (actual shear force per unit width) is then compared to the shear capacity. This process is performed for the two local directions of the reinforcement in each element; when defining sections using integration rules and multiple sets of material properties, it is important that each set of material properties referenced within the same section has the same AOPT and orientation data. Note that the shear demand and axial load (used in calculation of the shear capacity) are summed across the integration points within the section; the same values of capacity, demand, and difference between capacity and demand are then written to all the integration points.

**Thermal expansion:**

By default, thermal expansion properties from EC2 are used. If no temperatures are defined in the model, properties for 20deg C are used. For TYPEC = 3, 6 or 9, and TYPER = 5, there is no thermal expansion by default, and the properties do not vary with temperature. The user may override the default thermal expansion behaviour by defining curves of thermal expansion coefficient versus temperature (LCALPC, LCALPR). These apply no matter what types TYPEC and TYPER have been selected.

**Output:**

“Plastic Strain” is the maximum of the plastic strains in the reinforcement in the two local directions.

**Element deletion:**

Elements are deleted from the calculation when all of their integration points have reached the erosion criterion:

Concrete crack opening strain > ERODET  
or Concrete compressive strain >  $\epsilon_{c\_erode}$

where  $\epsilon_{c\_erode} = ERODEC + \epsilon_{csp}$  with  $\epsilon_{csp}$  the strain at which the stress-strain relation falls to zero.

Reinforcement plastic strain >  $\epsilon_{r\_erode}$

where  $\epsilon_{r\_erode} = ERODER + \epsilon_{rsp}$  with  $\epsilon_{rsp}$  the strain at which the stress-strain relation falls to zero, or if LCRSU > 0  $\epsilon_{rsp}$  is assumed to be 2.0.

If the material is smeared concrete/reinforcement, i.e.  $0 < \max(\text{FRACRX}, \text{FRACRY}) < 1$ , the erosion criteria must be met for both concrete and reinforcement before erosion can occur.

Extra history variables may be requested for shell elements (NEIPS on \*DATABASE\_EXTENT\_BINARY), which have the following meaning:

- |                |     |  |
|----------------|-----|--|
| Extra Variable | 1:  | Current crack opening strain (if two cracks are present, max of the two)   |
| Extra Variable | 2:  | Equivalent uniaxial strain for concrete compressive behaviour  |
| Extra Variable | 3:  | Number of cracks (0, 1 or 2)   |
| Extra Variable | 4:  | Temperature  |
| Extra Variable | 5:  | Thermal strain   |
| Extra Variable | 6:  | Current crack opening strain – first crack to form   |
| Extra Variable | 7:  | Current crack opening strain – crack at 90 degrees to first crack  |
| Extra Variable | 8:  | Max crack opening strain – first crack to form   |
| Extra Variable | 9:  | Max crack opening strain – crack at 90 degrees to first crack  |
| Extra Variable | 10: | Maximum difference (shear demand minus capacity) that has occurred so far, in either of the two reinforcement directions |
| Extra Variable | 11: | Maximum difference (shear demand minus capacity) that has occurred so far, in reinforcement $x$ -direction               |
| Extra Variable | 12: | Maximum difference (shear demand minus capacity) that has occurred so far, in reinforcement $y$ -direction               |
| Extra Variable | 13: | Current shear demand minus capacity, in reinforcement $x$ -direction   |

- Extra Variable 14: Current shear demand minus capacity, in reinforcement  $y$ -direction
- Extra Variable 15: Current shear capacity  $V_{cx}$ , in reinforcement  $x$ -direction
- Extra Variable 16: Current shear capacity  $V_{cy}$ , in reinforcement  $y$ -direction
- Extra Variable 17: Current shear demand  $V_x$ , in reinforcement  $x$ -direction
- Extra Variable 18: Current shear demand  $V_y$ , in reinforcement  $y$ -direction
- Extra Variable 19: Maximum shear demand that has occurred so far, in reinforcement  $x$ -direction
- Extra Variable 20: Maximum shear demand) that has occurred so far, in reinforcement  $y$ -direction
- Extra Variable 21: Current strain in reinforcement ( $x$ -direction)
- Extra Variable 22: Current strain in reinforcement ( $y$ -direction)
- Extra Variable 23: Engineering shear strain (slip) across first crack
- Extra Variable 24: Engineering shear strain (slip) across second crack
- Extra Variable 25:  $x$ -stress in concrete (element local axes)
- Extra Variable 26:  $y$ -stress in concrete (element local axes)
- Extra Variable 27:  $xy$ -stress in concrete (element local axes)
- Extra Variable 28:  $yz$ -stress in concrete (element local axes)
- Extra Variable 29:  $xz$ -Stress in concrete (element local axes)
- Extra Variable 30: Reinforcement stress ( $a$ -direction)
- Extra Variable 31: Reinforcement stress ( $b$ -direction)
- Extra Variable 32: Current shear demand  $V_{max}$
- Extra Variable 33: Maximum  $V_{max}$  that has occurred so far
- Extra Variable 34: Current shear capacity  $V_{c\theta}$
- Extra Variable 35: Excess shear:  $V_{max} - V_{c\theta}$
- Extra Variable 36: Maximum excess shear that has occurred so far

In the above list  $V_{max}$  is given by

$$V_{max} = \sqrt{V_x^2 + V_y^2}$$

Where  $V_x$  and  $V_y$  is the shear demand reinforcement in  $x$  and  $y$  directions respectively. Additionally,

$$V_{c\theta} = \frac{V_{\max}}{\sqrt{\left(\frac{V_x}{V_{cx}}\right)^2 + \left(\frac{V_y}{V_{cy}}\right)^2}}$$

where  $V_{cx}$ ,  $V_{cy}$  are the shear capacities in the  $x$  and  $y$  directions.

Note that the concrete stress history variables are stored in element local axes irrespective of AOPT, i.e. local  $x$  is always the direction from node 1 to node 2. The reinforcement stresses are in the reinforcement directions; these do take account of AOPT.

MAXINT (shells) and/or BEAMIP (beams) on \*DATABASE\_EXTENT\_BINARY may be set to the maximum number of integration points, so that results for all integration points can be plotted separately.





**Plane Cards.** Repeat for each plane (maximum 6 planes).

Card 4	1	2	3	4	5	6	7	8
Variable	DIP	DIPANG	CPLANE	FRPLANE	TPLANE	SHRMAX	LOCAL	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	1.e20	0.0	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
GMOD	Elastic shear modulus
RNU	Poisson's ratio
PHI	Angle of friction (radians)
CVAL	Cohesion value (shear strength at zero normal stress)
PSI	Dilation angle (radians)
NOVOID	Flag = 1 to switch off voiding behavior (see notes)
NPLANES	Number of joint planes (maximum 6)
LCCPDR	Load curve for extra cohesion for parent material (dynamic relaxation)
LCCPT	Load curve for extra cohesion for parent material (transient)
LCCJDR	Load curve for extra cohesion for joints (dynamic relaxation)
LCCJT	Load curve for extra cohesion for joints (transient)
LCSFAC	Load curve giving factor on strength vs. time
GMODDP	Z-coordinate at which GMOD and CVAL are correct
GMODGR	Gradient of GMOD versus z-coordinate (usually negative)

VARIABLE	DESCRIPTION
LCGMEP	Load curve of GMOD versus plastic strain (overrides GMODGR)
LCPHIEP	Load curve of PHI versus plastic strain
LCPSIEP	Load curve of PSI versus plastic strain
LCGMST	(Leave blank)
CVALGR	Gradient of CVAL versus z-coordinate (usually negative)
ANISO	Factor applied to elastic shear stiffness in global XZ and YZ planes
DIP	Angle of the plane in degrees below the horizontal
DIPANG	Plan view angle (degrees) of downhill vector drawn on the plane
CPLANE	Cohesion for shear behavior on plane
PHPLANE	Friction angle for shear behavior on plane (degrees)
TPLANE	Tensile strength across plane (generally zero or very small)
SHRMAX	Max shear stress on plane (upper limit, independent of compression)
LOCAL	EQ.0: DIP and DIPANG are with respect to the global axes EQ.1: DIP and DIPANG are with respect to the local element axes

**Remarks:**

- The material has a Mohr Coulomb yield surface, given by  $\tau_{\max} = C + \sigma_n \tan(\text{PHI})$ , where  $\tau_{\max}$  = maximum shear stress on any plane,  $\sigma_n$  = normal stress on that plane (positive in compression),  $C$  = cohesion,  $\text{PHI}$  = friction angle. The plastic potential function is of the form  $\beta \sigma_k - \sigma_l + \text{constant}$ , where  $\sigma_k$  = maximum principal stress,  $\sigma_l$  = minimum principal stress, and  $\beta = \frac{1 + \sin(\text{PSI})}{1 - \sin(\text{PSI})}$ .
- The tensile strength of the material is given by  $\sigma_{\max} = \frac{C}{\tan(\text{PHI})}$  where  $C$  is the cohesion. After the material reaches its tensile strength, further tensile straining leads to volumetric voiding; the voiding is reversible if the strain is reversed.
- If depth-dependent properties are used, the model must be oriented with the z-axis in the upward direction.

4. Plastic strain is defined as  $\sqrt{\frac{2}{3}\varepsilon_{p_{ij}}\varepsilon_{p_{ij}}}$ , i.e. the same way as for other elasto-plastic material models.
5. Friction and dilation angles PHI and PSI may vary with plastic strain, to model heavily consolidated materials under large shear strains – as the strain increases, the dilation angle typically reduces to zero and the friction angle to a lower, pre-consolidation value.
6. For similar reasons, the shear modulus may reduce with plastic strain, but this option may sometimes give unstable results.
7. The load curves LCCPDR, LCCPT, LCCJDR, LCCJT allow extra cohesion to be specified as a function of time. The cohesion is additional to that specified in the material parameters. This is intended for use during the initial stages of an analysis to allow application of gravity or other loads without cracking or yielding, and for the cracking or yielding then to be introduced in a controlled manner. This is done by specifying extra cohesion that exceeds the expected stresses initially, then declining to zero. If no curves are specified, no extra cohesion is applied.
8. The load curve for factor on strength applies simultaneously to the cohesion and tan(friction angle) of parent material and all joints. This feature is intended for reducing the strength of the material gradually, to explore factors of safety. If no curve is present, a constant factor of 1 is assumed. Values much greater than 1.0 may cause problems with stability.
9. The anisotropic factor ANISO applies the elastic shear stiffness in the global XZ and YZ planes. It can be used only in pure Mohr-Coulomb mode (NPLANES = 0).
10. For friction angle greater than zero, the Mohr Coulomb yield surface implies a tensile pressure limit equal to  $CVAL/\tan(\text{PHI})$ . The default behaviour is that voids develop in the material when this pressure limit is reached, and the pressure will never become more tensile than the pressure limit. If  $\text{NOVOID} = 1$ , the tensile pressure limit is not applied. Stress states in which the pressure is more tensile than  $CVAL/\tan(\text{PHI})$  are permitted, but will be purely hydrostatic with no shear stress.  $\text{NOVOID}$  is recommended in Multi-Material ALE simulations, in which the development of voids or air space is already accounted for by the Multi-Material ALE.
11. To model soil, set  $\text{NJOINT} = 0$ . The joints are to allow modeling of rock, and are treated identically to those of `*MAT_JOINTED_ROCK`.
12. The joint plane orientations are defined by the angle of a “downhill vector” drawn on the plane, i.e. the vector is oriented within the plane to obtain the maximum possible downhill angle. DIP is the angle of this line below the horizontal. DI-

PANG is the plan-view angle of the line (pointing down hill) measured clockwise from the global Y-axis about the global Z-axis.

13. Joint planes would generally be defined in the global axis system if they are taken from survey data. However, the material model can also be used to represent masonry, in which case the weak planes represent the cement and lie parallel to the local element axes.
14. The joint planes rotate with the rigid body motion of the elements, irrespective of whether their initial definitions are in the global or local axis system.
15. Extra variables for plotting. By setting NEIPH on \*DATABASE\_EXTENT\_BINARY to 27, the following variables can be plotted in Oasys Ltd. Post Processors D3-PLOT, T/HIS and LS-PrePost:

<i>Variable(s)</i>	<i>Description</i>
1	mobilized strength fraction for base material
2	volumetric void strain
3	maximum stress overshoot during plastic calculation
4 – 9	crack opening strain for planes 1 - 6
10 - 15	crack accumulated engineering shear strain for planes 1 - 6
16 - 20	current shear utilization for planes 1 - 6
21 - 27	maximum shear utilization to date for planes 1 – 6
33	elastic shear modulus (for checking depth-dependent input)
34	cohesion (for checking depth-dependent input)

**\*MAT\_RC\_BEAM**

This is Material Type 174, for Hughes-Liu beam elements only. The material model can represent plain concrete only, reinforcing steel only, or a smeared combination of concrete and reinforcement. The main emphasis of this material model is the cyclic behavior – it is intended primarily for seismic analysis.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EUNL	PR	FC	EC1	EC50	RESID
Type	A8	F	F	F	F	F	F	F
Default	none	none	See Remarks	0.0	none	0.0022	See Remarks	0.2

Card 2	1	2	3	4	5	6	7	8
Variable	FT	UNITC	(blank)	(blank)	(blank)	ESOFT	LCHAR	OUTPUT
Type	F	F	F	F	F	F	F	F
Default	See Remarks	1.0	none	none	none	See Remarks	none	0

Card 3	1	2	3	4	5	6	7	8
Variable	FRACR	YMREIN	PRREIN	SYREIN	SUREIN	ESHR	EUR	RREINF
Type	F	F	F	F	F	F	F	F
Default	0.0	none	0.0	0.0	SYREIN	0.03	0.2	4.0

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label not exceeding 8 characters must be specified.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RO	Mass density
EUNL	Initial unloading elastic modulus (See Remarks).
PR	Poisson's ratio.
FC	Cylinder strength (stress units)
EC1	Strain at which stress FC is reached.
EC50	Strain at which the stress has dropped to 50% FC
RESID	Residual strength factor
FT	Maximum tensile stress
UNITC	Factor to convert stress units to MPa (See Remarks)
ESOFT	Slope of stress-strain curve post-cracking in tension
LCHAR	Characteristic length for strain-softening behavior
OUTPUT	Output flag controlling what is written as "plastic strain" EQ.0.0: Curvature EQ.1.0: "High-tide" plastic strain in reinforcement
FRACR	Fraction of reinforcement (e.g. for 1% reinforcement FRACR = 0.01)
YMREIN	Young's Modulus of reinforcement
PRREIN	Poisson's Ratio of reinforcement
SYREIN	Yield stress of reinforcement
SUREIN	Ultimate stress of reinforcement
ESHR	Strain at which reinforcement begins to harden
EUR	Strain at which reinforcement reaches ultimate stress
R_REINF	Dimensionless Ramberg-Osgood parameter r. If zero, a default value $r = 4.0$ will be used. If set to -1, parameters will be calculated from Kent & Park formulae. (See Remarks)

**Creating sections for reinforced concrete beams:**

This material model can be used to represent unreinforced concrete (FRACR = 0), steel (FRACR = 1), or reinforced concrete with evenly distributed reinforcement ( $0 < \text{FRACR} < 1$ ).

Alternatively, use \*INTEGRATION\_BEAM to define the section. A new option in allows the user to define a Part ID for each integration point, similar to the facility already available with \*INTEGRATION\_SHELL. All parts referred to by one integration rule must have the same material type, but can have different material properties. Create one Part for concrete, and another for steel. These Parts should reference Materials, both of type \*MAT\_RC\_BEAM, one with FRACR = 0, the other with FRACR = 1. Then, by assigning one or other of these Part Ids to each integration point the reinforcement can be applied to the correct locations within the section of the beam.

**Concrete:**

In monotonic compression, the approach of Park and Kent, as described in Park & Paulay [1975] is used. The material follows a parabolic stress-strain curve up to a maximum stress equal to the cylinder strength FC; thereafter the strength decays linearly with strain until the residual strength is reached. Default values for some material parameters will be calculated automatically as follows:

$$EC50 = \frac{(3 + 0.29FC)}{145FC - 1000}$$

where FC is in MPa as per Park and Kent test data.

$$EUNL = \text{initial tangent slope} = \frac{2FC}{EC1}$$

User-defined values for EUNL lower than this are not permitted, but higher values may be defined if desired.

$$FT = 1.4 \left( \frac{FC}{10} \right)^{\frac{2}{3}}$$

where FC is in MPa as per Park and Kent test data.

$$ESOFT = EUNL$$

User-defined values higher than EUNL are not permitted.

UNITC is used only to calculate default values for the above parameters from FC.

Strain-softening behavior tends to lead to deformations being concentrated in one element, and hence the overall force-deflection behavior of the structure can be mesh-size-dependent if the softening is characterized by strain. To avoid this, a characteristic length (LCHAR) may be defined. This is the length of specimen (or element) that would exhibit the defined monotonic stress-strain relationship. LS-DYNA adjusts the stress-strain



relationship after ultimate load for each element, such that all elements irrespective of their length will show the same deflection during strain softening (i.e. between ultimate load and residual load). Therefore, although deformation will still be concentrated in one element, the load-deflection behavior should be the same irrespective of element size. For tensile behavior, ESOF is similarly scaled.

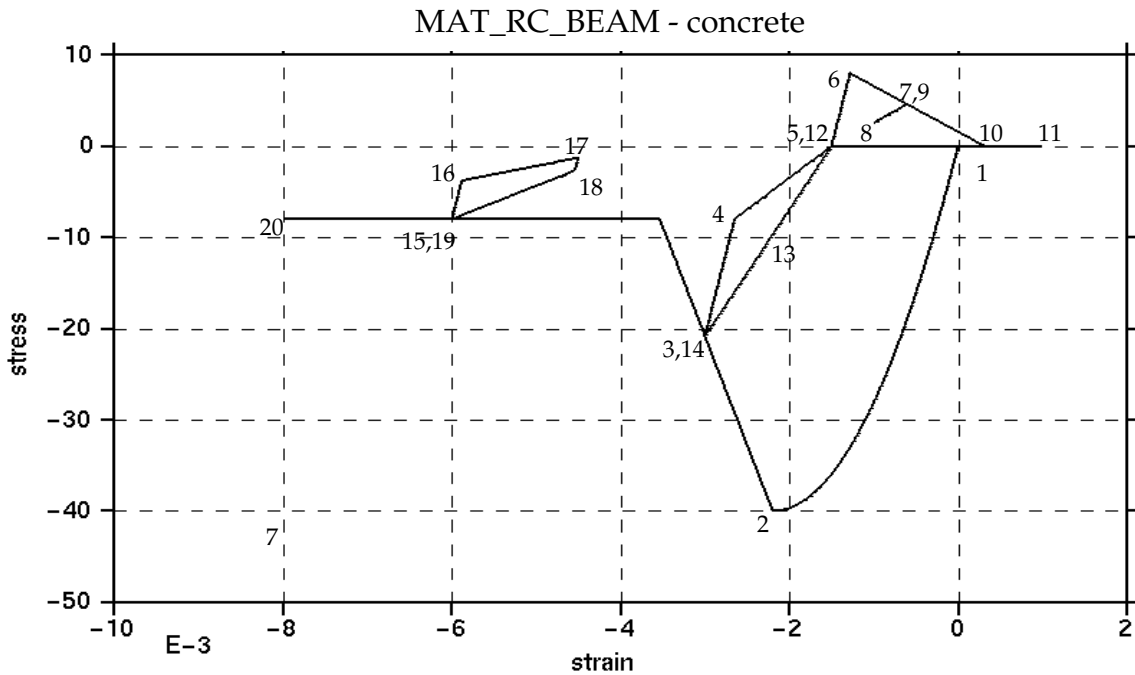


Figure M174-1

Cyclic behavior is broadly suggested by Blakeley and Park [1973] as described in Park & Paulay [1975]; the stress-strain response lies within the Park-Kent envelope, and is characterized by stiff initial unloading response at slope EUNL followed by a less stiff response if it unloads to less than half the current strength. Reloading stiffness degrades with increasing strain.

In tension, the stress rises linearly with strain until a tensile limit  $F_T$  is reached. Thereafter the stiffness and strength decays with increasing strain at a rate ESOF. The stiffness also decays such that unloading always returns to strain at which the stress most recently changed to tensile.

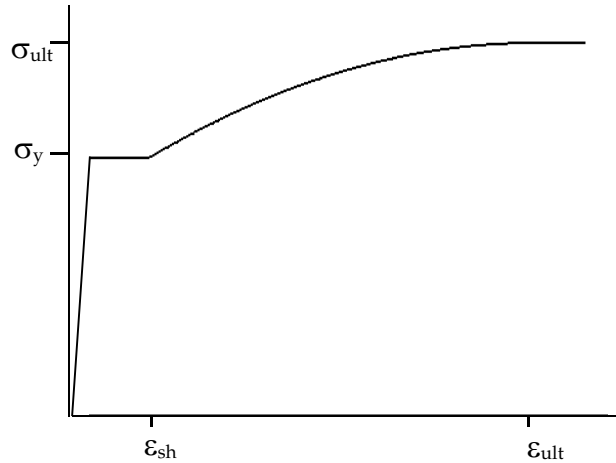


Figure M174-2

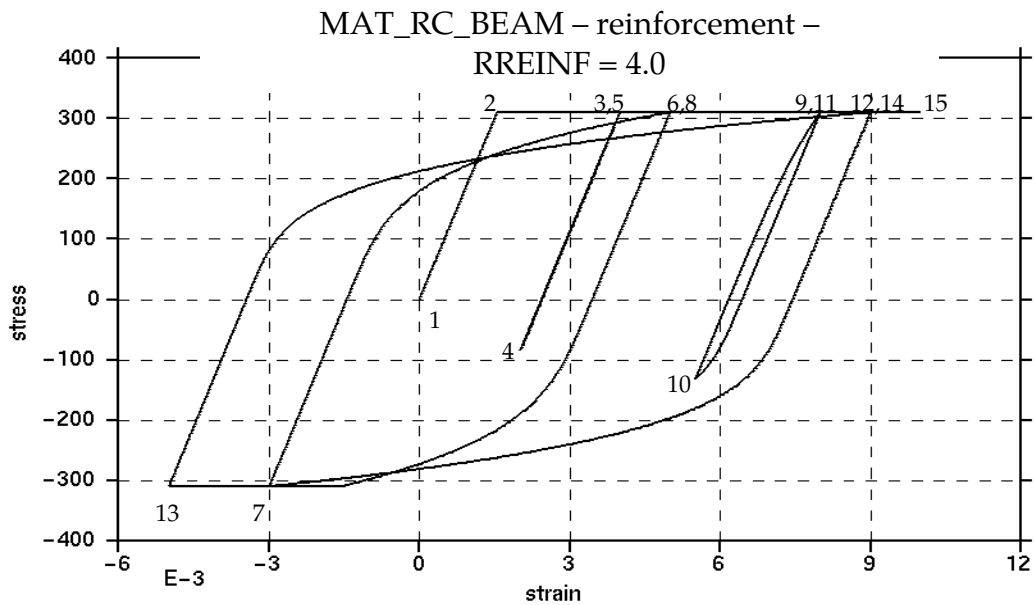


Figure M174-3

Monotonic loading of the reinforcement results in the stress-strain curve shown, which is parabolic between  $\epsilon_{sh}$  and  $\epsilon_{ult}$ . The same curve acts as an envelope on the hysteretic behavior, when the x-axis is cumulative plastic strain.

Unloading from the yielded condition is elastic until the load reverses. Thereafter, the Bauschinger Effect (reduction in stiffness at stresses less than yield during cyclic deformation) is represented by following a Ramberg-Osgood relationship until the yield stress is reached:

$$\epsilon - \epsilon_s = \left(\frac{\sigma}{E}\right) \left\{ 1 + \left(\frac{\sigma}{\sigma_{CH}}\right)^{r-1} \right\}$$

where  $\varepsilon$  and  $\sigma$  are strain and stress,  $\varepsilon_s$  is the strain at zero stress,  $E$  is Young's Modulus, and  $r$  and  $\sigma_{CH}$  are as defined below

Two options are given for calculation  $r$  and  $\sigma_{CH}$ , which is performed at each stress reversal:

1. If RREINF is input as -1,  $r$  and  $\sigma_{CH}$  are calculated internally from formulae given in Kent and Park. Parameter  $r$  depends on the number of stress reversals. Parameter  $\sigma_{CH}$  depends on the plastic strain that occurred between the previous two stress reversals. The formulae were statistically derived from experiments, but may not fit all circumstances. In particular, large differences in behavior may be caused by the presence or absence of small stress reversals such as could be caused by high frequency oscillations. Therefore, results might sometimes be unduly sensitive to small changes in the input data.
2. If RREINF is entered by the user or left blank,  $r$  is held constant while  $\sigma_{CH}$  is calculated on each reversal such that the Ramberg-Osgood curve meets the monotonic stress-strain curve at the point from which it last unloaded, e.g. points 6 and 8 are coincident in the graph below. The default setting RREINF = 4.0 gives similar hysteresis behavior to that described by Kent & Park but is unlikely to be so sensitive to small changes of input data.

### **Output:**

It is recommended to use BEAMIP on \*DATABASE\_EXTENT\_BINARY to request stress and strain output at the individual integration points. If this is done, for MAT\_RC\_BEAM only, element curvature is written to the output files in place of plastic strain. In the post-processor, select "plastic strain" to display curvature (whichever of the curvatures about local y and z axes has greatest absolute value will be plotted). Alternatively, select "axial strain" to display the total axial strain (elastic + plastic) at that integration point; this can be combined with axial stress to create hysteresis plots such as those shown above.

**\*MAT\_VISCOELASTIC\_THERMAL**

This is Material Type 175. This material model provides a general viscoelastic Maxwell model having up to 12 terms in the prony series expansion and is useful for modeling dense continuum rubbers and solid explosives. Either the coefficients of the prony series expansion or a relaxation curve may be specified to define the viscoelastic deviatoric and bulk behavior. Note that \*MAT\_GENERAL\_VISCOELASTIC (Material Type 76) has all the capability of \*MAT\_VISCOELASTIC\_THERMAL, and additionally offers more terms (18) in the prony series expansion and an optional scaling of material properties with moisture content.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	PCF	EF	TREF	A	B
Type	A8	F	F	F	F	F	F	F

If fitting is done from a relaxation curve, specify fitting parameters on card 2, *otherwise* if constants are set on Viscoelastic Constant Cards *LEAVE THIS CARD BLANK*.

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	NT	BSTART	TRAMP	LCIDK	NTK	BSTARTK	TRAMPK
Type	F	I	F	F	F	I	F	F

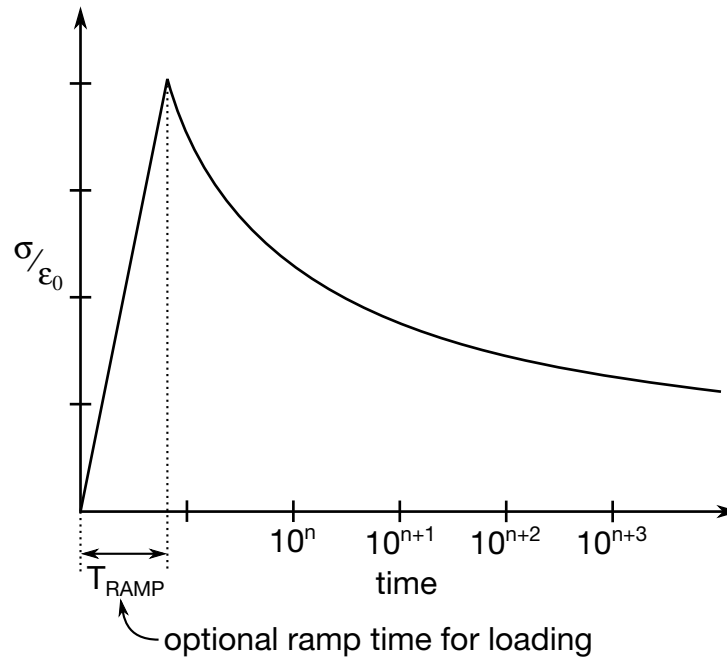
**Viscoelastic Constant Cards.** Up to 6 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 6 cards are used. These cards are not needed if relaxation data is defined. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included. If an elastic layer is defined you only need to define GI and KI (note in an elastic layer only one card is needed).

Optional	1	2	3	4	5	6	7	8
Variable	Gi	BETAi	Ki	BETAKi				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
BULK	Elastic bulk modulus.
PCF	Tensile pressure elimination flag for solid elements only. If set to unity tensile pressures are set to zero.
EF	Elastic flag: EQ.0: the later is viscoelastic EQ.1: the layer is elastic
TREF	Reference temperature for shift function (must be greater than zero).
A	Coefficient for the Arrhenius and the Williams-Landel-Ferry shift functions.
B	Coefficient for the Williams-Landel-Ferry shift function.
LCID	Load curve ID for deviatoric behavior if constants, $G_i$ , and $\beta_i$ are determined via a least squares fit. This relaxation curve is shown below.
NT	Number of terms in shear fit. If zero the default is 6. Fewer than NT terms will be used if the fit produces one or more negative shear moduli. Currently, the maximum number is set to 6.
BSTART	In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$ , $\beta_4$ is 10 times $\beta_3$ , and so on. If zero, BSTART is determined by an iterative trial and error scheme.
TRAMP	Optional ramp time for loading.
LCIDK	Load curve ID for bulk behavior if constants, $K_i$ , and $\beta\kappa_i$ are determined via a least squares fit. This relaxation curve is shown below.
NTK	Number of terms desired in bulk fit. If zero the default is 6. Currently, the maximum number is set to 6.

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BSTARTK	In the fit, $\beta\kappa_1$ is set to zero, $\beta\kappa_2$ is set to BSTARTK, $\beta\kappa_3$ is 10 times $\beta\kappa_2$ , $\beta\kappa_4$ is 10 times $\beta\kappa_3$ , and so on. If zero, BSTARTK is determined by an iterative trial and error scheme.
TRAMPK	Optional ramp time for bulk loading.
$G_i$	Optional shear relaxation modulus for the $i$ th term
BETA $i$	Optional shear decay constant for the $i$ th term
$K_i$	Optional bulk relaxation modulus for the $i$ th term
BETA $K_i$	Optional bulk decay constant for the $i$ th term



**Figure M175-1.** Relaxation curve. This curve defines stress versus time where time is defined on a logarithmic scale. For best results, the points defined in the load curve should be equally spaced on the logarithmic scale. Furthermore, the load curve should be smooth and defined in the positive quadrant. If nonphysical values are determined by least squares fit, LS-DYNA will terminate with an error message after the initialization phase is completed. If the ramp time for loading is included, then the relaxation which occurs during the loading phase is taken into account. This effect may or may not be important

### Remarks:

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  is the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^N G_m e^{-\beta_m t}$$

We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . An arbitrary number of terms, up to 6, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk moduli:

$$k(t) = \sum_{m=1}^N K_m e^{-\beta_k t}$$

The Arrhenius and Williams-Landel-Ferry (WLF) shift functions account for the effects of the temperature on the stress relaxation. A scaled time,  $t'$ ,

$$t' = \int_0^t \Phi(T) dt$$

is used in the relaxation function instead of the physical time. The Arrhenius shift function is

$$\Phi(T) = \exp \left[ -A \left( \frac{1}{T} - \frac{1}{T_{\text{REF}}} \right) \right]$$

and the Williams-Landel-Ferry shift function is

$$\Phi(T) = \exp \left( -A \frac{T - T_{\text{REF}}}{B + T - T_{\text{REF}}} \right)$$

If all three values ( $T_{\text{REF}}$ ,  $A$ , and  $B$ ) are not zero, the WLF function is used; the Arrhenius function is used if  $B$  is zero; and no scaling is applied if all three values are zero.





**Viscoelastic Constant Card 2.** Additional Card for LC1 = 0.

Card 4	1	2	3	4	5	6	7	8
Variable	G5	BETA5	G6	BETA6	G7	BETA7	G8	BETA8
Type	F	F	F	F	F	F	F	F

**Viscoelastic Constant Card 3.** Additional Card for LC1 = 0.

Card 5	1	2	3	4	5	6	7	8
Variable	G9	BETA9	G10	BETA10	G11	BETA11	G12	BETA12
Type	F	F	F	F	F	F	F	F

**Instantaneous Elastic Reponses Card.** Additional Card for LC2 = 0.

Card	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
K	Bulk modulus.
LC1	Load curve ID that defines the relaxation function in shear. This curve is used to fit the coefficients $G_i$ and $BETA_i$ . If zero, define the coefficients directly. The latter is recommended.

VARIABLE	DESCRIPTION
LC2	Load curve ID that defines the instantaneous elastic response in compression and tension. If zero, define the coefficients directly. <i>Symmetry is not assumed if only the tension side is define; therefore, defining the response in tension only, may lead to nonphysical behavior in compression. Also, this curve should give a softening response for increasing strain without any negative or zero slopes. A stiffening curve or one with negative slopes is generally unstable.</i>
N	Number of terms used in the Prony series, a number less than or equal to 6. This number should be equal to the number of decades of time covered by the experimental data. Define this number if LC1 is nonzero. Carefully check the fit in the d3hsp file to ensure that it is valid, since the least square fit is not always reliable.
GSTART	Starting value for least square fit. If zero, a default value is set equal to the inverse of the largest time in the experiment. Define this number if LC1 is nonzero.
M	Number of terms used to determine the instantaneous elastic response. This variable is ignored with the new formulation but is kept for compatibility with the previous input.
SO	Strain (logarithmic) output option to control what is written as component 7 to the d3plot database. (LS-PrePost always blindly labels this component as effective plastic strain.) The maximum values are updated for each element each time step: EQ.0.0: maximum principal strain that occurs during the calculation, EQ.1.0: maximum magnitude of the principal strain values that occurs during the calculation, EQ.2.0: maximum effective strain that occurs during the calculation.
E_MIN	Minimum strain used to generate the load curve from $C_i$ . The default range is -0.9 to 5.1. The computed solution will be more accurate if the user specifies the range used to fit the $C_i$ . Linear extrapolation is used outside the specified range.
E_MAX	Maximum strain used to generate the load curve from $C_i$ .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
K	Material failure parameter that controls the volume enclosed by the failure surface, see *MAT_SIMPLIFIED_RUBBER. LE.0.0: ignore failure criterion; GT.0.0: use actual K value for failure criterions.
GAMA1	Material failure parameter, see *MAT_SIMPLIFIED_RUBBER and <a href="#">Figure M181-1</a> .
GAMA2	Material failure parameter, see *MAT_SIMPLIFIED_RUBBER.
EH	Damage parameter, see *MAT_SIMPLIFIED_RUBBER.
FORM	Formulation of model. FORM = 0 gives the original model developed by Fung, which always relaxes to a zero stress state as time approaches infinity, and FORM = 1 gives the alternative model, which relaxes to the quasi-static elastic response. In general, the two formulations won't give the same responses. Formulation, FORM = -1, is an improvement on FORM = 0 where the instantaneous elastic response is used in the viscoelastic stress update, not just in the relaxation, as in FORM = 0. Consequently, the constants for the elastic response do not need to be scaled.
Gi	Coefficients of the relaxation function. The number of coefficients is currently limited to 6 although 12 may be read in to maintain compatibility with the previous formulation's input. Define these coefficients if LC1 is set to zero. At least 2 coefficients must be nonzero.
BETAi	Decay constants of the relaxation function. Define these coefficients if LC1 is set to zero. The number of coefficients is currently limited to 6 although 12 may be read in to maintain compatibility with the previous formulation's input.
Ci	Coefficients of the instantaneous elastic response in compression and tension. Define these coefficients only if LC2 is set to zero.

**Remarks:**

The equations for the original model (FORM = 0) are given as:

$$\sigma_V(t) = \int_0^t G(t - \tau) \frac{\partial \sigma_\varepsilon[\varepsilon(\tau)]}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \tau} d\tau$$

$$G(t) = \sum_{i=1}^n G_i e^{-\beta t}$$
$$\sigma_\varepsilon(\varepsilon) = \sum_{i=1}^k C_i \varepsilon^i$$

where  $G$  is the shear modulus. Effective strain (which can be written to the d3plot database) is calculated as follows:

$$\varepsilon^{\text{effective}} = \sqrt{\frac{2}{3} \varepsilon_{ij} \varepsilon_{ij}}$$

The polynomial for instantaneous elastic response should contain only odd terms if symmetric tension-compression response is desired.

The new model (FORM = 1) is based on the hyperelastic model used \*MAT\_SIMPLIFIED\_RUBBER assuming incompressibility. The one-dimensional expression for  $\sigma_\varepsilon$  generates the uniaxial stress-strain curve and an additional visco-elastic term is added on,

$$\sigma(\varepsilon, t) = \sigma_{SR}(\varepsilon) + \sigma_V(t)$$
$$\sigma_V(t) = \int_0^t G(t - \tau) \frac{\partial \varepsilon}{\partial \tau} d\tau$$

where the first term to the right of the equals sign is the hyperelastic stress and the second is the viscoelastic stress. Unlike the previous formulation, where the stress always relaxed to zero, the current formulation relaxes to the hyperelastic stress.

**\*MAT\_HILL\_FOAM**

Purpose: This is Material Type 177. This is a highly compressible foam based on the strain-energy function proposed by Hill [1979]; also see Storakers [1986]. Poisson's ratio effects are taken into account.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	N	MU	LCID	FITYPE	LCSR
Type	A8	F	F	F	F	I	I	I
Default	none	none	none	0	0	0	0	0

**Material Constant Card 1.** Additional card for LCID = 0.

Card 2	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F

**Material Constant Card 2.** Additional card for LCID = 0.

Card 3	1	2	3	4	5	6	7	8
Variable	B1	B2	B3	B4	B5	B6	B7	B8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	R	M						
Type	F	F						

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
K	Bulk modulus. This modulus is used for determining the contact interface stiffness.
N	Material constant. Define if LCID = 0 below; otherwise, N is fit from the load curve data. See equations below.
MU	Damping coefficient.
LCID	Load curve ID that defines the force per unit area versus the stretch ratio. This curve can be given for either uniaxial or biaxial data depending on FITTYPE.
FITTYPE	Type of fit: EQ.1: uniaxial data, EQ.2: biaxial data, EQ.3: pure shear data.
LCSR	Load curve ID that defines the uniaxial or biaxial stretch ratio (see FITTYPE) versus the transverse stretch ratio.
Ci	Material constants. See equations below. Define up to 8 coefficients if LCID = 0.
Bi	Material constants. See equations below. Define up to 8 coefficients if LCID = 0.
R	Mullins effect model $r$ coefficient
M	Mullins effect model $m$ coefficient

**Remarks:**

If load curve data is defined, the fit generated by LS-DYNA must be closely checked in the D3HSP output file. It may occur that the nonlinear least squares procedure in LS-DYNA, which is used to fit the data, is inadequate.

The Hill strain energy density function for this highly compressible foam is given by:

$$W = \sum_{j=1}^m \frac{C_j}{b_j} \left[ \lambda_1^{b_j} + \lambda_2^{b_j} + \lambda_3^{b_j} - 3 + \frac{1}{n} (J^{-nb_j} - 1) \right]$$

where  $C_j$ ,  $b_j$ , and  $n$  are material constants and  $J = \lambda_1 \lambda_2 \lambda_3$  represents the ratio of the deformed to the undeformed state. The constant  $m$  is internally set to 4. In case number of points in the curve is less than 8, then  $m$  is set to the number of points divided by 2. The principal Cauchy stresses are

$$t_i = \sum_{j=1}^m \frac{C_j}{J} \left[ \lambda_i^{b_j} - J^{-nb_j} \right] \quad i = 1, 2, 3$$

From the above equations the shear modulus is:

$$\mu = \frac{1}{2} \sum_{j=1}^m C_j b_j$$

and the bulk modulus is:

$$K = 2\mu \left( n + \frac{1}{3} \right)$$

The value for  $K$  defined in the input is used in the calculation of contact forces and for the material time step. Generally, this value should be equal to or greater than the  $K$  given in the above equation.



\*MAT\_VISCOELASTIC\_HILL\_FOAM

Purpose: This is Material Type 178. This is a highly compressible foam based on the strain-energy function proposed by Hill [1979]; also see Storakers [1986]. The extension to include large strain viscoelasticity is due to Feng and Hallquist [2002].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	N	MU	LCID	FITTYPE	LCSR
Type	A8	F	F	F	F	I	I	I
Default	none	none	none	0	0.05	0	0	0

Card 2	1	2	3	4	5	6	7	8
Variable	LCVE	NT	GSTART					
Type	I	F	F					
Default	0	6	1/TMAX					

**Material Constant Card 1.** Additional card for LCID = 0.

Card 3	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F

**Material Constant Card 2.** Additional card for LCID = 0.

Card 4	1	2	3	4	5	6	7	8
Variable	B1	B2	B3	B4	B5	B6	B7	B8
Type	F	F	F	F	F	F	F	F

**Viscoelastic Constant Cards.** Up to 12 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 12 cards are used.

Card 5	1	2	3	4	5	6	7	8
Variable	GI	BETAI						
Type	F	F						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
K	Bulk modulus. This modulus is used for determining the contact interface stiffness.
N	Material constant. Define if LCID = 0 below; otherwise, N is fit from the load curve data. See equations below.
MU	Damping coefficient (0.05 < recommended value < 0.50; default is 0.05).
LCID	Load curve ID that defines the force per unit area versus the stretch ratio. This curve can be given for either uniaxial or biaxial data depending on FITTYPE. Load curve LCSR below must also be defined.
FITTYPE	Type of fit: EQ.1: uniaxial data, EQ.2: biaxial data.
LCSR	Load curve ID that defines the uniaxial or biaxial stress ratio (see FITTYPE) versus the transverse stretch ratio.
LCVE	Optional load curve ID that defines the relaxation function in shear. This curve is used to fit the coefficients $G_i$ and $BETA_i$ . If zero, define the coefficients directly. The latter is recommended.

VARIABLE	DESCRIPTION
NT	Number of terms used to fit the Prony series, which is a number less than or equal to 12. This number should be equal to the number of decades of time covered by the experimental data. Define this number if LCVE is nonzero. Carefully check the fit in the D3HSP file to ensure that it is valid, since the least square fit is not always reliable.
GSTART	Starting value for least square fit. If zero, a default value is set equal to the inverse of the largest time in the experiment. Define this number if LC1 is nonzero, Ci, Material constants. See equations below. Define up to 8 coefficients.
Ci	Material constants. See equations below. Define up to 8 coefficients if LCID = 0.
Bi	Material constants. See equations below. Define up to 8 coefficients if LCID = 0.
GI	Optional shear relaxation modulus for the ith term
BETAI	Optional decay constant if ith term

### Remarks:

If load curve data is defined, the fit generated by LS-DYNA must be closely checked in the D3HSP output file. It may occur that the nonlinear least squares procedure in LS-DYNA, which is used to fit the data, is inadequate.

The Hill strain energy density function for this highly compressible foam is given by:

$$p^{n+1} = p^n e^{-\beta \cdot \Delta t} + K \dot{\epsilon}_{kk} \left( \frac{1 - e^{-\beta \cdot \Delta t}}{\beta} \right) \quad \text{where } \beta = |BETA|$$

where  $C_j$ ,  $b_j$ , and  $n$  are material constants and  $J = \lambda_1 \lambda_2 \lambda_3$  represents the ratio of the deformed to the undeformed state. The principal Cauchy stresses are

$$t_i = \sum_{j=1}^m \frac{C_j}{J} \left[ \lambda_i^{b_j} - J^{-n b_j} \right] \quad i = 1, 2, 3$$

From the above equations the shear modulus is:

$$\mu = \frac{1}{2} \sum_{j=1}^m C_j b_j$$

and the bulk modulus is:

$$K = 2\mu \left( n + \frac{1}{3} \right)$$

The value for  $K$  defined in the input is used in the calculation of contact forces and for the material time step. Generally, this value should be equal to or greater than the  $K$  given in the above equation.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl}(t - \tau) \frac{\partial E_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta t}$$

given by,

$$g(t) = \sum_{i=1}^n G_i e^{-\beta_i t}$$

This model is effectively a Maxwell fluid which consists of dampers and springs in series. We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . The viscoelastic behavior is optional and an arbitrary number of terms may be used.

**\*MAT\_LOW\_DENSITY\_SYNTHETIC\_FOAM\_{OPTION}**

This is Material Type 179 (and 180 if the ORTHO option below is active) for modeling rate independent low density foams, which have the property that the hysteresis in the loading-unloading curve is considerably reduced after the first loading cycle. In this material we assume that the loading-unloading curve is identical after the first cycle of loading is completed and that the damage is isotropic, i.e., the behavior after the first cycle of loading in the orthogonal directions also follows the second curve. The main application at this time is to model the observed behavior in the compressible synthetic foams that are used in some bumper designs. Tables may be used in place of load curves to account for strain rate effects.

Available options include:

<BLANK>

ORTHO

WITH\_FAILURE

ORTHO\_WITH\_FAILURE

If the foam develops orthotropic behavior, i.e., after the first loading and unloading cycle the material in the orthogonal directions are unaffected then the ORTHO option should be used. If the ORTHO option is active the directionality of the loading is stored. This option is requires additional storage to store the history variables related to the orthogonality and is slightly more expensive.

An optional failure criterion is included. A description of the failure model is provided below for material type 181, \*MAT\_SIMPLIFIED\_RUBBER/FOAM.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	LCID1	LCID2	HU	BETA	DAMP
Type	A8	F	F	F	F	F	F	F
Default						1.		0.05

**\*MAT\_179****\*MAT\_LOW\_DENSITY\_SYNTHETIC\_FOAM**

Card 2	1	2	3	4	5	6	7	8
Variable	SHAPE	FAIL	BVFLAG	ED	BETA1	KCON	REF	TC
Type	F	F	F	F	F	F	F	F
Default	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.E+20

Additional card for LCID1 < 0.

Card 3	1	2	3	4	5	6	7	8
Variable	RFLAG	DTRT						
Type	F	F						
Default	0.0	0.0						

Additional card for WITH\_FAILURE keyword option.

Card 4	1	2	3	4	5	6	7	8
Variable	K	GAMA1	GAMA2	EH				
Type	F	F	F	F				

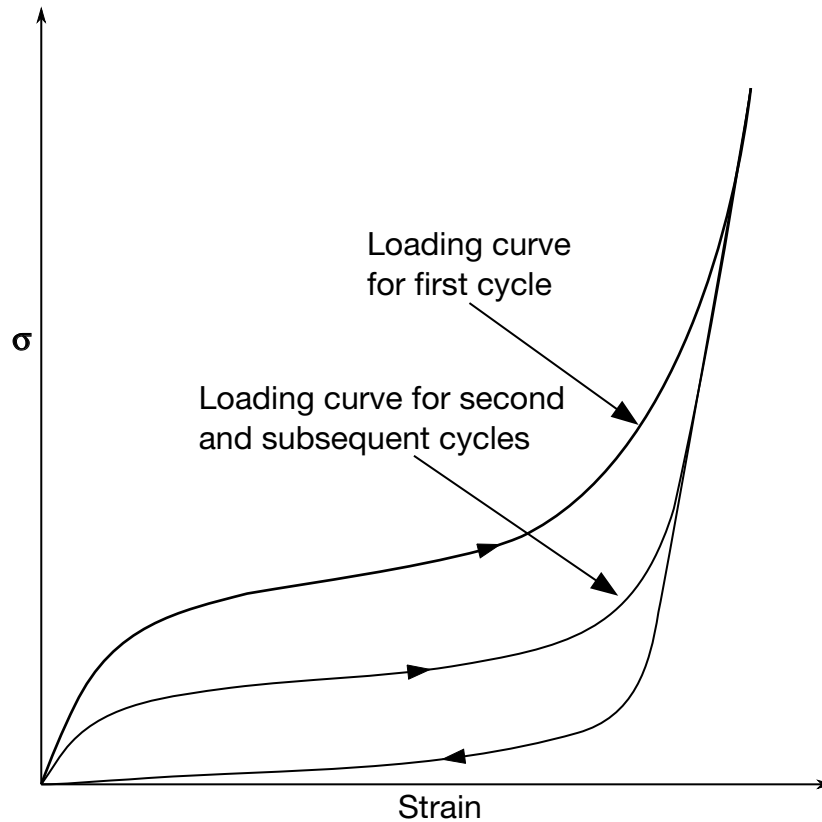
**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus. This modulus is used if the elongations are tensile as described for the *MAT_LOW_DENSITY_FOAM.

VARIABLE	DESCRIPTION
LCID1	Load curve or table ID: GT.0: Load curve ID, see *DEFINE_CURVE, for nominal stress versus strain for the undamaged material. LT.0: -LCID1 is Table ID, see *DEFINE_TABLE, for nominal stress versus strain for the undamaged material as a function of strain rate
LCID2	Load curve or table ID. The load curve ID, see *DEFINE_CURVE, defines the nominal stress versus strain for the damaged material. The table ID, see *DEFINE_TABLE, defines the nominal stress versus strain for the damaged material as a function of strain rate
HU	Hysteretic unloading factor between 0 and 1 (default = 1, i.e., no energy dissipation), see also <a href="#">Figure M179-1</a> .
BETA	$\beta$ , decay constant to model creep in unloading
DAMP	Viscous coefficient (.05 < recommended value <.50) to model damping effects. LT.0.0:  DAMP  is the load curve ID, which defines the damping constant as a function of the maximum strain in compression defined as: $\epsilon_{\max} = \max(1 - \lambda_1, 1 - \lambda_2, 1 - \lambda_3).$ In tension, the damping constant is set to the value corresponding to the strain at 0. The abscissa should be defined from 0 to 1.
SHAPE	Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor. Values less than one reduces the energy dissipation and greater than one increases dissipation, see also <a href="#">Figure M179-1</a>
FAIL	Failure option after cutoff stress is reached: EQ.0.0: tensile stress remains at cut-off value, EQ.1.0: tensile stress is reset to zero.
BVFLAG	Bulk viscosity activation flag, see remark below: EQ.0.0: no bulk viscosity (recommended), EQ.1.0: bulk viscosity active.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ED	Optional Young's relaxation modulus, $E_d$ , for rate effects. See comments below.
BETA1	Optional decay constant, $\beta_1$ .
KCON	Stiffness coefficient for contact interface stiffness. If undefined the maximum slope in stress vs. strain curve is used. When the maximum slope is taken for the contact, the time step size for this material is reduced for stability. In some cases $\Delta t$ may be significantly smaller, and defining a reasonable stiffness is recommended.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.
TC	Tension cut-off stress
RFLAG	Rate type for input: EQ.0.0: LCID1 and LCID2 should be input as functions of true strain rate EQ.1.0: LCID1 and LCID2 should be input as functions of engineering strain rate.
DTRT	Strain rate averaging flag: EQ.0.0: use weighted running average LT.0.0: average the last 11 values GT.0.0: average over the last DTRT time units.
K	Material failure parameter that controls the volume enclosed by the failure surface. LE.0.0: ignore failure criterion; GT.0.0: use actual K value for failure criterions.
GAMA1	Material failure parameter, see equations below and <a href="#">Figure M181-1</a> .
GAMA2	Material failure parameter, see equations below.





**Figure M179-1.** Loading and reloading curves.

VARIABLE	DESCRIPTION
EH	Damage parameter.

#### Remarks:

This model is based on \*MAT\_LOW\_DENSITY\_FOAM. The uniaxial response is shown below with a large shape factor and small hysteretic factor. If the shape factor is not used, the unloading will occur on the loading curve for the second and subsequent cycles.

The damage is defined as the ratio of the current volume strain to the maximum volume strain, and it is used to interpolate between the responses defined by LCID1 and LCID2.

HU defines a hysteretic scale factor that is applied to the stress interpolated from LCID1 and LCID2,

$$\sigma = \left[ HU + (1 - HU) \times \min \left( 1, \frac{e_{\text{int}}}{e_{\text{int}}^{\text{max}}} \right)^S \right] \sigma(\text{LCID1}, \text{LCID2})$$

where  $e_{\text{int}}$  is the internal energy and  $S$  is the shape factor. Setting HU to 1 results in a scale factor of 1. Setting HU close to zero scales the stress by the ratio of the internal energy to

**\*MAT\_179**

**\*MAT\_LOW\_DENSITY\_SYNTHETIC\_FOAM**

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the maximum internal energy raised to the power  $S$ , resulting in the stress being reduced when the strain is low.

**\*MAT\_SIMPLIFIED\_RUBBER/FOAM\_{OPTION}**

This is Material Type 181. This material model provides a rubber and foam model defined by a single uniaxial load curve or by a family of uniaxial curves at discrete strain rates. The definition of hysteretic unloading is optional and can be realized via a single uniaxial unloading curve or a two-parameter formulation (starting with 971 release R5). The foam formulation is triggered by defining a Poisson’s ratio. This material may be used with both shell and solid elements.

Available options include:

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WITH\_FAILURE

LOG\_LOG\_INTERPOLATION

When the WITH\_FAILURE keyword option is active, a strain based failure surface is defined suitable for incompressible polymers modeling failure in both tension and compression.

This material law has been developed at DaimlerChrysler, Sindelfingen, in collaboration with Paul Du Bois, LSTC, and Prof. Dave J. Benson, UCSD.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	KM	MU	G	SIGF	REF	PRTEN
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LC /TBID	TENSION	RTYPE	AVGOPT	PR/BETA
Type	F	F	F	F	F	F	F	F

Additional card required for WITH\_FAILURE option. Otherwise skip this card.

Card 3	1	2	3	4	5	6	7	8
Variable	K	GAMA1	GAMA2	EH				
Type	F	F	F	F				

**Optional Parameter Card.**

Card 4	1	2	3	4	5	6	7	8
Variable	LCUNLD	HU	SHAPE	STOL	VISCO			
Type	F	F	F	F	F			

**Optional Viscoelastic Constants Cards.** Up to 12 card in format 5 may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 12 cards are used.

Card 5	1	2	3	4	5	6	7	8
Variable	Gi	BETAi	VFLAG					
Type	F	F	I					
Default			0					

**VARIABLE**

**DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
KM	Linear bulk modulus.
MU	Damping coefficient (0.05 < recommended value < 0.50; default is 0.10).

VARIABLE	DESCRIPTION
G	Shear modulus for frequency independent damping. Frequency independent damping is based on a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250-1000 times greater than SIGF.
SIGF	Limit stress for frequency independent, frictional, damping.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.
PRTEN	The tensile Poisson's ratio for shells (optional). If PRTEN is zero, PR/BETA will serve as the Poisson's ratio for both tension and compression in shells. If PRTEN is nonzero, PR/BETA will serve only as the compressive Poisson's ratio for shells.
SGL	Specimen gauge length
SW	Specimen width
ST	Specimen thickness
LC/TBID	Load curve or table ID, see *DEFINE_TABLE, defining the force versus actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), then curve LC is also engineering stress versus engineering strain. If the table definition is used a family of curves are defined for discrete strain rates. The load curves should cover the complete range of expected loading, i.e., the smallest stretch ratio to the largest.
TENSION	Parameter that controls how the rate effects are treated. Applicable to the table definition. EQ.-1.0: rate effects are considered during tension and compression loading, but not during unloading, EQ.0.0: rate effects are considered for compressive loading only, EQ.1.0: rate effects are treated identically in tension and compression.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RTYPE	Strain rate type if a table is defined: EQ.0.0: true strain rate, EQ.1.0: engineering strain rate
AVGOPT	Averaging option determine strain rate to reduce numerical noise. LT.0.0:  AVGOPT  is a time window/interval over which the strain rates are averaged. EQ.0.0: simple average of twelve time steps, EQ.1.0: running average of last 12 averages.
PR/BETA	If the value is specified between 0 and 0.5 exclusive, i.e., $0 < PR < 0.50$ the number defined here is taken as Poisson's ratio. If zero, an incompressible rubber like behavior is assumed and a default value of 0.495 is used internally. If a Poisson's ratio of 0.0 is desired, input a small value for PR such as 0.001. When fully integrated solid elements are used and when a nonzero Poisson's ratio is specified, a foam material is assumed and selective-reduced integration is not used due to the compressibility. This is true even if PR approaches 0.500. If any other value excluding zero is defined, then BETA is taken as the absolute value of the given number and a nearly incompressible rubber like behavior is assumed. An incrementally updated mean viscous stress develops according to the equation: $p^{n+1} = p^n e^{-\beta \Delta t} + K_m \dot{\epsilon}_{kk} \left( \frac{1 - e^{-\beta \Delta t}}{\beta} \right),$ where $\beta =  \text{BETA} $ and $K_m = \text{KM}$ . The BETA parameter does not apply to highly compressible foam materials.
K	Material failure parameter that controls the volume enclosed by the failure surface. LE.0.0: ignore failure criterion; GT.0.0: use actual K value for failure criterions.
GAMA1	Material failure parameter, see equations below and Figure 181.1.
GAMA2	Material failure parameter, see equations below.
EH	Damage parameter.

VARIABLE	DESCRIPTION
LCUNLD	Load curve, see *DEFINE_CURVE, defining the force versus actual length during unloading. The unload curve should cover exactly the same range as LC or the load curves of TBID and its end points should have identical values, i.e., the combination of LC and LCUNLD or the first curve of TBID and LCUNLD describes a complete cycle of loading and unloading. See also material *MAT_083.
HU	Hysteretic unloading factor between 0 and 1 (default = 1., i.e. no energy dissipation), see also material *MAT_083 and <a href="#">Figure M57-1</a> . This option is ignored if LCUNLD is used.
SHAPE	Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor HU. Values less than one reduces the energy dissipation and greater than one increases dissipation, see also material *MAT_083 and <a href="#">Figure M57-1</a> .
STOL	Tolerance in stability check, see remarks.
VISCO	Flag to invoke visco-elastic formulation. The visco-elastic formulation does not apply to shell elements and will be ignored for shells. EQ.0.0: purely elastic; EQ.1.0: visco-elastic formulation (solids only).
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term. The $G_i$ and $BETA_i$ terms are used only for solid elements when VISCO = 1.
$BETA_i$	Optional decay constant if $i^{\text{th}}$ term
VFLAG	Type of viscoelasticity formulation. This appears only on the first line defining $G_i$ , $BETA_i$ , and VFLAG. If VFLAG = 0, the standard viscoelasticity formulation is used (the default), and if VFLAG = 1, the viscoelasticity formulation using the instantaneous elastic stress is used.

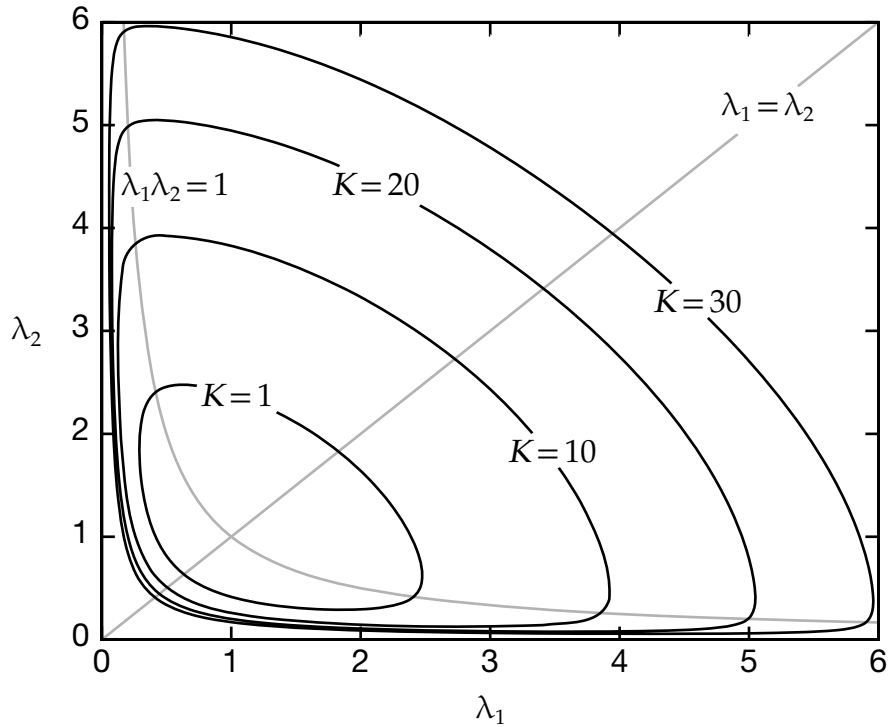
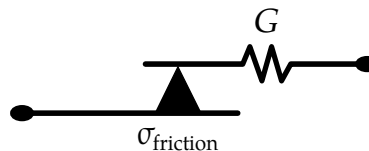


Figure M181-1. Failure surface for polymer for  $\Gamma_1 = 0$  and  $\Gamma_2 = 0.02$ .

**Remarks:**

The frequency independent damping is obtained by the having a spring and slider in series as shown in the following sketch:



The general failure criterion for polymers is proposed by Feng and Hallquist as

$$f(I_1, I_2, I_3) = (I_1 - 3) + \Gamma_1(I_1 - 3)^2 + \Gamma_2(I_2 - 3) = K$$

where  $K$  is a material parameter which controls the size enclosed by the failure surface, and  $I_1, I_2$  and  $I_3$  are the three invariants of right Cauchy-Green deformation tensor ( $\mathbf{C}$ )

$$I_1 = C_{ii} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

$$I_2 = \frac{1}{2}(C_{ii}C_{jj} - C_{ij}C_{ij}) = \lambda_1^2 \lambda_2^2 + \lambda_1^2 \lambda_3^2 + \lambda_2^2 \lambda_3^2$$

$$I_3 = \det(\mathbf{C}) = \lambda_1^2 \lambda_2^2 \lambda_3^2$$

with  $\lambda_i$  are the stretch ratios in three principal directions.

To avoid sudden failure and numerical difficulty, material failure, which is usually a time point, is modeled as a process of damage growth. In this case, the two threshold values are



chosen as  $(1 - h)K$  and  $K$ , where  $h$  (also called EH) is a small number chosen based on experimental results reflecting the range between damage initiation and material failure.

The damage is defined as function of  $f$ :

$$D = \begin{cases} 0 & \text{if } f \leq (1 - h)K \\ \frac{1}{2} \left[ 1 + \cos \frac{\pi(f - K)}{hK} \right] & \text{if } (1 - h)K < f < K \\ 1 & \text{if } f \geq K \end{cases}$$

This definition indicates that damage is first-order continuous. Under this definition, the tangent stiffness matrix will be continuous. The reduced stress considering damage effect is

$$\sigma_{ij} = (1 - D)\sigma_{ij}^o$$

where  $\sigma_{ij}^o$  is the undamaged stress. It is assumed that prior to final failure, material damage is recoverable. Once material failure occurs, damage will become permanent.

The LOG\_LOG\_INTERPOLATION option interpolates the strain rate effect in the table TBID using log-log interpolation.

Bad choices of curves for the stress-strain response may lead to an unstable model, and there is an option to check this to a certain tolerance level, see dimensionless parameter STOL. The check is done by examining the eigenvalues of the tangent modulus at selected stretch points and a warning message is issued if an eigenvalue is less than  $-\text{STOL} \times \text{BULK}$ , where BULK indicates the bulk modulus of the material. For  $\text{STOL} < 0$  the check is disabled, otherwise it should be chosen with care, a too small value may detect instabilities that are insignificant in practice. To avoid significant instabilities it is recommended to use smooth curves, at best the curves should be continuously differentiable, in fact for the incompressible case, a sufficient condition for stability is that the stress-stretch curve  $S(\lambda)$  can be written as

$$S(\lambda) = H(\lambda) - \frac{H\left(\frac{1}{\sqrt{\lambda}}\right)}{\lambda\sqrt{\lambda}}$$

where  $H(\lambda)$  is a function with  $H(1) = 0$  and  $H'(\lambda) > 0$ .

For solid elements, rate effects may also be taken into account through linear viscoelasticity by setting  $\text{VISCO} = 1.0$ . For  $\text{VFLAG} = 0$ , this is done through a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

or in terms of the second Piola-Kirchhoff stress,  $\{S_0\}$ , and Green's strain tensor,  $\{S_{RT}\}$ ,

$$S_{ij} = \int_0^t G_{ijkl}(t - \tau) \frac{\partial E_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

The relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta t}$$

given by,

$$g(t) = \sum_{i=1}^n G_i e^{-\beta_i t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . An arbitrary number of terms may be used.

For VFLAG = 1, the viscoelastic term is

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \sigma_{kl}^E}{\partial \tau} d\tau$$

where  $\sigma_{kl}^E$  is the instantaneous stress evaluated from the internal energy functional. The coefficients in the Prony series therefore correspond to normalized relaxation moduli instead of elastic moduli.

The Mooney-Rivlin rubber model (model 27) is obtained by specifying  $n = 1$ . In spite of the differences in formulations with model 27, we find that the results obtained with this model are nearly identical with those of material 27 as long as large values of Poisson's ratio are used.

**\*MAT\_SIMPLIFIED\_RUBBER\_WITH\_DAMAGE**

An available options includes:

**LOG\_LOG\_INTERPOLATION**

This is Material Type 183. This material model provides an incompressible rubber model defined by a single uniaxial load curve for loading (or a table if rate effects are considered) and a single uniaxial load curve for unloading. This model is similar to \*MAT\_SIMPLIFIED\_RUBBER/FOAM This material may be used with both shell and solid elements.

This material law has been developed at DaimlerChrysler, Sindelfingen, in collaboration with Paul Du Bois, LSTC, and Prof. Dave J. Benson, UCSD.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	MU	G	SIGF		
Type	A8	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LC / TBID	TENSION	RTYPE	AVGOPT	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	LCUNLD	REF	STOL					
Type	F	F	F					

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
K	Linear bulk modulus.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MU	Damping coefficient.
G	Shear modulus for frequency independent damping. Frequency independent damping is based of a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250-1000 times greater than SIGF.
SIGF	Limit stress for frequency independent, frictional, damping.
SGL	Specimen gauge length
SW	Specimen width
ST	Specimen thickness
LC/TBID	Load curve or table ID, see *DEFINE_TABLE, defining the force versus actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), then curve LC is also engineering stress versus engineering strain. If the table definition is used a family of curves are defined for discrete strain rates. The load curves should cover the complete range of expected loading, i.e., the smallest stretch ratio to the largest.
TENSION	Parameter that controls how the rate effects are treated. Applicable to the table definition. EQ.-1.0: rate effects are considered during tension and compression loading, but not during unloading, EQ.0.0: rate effects are considered for compressive loading only, EQ.1.0: rate effects are treated identically in tension and compression.
RTYPE	Strain rate type if a table is defined: EQ.0.0: true strain rate, EQ.1.0: engineering strain rate
AVGOPT	Averaging option determine strain rate to reduce numerical noise. EQ.0.0: simple average of twelve time steps, EQ.1.0: running 12 point average.

VARIABLE	DESCRIPTION
LCUNLD	Load curve, see *DEFINE_CURVE, defining the force versus actual change in the gauge length during unloading. The unload curve should cover exactly the same range as LC (or as the first curve of table TBID) and its end points should have identical values, i.e., the combination of LC (or as the first curve of table TBID) and LCUNLD describes a complete cycle of loading and unloading.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.
STOL	Tolerance in stability check, see <a href="#">remark 2</a> .

**Remarks:**

1. The LOG\_LOG\_INTERPOLATION option interpolates the strain rate effect in the table TBID using log-log interpolation.
2. Bad choice of curves for the stress-strain response may lead to an unstable model, and there is an option to check this to a certain tolerance level, see dimensionless parameter STOL. The check is done by examining the eigenvalues of the tangent modulus at selected stretch points and a warning message is issued if an eigenvalue is less than  $-STOL \times BULK$ , where BULK indicates the bulk modulus of the material. For  $STOL < 0$  the check is disabled, otherwise it should be chosen with care, a too small value may detect instabilities that are insignificant in practice. To avoid significant instabilities it is recommended to use smooth curves, at best the curves should be continuously differentiable, in fact for the incompressible case, a sufficient condition for stability is that the stress-stretch curve  $S(\lambda)$  can be written as

$$S(\lambda) = H(\lambda) - \frac{H(\frac{1}{\sqrt{\lambda}})}{\lambda\sqrt{\lambda}}$$

where  $H(\lambda)$  is a function with  $H(1) = 0$  and  $H'(\lambda) > 0$ .

**\*MAT\_COHESIVE\_ELASTIC**

This is Material Type 184. It is a simple cohesive elastic model for use with cohesive element formulations; see the variable ELFORM in \*SECTION\_SOLID and \*SECTION\_SHELL.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	ET	EN	FN_FAIL	FT_FAIL
Type	A8	F	F	F	F	F	F	F

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
ROFLG	Flag for whether density is specified per unit area or volume. ROFLG = 0 specified density per unit volume (default), and ROFLG = 1 specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero.
INTFAIL	The number of integration points required for the cohesive element to be deleted. If it is zero, the element won't be deleted even if it satisfies the failure criterion. The value of INTFAIL may range from 1 to 4, with 1 the recommended value.
ET	The stiffness in the plane of the cohesive element.
EN	The stiffness normal to the plane of the cohesive element.
FN_FAIL	The traction in the normal direction for tensile failure.
FT_FAIL	The traction in the tangential direction for shear failure.

**Remarks:**

This material cohesive model outputs three tractions having units of force per unit area into the d3plot database rather than the usual six stress components. The in plane shear traction along the 1-2 edge replaces the  $x$ -stress, the orthogonal in plane shear traction replaces the  $y$ -stress, and the traction in the normal direction replaces the  $z$ -stress.

**\*MAT\_COHESIVE\_TH**

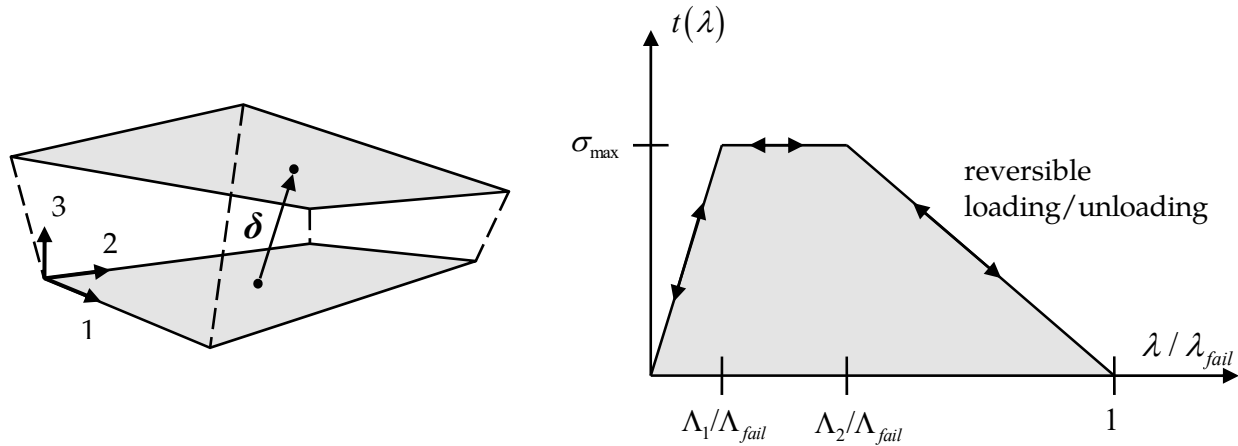
This is Material Type 185. It is a cohesive model by Tvergaard and Hutchinson [1992] for use with cohesive element formulations; see the variable ELFORM in \*SECTION\_SOLID and \*SECTION\_SHELL. The implementation is based on the description of the implementation in the Sandia National Laboratory code, Tahoe [2003].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	SIGMAX	NLS	TLS	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	LAMDA1	LAMDA2	LAMDAF	STFSF				
Type	F	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
ROFLG	Flag for whether density is specified per unit area or volume. ROFLG = 0 specified density per unit volume (default), and ROFLG = 1 specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero.
INTFAIL	The number of integration points required for the cohesive element to be deleted. If it is zero, the element won't be deleted even if it satisfies the failure criterion. The value of INTFAIL may range from 1 to 4, with 1 the recommended value.
SIGMAX	Peak traction.
NLS	Length scale (maximum separation) in the normal direction.
TLS	Length scale (maximum separation) in the tangential direction.
LAMDA1	Scaled distance to peak traction ( $\Lambda_1$ ).



**Figure M185-1.** Relative displacement and trilinear traction-separation law

VARIABLE	DESCRIPTION
LAMDA2	Scaled distance to beginning of softening ( $\Lambda_2$ ).
LAMDAAF	Scaled distance for failure ( $\Lambda_{fail}$ ).
STFSF	Penetration stiffness multiplier. The penetration stiffness, $PS$ , in terms of input parameters becomes:

$$PS = \frac{STFSF \times SIGMAX}{NLS \times \left(\frac{LAMDAA1}{LAMDAAF}\right)}$$

**Remarks:**

In this cohesive material model, a dimensionless separation measure  $\lambda$  is used, which grasps for the interaction between relative displacements in normal ( $\delta_3$  - mode I) and tangential ( $\delta_1, \delta_2$  - mode II) directions (see [Figure M185-1](#) left):

$$\lambda = \sqrt{\left(\frac{\delta_1}{TLS}\right)^2 + \left(\frac{\delta_2}{TLS}\right)^2 + \left(\frac{\langle\delta_3\rangle}{NLS}\right)^2}$$

where the Mc-Cauley bracket is used to distinguish between tension ( $\delta_3 \geq 0$ ) and compression ( $\delta_3 < 0$ ). NLS and TLS are critical values, representing the maximum separations in the interface in normal and tangential direction. For stress calculation, a trilinear traction-separation law is used, which is given by (see [Figure M185-1](#) right):



$$t(\lambda) = \begin{cases} \sigma_{\max} \frac{\lambda}{\Lambda_1/\Lambda_{\text{fail}}} & \lambda < \Lambda_1/\Lambda_{\text{fail}} \\ \sigma_{\max} & \Lambda_1/\Lambda_{\text{fail}} < \lambda < \Lambda_2/\Lambda_{\text{fail}} \\ \sigma_{\max} \frac{1-\lambda}{1-\Lambda_2/\Lambda_{\text{fail}}} & \Lambda_2/\Lambda_{\text{fail}} < \lambda < 1 \end{cases}$$

With these definitions, the traction drops to zero when  $\lambda = 1$ . Then, a potential  $\phi$  is defined as:

$$\phi(\delta_1, \delta_2, \delta_3) = \text{NLS} \times \int_0^\lambda t(\bar{\lambda}) d\bar{\lambda}$$

Finally, tangential components ( $t_1, t_2$ ) and normal component ( $t_3$ ) of the traction acting on the interface in the fracture process zone are given by:

$$t_{1,2} = \frac{\partial \phi}{\partial \delta_{1,2}} = \frac{t(\lambda)}{\lambda} \frac{\delta_{1,2}}{\text{TLS}} \frac{\text{NLS}}{\text{TLS}}, \quad t_3 = \frac{\partial \phi}{\partial \delta_3} = \frac{t(\lambda)}{\lambda} \frac{\delta_3}{\text{NLS}}$$

which in matrix notation is

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \frac{t(\lambda)}{\lambda} \begin{bmatrix} \frac{\text{NLS}}{\text{TLS}^2} & 0 & 0 \\ 0 & \frac{\text{NLS}}{\text{TLS}^2} & 0 \\ 0 & 0 & \frac{1}{\text{NLS}} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

In case of compression ( $\delta_3 < 0$ ), penetration is avoided by:

$$t_3 = \frac{\text{STFSF} \times \sigma_{\max}}{\text{NLS} \times \Lambda_1/\Lambda_{\text{fail}}} \delta_3$$

Loading and unloading follows the same path, i.e. this model is completely reversible.

This cohesive material model outputs three tractions having units of force per unit area into the D3PLOT database rather than the usual six stress components. The in plane shear traction  $t_1$  along the 1-2 edge replaces the x-stress, the orthogonal in plane shear traction  $t_2$  replaces the y-stress, and the traction in the normal direction  $t_3$  replaces the z-stress.

**\*MAT\_COHESIVE\_GENERAL**

This is Material Type 186 and can be used only with cohesive element formulations; see the variable ELFORM in \*SECTION\_SOLID and \*SECTION\_SHELL. The material model includes three general irreversible mixed-mode interaction cohesive formulations with arbitrary normalized traction-separation law given by a load curve (TSLC). These three formulations are differentiated via the type of effective separation parameter (TES). The interaction between fracture modes I and II is considered, and irreversible conditions are enforced via a damage formulation (unloading/reloading path pointing to/from the origin). See remarks for details.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	TES	TSLC	GIC	GIIC
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	XMU	T	S	STFSF	TSLC2			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
ROFLG	Flag for whether density is specified per unit area or volume. ROFLG = 0 specifies density per unit volume (default), and ROFLG = 1 specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero.
INTFAIL	Number of integration points required for a cohesive element to be deleted. If it is zero, the element will not be deleted even if it satisfies failure criterion. The value of INTFAIL may range from 1 to 4, with 1 the recommended value.

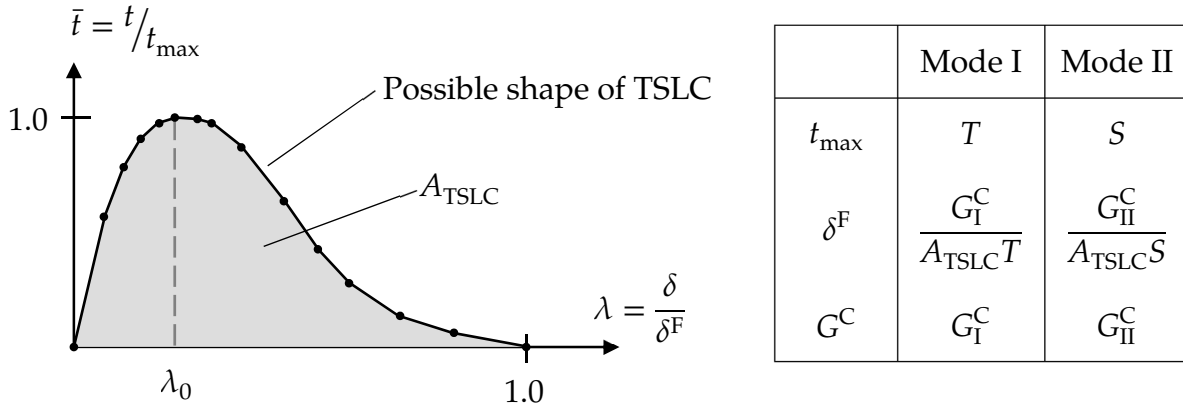


Figure M186-1. Normalized traction-separation law

VARIABLE	DESCRIPTION
TES	<p>Type of effective separation parameter (ESP).</p> <p>EQ.0.0 or 1.0: a dimensional separation measure is used. For the interaction between mode I and II, a mixed-mode propagation criterion is used. For TES = 0.0 this is a power-law, and for TES = 1.0 this is the Benzeggagh-Kenane law [1996]. See remarks below.</p> <p>EQ.2.0: a dimensionless separation measure is used, which grasps for the interaction between mode I displacements and mode II displacements (similar to MAT_185, but with damage and general traction-separation law). See remarks below.</p>
TSLC	<p>Normalized traction-separation load curve ID. The curve must be normalized in both coordinates and must contain at least three points: (0.0,0.0), (<math>\lambda_0</math>, 1.0), and (1.0,0.0), which represents the origin, the peak and the complete failure, respectively (see Figure M186-1). A platform can exist in the curve like the tri-linear TSLC (see MAT_185).</p>
GIC	Fracture toughness / energy release rate $G_I^C$ for mode I
GIIC	Fracture toughness / energy release rate $G_{II}^C$ for mode II
XMU	Exponent that appears in the power failure criterion (TES = 0.0) or the Benzeggagh-Kenane failure criterion (TES = 1.0). Recommended values for XMU are between 1.0 and 2.0.
T	Peak traction in normal direction (mode I)
S	Peak traction in tangential direction (mode II)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
STFSF	Penetration stiffness multiplier for compression. Factor = (1.0 + STFSF) is used to scale the compressive stiffness, i.e. no scaling is done with STFSF = 0.0 (recommended).
TSLC2	Normalized traction-separation load curve ID for Mode II. The curve must be normalized in both coordinates and must contain at least three points: (0.0, 0.0), ( $\lambda_0$ , 1.0), and (1.0, 0.0), which represents the origin, the peak and the complete failure, respectively (see <a href="#">Figure M186-1</a> ). If not specified, TSLC is used for Mode II behavior as well.

**Remarks:**

All three formulations have in common that the traction-separation behavior of this model is mainly given by  $G_I^c$  and T for normal mode I,  $G_{II}^c$  and S for tangential mode II and an arbitrary normalized traction-separation load curve for both modes (see [Figure M186-1](#)). The maximum (or failure) separations are then given by:

$$\delta_I^F = \frac{G_I^c}{A_{TSLC} \times T} \quad , \quad \delta_{II}^F = \frac{G_{II}^c}{A_{TSLC} \times S}$$

where  $A_{TSLC}$  is the area under the normalized traction-separation curve given with TSLC. If TSLC2 is defined

$$\delta_I^F = \frac{G_I^c}{A_{TSLC} \times T} \quad , \quad \delta_{II}^F = \frac{G_{II}^c}{A_{TSLC2} \times S}$$

Where  $A_{TSLC2}$  is the area under the normalized traction-separation curve given with TSLC2.

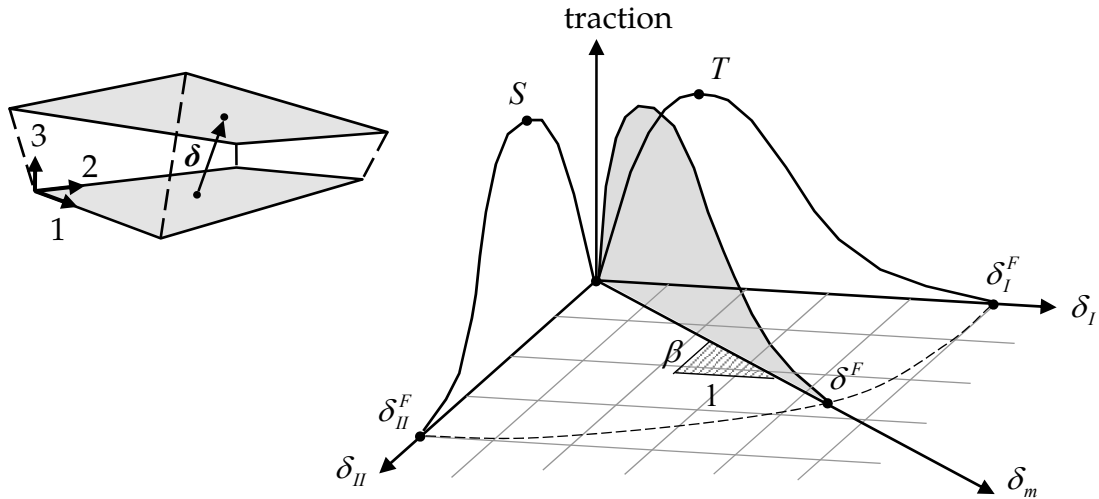


Figure M186-2. Mixed mode traction-separation law

**First and second formulation (TES = 0.0 and TES = 1.0):**

For mixed-mode behavior, three different formulations are possible (where default TES = 0.0 with XMU = 1.0 is recommended as a first try). Here, the total mixed-mode relative displacement  $\delta_m$  is defined as  $\delta_m = \sqrt{\delta_I^2 + \delta_{II}^2}$ , where  $\delta_I = \delta_3$  is the separation in normal direction (mode I) and  $\delta_{II} = \sqrt{\delta_1^2 + \delta_2^2}$  is the separation in tangential direction (mode II). See Figure M186-2. The ultimate mixed-mode displacement  $\delta^F$  (total failure) for the power law (TES = 0.0) is:

$$\delta^F = \frac{1 + \beta^2}{A_{TSLC}} \left[ \left( \frac{T}{G_I^c} \right)^{XMU} + \left( \frac{S \times \beta^2}{G_{II}^c} \right)^{XMU} \right]^{-\frac{1}{XMU}}$$

If TSLC2 is defined this changes to:

$$\delta^F = 1 + \beta^2 \left[ \left( \frac{A_{TSLC} \times T}{G_I^c} \right)^{XMU} + \left( \frac{A_{TSLC2} \times S \times \beta^2}{G_{II}^c} \right)^{XMU} \right]^{-\frac{1}{XMU}}$$

and alternatively for the Benzeggagh-Kenane law [1996] (TES = 1.0):

$$\delta^F = \frac{1 + \beta^2}{A_{TSLC}(T + S \times \beta^2)} \left[ G_I^c + (G_{II}^c - G_I^c) \left( \frac{S \times \beta^2}{T + S \times \beta^2} \right)^{XMU} \right]$$

If TSLC2 is defined this changes to:

$$\delta^F = \frac{1 + \beta^2}{A_{\text{TSLC}} \times T + A_{\text{TSLC2}} \times S \times \beta^2} \left[ G_I^c + (G_{II}^c - G_I^c) \left( \frac{A_{\text{TSLC2}} \times S \times \beta^2}{A_{\text{TSLC}} \times T + A_{\text{TSLC2}} \times S \times \beta^2} \right)^{\text{XMU}} \right]$$

where  $\beta = \delta_{II}/\delta_I$  is the “mode mixity”. The larger the exponent  $\text{XMU}$  is chosen, the larger the fracture toughness in mixed-mode situations will be. In this model, damage of the interface is considered, i.e. irreversible conditions are enforced with loading/unloading paths coming from/pointing to the origin. This formulation is similar to `MAT_COHESIVE_MIXED_MODE` (`MAT_138`), but with the arbitrary traction-separation law `TSLC`.

### Third formulation (TES = 2.0):

Here, a dimensionless effective separation parameter  $\lambda$  is used, which grasps for the interaction between relative displacements in normal ( $\delta_3$  - mode I) and tangential ( $\delta_1, \delta_2$  - mode II) directions:

$$\lambda = \sqrt{\left( \frac{\delta_1}{\delta_{II}^F} \right)^2 + \left( \frac{\delta_2}{\delta_{II}^F} \right)^2 + \left\langle \frac{\delta_3}{\delta_I^F} \right\rangle^2}$$

where the Mc-Cauley bracket is used to distinguish between tension ( $\delta_3 \geq 0$ ) and compression ( $\delta_3 < 0$ ).  $\delta_I^F$  and  $\delta_{II}^F$  are critical values, representing the maximum separations in the interface in normal and tangential direction. For stress calculation, the normalized traction-separation load curve `TSLC` is used:  $t = t_{\text{max}} \times \bar{t}(\lambda)$ . This formulation is similar to `MAT_COHESIVE_TH` (`MAT_185`), but with the arbitrary traction-separation law and a damage formulation (i.e. irreversible conditions are enforced with loading/unloading paths coming from/pointing to the orig

**\*MAT\_SAMP-1**

Purpose: This is Material Type 187 (Semi-Analytical Model for Polymers). This material model uses an isotropic C-1 smooth yield surface for the description of non-reinforced plastics. Details of the implementation are given in [Kolling, Haufe, Feucht and Du Bois 2005].

This material law has been developed at DaimlerChrysler, Sindelfingen, in collaboration with Paul Du Bois and Dynamore, Stuttgart.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	GMOD	EMOD	NUE	RBCFAC	NUMINT
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	LCID-T	LCID-C	LCID-S	LCID-B	NUEP	LCID-P		INCDAM
Type	I	I	I	I	F	I		

Card 3	1	2	3	4	5	6	7	8
Variable	LCID-D	EPFAIL	DEPRPT	LCID_TRI	LCID_LC			
Type	I	F	F	I	I			

Card 4	1	2	3	4	5	6	7	8
Variable	MITER	MIPS		INCFail	ICONV	ASAF		
Type	I	I		I	I	F		

**Optional Card.**

Card 5	1	2	3	4	5	6	7	8
Variable	LCEMOD	BETA	FILT					
Type	F	F	F					

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
BULK	Bulk modulus, used by LS-DYNA in the time step calculation
GMOD	Shear modulus, used by LS-DYNA in the time step calculation
EMOD	Young's modulus
NUE	Poisson ratio



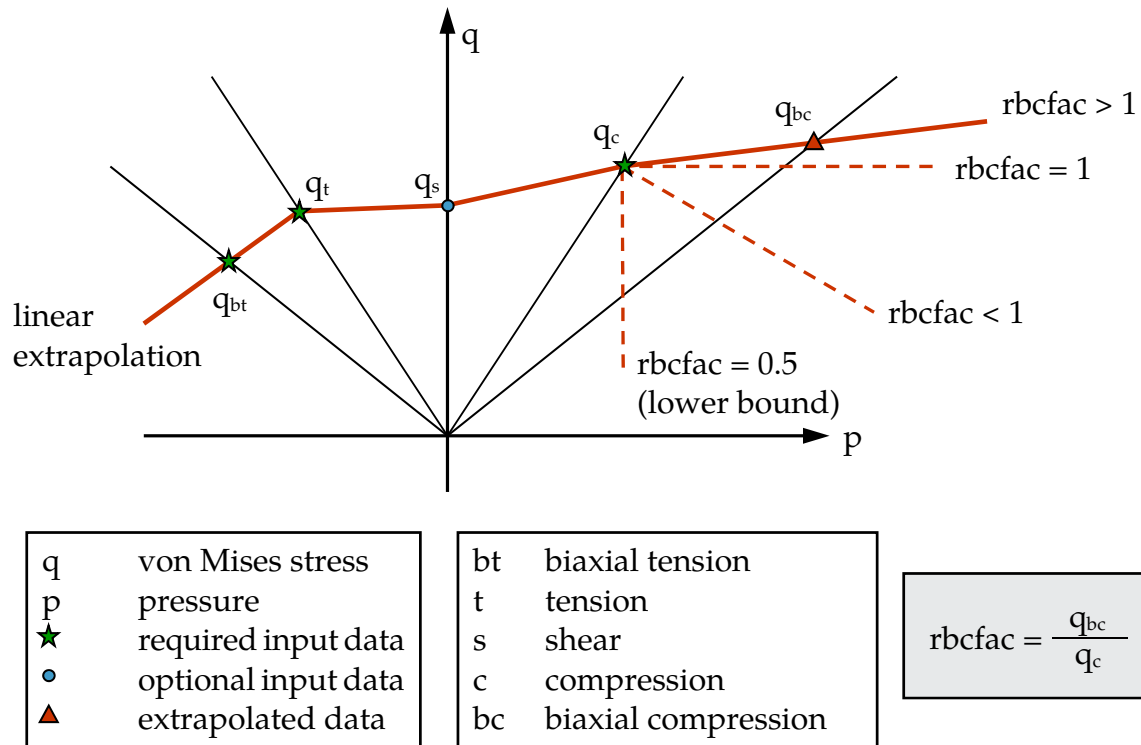
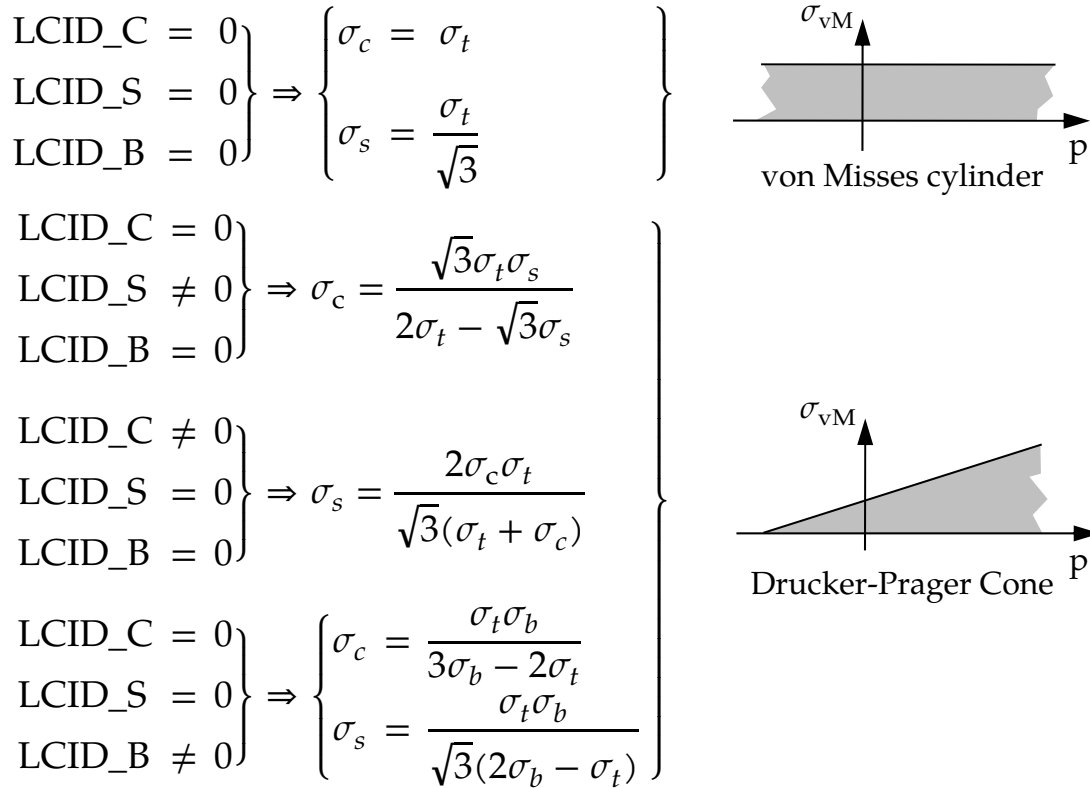


Figure M187-1. von Mises stress as a function of pressure

VARIABLE	DESCRIPTION
RBCFAC	Ratio of yield in biaxial compression vs. yield in uniaxial compression. If RBCFAC is nonzero and all four curves LCID-T, LCID-C, LCID-S, and LCID-B are defined, a piecewise-linear yield surface as shown in Figure M187-1 is activated. See Remark 3. Default is 0.
NUMINT	Number of integration points which must fail before the element is deleted. This option is available for shells and solids.  LT.0.0:  NUMINT  is percentage of integration points/layers which must fail before shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells.

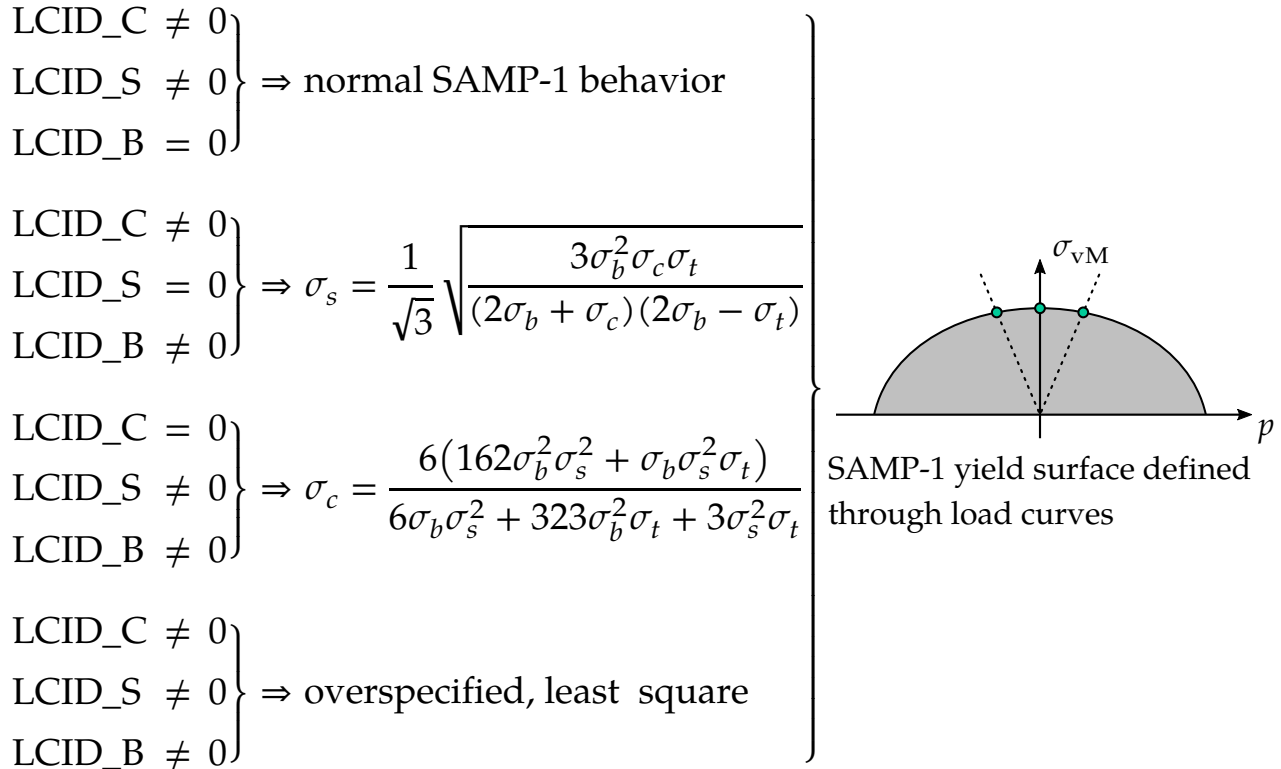
<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCID-T	Load curve or table ID giving the yield stress as a function of plastic strain, these curves should be obtained from quasi-static and (optionally) dynamic uniaxial tensile tests, this input is mandatory and the material model will not work unless at least one tensile stress-strain curve is given. If LCID-T is a table ID, the table values are plastic strain rates, and a curve of yield stress versus plastic strain must be given for each of those strain rates. If the first value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of plastic strain rate. When the highest plastic strain rate is several orders of magnitude greater than the lowest strain rate, it is recommended that the natural log of plastic strain rate be input in the table. See Remark 4.
LCID-C	Load curve ID giving the yield stress as a function of plastic strain, this curve should be obtained from a quasi-static uniaxial compression test, this input is optional.
LCID-S	Load curve ID giving the yield stress as a function of plastic strain, this curve should be obtained from a quasi-static shear test, this input is optional.
LCID-B	Load curve ID giving the yield stress as a function of plastic strain, this curve should be obtained from a quasi-static biaxial tensile test, this input is optional.
NUEP	Plastic Poisson's ratio: an estimated ratio of transversal to longitudinal plastic rate of deformation under uniaxial loading should be given.
LCID-P	Load curve ID giving the plastic Poisson's ratio as a function of plastic strain during uniaxial tensile and uniaxial compressive testing. The plastic strain on the abscissa is negative for compression and positive for tension. It is important to cover both tension and compression. If LCID-P is given, the constant value of plastic Poisson's ratio NUEP is ignored.
INCDAM	Flag to control the damage evolution as a function of triaxiality. EQ.0: damage evolution is independent of the triaxialty. EQ.1: an incremental formulation is used to compute the damage.

VARIABLE	DESCRIPTION
LCID-D	Load curve ID giving the damage parameter as a function of equivalent plastic strain during uniaxial tensile testing. By default this option assumes that effective (i.e. undamaged) yield values are used in the load curves LCID-T, LCID-C, LCID-S and LCID-B. If LCID-D is given a negative value, true (i.e. damaged) yield stress values can be used. In this case an automatic stress-strain recalibration (ASSR) algorithm is activated. The damage value must be defined in the range $0 \leq d < 1$ . If EPFAIL and DEPRPT are given, the curve is used only until the effective plastic strain reaches EPFAIL.
EPFAIL	This parameter is the equivalent plastic strain at failure. If EPFAIL is given as a negative integer, a load curve is expected that defines EPFAIL as a function of the plastic strain rate. Default value is $10^5$ .
DEPRPT	Increment of equivalent plastic strain between failure point and rupture point. Stresses will fade out to zero between EPFAIL and EPFAIL+DEPRPT. If DEPRPT is given a negative value a curve definition is expected where DEPRPT is defined as function of the triaxiality.
LCID_TRI	Load curve that specifies a factor that works multiplicatively on the value of EPFAIL depending on the triaxiality (i.e. pressure/sigma <sub>vm</sub> ). For a triaxiality of -1/3 a value of 1.0 should be specified.
LCID_LC	Load curve that specifies a factor that works multiplicatively on the value of EPFAIL depending on a characteristic element length, defined as the average length of spatial diagonals.
MITER	Maximum number of iterations in the cutting plane algorithm, default is set to 400
MIPS	Maximum number of iterations in the secant iteration performed to enforce plane stress (shell elements only), default set to 10
INCFAIL	Flag to control the failure evolution as a function of triaxiality. EQ.0: Failure evolution is independent of the triaxiality. EQ.1: Incremental formulation is used to compute the failure value. EQ.-1: the failure model is deactivated.



**Figure M187-2.** Fewer than 3 load curves

VARIABLE	DESCRIPTION
ICONV	Formulation flag: EQ.0: default EQ.1: yield surface is internally modified by increasing the shear yield until a convex yield surface is achieved.
ASAF	Safety factor, used only if ICONV = 1, values between 1 and 2 can improve convergence, however the shear yield will be artificially increased if this option is used, default is set to 1.
LCEMOD	Load curve ID defining Young's modulus as function of effective strain rate.
BETA	Decay constant in viscoelastic law: $\dot{\sigma}(t) = -\beta \bullet \sigma(t) + E(\dot{\epsilon}(t)) \bullet \dot{\epsilon}(t)$
FILT	Factor for strain rate filtering: $\dot{\epsilon}_{n+1}^{avg} = (1 - \text{FILT}) \bullet \dot{\epsilon}_{n+1}^{curr} + \text{FILT} \bullet \dot{\epsilon}_n^{avg}$



**Figure M187-3.** Three or more load curves

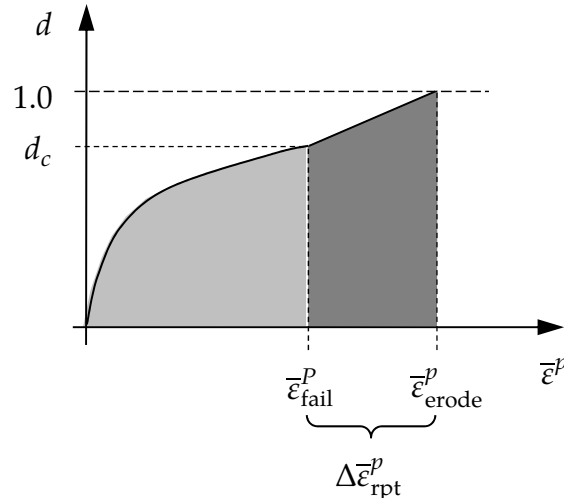
**Load curves:**

Material SAMP-1 uses three yield curves internally to evaluate a quadratic yield surface. \*MAT\_SAMP-1 accepts four different kinds of yield curves, LCID\_T, LCID\_C, LCID\_S, and LCID\_B where data from tension tests (LCID\_T) is always required, but the others are optional. If fewer than three curves are defined, as indicated by setting the missing load curve IDs to 0, the remaining curves are generated internally.

1. **Fewer than 3 load curves.** In the case of fewer than 3 load curves, a linear yield surface in the invariant space spanned by the pressure and the von Mises stress is generated using the available data. See [figure M187-2](#).
2. **Three or more load curves.** See [figure M187-3](#).

**Remarks:**

1. **Damage.** If the LCID\_D is given, then a damage curve as a function of equivalent plastic strains acting on the stresses is defined as shown in [Figure M187-4](#).



**Figure M187-4.** EPFAIL and DEPRPT defined the failure and fading behavior of a single element.

Since the damaging curve acts on the yield values, the inelastic results are effected by the damage curve. As a means to circumvent this, the load curve LCID-D may be given a negative ID. This will lead to an internal conversion of from nominal to effective stresses (ASSR). While this conversion is possible for some combinations of yield curve definitions, plastic Poisson's ratio and damage curves, the corresponding inverse problem cannot be solved for all combinations. An error message is provided in this case.

2. **Unsolvable Yield Surface Case.** Since the generality of arbitrary curve inputs allows unsolvable yield surfaces, SAMP may modify curves internally. This will always lead to warning messages at the beginning of the simulation run. One modification that is not allowed are negative tangents of the last two data points of any of the yield curves.
3. **RBCFAC.** If RBCFAC is nonzero and curves LCID-T, LCID-C, LCID-S, and LCID-B are specified, the yield surface in  $I_1$ - $\sigma_{vm}$  -stress space is constructed such that a piecewise-linear yield surface is activated. This option can help promote convergence of the plasticity algorithm. [Figure M187-1](#) illustrates the effect of RBCFAC on behavior in biaxial compression.
4. **Dynamic Amplification Factor for Yield Stress.** If LCID-T is given as a table specifying strain-rate scaling of the yield stress, then the compressive, shear and biaxial yield stresses are computed by multiplying their respective static values by dynamic amplification factor (dynamic/static ratio) of the tensile yield stress.

5. History Variables.

#	<i>Interpretation</i>
2	plastic strain in tension/compression
3	plastic strain in shear
4	biaxial plastic strain
5	damage
6	volumetric plastic strain
16	plastic strain rate in tension/compression
17	plastic strain rate in shear
18	biaxial plastic strain rate





Optional card 5

Card 5	1	2	3	4	5	6	7	8
Variable	CRPLAW							
Type	F							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus
PR	Poisson's ratio
SIGY	Initial yield stress
ALPHA	Thermal expansion coefficient
LCSS	Load curve ID or Table ID. The load curve ID defines effective stress versus effective plastic strain. The table ID defines for each temperature value a load curve ID giving the stress versus effective plastic strain for that temperature. The stress versus effective plastic strain curve for the lowest value of temperature is used if the temperature falls below the minimum value. Likewise, the stress versus effective plastic strain curve for the highest value of temperature is used if the temperature exceeds the maximum value. Card 2 is ignored with this option.
REFTEM	Reference temperature that defines thermal expansion coefficient
QR1	Isotropic hardening parameter $Q_{r1}$
CR1	Isotropic hardening parameter $C_{r1}$
QR2	Isotropic hardening parameter $Q_{r2}$
CR2	Isotropic hardening parameter $C_{r2}$
QX1	Kinematic hardening parameter $Q_{\chi1}$
CX1	Kinematic hardening parameter $C_{\chi1}$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
QX2	Kinematic hardening parameter $Q_{\chi 2}$
CX2	Kinematic hardening parameter $C_{\chi 2}$
C	Viscous material parameter $C$
P	Viscous material parameter $P$
LCE	Load curve for scaling Young's modulus as a function of temperature
LCPR	Load curve for scaling Poisson's ratio as a function of temperature
LCSIGY	Load curve for scaling initial yield stress as a function of temperature
LCQR	Load curve for scaling the isotropic hardening parameters QR1 and QR2 or the stress given by the load curve LCSS as a function of temperature
LCQX	Load curve for scaling the kinematic hardening parameters QX1 and QX2 as a function of temperature
LCALPH	Load curve for scaling the thermal expansion coefficient as a function of temperature
LCC	Load curve for scaling the viscous material parameter $C$ as a function of temperature
LCP	Load curve for scaling the viscous material parameter $P$ as a function of temperature
LCCR	Load curve for scaling the isotropic hardening parameters CR1 and CR2 as a function of temperature
LCCX	Load curve for scaling the kinematic hardening parameters CX1 and CX2 as a function of temperature
CRPA	Creep law parameter $A$ GT.0.0: Constant value LT.0.0: Load curve ID = (-CRPA) which defines $A$ as a function of temperature, $A(T)$ .

VARIABLE	DESCRIPTION
CRPB	Creep law parameter $B$ GT.0.0: Constant value LT.0.0: Load curve ID = (-CRPB) which defines $B$ as a function of temperature, $B(T)$ .
CRPQ	Creep law parameter $Q = E/R$ where $E$ is the activation energy and $R$ is the universal gas constant. GT.0.0: Constant value LT.0.0: Load curve ID = (-CRPQ) which defines $Q$ as a function of temperature, $Q(T)$ .
CRPM	Creep law parameter $m$ GT.0.0: Constant value LT.0.0: Load curve ID = (-CRPM) which defines $m$ as a function of temperature, $m(T)$ .
CRPLAW	Creep law definition (see Remarks): EQ.0.0: Garafalo's hyperbolic sine law (default). EQ.1.0: Norton's power law.

**Remarks:**

If LCSS is not given any value the uniaxial stress-strain curve has the form

$$\sigma(\epsilon_{\text{eff}}^p) = \sigma_0 + Q_{r1}[1 - \exp(-C_{r1}\epsilon_{\text{eff}}^p)] + Q_{r2}[1 - \exp(-C_{r2}\epsilon_{\text{eff}}^p)] \\ + Q_{\chi1}[1 - \exp(-C_{\chi1}\epsilon_{\text{eff}}^p)] + Q_{\chi2}[1 - \exp(-C_{\chi2}\epsilon_{\text{eff}}^p)].$$

Viscous effects are accounted for using the Cowper-Symonds model, which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\epsilon}_{\text{eff}}^p}{C}\right)^{1/p}.$$

For CRPLAW = 0, the steady-state creep strain rate of Garafalo's hyperbolic sine equation is given by

$$\dot{\epsilon}^c = A[\sinh(B\tau^e)]^m \exp\left(-\frac{Q}{T}\right).$$

For CRPLAW = 1, the steady-state creep strain rate is given by Norton's power law equation:

$$\dot{\epsilon}^c = A(\tau^e)^B t^m.$$

In the above,  $\tau^e$  is the effective elastic stress in the von Mises sense,  $T$  is the temperature and  $t$  is the time. The following is a schematic overview of the resulting stress update. The multiaxial creep strain increment is given by

$$\Delta\epsilon^c = \Delta\epsilon^c \frac{3\tau^e}{2\tau^e}$$

where  $\tau^e$  is the elastic deviatoric stress tensor. Similarly the plastic and thermal strain increments are given by

$$\Delta\epsilon^p = \Delta\epsilon^p \frac{3\tau^e}{2\tau^e}$$

$$\Delta\epsilon^T = \alpha_{t+\Delta t}(T - T_{\text{ref}})\mathbf{I} - \epsilon_t^T$$

where  $\alpha$  is the thermal expansion coefficient (note the definition compared to that of other materials). Adding it all together, the stress update is given by

$$\sigma_{t+\Delta t} = \mathbf{C}_{t+\Delta t}(\epsilon_t^e + \Delta\epsilon - \Delta\epsilon^p - \Delta\epsilon^c - \Delta\epsilon^T)$$

The plasticity is isotropic or kinematic but with a von Mises yield criterion, the subscript in the equation above indicates the simulation time of evaluation. Internally, this stress update requires the solution of a nonlinear equation in the effective stress, the viscoelastic strain increment and potentially the plastic strain increment.

**\*MAT\_ANISOTROPIC\_THERMOELASTIC**

This is Material Type 189. This model characterizes elastic materials whose elastic properties are temperature-dependent.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	TA1	TA2	TA3	TA4	TA5	TA6
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	C11	C12	C13	C14	C15	C16	C22	C23
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C24	C25	C26	C33	C34	C35	C36	C44
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	C45	C46	C55	C56	C66	TGE	TREF	AOPT
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	F	

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	REF
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
TAi	Curve IDs defining the coefficients of thermal expansion for the six components of strain tensor as function of temperature.
CIJ	Curve IDs defining the 6×6 symmetric constitutive matrix in material coordinate system as function of temperature. Note that 1 corresponds to the <i>a</i> material direction, 2 to the <i>b</i> material direction, and 3 to the <i>c</i> material direction.
TGE	Curve ID defining the structural damping coefficient as function of temperature.
TREF	Reference temperature for the calculation of thermal loads or the definition of thermal expansion coefficients.
AOPT	Material axes option, (see MAT_ANISOTROPIC_ELASTIC/MAT_002 for a complete description.) <ul style="list-style-type: none"> <li>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</li> <li>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction.</li> <li>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</li> <li>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <i>v</i> with the element nor-</li> </ul>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	mal. EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector $\mathbf{v}$ , and an originating point, $P$ , which define the centerline axis. LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
XP, YP, ZP	XP, YP, ZP define coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	a1, a2, a3 define components of vector $\mathbf{a}$ for AOPT = 2.
MACF	Material axis change flag for brick elements (see MAT_002 for a complete description.)
D1, D2, D3	d1, d2, d3 define components of vector $\mathbf{d}$ for AOPT = 2.
V1, V2, V3	v1, v2, v3 define components of vector $\mathbf{v}$ for AOPT = 3 and 4.
BETA	Material angle in degrees for AOPT = 3, may be overwritten on the element card, see *ELEMENT_SOLID_ORTHO.
REF	Use initial geometry to initialize the stress tensor (see MAT_002 for a complete description.)

**\*MAT\_190****\*MAT\_FLD\_3-PARAMETER\_BARLAT****\*MAT\_FLD\_3-PARAMETER\_BARLAT**

This is Material Type 190. This model was developed by Barlat and Lian [1989] for modeling sheets with anisotropic materials under plane stress conditions. This material allows the use of the Lankford parameters for the definition of the anisotropy. This particular development is due to Barlat and Lian [1989]. It has been modified to include a failure criterion based on the Forming Limit Diagram. The curve can be input as a load curve, or calculated based on the n-value and sheet thickness.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	HR	P1	P2	ITER
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	M	R00	R45	R90	LCID	E0	SPI	P3
Type	F	F	F	F	I	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	C	P	FLDCID	RN	RT	FLDSAFE	FLDNIPF
Type	F	F	F	I	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		



Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's modulus, $E$
PR	Poisson's ratio, $\nu$
HR	Hardening rule: EQ.1.0: linear (default) EQ.2.0: exponential (Swift) EQ.3.0: load curve EQ.4.0: exponential (Voce) EQ.5.0: exponential (Gosh) EQ.6.0: exponential (Hockett-Sherby)
P1	Material parameter: HR.EQ.1.0: Tangent modulus HR.EQ.2.0: $k$ , strength coefficient for Swift exponential hardening HR.EQ.4.0: $a$ , coefficient for Voce exponential hardening HR.EQ.5.0: $k$ , strength coefficient for Gosh exponential hardening HR.EQ.6.0: $a$ , coefficient for Hockett-Sherby exponential hardening

<b>VARIABLE</b>	<b>DESCRIPTION</b>
P2	Material parameter: HR.EQ.1.0: Yield stress HR.EQ.2.0: $n$ , exponent for Swift exponential hardening HR.EQ.4.0: $c$ , coefficient for Voce exponential hardening HR.EQ.5.0: $n$ , exponent for Gosh exponential hardening HR.EQ.6.0: $c$ , coefficient for Hocket-Sherby exponential hardening
ITER	Iteration flag for speed: ITER.EQ.0.0: fully iterative ITER.EQ.1.0: fixed at three iterations Generally, ITER = 0 is recommended. However, ITER = 1 is somewhat faster and may give acceptable results in most problems.
M	$m$ , exponent in Barlat's yield surface
R00	$R_{00}$ , Lankford parameter determined from experiments
R45	$R_{45}$ , Lankford parameter determined from experiments
R90	$R_{90}$ , Lankford parameter determined from experiments
LCID	load curve ID for the load curve hardening rule
E0	Material parameter HR.EQ.2.0: $\epsilon_0$ for determining initial yield stress for Swift exponential hardening. (Default = 0.0) HR.EQ.4.0: $b$ , coefficient for Voce exponential hardening HR.EQ.5.0: $\epsilon_0$ for determining initial yield stress for Gosh exponential hardening. (Default = 0.0) HR.EQ.6.0: $b$ , coefficient for Hocket-Sherby exponential hardening

VARIABLE	DESCRIPTION
SPI	<p>If <math>\varepsilon_0</math> is zero above and HR.EQ.2.0. (Default = 0.0)</p> <p>EQ.0.0: <math>\varepsilon_0 = (E/k)^{1/(n-1)}</math></p> <p>LE.0.2: <math>\varepsilon_0 = \text{SPI}</math></p> <p>GT.0.2: <math>\varepsilon_0 = (\text{SPI}/k)^{1/n}</math></p>
P3	<p>Material parameter:</p> <p>HR.EQ.5.0: <math>p</math>, parameter for Gosh exponential hardening</p> <p>HR.EQ.6.0: <math>n</math>, exponent for Hocket-Sherby exponential hardening</p>
AOPT	<p>Material axes option (see MAT_OPTION TROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>v</math> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
C	C in Cowper-Symonds strain rate model
P	$p$ in Cowper-Symonds strain rate model, $p = 0.0$ for no strain rate effects

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FLDCID	Load curve ID defining the Forming Limit Diagram. Minor engineering strains in percent are defined as abscissa values and Major engineering strains in percent are defined as ordinate values. The forming limit diagram is shown in <a href="#">Figure M39-1</a> . In defining the curve list pairs of minor and major strains starting with the left most point and ending with the right most point, see *DEFINE_-CURVE.
RN	Hardening exponent equivalent to the n-value in a power law hardening law. If the parameter FLDCID is not defined, this value in combination with the value RT can be used to calculate a forming limit curve to allow for failure.
RT	Sheet thickness used for calculating a forming limit curve. This value does not override the sheet thickness in any way. It is only used in conjunction with the parameter RN to calculate a forming limit curve if the parameter FLDCID is not defined.
FLDSAFE	A safety offset of the forming limit curve. This value should be input as a percentage (ex. 10 not 0.10). This safety margin will be applied to the forming limit curve defined by FLDCID or the curve calculated by RN and RT.
FLDNIPF	Numerical integration points failure treatment.  GT.0.0: The number of element integration points that must fail before the element is deleted. By default, if one integration point has strains above the forming limit curve, the element is flagged for deletion.  LT.0.0: The element is deleted when all integration points within a relative distance of -FLDNIPF from the mid surface have failed (value between -1.0 and 0.0).
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3.
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

**Remarks:**

See material 36 for the theoretical basis.

The forming limit curve can be input directly as a curve by specifying a load curve id with the parameter FLDCID. When defining such a curve, the major and minor strains must be input as percentages.

Alternatively, the parameters RN and RT can be used to calculate a forming limit curve. The use of RN and RT is not recommended for non-ferrous materials. RN and RT are ignored if a non-zero FLDCID is defined.

The first history variable is the maximum strain ratio defined by:

$$\frac{\varepsilon_{\text{major\_workpiece}}}{\varepsilon_{\text{major}_{fd}}}$$

corresponding to  $\varepsilon_{\text{minor\_workpiece}}$ . A value between 0 and 1 indicates that the strains lie below the forming limit curve. Values above 1 indicate that the strains are above the forming limit curve.



This card 3 format is used when FTYPE = 2.

Card 3	1	2	3	4	5	6	7	8
Variable	SIGY	D	W	TF	TW			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

This card 3 format is used when FTYPE = 4.

Card 3	1	2	3	4	5	6	7	8
Variable	PHI_T	PHI_C	PHI_B					
Type	F	F	F					
Default	0.8	0.85	0.9					

This card 3 format is used when FTYPE = 5.

Card 3	1	2	3	4	5	6	7	8
Variable	ALPHA	BETA	GAMMA	DELTA	PHI_T	PHI_C	PHI_B	
Type	F	F	F	F	F	F	F	
Default	none	none	1.4	none	1.0	1.0	1.0	

**FEMA limits Card 1.** Additional card for IFEMA > 0.

Card 4	1	2	3	4	5	6	7	8
Variable	PR1	PR2	PR3	PR4				
Type	F	F	F	F				
Default	0	0	0	0				

**FEMA limits Card 2.** Additional card for IFEMA = 2.

Card 5	1	2	3	4	5	6	7	8
Variable	TS1	TS2	TS3	TS4	CS1	CS2	CS3	CS4
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	TS1	TS2	TS3	TS4

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
ASFLAG	Axial strain definition for force-strain curves, degradation and FEMA output: EQ.0.0: true (log) total strain EQ.1.0: change in length EQ.2.0: nominal total strain EQ.3.0: FEMA plastic strain (= nominal total strain minus elastic strain)



<b>VARIABLE</b>	<b>DESCRIPTION</b>
FTYPE	Formulation type for interaction EQ.1: Parabolic coefficients, axial load and biaxial bending (default). EQ.2: Japanese code, axial force and major axis bending. EQ.4: AISC utilization calculation but no yielding EQ.5: AS4100 utilization calculation but no yielding
DEGRADE	Flag for degrading moment behavior (see Remarks) EQ.0: Behavior as in previous versions EQ.1: Fatigue-type degrading moment-rotation behavior EQ.2: FEMA-type degrading moment-rotation behavior
IFEMA	Flag for input of FEMA thresholds EQ.0: No input EQ.1: Input of rotation thresholds only EQ.2: Input of rotation and axial strain thresholds
LCPMS	Load curve ID giving plastic moment vs. Plastic rotation at node 2 about local s-axis. See *DEFINE_CURVE.
SFS	Scale factor on s-moment at node 2.
LCPMT	Load curve ID giving plastic moment vs. Plastic rotation at node 2 about local t-axis. See *DEFINE_CURVE.
SFT	Scale factor on t-moment at node 2.
LCAT	Load curve ID giving axial tensile yield force vs. total tensile (elastic + plastic) strain or vs. elongation. See AOPT above. All values are positive. See *DEFINE_CURVE.
SFAT	Scale factor on axial tensile force.
LCAC	Load curve ID giving compressive yield force vs. total compressive (elastic + plastic) strain or vs. elongation. See AOPT above. All values are positive. See *DEFINE_CURVE.
SFAC	Scale factor on axial tensile force.
ALPHA	Parameter to define yield surface.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	Parameter to define yield surface.
GAMMA	Parameter to define yield surface.
DELTA	Parameter to define yield surface.
A	Parameter to define yield surface.
B	Parameter to define yield surface.
FOFFS	Force offset for yield surface (see Remarks).
SIGY	Yield stress of material.
D	Depth of section used to calculate interaction curve.
W	Width of section used to calculate interaction curve.
TF	Flange thickness of section used to calculate interaction curve.
TW	Web thickness used to calculate interaction curve.
PHI_T	Factor on tensile capacity, $\phi_t$
PHI_C	Factor on compression capacity, $\phi_c$
PHI_B	Factor on bending capacity, $\phi_b$
PR1 - PR4	Plastic rotation thresholds 1 to 4
TS1 - TS4	Tensile axial strain thresholds 1 to 4
CS1 - CS4	Compressive axial strain thresholds 1 to 4

**Remarks:**

Yield surface for formulation type 1 is of the form:

$$\psi = \left( \frac{M_s}{M_{ys}} \right)^\alpha + \left( \frac{M_t}{M_{yt}} \right)^\beta + A \left( \frac{F}{F_y} \right)^\gamma + B \left( \frac{F}{F_y} \right)^\delta - 1$$

Where,

$M_s, M_t$  = moments about local s and t axes

$M_{ys}, M_{yt}$  = current yield moments

$F$  = axial force

$F_y$  = Yield force; LCAC in compression or LCAT in tension

$\alpha, \beta, \gamma, \delta$  = Input parameters; must be greater than or equal to 1.1

$A, B$  = input parameters

If  $\alpha, \beta, \gamma, \delta, A$  and  $B$  are all set to zero then the following default values are used:

ALPHA	=	2.0
BETA	=	2.0
GAMMA	=	2.0
DELTA	=	4.0
A	=	2.0
B	=	-1.0

FOFFS offsets the yield surface parallel to the axial force axis. It is the compressive axial force at which the maximum bending moment capacity about the local s-axis (determined by LCPMS and SFS), and that about the local t-axis (determined by LCPMT and SFT), occur. For steel beams and columns, the value of FOFFS is usually zero. For reinforced concrete beams, columns and shear walls, the maximum bending moment capacity occurs corresponding to a certain compressive axial force, FOFFS. The value of FOFFS can be input as either positive or negative. Internally, LS-DYNA converts FOFFS to, and regards compressive axial force as, negative.

Interaction surface FTYPE 4 calculates a utilisation parameter using the yield force and moment data given on card 2, but the elements remain elastic even when the forces or moments exceed yield values. This is done for consistency with the design code OBE AISC LRFD (2000). The utilisation calculation is as follows:

$$\text{Utilisation} = \frac{K_1 F}{\phi F_y} + \frac{K_2}{\phi_b} \left( \frac{M_s}{M_{ys}} + \frac{M_t}{M_{yt}} \right)$$

where,

$M_s, M_t, M_{ys}, M_{yt}, F_y$  are defined as in the preceding equation

and,

$\phi$  = from PHI\_T under tension; PHI\_C under compression

$\phi_b$  = take from PHI\_B

$$K_1 = \begin{cases} 0.5 & \frac{F}{\phi F_y} < 0.2 \\ 1.0 & \frac{F}{\phi F_y} \geq 0.2 \end{cases}$$

$$K_2 = \begin{cases} 1.0 & \frac{F}{\phi F_y} < 0.2 \\ 8/9 & \frac{F}{\phi F_y} \geq 0.2 \end{cases}$$

Interaction surface FTYPE 5 is similar to type 4 (calculates a utilisation parameter using the yield data, but the elements do not yield). The equations are taken from Australian code AS4100. The user must select appropriate values of  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  using the various clauses of Section 8 of AS4100. It is assumed that the local s-axis is the major axis for bending.

$$\text{Utilisation} = \max(U_1, U_2, U_3, U_4, U_5)$$

$$U_1 = \frac{F}{\beta \phi_c F_{yc}} \quad \text{used for members in compression}$$

$$U_2 = \frac{F}{\phi_t F_{yt}} \quad \text{used for members in tension}$$

$$U_3 = \left[ \frac{M_s}{K_2 \phi_b M_{ys}} \right]^\gamma + \left[ \frac{M_t}{K_1 \phi_b M_{yt}} \right]^\gamma \quad \text{used for members in compression}$$

$$U_4 = \left[ \frac{M_s}{K_4 \phi_b M_{ys}} \right]^\gamma + \left[ \frac{M_t}{K_3 \phi_b M_{yt}} \right]^\gamma \quad \text{used for members in tension}$$

$$U_5 = \frac{F}{\phi_c F_{yc}} + \frac{M_s}{\phi_b M_{ys}} + \frac{M_t}{\phi_b M_{yt}} \quad \text{used for all members}$$

where,

$M_s, M_t, F, M_{ys}, M_{yt}, F_{yt}, F_{yc}$  are as defined above

and,

$$K_1 = 1.0 - \frac{F}{\beta \phi_c F_{yc}}$$

$$K_2 = \min \left[ K_1, \alpha \left( 1.0 - \frac{F}{\delta \phi_c F_{yc}} \right) \right]$$

$$K_3 = 1.0 - \frac{F}{\phi_t F_{yt}}$$

$$K_4 = \min \left[ K_3, \alpha \left( 1.0 + \frac{F}{\phi_t F_{yt}} \right) \right]$$

where

$K_1, K_2, K_3, K_4$  are subject to a minimum value of  $10^{-6}$ ,

$\alpha, \beta, \gamma, \delta, \phi_t, \phi_c, \phi_b$  are input parameters

The option for degrading moment behavior changes the meaning of the plastic moment-rotation curve as follows:

If  $DEGRAD = 0$  (not recommended), the x-axis points on the curve represent current plastic rotation (i.e. total rotation minus the elastic component of rotation). This quantity can be positive or negative depending on the direction of rotation; during hysteresis the behavior will repeatedly follow backwards and forwards along the same curve. The curve should include negative and positive rotation and moment values. This option is retained so that results from existing models will be unchanged.

If  $DEGRAD = 1$ , the x-axis points represent cumulative absolute plastic rotation. This quantity is always positive, and increases whenever there is plastic rotation in either direction. Thus, during hysteresis, the yield moments are taken from points in the input curve with increasingly positive rotation. If the curve shows a degrading behavior (reducing moment with rotation), then, once degraded by plastic rotation, the yield moment can never recover to its initial value. This option can be thought of as having “fatigue-type” hysteretic damage behavior, where all plastic cycles contribute to the total damage.

If  $DEGRAD = 2$ , the x-axis points represent the high-tide value (always positive) of the plastic rotation. This quantity increases only when the absolute value of plastic rotation exceeds the previously recorded maximum. If smaller cycles follow a larger cycle, the plastic moment during the small cycles will be constant, since the high-tide plastic rotation is not altered by the small cycles. Degrading moment-rotation behavior is possible. This option can be thought of as showing rotation-controlled damage, and follows the FEMA approach for treating fracturing joints.

$DEGRAD$  applies also to the axial behavior. The same options are available as for rotation:  $DEGRAD = 0$  gives unchanged behavior from previous versions;  $DEGRAD = 1$  gives a fatigue-type behavior using cumulative plastic strain; and  $DEGRAD = 2$  gives FEMA-type behavior, where the axial load capacity depends on the high-tide tensile and compressive strains. The definition of strain for this purpose is according to AOPT on Card 1 – it is expected that  $AOPT = 2$  will be used with  $DEGRAD = 2$ . The “axial strain” variable plotted by post-processors is the variable defined by AOPT.

The output variables plotted as “plastic rotation” have special meanings for this material model as follows – note that hinges form only at Node 2:

“Plastic rotation at End 1” is really a high-tide mark of absolute plastic rotation at Node 2, defined as follows:

1. Current plastic rotation is the total rotation minus the elastic component of rotation.
2. Take the absolute value of the current plastic rotation, and record the maximum achieved up to the current time. This is the high-tide mark of plastic rotation.

If  $DEGRAD = 0$ , “Plastic rotation at End 2” is the current plastic rotation at Node 2.

If DEGRAD = 1 or 2, "Plastic rotation at End 2" is the current total rotation at Node 2.

The total rotation is a more intuitively understood parameter, e.g. for plotting hysteresis loops. However, with DEGRAD = 0, the previous meaning of that output variable has been retained such that results from existing models are unchanged.

FEMA thresholds are the plastic rotations at which the element is deemed to have passed from one category to the next, e.g. "Elastic", "Immediate Occupancy", "Life Safe", etc. The high-tide plastic rotation (maximum of Y and Z) is checked against the user-defined limits FEMA1, FEMA2, etc. The output flag is then set to 0, 1, 2, 3, or 4: 0 means that the rotation is less than FEMA1; 1 means that the rotation is between FEMA1 and FEMA2, and so on. By contouring this flag, it is possible to see quickly which joints have passed critical thresholds.

For this material model, special output parameters are written to the d3plot and d3thdt files. The number of output parameters for beam elements is automatically increased to 20 (in addition to the six standard resultants) when parts of this material type are present. Some post-processors may interpret this data as if the elements were integrated beams with 4 integration points. Depending on the post-processor used, the data may be accessed as follows:

Extra variable 16 (or Integration point 4 Axial Stress):	FEMA rotation flag
Extra variable 17 (or Integration point 4 XY Shear Stress):	Current utilization
Extra variable 18 (or Integration point 4 ZX Shear Stress):	Maximum utilization to date
Extra variable 20 (or Integration point 4 Axial Strain):	FEMA axial flag

"Utilization" is the yield parameter, where 1.0 is on the yield surface.

\*MAT\_SOIL\_BRICK

Purpose: This is Material Type 192. It is intended for modeling over-consolidated clay.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	RLAMDA	RKAPPA	RIOTA	RBETA1	RBETA2	RMU
Type	A8	F	F	F	F	F	F	F
Default								1.0

Card 2	1	2	3	4	5	6	7	8
Variable	RNU	RLCID	TOL	PGCL	SUB-INC	BLK	GRAV	THEORY
Type	F	F	F	F	F	F	F	I
Default			0.0005				9.807	0

Additional card for THEORY > 0.

Card 3	1	2	3	4	5	6	7	8
Variable	RVHHH	XSICRIT	ALPHA	RVH	RNU21	ANISO_4		
Type	F	F	F	F	F	F		
Default	0	0	0	0	0	0		

**VARIABLE**

**DESCRIPTION**

- MID Material identification. A unique number or label not exceeding 8 characters must be specified.
- RO Mass density
- RLAMDA Material coefficient

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RKAPPA	Material coefficient
RIOTA	Material coefficient
RBETA1	Material coefficient
RBETA2	Material coefficient
RMU	Shape factor coefficient. This parameter will modify the shape of the yield surface used. 1.0 implies a von Mises type surface, but 1.1 to 1.25 is more indicative of soils. The default value is 1.0.
RNU	Poisson's ratio
RLCID	Load curve identification number referring to a curve defining up to 10 pairs of 'string-length' vs G/Gmax points.
TOL	User defined tolerance for convergence checking. Default value is set to 0.02.
PGCL	Pre-consolidation ground level. This parameter defines the maximum surface level (relative to $z = 0.0$ in the model) of the soil throughout geological history. This is used calculate the maximum over burden pressure on the soil elements.
SUB-INC	User defined strain increment size. This is the maximum strain increment that the material model can normally cope with. If the value is exceeded a warning is echoed to the d3hsp file.
BLK	The elastic bulk stiffness of the soil. This is used for the contact stiffness only.
GRAV	The gravitational acceleration. This is used to calculate the element stresses due the overlying soil. Default is set to $9.807 \text{ m/s}^2$ .
THEORY	Version of material subroutines used (See Remarks). EQ.0: 1995 version, vectorized (Default) EQ.4: 2003 version, unvectorized
RVHHH	Anisotropy ratio $G_{vh} / G_{hh}$ (default = Isotropic behavior)
XSICRIT	Anisotropy parameter
ALPHA	Anisotropy parameter



VARIABLE	DESCRIPTION
RVH	Anisotropy ratio $E_v / E_h$
RNU21	Anisotropy ratio $\nu_2 / \nu_1$
ANISO_4	Anisotropy parameter

**Remarks:**

1. This material type requires that the model is oriented such that the z-axis is defined in the upward direction. Compressive initial stress must be defined, e.g. using \*INITIAL\_STRESS\_SOLID or \*INITIAL\_STRESS\_DEPTH.

The recommended unit system is kN, meters, seconds, tonnes. There are some built-in defaults that assume stress units of KN/m<sup>2</sup>.

Over-consolidated clays have suffered previous loading to higher stress levels than are present at the start of the analysis. This could have occurred due to ice sheets during previous ice ages, or the presence of soil or rock that has subsequently been eroded. The maximum vertical stress during that time is assumed to be:

$$\sigma_{VMAX} = RO \times GRAV \times (PGCL - Z_{el})$$

where

RO, GRAV, and PGCL = input parameters

$Z_{el}$  = z coordinate of center of element

Since that time, the material has been unloaded until the vertical stress equals the user-defined initial vertical stress. The previous load/unload history has a significant effect on subsequent behavior, e.g. the horizontal stress in an over-consolidated clay may be greater than the vertical stress.

This material model creates a load/unload cycle for a sample element of each material of this type, stores in a scratch file the horizontal stress and history variables as a function of the vertical stress, and interpolates these quantities from the defined initial vertical stress for each element. Therefore the initial horizontal stress seen in the output files will be different from the input initial horizontal stress.

This material model is developed for a Geotechnical FE program (Oasys Ltd.'s SAFE) written by Arup. The default THEORY = 0 gives a vectorized version ported from SAFE in the 1990's. Since then the material model has been developed further in SAFE; the most recent porting is accessed using THEORY = 4 (recommended); however, this version is not vectorized and will run more slowly on most computer platforms.

2. The shape factor for a typical soil would be 1.25. Do not use values higher than 1.35.

**\*MAT\_DRUCKER\_PRAGER**

Purpose: This is Material Type 193. This material enables soil to be modeled effectively. The parameters used to define the yield surface are familiar geotechnical parameters (i.e. angle of friction). The modified Drucker-Prager yield surface is used in this material model enabling the shape of the surface to be distorted into a more realistic definition for soils.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	GMOD	RNU	RKF	PHI	CVAL	PSI
Type	A8	F	F	F	F	F	F	F
Default					1.0			0.0

Card 2	1	2	3	4	5	6	7	8
Variable	STR_LIM							
Type	F							
Default	0.005							

Card 3	1	2	3	4	5	6	7	8
Variable	GMODDP	PHIDP	CVALDP	PSIDP	GMODGR	PHIGR	CVALGR	PSIGR
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**

MID Material identification. A unique number or label not exceeding 8 characters must be specified.

RO Mass density

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GMOD	Elastic shear modulus
RNU	Poisson's ratio
RKF	Failure surface shape parameter
PHI	Angle of friction (radians)
CVAL	Cohesion value
PSI	Dilation angle (radians)
STR_LIM	Minimum shear strength of material is given by STR_LIM*CVAL
GMODDP	Depth at which shear modulus (GMOD) is correct
PHIDP	Depth at which angle of friction (PHI) is correct
CVALDP	Depth at which cohesion value (CVAL) is correct
PSIDP	Depth at which dilation angle (PSI) is correct
GMODGR	Gradient at which shear modulus (GMOD) increases with depth
PHIGR	Gradient at which friction angle (PHI) increases with depth
CVALGR	Gradient at which cohesion value (CVAL) increases with depth
PSIGR	Gradient at which dilation angle (PSI) increases with depth

**Remarks:**

1. This material type requires that the model is oriented such that the z-axis is defined in the upward direction. The key parameters are defined such that may vary with depth (i.e. the z-axis).
2. The shape factor for a typical soil would be 0.8, but should not be pushed further than 0.75.
3. If STR\_LIM is set to less than 0.005, the value is reset to 0.005.
4. The yield function is defined as:

$$t - p.\tan\beta - d = 0$$

where:

$p = \text{hydrostatic stress} = J_1/3$

$t = (q/2)\{a - b(r/q)^3\}$

$q = \text{Von Mises stress} = \sqrt{3J_2}$

$a = 1 + 1/K$

$b = 1 - 1/K$

$K = \text{input parameter RKF}$

$r = (27 J_3/2)^{1/3}$

$J_2, J_3 = \text{second and third deviatoric stress invariants}$

$\tan\beta = 6 \sin\varphi / (3 - \sin\varphi)$

$d = 6 C \cos\varphi / (3 - \sin\varphi)$

$\varphi = \text{input parameter PHI}$

$C = \text{input parameter CVAL}$



Card 3	1	2	3	4	5	6	7	8
Variable	A	B	C	D	E	F		
Type	F	F	F	F	F	F		
Default	0.05	0.55	0.125	0.66	0.25	1.0		

Card 4	1	2	3	4	5	6	7	8
Variable	Y1	Y2	Y3	Y4	Y5			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	0.0	0.0			

Card 5	1	2	3	4	5	6	7	8
Variable	T1	T2	T3	T4	T5			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	0.0	0.0			

Card 6	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							
Default	0.0							

**\*MAT\_194****\*MAT\_RC\_SHEAR\_WALL**

Card 7	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

Card 8	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
E	Young's Modulus
PR	Poisson's Ratio
TMAX	Ultimate in-plane shear stress. If set to zero, LS-DYNA will calculate TMAX based on the formulae in the Uniform Building Code, using the data on card 2. See Remarks.
FC	Unconfined Compressive Strength of concrete (used in the calculation of ultimate shear stress; crushing behavior is not modeled)
PREF	Percent reinforcement, e.g. if 1.2% reinforcement, enter 1.2
FYIELD	Yield stress of reinforcement
SIG0	Overburden stress (in-plane compressive stress) - used in the calculation of ultimate shear stress. Usually sig0 is left as zero.



VARIABLE	DESCRIPTION
UCONV	<p>Unit conversion factor. UCONV is expected to be set such that,</p> $UCONV = \sqrt{1.0 \text{ PSI in the model's stress units.}}$ <p>This used to convert the ultimate tensile stress of concrete which is expressed as <math>\sqrt{FC}</math> where FC is given in PSI. Therefore a unit conversion factor of <math>\sqrt{\text{PSI}/\text{Stress Unit}}</math> is required. Examples:</p> $UCONV = 83.3 = \sqrt{6894} \text{ if stress unit is N/m}^2$ $UCONV = 0.083 \text{ if stress unit is MN/m}^2 \text{ or N/mm}^2$
ALPHA	Shear span factor - see below.
FT	Cracking stress in direct tension - see notes below. Default is 8% of the cylinder strength.
ERIENF	Young's Modulus of reinforcement. Used in calculation of post-cracked stiffness - see notes below.
A	Hysteresis constants determining the shape of the hysteresis loops.
B	Hysteresis constants determining the shape of the hysteresis loops.
C	Hysteresis constants determining the shape of the hysteresis loops.
D	Hysteresis constants determining the shape of the hysteresis loops.
E	Hysteresis constants determining the shape of the hysteresis loops.
F	Strength degradation factor. After the ultimate shear stress has been achieved, F multiplies the maximum shear stress from the curve for subsequent reloading. F = 1.0 implies no strength degradation (default). F = 0.5 implies that the strength is halved for subsequent reloading.
Y1, Y2, ..., Y5	Engineering shear strain points on stress-strain curve. By default these are calculated from the values on card 1. See below for more guidance.
T1, T2, ..., T5	Shear stress points on stress-strain curve. By default these are calculated from the values on card 1. See below for more guidance.
AOPT	<p>Material axes option:</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>, and then rotated</p>

VARIABLE	DESCRIPTION
	<p>about the shell element normal by the angle BETA. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: applicable to shell elements only. This option determines locally orthotropic material axes by offsetting the material axes by an angle to be specified from a line in the plane of the shell determined by taking the cross product of the vector <math>\mathbf{v}</math> defined below with the shell normal vector.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

**Remarks:**

The element is linear elastic except for in-plane shear and tensile cracking effects. Crushing due to direct compressive stresses are modeled only insofar as there is an in-plane shear stress component. It is not recommended that this model be used where nonlinear response to direct compressive or loads is important.

Note that the in-plane shear stress  $t_{xy}$  is defined as the shear stress in the element's local  $x$ - $y$  plane. This is not necessarily equal to the maximum shear stress in the plane: for example, if the principal stresses are at 45 degrees to the local axes,  $t_{xy}$  is zero. Therefore it is important to ensure that the local axes are appropriate - for a shear wall the local axes should be vertical or horizontal. By default, local  $x$  points from node 1 to node 2 of the element. It is possible to change the local axes by using AOPT > 0.

If TMAX is set to zero, the ultimate shear stress is calculated using a formula in the Uniform Building Code 1997, section 1921.6.5:

$$TMAX_{UBC} = UCONV \times ALPHA \times \sqrt{FC} + RO \times FY$$

where,

UCONV = unit conversion factor, see varriable list

ALPHA = aspect ratio

= 2.0 for  $h/l \in (2.0, \infty)$  increases linearly to 3.0 for  $h/l \in (2.0, 1.5)$

FC = unconfined compressive strength of concrete

RO = fraction of reinforcement

= (percent reinforcement)/100

FY = yield stress of reinforcement

To this we add shear stress due to the overburden to obtain the ultimate shear stress:

$$TMAX_{UBC} = TMAX_{UBC} + SIG0$$

where

SIG0 = in plane compressive stress under static equilibrium conditions

The UBC formula for ultimate shear stress is generally conservative (predicts that the wall is weaker than shown in test), sometimes by 50% or more. A less conservative formula is that of Fukuzawa:

$$TMAX = \max \left[ \left( 0.4 + \frac{A_c}{A_w} \right), 1 \right] \times 2.7 \times \left( 1.9 + \frac{M}{L_v} \right) \times UCONV + \sqrt{FC} + 0.5 \\ \times RO \times FY + SIG0$$

where

$A_c$  = Cross-sectional area of stiffening features such as columns or flanges

$A_w$  = Cross-sectional area of wall

$M/L_v$  = Aspect ratio of wall height/length

Other terms are as above. This formula is not included in the material model: TMAX should be calculated by hand and entered on Card 1 if the Fukuzawa formula is required.

It should be noted that none of the available formulae, including Fukuzawa, predict the ultimate shear stress accurately for all situations. Variance from the experimental results can be as great as 50%.

The shear stress vs shear strain curve is then constructed automatically as follows, using the algorithm of Fukuzawa extended by Arup:

1. Assume ultimate engineering shear strain,  $\gamma_u = 0.0048$
2. First point on curve, corresponding to concrete cracking, is at

$$\left(0.3 \times \frac{TMAX}{G}, 0.3 \times TMAX\right),$$

where  $G$  is the elastic shear modulus given by

$$G = \frac{E}{2(1 + \nu)}.$$

3. Second point, corresponding to the reinforcement yield, is at  
 $(0.5 \times \gamma_u, 0.8 \times TMAX)$ .
4. Third point, corresponding to the ultimate strength, is at  
 $(\gamma_u, TMAX)$ .
5. Fourth point, corresponding to the onset of strength reduction, is at  
 $(2\gamma_u, TMAX)$ .
6. Fifth point, corresponding to failure is at  
 $(3\gamma_u, 0.6 \times TMAX)$ .

After failure, the shear stress drops to zero. The curve points can be entered by the user if desired, in which case they over-ride the automatically calculated curve. However, it is anticipated that in most cases the default curve will be preferred due to ease of input.

Hysteresis follows the algorithm of Shiga as for the squat shear wall spring (see \*MAT\_SPRING\_SQUAT\_SHEARWALL). The hysteresis constants which are defined in fields A, B, C, D, and E can be entered by the user if desired, but it is generally recommended that the default values be used.

Cracking in tension is checked for the local  $x$  and  $y$  directions only – this is calculated separately from the in-plane shear. A trilinear response is assumed, with turning points at concrete cracking and reinforcement yielding. The three regimes are:

1. Pre-cracking, linear elastic response is assumed using the overall Young's Modulus on Card 1.
2. **Cracking occurs in the local  $x$  or  $y$  directions when the tensile stress in that direction exceeds the concrete tensile strength  $F_T$  (if not input on Card 2, this defaults to 8% of the compressive strength  $F_C$ ).** Post-cracking, a linear stress-strain response is assumed up to reinforcement yield at a strain defined by reinforcement yield stress divided by reinforcement Young's Modulus.
3. Post-yield, a constant stress is assumed (no work hardening).

Unloading returns to the origin of the stress-strain curve.

For compressive strains the response is always linear elastic using the overall Young's Modulus on Card 1.

If insufficient data is entered, no cracking occurs in the model. As a minimum, FC and FY are needed.

Extra variables are available for post-processing as follows:

Extra variable 1: Current engineering shear strain

Extra variable 2: Shear status: 0, 1, 2, 3, 4, or 5– see below

Extra variable 3: Maximum direct strain so far in local  $x$  direction (for tensile cracking)

Extra variable 4: Maximum direct strain so far in local  $y$  direction (for tensile cracking)

Extra variable 5: Tensile status: 0, 1 or 2 = elastic, cracked, or yielded respectively.

The shear status shows how far along the shear stress-strain curve each element has progressed, e.g. status 2 means that the element has passed the second point on the curve. These status levels correspond to performance criteria in building design codes such as FEMA.

**\*MAT\_195****\*MAT\_CONCRETE\_BEAM****\*MAT\_CONCRETE\_BEAM**

This is Material Type 195 for beam elements. An elasto-plastic material with an arbitrary stress versus strain curve and arbitrary strain rate dependency can be defined. See also Remark below. Also, failure based on a plastic strain or a minimum time step size can be defined.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ETAN	FAIL	TDEL
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	10.E+20	10.E+20

Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR				
Type	F	F	F	F				
Default	0	0	0	0				

Card 3	1	2	3	4	5	6	7	8
Variable	NOTEN	TENCUT	SDR					
Type	I	F	F					
Default	0	E15.0	0.0					

**VARIABLE****DESCRIPTION**

MID Material identification. A unique number or label not exceeding 8 characters must be specified.

RO Mass density.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
E	Young's modulus.
PR	Poisson's ratio.
SIGY	Yield stress.
ETAN	Tangent modulus, ignored if (LCSS.GT.0) is defined.
FAIL	Failure flag. LT.0.0: user defined failure subroutine is called to determine failure EQ.0.0: failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic element deletion.
C	Strain rate parameter, C, see formula below.
P	Strain rate parameter, P, see formula below.
LCSS	Load curve ID or Table ID. Load curve ID defining effective stress versus effective plastic strain. If defined EPS1-EPS8 and ES1-ES8 are ignored. The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See <a href="#">Figure M16-1</a> stress versus effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress versus effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters: C and P;
LCSR	Load curve ID defining strain rate scaling effect on yield stress.
NOTEN	No-tension flag, EQ.0: beam takes tension, EQ.1: beam takes no tension, EQ.2: beam takes tension up to value given by TENCUT.
TENCUT	Tension cutoff value.

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
SDR	Stiffness degradation factor.

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**Remarks:**

The stress strain behavior may be treated by a bilinear stress strain curve by defining the tangent modulus, ETAN. An effective stress versus effective plastic strain curve (LCSS) may be input instead of defining ETAN. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate.  $\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$ .

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor versus strain rate is defined.
3. If different stress versus strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used.



**\*MAT\_GENERAL\_SPRING\_DISCRETE\_BEAM**

This is Material Type 196. This model permits elastic and elastoplastic springs with damping to be represented with a discrete beam element of type 6 by using six springs each acting about one of the six local degrees-of-freedom. For elastic behavior, a load curve defines force or moment versus displacement or rotation. For inelastic behavior, a load curve defines yield force or moment versus plastic deflection or rotation, which can vary in tension and compression.

The two nodes defining a beam may be coincident to give a zero length beam, or offset to give a finite length beam. For finite length discrete beams the absolute value of the variable SCOOR in the SECTION\_BEAM input should be set to a value of 2.0, which causes the local r-axis to be aligned along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad is used to orient the beam for the directional springs.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO						
Type	A8	F						

**Degree of Freedom Card Pairs.** For each active degree of freedom include a pair of cards 2 and 3. This data is terminated by the next keyword ("\*") card or when all six degrees of freedom have been specified.

Card 2	1	2	3	4	5	6	7	8
Variable	DOF	TYPE	K	D	CDF	TDF		
Type	I	I	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	FLCID	HLCID	C1	C2	DLE	GLCID		
Type	F	F	F	F	F	I		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density, see also volume in *SECTION_BEAM definition.
DOF	Active degree-of-freedom, a number between 1 and 6 inclusive. Each value of DOF can only be used once. The active degree-of-freedom is measured in the local coordinate system for the discrete beam element.
TYPE	The default behavior is elastic. For inelastic behavior input 1.
K	Elastic loading/unloading stiffness. This is required input for inelastic behavior.
D	Optional viscous damping coefficient.
CDF	Compressive displacement at failure. Input as a positive number. After failure, no forces are carried. This option does not apply to zero length springs. EQ.0.0: inactive.
TDF	Tensile displacement at failure. After failure, no forces are carried. EQ.0.0: inactive.
FLCID	Load curve ID, see *DEFINE_CURVE. For option TYPE = 0, this curve defines force or moment versus deflection for nonlinear elastic behavior. For option TYPE = 1, this curve defines the yield force versus plastic deflection. If the abscissa of the first point of the curve is 0. the force magnitude is identical in tension and compression, i.e., only the sign changes. If not, the yield stress in the compression is used when the spring force is negative. The plastic displacement increases monotonically in this implementation. The load curve is required input.
HLCID	Load curve ID, see *DEFINE_CURVE, defining force versus relative velocity (Optional). If the origin of the curve is at (0,0) the force magnitude is identical for a given magnitude of the relative velocity, i.e., only the sign changes.
C1	Damping coefficient.
C2	Damping coefficient

VARIABLE	DESCRIPTION
DLE	Factor to scale time units.
GLCID	Optional load curve ID, see *DEFINE_CURVE, defining a scale factor versus deflection for load curve ID, HLCID. If zero, a scale factor of unity is assumed.

**Remarks:**

If TYPE = 0, elastic behavior is obtained. In this case, if the linear spring stiffness is used, the force,  $F$ , is given by:

$$F = K \times \Delta L + D \times \Delta \dot{L}$$

but if the load curve ID is specified, the force is then given by:

$$F = K f(\Delta L) \left[ 1 + C1 \times \Delta \dot{L} + C2 \times \text{sgn}(\Delta \dot{L}) \ln \left( \max \left\{ 1, \frac{|\Delta \dot{L}|}{DLE} \right\} \right) \right] + D \times \Delta \dot{L} + g(\Delta L) h(\Delta \dot{L})$$

In these equations,  $\Delta L$  is the change in length

$$\Delta L = \text{current length} - \text{initial length}$$

For the first three degrees of freedom the parameters on cards 2 and 3 have dimensions as shown below. Being angular in nature, the next three degrees of freedom involve moment instead of force and angle instead of length, but are otherwise identical.

$$[K] = \begin{cases} \frac{[\text{force}]}{[\text{length}]} & \text{FLCID} = 0 \\ \text{unitless} & \text{FLCID} > 0 \end{cases}$$

$$[D] = \frac{[\text{force}]}{[\text{velocity}]} = \frac{[\text{force}][\text{time}]}{[\text{length}]}$$

$$[\text{FLCID}] = [\text{GLCID}] = ([\text{length}], [\text{force}])$$

$$[\text{HLCID}] = ([\text{velocity}], [\text{force}])$$

$$[C1] = \frac{[\text{time}]}{[\text{length}]}$$

$$[C2] = \text{unitless}$$

$$[DLE] = \frac{[\text{length}]}{[\text{time}]}$$

If TYPE = 1, inelastic behavior is obtained. In this case, the yield force is taken from the load curve:

$$F^Y = F_y(\Delta L^{\text{plastic}})$$

where  $L^{\text{plastic}}$  is the plastic deflection. A trial force is computed as:

$$F^T = F^n + K \times \Delta \dot{L}(\Delta t)$$

and is checked against the yield force to determine  $F$ :

$$F = \begin{cases} F^Y & \text{if } F^T > F^Y \\ F^T & \text{if } F^T \leq F^Y \end{cases}$$

The final force, which includes rate effects and damping, is given by:

$$F^{n+1} = F \times \left[ 1 + C1 \times \Delta \dot{L} + C2 \times \text{sgn}(\Delta \dot{L}) \ln \left( \max \left\{ 1, \frac{|\Delta \dot{L}|}{\text{DLE}} \right\} \right) \right] + D \times \Delta \dot{L} + g(\Delta L)h(\Delta \dot{L})$$

Unless the origin of the curve starts at (0,0), the negative part of the curve is used when the spring force is negative where the negative of the plastic displacement is used to interpolate,  $F_y$ . The positive part of the curve is used whenever the force is positive.

The cross sectional area is defined on the section card for the discrete beam elements, See \*SECTION\_BEAM. The square root of this area is used as the contact thickness offset if these elements are included in the contact treatment.

**\*MAT\_SEISMIC\_ISOLATOR**

This is Material Type 197 for discrete beam elements. Sliding (pendulum) and elastomeric seismic isolation bearings can be modeled, applying bi-directional coupled plasticity theory. The hysteretic behavior was proposed by Wen [1976] and Park, Wen, and Ang [1986]. The sliding bearing behavior is recommended by Zayas, Low and Mahin [1990]. The algorithm used for implementation was presented by Nagarajaiah, Reinhorn, and Constantinou [1991]. Further options for tension-carrying friction bearings are as recommended by Roussis and Constantinou [2006]. Element formulation type 6 must be used. Local axes are defined on \*SECTION\_BEAM; the default is the global axis system. It is expected that the local z-axis will be vertical. On \*SECTION\_BEAM SCOR must be set to zero when using this material model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	A	BETA	GAMMA	DISPY	STIFFV	ITYPE
Type	A8	F	F	F	F	F	F	I
Default	none	none	1.0	0.5	0.5	0.0	0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	PRELOAD	DAMP	MX1	MX2	MY1	MY2		
Type	F	F	F	F	F	F		
Default	0	1.0	0	0	0	0		

**\*MAT\_197****\*MAT\_SEISMIC\_ISOLATOR**

**Sliding Isolator Card.** This card is used for ITYPE = 0 or 2. Leave this card *blank* for elastomeric isolator (TYPE = 1).

Card 3	1	2	3	4	5	6	7	8
Variable	FMAX	DELF	AFRIC	RADX	RADY	RADB	STIFFL	STIFFTS
Type	F	F	F	F	F	F	F	F
Default	0	0	0	1.0e20	1.0e20	1.0e20	STIFFV	0

Card 4 for ITYPE = 1 or 2. leave blank for sliding isolator ITYPE = 0:

Card 4	1	2	3	4	5	6	7	8
Variable	FORCEY	ALPHA	STIFFT	DFAIL	FMAXYC	FMAXXT	FMAXYT	YLOCK
Type	F	F	F	F	F	F	F	F
Default	0	0	0.5 × STIFFV	1.0e20	FMAX	FMAX	FMAX	0.0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
A	Nondimensional variable - see below
GAMMA	Nondimensional variable - see below
BETA	Nondimensional variable - see below
DISPY	Yield displacement (length units - must be > 0.0)
STIFFV	Vertical stiffness (force/length units)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ITYPE	Type: EQ.0: sliding (spherical or cylindrical) EQ.1: elastomeric EQ.2: sliding (two perpendicular curved beams)
PRELOAD	Vertical preload not explicitly modeled (force units)
DAMP	Damping ratio (nondimensional)
MX1, MX2	Moment factor at ends 1 and 2 in local X-direction
MY1, MY2	Moment factor at ends 1 and 2 in local Y-direction
FMAX	Maximum friction coefficient (dynamic)
DELFC	Difference between maximum friction and static friction coefficient
AFRIC	Velocity multiplier in sliding friction equation (time/length units)
RADX	Radius for sliding in local X direction
RADY	Radius for sliding in local Y direction
RADB	Radius of retaining ring
STIFFL	Stiffness for lateral contact against the retaining ring
STIFFTS	Stiffness for tensile vertical response (sliding isolator - default = 0)
FORCEY	Yield force. Used for elastomeric type (ITYPE = 1). Leave blank for sliding type (0, and 2).
ALPHA	Ratio of postyielding stiffness to preyielding stiffness. Used for elastomeric type (ITYPE = 1). Leave blank for sliding type (0, and 2).
STIFFT	Stiffness for tensile vertical response (elastomeric isolator). Used for elastomeric type (ITYPE = 1). Leave blank for sliding type (0, and 2).
DFAIL	Lateral displacement at which the isolator fails. Used for elastomeric type (ITYPE = 1). Leave blank for sliding type (0, and 2).
FMAXYC	Max friction coefficient (dynamic) for local Y-axis (compression). Used for ITYPE = 2. Leave blank for ITYPE = 0 or 1.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FMAXXT	Max friction coefficient (dynamic) for local X-axis (tension). Used for ITYPE = 2. Leave blank for ITYPE = 0 or 1.
FMAXYT	Max friction coefficient (dynamic) for local Y-axis (tension). Used for ITYPE = 2. Leave blank for ITYPE = 0 or 1.
YLOCK	Stiffness locking the local Y-displacement (optional -single-axis sliding). Used for ITYPE = 2. Leave blank for ITYPE = 0 or 1.

**Remarks:**

The horizontal behavior of both types is governed by plastic history variables  $Z_x$ ,  $Z_y$  that evolve according to equations given in the reference;  $A$ ,  $\gamma$  and  $\beta$  and the yield displacement are the input parameters for this. The intention is to provide smooth build-up, rotation and reversal of forces in response to bidirectional displacement histories in the horizontal plane. The theoretical model has been correlated to experiments on seismic isolators.

The RADX, RADY inputs for the sliding isolator are optional. If left blank, the sliding surface is assumed to be flat. A cylindrical surface is obtained by defining either RADX or RADY; a spherical surface can be defined by setting RADX = RADY. The effect of the curved surface is to add a restoring force proportional to the horizontal displacement from the center. As seen in elevation, the top of the isolator will follow a curved trajectory, lifting as it displaces away from the center.

The vertical behavior for all types is linear elastic, but with different stiffnesses for tension and compression. By default, the tensile stiffness is zero for the sliding types.

The vertical behavior for the elastomeric type is linear elastic; in the case of uplift, the tensile stiffness will be different to the compressive stiffness. For the sliding type, compression is treated as linear elastic but no tension can be carried.

Vertical preload can be modeled either explicitly (for example, by defining gravity), or by using the PRELOAD input. PRELOAD does not lead to any application of vertical force to the model. It is added to the compression in the element before calculating the friction force and tensile/compressive vertical behavior.

ITYPE = 0 is used to model a single (spherical) pendulum bearing. Triple pendulum bearings can be modelled using three of these elements in series, following the method described by Fenz and Constantinou 2008. The input properties for the three elements (given by  $\bar{R}_{eff1}$ ,  $\bar{\mu}_1$ ,  $\bar{d}_1$ ,  $\bar{a}_1$ , etc) are calculated from the properties of the actual triple bearing (given by  $R_{eff1}$ ,  $\mu_1$ ,  $d_1$ ,  $a_1$ , etc) as follows:



ITYPE = 2 is intended to model uplift-prevention sliding isolators that consist of two perpendicular curved beams joined by a connector that can slide in slots on both beams. The beams are aligned in the local X and Y axes respectively. The vertical displacement is the sum of the displacements induced by the respective curvatures and slider displacements along the two beams. Single-axis sliding is obtained by using YLOCK to lock the local-Y displacement. To resist uplift, STIFFTS must be defined (recommended value: same as STIFFV). This isolator type allows different friction coefficients on each beam, and different values in tension and compression. The total friction, taking into account sliding velocity and the friction history functions, is first calculated using FMAX and then scaled by FMAXXT/FMAX etc as appropriate. For this reason, FMAX should not be zero.

DAMP is the fraction of critical damping for free vertical vibration of the isolator, based on the mass of the isolator (including any attached lumped masses) and its vertical stiffness. The viscosity is reduced automatically if it would otherwise infringe numerical stability. Damping is generally recommended: oscillations in the vertical force would have a direct effect on friction forces in sliding isolators; for isolators with curved surfaces, vertical oscillations can be excited as the isolator slides up and down the curved surface. It may occasionally be necessary to increase DAMP if these oscillations become significant.

This element has no rotational stiffness - a pin joint is assumed. However, if required, moments can be generated according to the vertical load multiplied by the lateral displacement of the isolator. The moment about the local X-axis (i.e. the moment that is dependent on lateral displacement in the local Y-direction) is reacted on nodes 1 and 2 of the element in the proportions MX1 and MX2 respectively. Similarly, moments about the local Y-axis are reacted in the proportions MY1, MY2. These inputs effectively determine the location of the pin joint.

For example, a pin at the base of the column could be modeled by setting  $MX1 = MY1 = 1.0$ ,  $MX2 = MY2 = 0.0$  and ensuring that node 1 is on the foundation, node 2 at the base of the column - then all the moment is transferred to the foundation. For the same model,  $MX1 = MY1 = 0.0$ ,  $MX2 = MY2 = 1.0$  would imply a pin at the top of the foundation - all the moment is transferred to the column. Some isolator designs have the pin at the bottom for moments about one horizontal axis, and at the top for the other axis - these can be modeled by setting  $MX1 = MY2 = 1.0$ ,  $MX2 = MY1 = 0.0$ . It is expected that all  $MX1,2$ , etc lie between 0 and 1, and that  $MX1+MX2 = 1.0$  (or both can be zero) - e.g.  $MX1 = MX2 = 0.5$  is permitted - but no error checks are performed to ensure this; similarly for  $MY1 + MY2$ .

Density should be set to a reasonable value, say 2000 to 8000 kg/m<sup>3</sup>. The element mass will be calculated as density x volume (volume is entered on \*SECTION\_BEAM).

Note on values for \*SECTION\_BEAM:

1. Set ELFORM to 6 (discrete beam)
2. VOL (the element volume) might typically be set to 0.1m<sup>3</sup>

3. INER needs to be non-zero (say 1.0) but the value has no effect on the solution since the element has no rotational stiffness.
4. CID can be left blank if the isolator is aligned in the global coordinate system, otherwise a coordinate system should be referenced.
5. By default, the isolator will be assumed to rotate with the average rotation of its two nodes. If the base of the column rotates slightly the isolator will no longer be perfectly horizontal: this can cause unexpected vertical displacements coupled with the horizontal motion. To avoid this, rotation of the local axes of the isolator can be eliminated by setting RRCON, SRCON and TRCON to 1.0. This does not introduce any rotational restraint to the model, it only prevents the orientation of the isolator from changing as the model deforms.
6. SCOOR must be set to zero.
7. All other parameters on \*SECTION\_BEAM can be left blank.

Post-processing note: as with other discrete beam material models, the force described in post-processors as "Axial" is really the force in the local X-direction; "Y-Shear" is really the force in the local Y-direction; and "Z-Shear" is really the force in the local Z-direction.



Repeat Card 4 for each plane (maximum 3 planes):

Card 4	1	2	3	4	5	6	7	8
Variable	DIP	STRIKE	CPLANE	FRPLANE	TPLANE	SHRMAX	LOCAL	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	1.e20	0.0	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
GMOD	Elastic shear modulus
RNU	Poisson's ratio
RKF	Failure surface shape parameter
PHI	Angle of friction (radians)
CVAL	Cohesion value
PSI	Dilation angle (radians)
STR_LIM	Minimum shear strength of material is given by STR_LIM*CVAL
NPLANES	Number of joint planes (maximum 3)
ELASTIC	Flag = 1 for elastic behavior only
LCCPDR	Load curve for extra cohesion for parent material (dynamic relaxation)
LCCPT	Load curve for extra cohesion for parent material (transient)
LCCJDR	Load curve for extra cohesion for joints (dynamic relaxation)
LCCJT	Load curve for extra cohesion for joints (transient)
LCSFAC	Load curve giving factor on strength vs time

VARIABLE	DESCRIPTION
GMODDP	Depth at which shear modulus (GMOD) is correct
PHIDP	Depth at which angle of friction (PHI) is correct
CVALDP	Depth at which cohesion value (CVAL) is correct
PSIDP	Depth at which dilation angle (PSI) is correct
GMODGR	Gradient at which shear modulus (GMOD) increases with depth
PHIGR	Gradient at which friction angle (PHI) increases with depth
CVALGR	Gradient at which cohesion value (CVAL) increases with depth
PSIGR	Gradient at which dilation angle (PSI) increases with depth
DIP	Angle of the plane in degrees below the horizontal
DIPANG	Plan view angle (degrees) of downhill vector drawn on the plane
CPLANE	Cohesion for shear behavior on plane
PHPLANE	Friction angle for shear behavior on plane (degrees)
TPLANE	Tensile strength across plane (generally zero or very small)
SHRMAX	Max shear stress on plane (upper limit, independent of compression)
LOCAL	EQ.0: DIP and DIPANG are with respect to the global axes

**Remarks:**

1. The joint plane orientations are defined by the angle of a “downhill vector” drawn on the plane, i.e. the vector is oriented within the plane to obtain the maximum possible downhill angle. DIP is the angle of this line below the horizontal. DIPANG is the plan-view angle of the line (pointing down hill) measured clockwise from the global Y-axis about the global Z-axis.
2. The joint planes rotate with the rigid body motion of the elements, irrespective of whether their initial definitions are in the global or local axis system.
3. The full facilities of the modified Drucker Prager model for the matrix material can be used – see description of Material type 193. Alternatively, to speed up the calculation, the ELASTIC flag can be set to 1, in which case the yield surface will not

be considered and only RO, GMOD, RNU, GMODDP, GMODGR and the joint planes will be used.

4. This material type requires that the model is oriented such that the z-axis is defined in the upward direction. The key parameters are defined such that may vary with depth (i.e. the z-axis)
5. The shape factor for a typical soil would be 0.8, but should not be pushed further than 0.75.
6. If STR\_LIM is set to less than 0.005, the value is reset to 0.005.
7. A correction has been introduced into the Drucker Prager model, such that the yield surface never infringes the Mohr-Coulomb criterion. This means that the model does not give the same results as a "pure" Drucker Prager model.
8. The load curves LCCPDR, LCCPT, LCCJDR, LCCJT allow additional cohesion to be specified as a function of time. The cohesion is additional to that specified in the material parameters. This is intended for use during the initial stages of an analysis to allow application of gravity or other loads without cracking or yielding, and for the cracking or yielding then to be introduced in a controlled manner. This is done by specifying extra cohesion that exceeds the expected stresses initially, then declining to zero. If no curves are specified, no extra cohesion is applied.
9. The load curve for factor on strength applies simultaneously to the cohesion and tan (friction angle) of parent material and all joints. This feature is intended for reducing the strength of the material gradually, to explore factors of safety. If no curve is present, a constant factor of 1 is assumed. Values much greater than 1.0 may cause problems with stability.
10. Extra variables for plotting. By setting NEIPH on \*DATABASE\_EXTENT\_BINARY to 15, the following variables can be plotted in D3PLOT and T/HIS:
  - Extra Variable 1: mobilized strength fraction for base material
  - Extra Variable 2: rk0 for base material
  - Extra Variable 3: rlamda for base material
  - Extra Variable 4: crack opening strain for plane 1
  - Extra Variable 5: crack opening strain for plane 2
  - Extra Variable 6: crack opening strain for plane 3
  - Extra Variable 7: crack accumulated engineering shear strain for plane 1
  - Extra Variable 8: crack accumulated engineering shear strain for plane 2
  - Extra Variable 9: crack accumulated engineering shear strain for plane 3
  - Extra Variable 10: current shear utilization for plane 1
  - Extra Variable 11: current shear utilization for plane 2
  - Extra Variable 12: current shear utilization for plane 3
  - Extra Variable 13: maximum shear utilization to date for plane 1

Extra Variable 14: maximum shear utilization to date for plane 2

Extra Variable 15: maximum shear utilization to date for plane 3

11. Joint planes would generally be defined in the global axis system if they are taken from survey data. However, the material model can also be used to represent masonry, in which case the weak planes represent the cement and lie parallel to the local element axes.

**\*MAT\_STEEL\_EC3**

This is Material Type 202. Tables and formulae from Eurocode 3 are used to derive the mechanical properties and their variation with temperature, although these can be overridden by user-defined curves. It is currently available only for Hughes-Liu beam elements. Warning, this material is still under development and should be used with caution.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY			
Type	A8	F	F	F	F			
Default	none	none	none	none	none			

Card 2	1	2	3	4	5	6	7	8
Variable	LC_E	LC_PR	LC_AL	TBL_SS	LC_FS			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

Card 3 *must* be included but left blank.

Card 3	1	2	3	4	5	6	7	8
Variable								
Type								
Default								

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label not exceeding 8 characters must be specified.

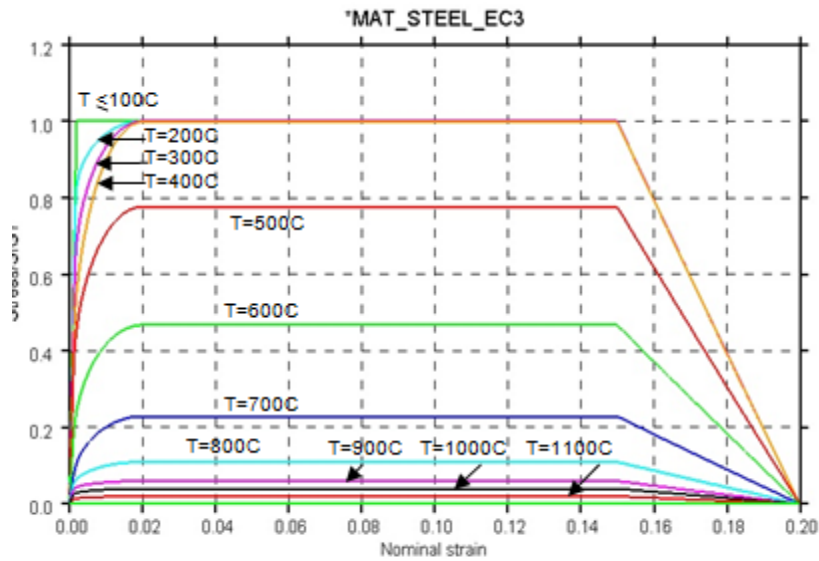


<b>VARIABLE</b>	<b>DESCRIPTION</b>
RO	Mass density.
E	Young's modulus – a reasonable value must be provided even if LC_E is also input. See notes.
PR	Poisson's ratio.
SIGY	Initial yield stress, $\sigma_{y0}$ .
LC_E	Optional Load curve ID: Young's Modulus vs Temperature (overrides E and factors from EC3).
LC_PR	Optional Load curve ID: Poisson's Ratio vs Temperature (overrides PR).
LC_AL	Optional Load curve ID: alpha vs temperature (over-rides thermal expansion data from EC3).
TBL_SS	Optional Table ID containing stress-strain curves at different temperatures (overrides curves from EC3).
LC_FS	Optional Load curve ID: failure strain vs temperature.

**Remarks:**

1. This material model is intended for modelling structural steel in fires.
2. By default, only E, PR and SIGY have to be defined. Eurocode 3 (EC3) Section 3.2 specifies the stress-strain behaviour of carbon steels at temperatures between 20C and 1200C. The stress-strain curves given in EC3 are scaled within the material model such that the maximum stress at low temperatures is SIGY, see graph below.
3. By default, the Young's Modulus E will be scaled by a factor which is a function of temperature as specified in EC3. The factor is 1.0 at low temperature.
4. By default, the thermal expansion coefficient as a function of temperature will be as specified in EC3 Section 3.4.1.1.
5. LC\_E, LC\_PR and LC\_AL are optional; they should have temperature on the x-axis and the material property on the y-axis, with the points in order of increasing temperature. If present (i.e. non-zero) they over-ride E, PR, and the relationships from EC3. However, a reasonable value for E should always be included, since these values will be used for purposes such as contact stiffness calculation.

- 6. TBL\_SS is optional. If present, TBL\_SS must be the ID of a \*DEFINE\_TABLE. TBL\_SS overrides SIGY and the stress-strain relationships from EC3. The field VALUE on the \*DEFINE\_TABLE should contain the temperature at which each stress-strain curve is applicable; the temperatures should be in ascending order. The curves that follow the temperature values have (true) plastic strain on the x-axis, (true) yield stress on the y-axis as per other LS-DYNA elasto-plastic material models. As with all instances of \*DEFINE TABLE, the curves containing the stress-strain data must immediately follow the \*DEFINE\_TABLE input data and must be in the correct order (i.e. the same order as the temperatures).
  
- 7. Temperature can be defined by any of the \*LOAD\_THERMAL methods. The temperature does not have to start at zero: the initial temperature will be taken as a reference temperature for each element, so non-zero initial temperatures will not cause thermal shock effects.



**Figure M202-1.**

**\*MAT\_HYSTERETIC\_REINFORCEMENT**

This is Material Type 203 in LS-DYNA. It is intended as an alternative reinforcement model for layered reinforced concrete shell elements, for use in seismic analysis where the nonlinear hysteretic behaviour of the reinforcement is important. \*PART\_COMPOSITE or \*INTEGRATION\_BEAM should be used to define some integration points as a part made of \*MAT\_HYSTERETIC\_REINFORCEMENT, while other integration points have concrete properties using \*MAT\_CONCRETE\_EC2. When using beam elements, ELFORM = 1 is required.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	YM	PR	SIGY	LAMDA	SBUCK	POWER
Type	I	F	F	F	F	F	F	F
Default	none	0.0	0.0	0.0	0.0	0.0	SIGY	0.5

Card 2	1	2	3	4	5	6	7	
Variable	FRACX	FRACY	LCTEN	LCCOMP	AOPT	EBU	DOWNSL	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.1	

Card 3	1	2	3	4	5	6	7	
Variable	DBAR	FCDOW	LCHARD	UNITC	UNITL			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	1.0	1.0			

**\*MAT\_203****\*MAY\_HYSTERETIC\_REINFORCEMENT**

Card 4	1	2	3	4	5	6	7	8
Variable	EPDAM1	EPDAM2	DRESID					
Type	F	F	F					
Default	0.0	0.0	0.0					

Additional Card for AOPT  $\neq$  0.

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

Additional Card for AOPT  $\neq$  0.

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number has to be chosen.
RO	Mass density.
YM	Young's Modulus
PR	Poisson's Ratio
SIGY	Yield stress

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LAMDA	Slenderness ratio
SBUCK	Initial buckling stress (should be positive)
POWER	Power law for Bauschinger effect (non-dimensional)
FRACX	Fraction of reinforcement at this integration point in local $x$ direction
FRACY	Fraction of reinforcement at this integration point in local $y$ direction
LCTEN	Optional curve providing the factor on SIGY versus plastic strain (tension)
LCCOMP	Optional curve providing the factor on SBUCK versus plastic strain (compression)
AOPT	Option for local axis alignment – see material type 2
EBU	Optional buckling strain (if defined, overrides LAMBDA)
DOWNSL	Initial down-slope of buckling curve as a fraction of YM (dimensionless)
DBAR	Reinforcement bar diameter used for dowel action. See remarks.
FCDOW	Concrete compressive strength used for dowel action. See notes. This field has units of stress
LCHARD	Characteristic length for dowel action (length units)
UNITC	Factor to convert model stress units to MPa, e.g. if model units are Newtons and meters, $UNITC = 10^{-6}$ , $[UNITC] = 1/[STRESS]$ .
UNITL	Factor to convert model length units to millimeters, e.g. if model units are meters, $UNITL = 1000$ , $[UNITL] = 1/[LENGTH]$ .
EPDAM1	Accumulated plastic strain at which hysteretic damage begins
EPDAM2	Accumulated plastic strain at which hysteretic damage is complete
DRESID	Residual factor remaining after hysteretic damage
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1 and 4 (see <a href="#">MAT 2</a> )
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2 (see <a href="#">MAT 2</a> )

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3 and 4 (see <a href="#">MAT 2</a> )
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2 (see <a href="#">MAT 2</a> )

**Remarks:**

Reinforcement is treated as bars, acting independently in the local material  $x$  and  $y$  directions. By default, the local material  $x$ -axis is the element  $x$ -axis (parallel to the line from Node 1 to Node 2), but this may be overridden using AOPT or Element Beta angles.

The reinforced concrete section should be defined using \*INTEGRATION\_SHELL, with some integration points being reinforcement (using this material model) and others being concrete (using for example \*MAT\_CONCRETE\_EC2). By default, strains in directions other than the local  $x$  and  $y$  are unresisted, so this material model should not be used alone (without concrete). The area fractions of reinforcement in the local  $x$  and  $y$  directions at each integration point are given by the area-weighting for the integration point on \*INTEGRATION\_SHELL times the fractions FRACX and FRACY.

The tensile response is elastic perfectly plastic, using yield stress SIGY. Optionally, load curves may be used to describe the stress-strain response in tension (LCTEN) and compression (LCCOMP). Either, neither or both curves may be defined. If present, LCTEN overrides the perfectly-plastic tensile response, and LCCOMP overrides the buckling curve. The tensile and compressive plastic strains are considered independent of each other.

Bar buckling may be defined either using the slenderness ratio LAMDA, or by setting the initial buckling strain EBU and down-slope DOWNSL. If neither are defined, the bars simply yield in compression. If both are defined, the buckling behaviour defined by EBU and DOWNSL overrides LAMDA.

The slenderness ratio LAMDA determines buckling behaviour and is defined as,

$$\frac{kL}{r'}$$

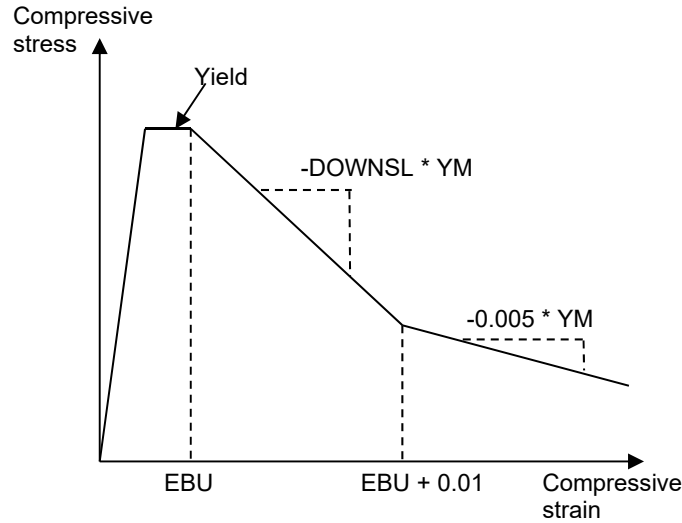
Where,  $k$  depends on end conditions, and

$L$  = unsupported length of reinforcement bars

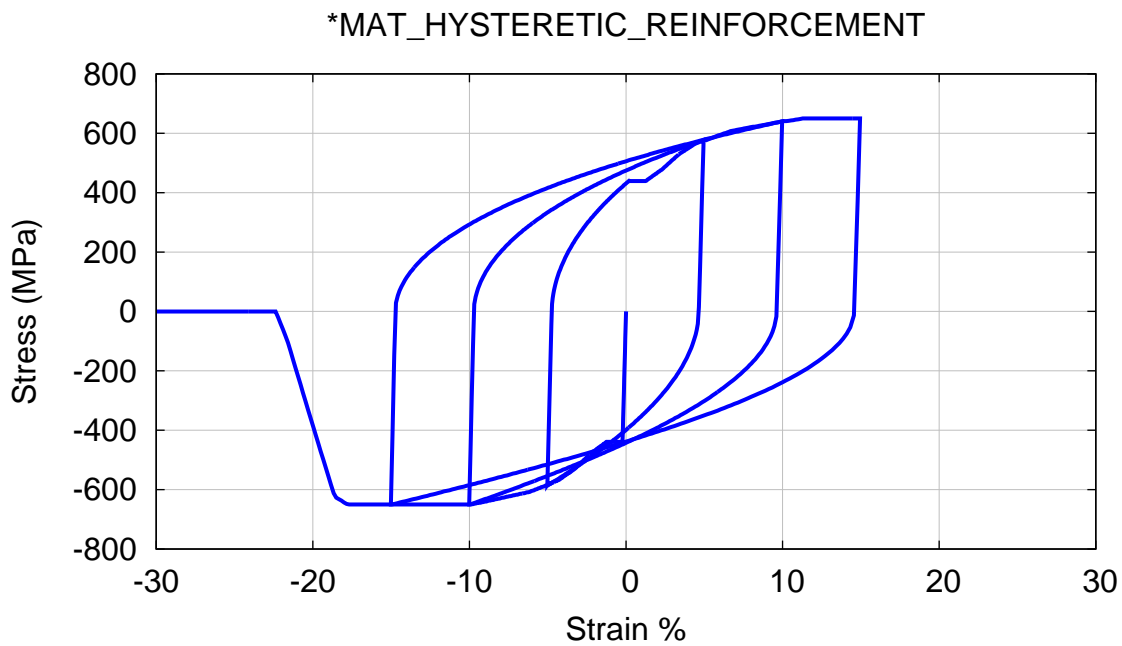
$r$  = radius of gyration which for round bars is equal to (bar radius)/ $\sqrt{2}$ .

It is expected that users will determine LAMDA accounting for the expected crack spacing.

The alternative buckling behaviour defined by EBU and DOWNSL is shown below.



Reloading after change of load direction follows a Bauschinger-type curve, leading to the hysteresis response shown below:



**Damage modelling:**

Two types of damage accumulation may be modelled. Damage based on ductility (strain) can be modelled using the curves LCTEN and LCCOMP – at high strain, these curves would show reducing stress with increasing strain.

Damage based on hysteretic energy accumulation can be modelled using the parameters EPDAM1, EPDAM2 and DRESID. The damage is a function of accumulated plastic strain: for this purpose, plastic strain increments are always treated as positive in both tension and

compression, and buckling strain also counts towards the accumulated plastic strain. The material has its full stiffness and strength until the accumulated plastic strain reaches EPDAM1. Between plastic strains EPDAM1 and EPDAM2 the stiffness and strength fall linearly with accumulated plastic strain, reaching a factor DRESID at plastic strain EPDAM2.

### **Dowel Action:**

The data on Card 3 defines the shear stiffness and strength, and is optional. Shear resistance is assumed to occur by dowel action. The bars bend locally to the crack and crush the concrete. An elastic-perfectly-plastic relation is assumed for all shear components (in-plane and through-thickness). The assumed (smeared) shear modulus and yield stress applicable to the reinforcement bar cross-sectional area are as follows, based on formulae derived from experimental data by El-Ariss, Soroushian, and Dulacska:

$$G[\text{MPa}] = 8.02E^{0.25}F_c^{0.375}L_{\text{char}}D_b^{0.75}$$

$$\tau_y = 1.62\sqrt{F_cS_y}$$

where,

$E$  = steel Youngs Modulus in MPa

$F_c$  = compressive strength of concrete in MPa

$L_{\text{char}}$  = characteristic length of shear deformation in mm

$D_b$  = bar diameter in mm

$S_y$  = steel yield stress in MPa.

The input parameters should be given in model units, e.g. DBAR and LCHAR are in model length units, FCDOW is in model stress units. These will be converted internally using UNITL and UNITC.

### **Output:**

The output stresses, as for all other LS-DYNA material models, are by default in the global coordinate system. They are scaled by the reinforcement fractions FRACX, FRACY. The plastic strain output is the accumulated plastic strain (increments always treated as positive), and is the greater such value of the two local directions. Extra history variables are available as follows:

- Extra variable 1: Reinforcement stress in local  $x$  direction (not scaled by FRACX)
- Extra variable 2: Reinforcement stress in local  $y$  direction (not scaled by FRACY)
- Extra variable 3: Total strain in local  $x$  direction
- Extra variable 4: Total strain in local  $y$  direction
- Extra variable 5: Accumulated plastic strain in local  $x$  direction



Extra variable 6: Accumulated plastic strain in local  $y$  direction  
Extra variable 7: Shear stress (dowel action) in local  $xy$   
Extra variable 8: Shear stress (dowel action) in local  $xz$   
Extra variable 9: Shear stress (dowel action) in local  $yz$

**\*MAT\_BOLT\_BEAM**

This is Material Type 208 for use with beam elements using ELFORM = 6 (Discrete Beam). The beam elements must have nonzero initial length so that the directions in which tension and compression act can be distinguished. See notes below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	KAX	KSHR	blank	blank	FPRE	TRAMP
Type	A8	F	F	F			F	F
Default	none	none	0.0	0.0			0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	LCAX	LCSHR	FRIC	CLEAR	DAFAIL	DRFAIL	DAMAG	TOPRE
Type	I	I	F	F	F	F	F	F
Default	0	0	0.0	0.0	1.E20	1.E20	0.1	0.0

## Card 3

Card 3	1	2	3	4	5	6	7	8
Variable	DACFAIL							
Type	F							
Default	1.E20							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
KAX	Axial elastic stiffness (Force/Length units).
KSHR	Shear elastic stiffness (Force/Length units).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FPRE	Preload force.
TRAMP	Time duration during which preload is ramped up.
LCAX	Load curve giving axial load versus plastic displacement (x-axis = displacement (length units), y-axis = force).
LCSHR	Load curve ID or table ID giving lateral load versus plastic displacement (x-axis - displacement (length units), y-axis - force). In the table case, each curve in the table represents lateral load versus displacement at a given (current) axial load, i.e. the values in the table are axial forces.
FRIC	Friction coefficient resisting sliding of bolt head/nut (non-dimensional).
CLEAR	Radial clearance (gap between bolt shank and the inner diameter of the hole) (length units).
DAFAIL	Axial tensile displacement at which failure is initiated (length units).
DRFAIL	Radial displacement at which failure is initiated (excludes clearance).
DAMAG	Failure is completed at (DAFAIL or DRFAIL or DACFAIL)*(1+DAMAG).
TOPRE	Time at which preload application begins.
DACFAIL	Axial compressive displacement at which failure is initiated (positive value, length units)

**Remarks:**

The element represents a bolted joint. The nodes of the beam should be thought of as representing the points at the centers of the holes in the plates that are joined by the bolt. It is expected that  $SCOOR = 0$  on **\*SECTION\_BEAM**. This is contrary to the normal rules for non-zero-length discrete beams.

The axial direction is initially the line connecting node 1 to node 2. The axial response is tensile-only. Instead of generating a compressive axial load, it is assumed that a gap would develop between the bolt head (or nut) and the surface of the plate. Contact between the bolted surfaces must be modelled separately, e.g. using **\*CONTACT**.

Curves LCAX, LCSHR give yield force versus plastic displacement for the axial and shear directions. The force increments are calculated from the elastic stiffnesses, subject

to the yield force limits given by the curves.

CLEAR allows the bolt to slide in shear, resisted by friction between bolt head/nut and the surfaces of the plates, from the initial position at the center of the hole. CLEAR is the total sliding shear displacement before contact occurs between the bolt shank and the inside surface of the hole. Sliding shear displacement is not included in the displacement used for LCSHR; LCSHR is intended to represent the behaviour after the bolt shank contacts the edge of the hole.

Output: beam "axial" or "X" force is the axial force in the beam. "shear-Y" and "shear-Z" are the shear forces.

Other output is written to the d3plot and d3thdt files in the places where post-processors expect to find the stress and strain at the first two integration points for integrated beams.

<u>Post-Processing data component</u>	<u>Actual meaning</u>
Int. Pt 1, Axial Stress	Change of length
Int Pt 1, XY Shear stress	Sliding shear displacement in local Y
Int Pt 1, ZX Shear stress	Sliding shear displacement in local Z
Int Pt 1, Plastic strain	Resultant shear sliding displacement
Int Pt 1, Axial strain	Axial plastic displacement
Int. Pt 2, Axial Stress	Shear plastic displacement excluding sliding
Int Pt 2, XY Shear stress	-
Int Pt 2, ZX Shear stress	-
Int Pt 2, Plastic strain	-
Int Pt 2, Axial strain	-

\*MAT\_SPR\_JLR

This is Material Type 211. This material model was written for Self-Piercing Rivets (SPR) connecting aluminium sheets. It intended that each SPR is modelled by a single hexahedral (8-node solid) element, fixed to the sheet either by direct meshing or by tied contact. Pre- and post-processing methods are the same as for solid-element Spotwelds using \*MAT\_SPOTWELD. On \*SECTION\_SOLID, set ELFORM = 1.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	HELAS	TELAS		
Type	A8	F	F	F	F	F		
Default	none	none	none	none	0	0		

Cards 2 and 3 define "Head" end of SPR inputs

Card 2	1	2	3	4	5	6	7	8
Variable	LCAXH	LCSHH	LCBMH	SFAXH	SFSHH	SFBMH		
Type	F	F	F	F	F	F		
Default	none	none	none	1	1	1		

Card 3	1	2	3	4	5	6	7	8
Variable	DFAKH	DFSHH	RFBMH	DMFAXH	DMFSHH	DMFBMH		
Type	F	F	F	F	F	F		
Default	see notes	see notes	see notes	0.1	0.1	0.1		

Cards 4 and 5 define "Tail" end of SPR inputs

Card 4	1	2	3	4	5	6	7	8
Variable	LCAXT	LCSHT	LCBMT	SFAXT	SFSHT	SBFMT		
Type	F	F	F	F	F	F		
Default	none	none	none	1	1	1		

Card 5	1	2	3	4	5	6	7	8
Variable	DFAXT	DFSHT	RFBMT	DFMAXT	DMFSHT	DMFBMT		
Type	F	F	F	F	F	F		
Default	see notes	see notes	see notes	0.1	0.1	0.1		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus, used only for contact stiffness calculation.
PR	Poisson's ratio, used only for contact stiffness calculation.
HELAS	SPR head end behaviour flag: EQ.0.0: Nonlinear. EQ.1.0: Elastic (Use first two points on load curves).
TELAS	SPR tail end behaviour flag: EQ.0.0: Nonlinear. EQ.1.0: Elastic (Use first two points on load curves).
LCAXH	Load curve ID, see *DEFINE_CURVE, giving axial force versus deformation (head).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCSHH	Load curve ID, see *DEFINE_CURVE, giving shear force versus deformation (head).
LCBMH	Load curve ID, see *DEFINE_CURVE, giving moment versus rotation (head).
SFAXH	Scale factor on axial force from curve LCAXH.
SFSHH	Scale factor on shear force from curve LCSHH.
SFBMH	Scale factor on bending moment from curve LCBMH.
DFAXH	Optional displacement to start of softening in axial load (head).
DFSHH	Optional displacement to start of softening in shear load (head).
RFBMH	Optional rotation (radians) to start of bending moment softening (head).
DMFAXH	Scale factor on DFAXH.
DMFSHH	Scale factor on FFSHH.
DMFBMH	Scale factor on RFBMH.
LCAXT	Load curve ID, see *DEFINE_CURVE, giving axial force versus deformation (tail).
LCSHT	Load curve ID, see *DEFINE_CURVE, giving shear force versus deformation (tail).
LCBMT	Load curve ID, see *DEFINE_CURVE, giving moment versus rotation (tail).
SFAXT	Scale factor on axial force from curve LCAXT.
SFSHT	Scale factor on shear force from curve LCSHT.
SFBMT	Scale factor on bending moment from curve LCBMT.
DFAXT	Optional displacement to start of softening in axial load (tail).
DFSHT	Optional displacement to start of softening in shear load (tail).
RFBMT	Optional rotation (radians) to start of bending moment softening (tail).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
DMFAXT	Scale factor on DFAXT.
DMFSHT	Scale factor on FFSHT.
DMFBMT	Scale factor on RFBMT.

**Remarks:**

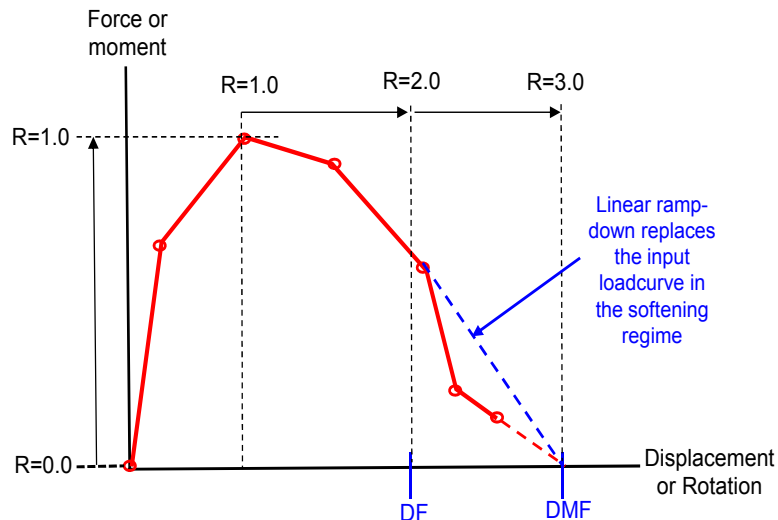
1. "Head" is the end of the SPR that fully perforates a sheet. "Tail" is the end that is embedded within the thickness of a sheet.
2. E and PR are used only to calculate contact stiffness. They are not used by the material model.
3. Deformation is in length units and is on the x-axis. Force is on the y-axis. Rotation is in radians, on the x-axis. Moment is on the y-axis.
4. All the load curves are expected to start at (0,0). "Deformation" means the total deformation including both elastic and plastic components, and similarly for rotation.
5. A "high tide" algorithm is used to determine the deformation or rotation to be used as the x-axis of the load curves when looking up the current yield force or moment. The "high tide" is the greatest displacement or rotation that has occurred so far during the analysis.
6. The first two points of the curve define the elastic stiffness, which is used for unloading.
7. If HELAS > 0, the remainder of the head load curves after the first two points is ignored and no softening or failure occurs. Similarly for TELAS and the tail load curves.
8. The sheet planes are defined at the head by the quadrilateral defined by nodes N1-N2-N3-N4 of the solid element; and at the tail by the quadrilateral defined by nodes N5-N6-N7-N8.
9. The tail of the SPR is defined as a point in the tail sheet plane, initially at the centre of the element face. The head of the SPR is initially at the centre of the head sheet plane. Thus the axis of the SPR would typically be coincident with the solid element local z-axis if the solid is a cuboid. It is the user's responsibility to ensure that each solid element is oriented correctly.



10. During the analysis, the head and tail will always remain in the plane of the sheet, but may move away from the centres of the sheet planes if the shear forces in these planes are sufficient.
11. The SPR axis is defined as the line joining the tail to the head.
12. Axial deformation is defined as change of length of the line between the tail and head of the SPR. This line also defines the direction in which the axial force is applied.
13. Shear deformation is defined as motion of the tail and head points, in the sheet planes. This deformation is not necessarily perpendicular to axial deformation. Shear forces in these planes are controlled by the load curves LCSHT and LCSHH.
14. Rotation at the tail is defined as rotation of the tail-to-head line relative to the normal of the tail sheet plane; and for the head, relative to the normal of the head sheet plane.
15. Displacement/rotation to start of softening (DFAXH, DFSHH, etc): if non-zero values are input, these must be within the abscissa values of the relevant curve, such that the curve force/moment value is greater than zero at the defined start of softening.
16. Although ELFORM = 1 is used in the input data, \*MAT\_SPR\_JLR is really a separate unique element formulation. The usual stress/force and hourglass calculations are bypassed, and deformations and nodal forces are calculated by a method unique to \*MAT\_SPR\_JLR; for example, a single \*MAT\_SPR\_JLR element can carry bending loads.
17. \*HOURGLASS inputs are irrelevant to \*MAT\_SPR\_JLR.
18. It is essential that the nodes N1 to N4 are fixed to the head sheet (e.g. by direct meshing or tied contact): the element has no stiffness to resist relative motion of nodes N1 to N4 in the plane of the head sheet. Similarly, nodes N5 to N8 must be fixed to the tail sheet.
19. Output to SWFORC file works in the same way as for Spotwelds. Although inside the material model the load curves LCSHT and LCSHH control “shear” forces in the sheet planes, in the SWFORC file the quoted shear force is the force normal to the axis of the SPR.
20. Before an element fails, it enters a “softening” regime in which the forces, moments and stiffnesses are ramped down as displacement increases (this avoids sudden shocks when the element is deleted). For example, for axial loading at the head, softening begins when the maximum axial displacement exceeds DFAXH. As the displacement increases beyond that point, the load curve will be ignored for that

deformation component. The forces, moments and stiffnesses are ramped down linearly with increasing displacement and reach zero at displacement =  $DFAXH \cdot (1 + DMFAXH)$  when the element is deleted. The softening factor scales all the force and moment components at both head and tail. Thus all the force and moment components are reduced when any one displacement component enters the softening regime. For example if  $DFAXT = 3.0\text{mm}$ , and  $DMFAXT = 0.1$ , then softening begins when axial displacement of the head reaches  $3.0\text{mm}$  and final failure occurs at  $3.3\text{mm}$ .

21. If the inputs  $DFAXT$  etc are left blank or zero, they will be calculated internally as follows:
  - a) Final failure will occur at the displacement or rotation ( $DFAIL$ ) at which the load curve reaches zero (determined if necessary by extrapolation from the last two points).
  - b) Displacement or rotation at which softening begins is then back-calculated, for example  $DFAXT = DFAIL / (1 + DMGAXT)$ .
  - c) If  $DMGAXT$  was left blank or zero, it defaults to 0.1.
  - d) If the load curve does not drop to zero, and the final two points have a zero or positive gradient, no failure or softening will be caused by that displacement component.
22. Output stresses (in the  $d3plot$  and time-history output files) are set to zero.
23. The output variable "displacement ratio" (or rotation ratio for bending),  $R$ , is defined as follows. See also the [Figure M211-1](#).
  - a)  $R = 0$  to 1: The maximum force or moment on the input curve has not yet been reached.  $R$  is proportional to the maximum force or moment reached so far, with 1.0 being the point of maximum force or moment on the input curve.
  - b)  $R = 1$  to 2: The element has passed the point of maximum force but has not yet entered the softening regime.  $R$  rises linearly with displacement (or rotation) from 1.0 when maximum force occurs to 2.0 when softening begins.
  - c)  $R = 2$  to 3: Softening is occurring.  $R$  rises linearly with displacement from 2.0 at the onset of softening to 3.0 when the element is deleted.



**Figure [M211-1].** Output variable “displacement ratio” (or rotation ratio for bending)

24. Displacement (or Rotation) Ratio is calculated separately for axial, shear and bending at the tail and head (see “Extra History Variables” below). The output listed by post-processors as “plastic strain” is actually the maximum displacement or rotation ratio of any displacement or rotation component at head or tail. This same variable is also output as “Failure” in the spotweld data in the swforc file (or the swforc section of the binout file).
25. Output extra history variables:
  - 1 Failure time (used for SWFORC file)
  - 2 (Softening factor used internally to prevent abrupt failure)
  - 3 Displacement ratio – axial, head
  - 4 Displacement ratio – axial, tail
  - 5 Displacement ratio – shear, head
  - 6 Displacement ratio – shear, tail
  - 7 Rotation ratio – bending, head
  - 8 Rotation ratio – bending, tail
  - 9 (Used for SWFORC output)
  - 10 Shear force in “beam” x-axis
  - 11 Shear force in “beam” y-axis
  - 12 Axial force in “beam” z-axis (along “beam”)
  - 13 Moment about “beam” x-axis at head
  - 14 Moment about “beam” y-axis at head
  - 15 Moment about “beam” z-axis at head (torsion – should be zero)
  - 16 “Beam” length
  - 17 Moment about “beam” x-axis at tail
  - 18 Moment about “beam” y-axis at tail
  - 19 Moment about “beam” z-axis at tail (torsion – should be zero)

- 20Isoparametric coordinate of head of "beam" (s)
- 21Isoparametric coordinate of head of "beam" (t)
- 22Isoparametric coordinate of tail of "beam" (s)
- 23Isoparametric coordinate of tail of "beam" (t)
- 24Timestep
- 25Plastic displacement, axial, head
- 26Plastic displacement, axial, tail
- 27Plastic rotation, head
- 28Plastic rotation, tail
- 29Plastic displacement, shear in sheet axes, head
- 30Plastic displacement, shear in sheet axes, tail
- 31Beam x-axis (global x component)
- 32Beam x-axis (global y component)
- 33Beam x-axis (global z component)
- 34Shear displacement, local x, head
- 35Shear displacement, local y, head
- 36Shear displacement, local x, tail
- 37Shear displacement, local y, tail
- 38Total displacement – axial
- 39Current rotation (radians) – head, local X direction
- 40Current rotation (radians) – head, local Y direction
- 41Current rotation (radians) – tail, local X direction
- 42Current rotation (radians) – tail, local Y direction

\*MAT\_DRY\_FABRIC

This is Material Type 214. This material model can be used to model high strength woven fabrics, such as Kevlar® 49, with transverse orthotropic behavior for use in structural systems where high energy absorption is required (Bansal et al., Naik et al., Stahlecker et al.). The major applications of the model are for the materials used in propulsion engine containment system, body armor and personal protections.

Woven dry fabrics are described in terms of two principal material directions, longitudinal warp and transverse fill yarns. The primary failure mode in these materials is the breaking of either transverse or longitudinal yarn. An equivalent continuum formulation is used and an element is designated as having failed when it reaches some critical value for strain in either directions. A linearized approximation to a typical stress-strain curve is shown in Figure M214-1, and to a typical engineering shear stress-strain curve is shown in the figure corresponding to the GABi field in the variable list. Note that the principal directions are labeled a for the warp and b for the fill, and the direction c is orthogonal to a and b.

The material model is available for membrane elements and it is recommended to use a double precision version of LS-DYNA.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	GAB1	GAB2	GAB3	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	GBC	GCA	GAMAB1	GAMAB2				
Type	F	F	F	F				

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT		XP	YP	ZP	A1	A2	A3
Type	F		F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

Card 5	1	2	3	4	5	6	7	8
Variable	EACRF	EBCRF	EACRP	EBCRP				
Type	F	F	F	F				
Remarks	2	2						

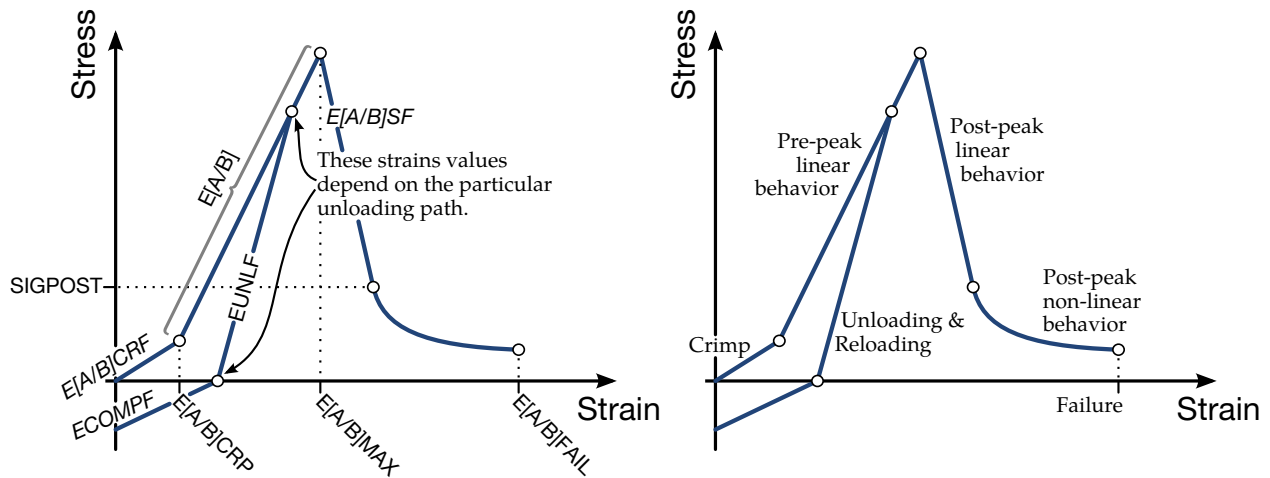
Card 6	1	2	3	4	5	6	7	8
Variable	EASF	EBSF	EUNLF	ECOMF	EAMAX	EBMAX	SIGPOST	
Type	F	F	F	F	F	F	F	
Remarks	2	2	2	2				

Card 7	1	2	3	4	5	6	7	8
Variable	CCE	PCE	CSE	PSE	DFAC	EMAX	EAFAIL	EBFAIL
Type	F	F	F	F	F	F	F	F
Remarks	1	1	1	1	3	4	4	4

**VARIABLE****DESCRIPTION**

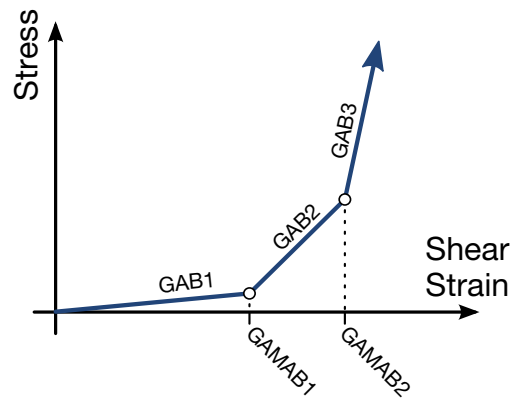
MID Material identification. A unique number or label not exceeding 8 characters must be specified.

RO Continuum equivalent mass density.



**Figure M214-1.** Stress – Strain curve for \*MAT\_DRY\_FABRIC. This curve models the force-response in the longitudinal and transverse directions.

VARIABLE	DESCRIPTION
EA	Modulus of elasticity in the longitudinal (warp) direction, which corresponds to the slope of segment AB in Figure M214-1.
EB	Modulus of elasticity in the transverse (fill) direction, which corresponds to the slope of segment of AB Figure M214-1.
GAB <sub>i</sub> / GAMAB <sub>i</sub>	Shear stress-strain behavior is modeled as piecewise linear in three segments. See the figure to the right. The shear moduli GAB <sub>i</sub> correspond to the slope of the <i>i</i> <sup>th</sup> segment. The start and end points for the segments are specified in the GAMAB[1-2] fields.
GBC	$G_{bc}$ , Shear modulus in <i>bc</i> direction.
GCA	$G_{ca}$ , Shear modulus in <i>ca</i> direction.
AOPT	Material axes option. See *MAT_OPTIONTROPIC_ELASTIC for a more complete description:



EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with \*DEFINE\_COORDINATE\_NODES, and then rotated about the element normal by an angle BETA.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
XP, YP, ZP	Components of vector <b>x</b> .
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3.
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.
EACRF	Factor for crimp region modulus of elasticity in longitudinal direction (See <a href="#">Figure M214-1</a> ): $E_{a,crimp} = E_{a,crimpfac}E, \quad E_{a,crimpfac} = EACRF$
EBCRF	Factor for crimp region modulus of elasticity in transverse direction (See <a href="#">Figure M214-1</a> ): $E_{b,crimp} = E_{b,crimpfac}E, \quad E_{b,crimpfac} = EBCRF$
EACRP	Crimp strain in longitudinal direction (See <a href="#">Figure M214-1</a> ): $\epsilon_{a,crimp}$
EBCRP	Crimp strain in transverse direction (See <a href="#">Figure M214-1</a> ): $\epsilon_{b,crimp}$



VARIABLE	DESCRIPTION
EASF	Factor for post-peak region modulus of elasticity in longitudinal direction (see <a href="#">Figure M214-1</a> ): $E_{a,\text{soft}} = E_{a,\text{softfac}}E, \quad E_{a,\text{softfac}} = \text{EASF}$
EBSF	Factor for post-peak region modulus of elasticity in transverse direction (see <a href="#">Figure M214-1</a> ): $E_{b,\text{soft}} = E_{b,\text{softfac}}E, \quad E_{b,\text{softfac}} = \text{EBSF}$
EUNLF	Factor for unloading modulus of elasticity (See <a href="#">Figure M214-1</a> ): $E_{\text{unload}} = E_{\text{unloadfac}}E, \quad E_{\text{unloadfac}} = \text{EUNLF}$
ECOMPF	Factor for compression zone modulus of elasticity (see <a href="#">Figure M214-1</a> ): $E_{\text{comp}} = E_{\text{compfac}}E, \quad E_{\text{compfac}} = \text{ECOMPF}$
EAMAX	Strain at peak stress in longitudinal direction (see <a href="#">Figure M214-1</a> ): $\varepsilon_{a,\text{max}}$
EBMAX	Strain at peak stress in transverse direction (see <a href="#">Figure M214-1</a> ): $\varepsilon_{b,\text{max}}$
SIGPOST	Stress value in post-peak region at which nonlinear behavior begins (see <a href="#">Figure M214-1</a> ): $\sigma_{\text{post}}$
CCE	Strain rate parameter $C$ , Cowper-Symonds factor for modulus. If zero, rate effects are not considered.
PCE	Strain rate parameter $P$ , Cowper-Symonds factor for modulus. If zero, rate effects are not considered.
CSE	Strain rate parameter $C$ , Cowper-Symonds factor for stress to peak / failure. If zero, rate effects are not considered.
PSE	Strain rate parameter $P$ , Cowper-Symonds factor for stress to peak / failure. If zero, rate effects are not considered.
DFAC	Damage factor: $d_{\text{fac}}$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EMAX	Erosion strain of element: $\epsilon_{\max}$
EAFAIL	Erosion strain in longitudinal direction (see <a href="#">Figure M214-1</a> ): $\epsilon_{a,\text{fail}}$
EBFAIL	Erosion strain in transverse direction (see <a href="#">Figure M214-1</a> ): $\epsilon_{b,\text{fail}}$

**Remarks:**

1. Strain rate effects are accounted for using a Cowper-Symonds model which scales the stress according to the strain rate:

$$\sigma^{\text{adj}} = \sigma \left( 1 + \frac{\dot{\epsilon}}{C} \right)^{\frac{1}{P}}$$

In the above equation  $\sigma$  is the quasi-static stress,  $\sigma^{\text{adj}}$  is the adjusted stress accounting for strain rate  $\dot{\epsilon}$ ,  $C$  (CCE) and  $P$  (PCE) are the Cowper-Symonds factors and have to be determined experimentally for each material.

The model captures the non-linear strain rate effects in many materials. With its less than unity exponent,  $1/p$ , this model captures the rapid increase in material properties at low strain rate, while increasing less rapidly at very high strain rates. Because stress is a function of strain rate the elastic stiffness also is:

$$\mathbf{E}^{\text{adj}} = \mathbf{E} \left( 1 + \frac{\dot{\epsilon}}{C} \right)^{\frac{1}{P}}$$

where  $\mathbf{E}^{\text{adj}}$  is the adjusted elastic stiffness. Additionally, the strains to peak and strains to failure are assumed to follow a Cowper-Symonds model with, *possibly different*, constants

$$\epsilon^{\text{adj}} = \epsilon \left( 1 + \frac{\dot{\epsilon}}{C_s} \right)^{\frac{1}{P_s}}$$

where,  $\epsilon^{\text{adj}}$  is the adjusted effective strain to peak stress or strain to failure, and  $C_s$  and  $P_s$  are CSE and PSE respectively.

2. When strained beyond the peak stress, the stress decreases linearly until it attains a value equal to SIGPOST, at which point the stress-strain relation becomes nonlinear. In the non-linear region the stress is given by

$$\sigma = \sigma_{\text{post}} \left[ 1 - \left( \frac{\varepsilon - \varepsilon_{[a/b],\text{post}}}{\varepsilon_{[a/b],\text{fail}} - \varepsilon_{[a/b],\text{post}}} \right)^{d_{\text{fac}}} \right]$$

where  $\sigma_{\text{post}}$  and  $\varepsilon_{\text{post}}$  are, respectively, the stress and strain demarcating the onset of nonlinear behavior. The value of SIGPOST is the same in both the transverse and longitudinal directions, whereas  $\varepsilon_{a,\text{post}}$  and  $\varepsilon_{b,\text{post}}$  depend on direction and are derived internally from EASF, EBSF, and SIGPOST. The failure strain,  $\varepsilon_{[a/b],\text{fail}}$ , specifies the onset of failure and differs in the longitudinal and transverse directions. Lastly the exponent,  $d_{\text{fac}}$ , determines the shape of nonlinear stress-strain curve between  $\varepsilon_{\text{post}}$  and  $\varepsilon_{[a/b],\text{fail}}$ .

3. The element is eroded if either (a) or (b) is satisfied:

a)  $\varepsilon_a > \varepsilon_{a,\text{fail}}$  and  $\varepsilon_b > \varepsilon_{b,\text{fail}}$

b)  $\varepsilon_a > \varepsilon_{\text{max}}$  and  $\varepsilon_b > \varepsilon_{\text{max}}$ .

**\*MAT\_4A\_MICROMECH**

This is Material Type 215. A micromechanical material that distinguishes between a fiber/inclusion and a matrix material, developed by 4a engineering GmbH. It is available for the explicit code for shell, thick shell and solid elements. Useful hints and input example can be found in [1]. More theory and application notes will be provided soon in [2].

The material is intended for anisotropic composite materials, especially for short (**SFRT**) and long fiber thermoplastics (**LFRT**). The matrix behavior is described by an isotropic elasto-viscoplastic von Mises model. The fiber/inclusion behavior is transversal isotropic elastic. This also allows to use this material model for classical endless fiber **composites**. The inelastic homogenization for describing the composite deformation behavior is based on:

- Mori Tanaka Meanfield Theory [3,4]
- ellipsoidal inclusions using Eshelby's solution [5,6]
- orientation averaging [7]
- a linear fitted closure approximation to determine the 4<sup>th</sup> order fiber orientation tensor out of the user provided 2<sup>nd</sup> order fiber orientation tensor.

The core functionality to calculate the thermo-elastic composite properties can be also found in the software product 4a micromec [8].

Failure/Damage of the composite can be currently considered by

- a ductile damage initiation and evolution model for the matrix (DIEM)
- fiber failure may be considered with a maximum stress criterion.

More details on the material characterization can be found in [9] and [10].

The (fiber) orientation can be defined either for the whole material using CARD 2 and 3 or elementwise using **\*ELEMENT\_(T)SHELL\_BETA** or **\*ELEMENT\_SOLID\_ORTHO**. The mechanical properties of SFRT and LFRT in injection molded parts are highly influenced through the manufacturing process. By mapping the fiber orientation from the process simulation to the structural analysis the local anisotropy can be considered [11,12]. The fiber orientation, length and volume fraction can therefore as well be defined for each integration point by using **\*INITIAL\_STRESS\_(T)SHELL(SOLID)** [2]. Details on the history variables that can be initialized (Extravars. 9-18) can be found in the output section.



**\*MAT\_215****\*MAT\_4A\_MICROMECH**

Card 4	1	2	3	4	5	6	7	8
Variable	FVF		FL	FD		A11	A22	
Type	F		F	F		F	F	
Default	0.0		0.0	1.0		1.0	0.0	

## Parameter for fiber/inclusion material

Card 5	1	2	3	4	5	6	7	8
Variable	ROF	EL	ET	GLT	PRTL	PRTT		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

Card 6	1	2	3	4	5	6	7	8
Variable	XT						SLIMXT	NCYRED
Type	F						F	F
Default	0.0						0.0	10

## Parameter for matrix material

Card 7	1	2	3	4	5	6	7	8
Variable	ROM	E	PR					
Type	F	F	F					
Default	0.0	0.0	0.0					

Card 8	1	2	3	4	5	6	7	8
Variable	SIGYT	ETANT			EPS0	C		
Type	F	F			F	F		
Default	0.0	0.0			0.0	0.0		

Card 9	1	2	3	4	5	6	7	8
Variable	LCIDT				LCDI	UPF		
Type	F				F	F		
Default	0				0	0.0		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
MMOPT	Option to define micromechanical material behavior EQ.0.0: elastic EQ.1.0: elastic-plastic
BUPD	Tolerance for update of Strain-Concentration Tensor

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FAILM	<p>Option for matrix failure – ductile DIEM-Model.            (see *MAT_ADD_EROSION – Damage Initiation and Evolution Criteria with ductile damage initialization (DITYP.EQ.0) based on stress triaxiality and a linear damage evolution (DETYP.EQ.0) type)</p> <p>LT.0:  FAILM  Effective plastic matrix strain at failure. When the matrix plastic strain reaches this value, the element is deleted from the calculation.</p> <p>EQ.0: only visualization (triaxiality of matrix stresses)</p> <p>EQ.1: active DIEM (triaxiality of matrix stresses)</p> <p>EQ.10: only visualization (triaxiality of composite stresses)</p> <p>EQ.11: active DIEM (triaxiality of composite stresses)</p>
FAILF	<p>Option for fiber failure</p> <p>EQ.0: only visualization (equivalent fiber stresses)</p> <p>EQ.1: active (equivalent fiber stresses)</p>
NUMINT	<p>Number of failed integration points prior to element deletion.</p> <p>LT.0.0: Only for shells.  NUMINT  is the percentage of integration points which must exceed the failure criterion before element fails. For shell formulations with 4 integration points per layer, the layer is considered failed if any of the integration points in the layer fails.</p>
<b>Parameter for fiber orientation</b>	
AOPT	See *MAT_002 (fiber orientation information may be overwritten using *INITIAL_STRESS_(T)SHELL/SOLID)
MACF	<p>Material axes change flag for solid elements:</p> <p>EQ.1: No change, default,</p> <p>EQ.2: switch material axes a and b,</p> <p>EQ.3: switch material axes a and c,</p> <p>EQ.4: switch material axes b and c.</p>
XP, YP, ZP	Define coordinates of point <b>p</b> for AOPT = 1 and 4.
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Define components of vector <b>v</b> for AOPT = 3 and 4.
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 3, may be overwritten on the element card, see *ELEMENT_(T)SHELL_BETA or *ELEMENT_SOLID_ORTHO.
FVF	Fiber-Volume-Fraction GT.0: Fiber-Volume-Fraction LT.0:  FVF  Fiber-Mass-Fraction
FL	Fiber length - if FD = 1 then FL = aspect ratio (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID)
FD	Fiber diameter (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID)
A11	Value of first principal fiber orientation (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID).
A22	Value of second principal fiber orientation (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID).

**Parameter for fiber/inclusion material**

ROF	Mass density of fiber
EL	$E_L$ , Young's modulus of fiber – longitudinal direction.
ET	$E_T$ , Young's modulus of fiber – transverse direction.
GLT	$G_{LT}$ , Shear modulus LT
PRTL	$\nu_{TL}$ , Poisson's ratio TL
PRTT	$\nu_{TT}$ , Poisson's ratio TT
XT	Fiber tensile strength – longitudinal direction.
SLIMXT	Factor to determine the minimum stress limit in the fiber after stress maximum (fiber tension)

NCYRED            Number of cycles for stress reduction from maximum to minimum (fiber tension)

**Parameter for matrix material**

ROM                Mass density of matrix.

E                    Young's modulus of matrix.

PR                  Poisson's ratio of matrix.

SIGYT              Yield stress of matrix in tension

ETANT              Tangent modulus of matrix in tension, ignore if (LCST.GT.0.) is defined.

EPS0                Quasi-static threshold strain rate (Johnson-Cook model) for bi-linear hardening

C                    Johnson-Cook constant for bi-linear hardening

LCIDT              Load curve ID or Table ID for defining effective stress versus effective plastic strain in tension of matrix material (Table to include strain-rate effects, viscoplastic formulation)

LCDI                Damage initiation parameter (ductile)

                      shells: Load curve ID representing plastic strain at onset of damage as function of stress triaxiality.  
                          or  
                          Table ID representing plastic strain at onset of damage as function of stress triaxiality and plastic strain rate.

                      solids: Load curve ID representing plastic strain at onset of damage as function of stress triaxiality.  
                          or  
                          Table ID representing plastic strain at onset of damage as function of stress triaxiality and lode angle.  
                          or  
                          Table3D ID representing plastic strain at onset of damage as function of stress triaxiality, lode angle and plastic strain rate.

UPF                      Damage evolution parameter

                            GT.0.0: plastic displacement at failure,  $u_f^p$

                            LT.0.0: |UPF| is a table ID for  $u_f^p$  as a function of triaxiality and damage

**Output:**

“Plastic Strain” is the equivalent plastic strain in the matrix.  
 Extra history variables may be requested for (t)shell (NEIPS) and solid (NEIPH) elements on \*DATABASE\_EXTENT\_BINARY. Extra history variables 1-8 are intended for post processing, 9-18 for initialization with \*INITIAL\_STRESS\_(T)SHELL/SOLID. They have the following meaning:

<b>Extravar.</b>	<b>DESCRIPTION</b>
1	effs - equivalent plastic strain rate of matrix
2	eta - triaxiality of matrix ... $\eta = -\frac{p}{q}$
3	xi - lode parameter of matrix ... $\xi = -\frac{27 \bullet J_3}{2 \bullet q}$
4	dM - Damage initiation d of matrix (Ductile Criteria)
5	DM - Damage evolution D of matrix
6	RFF - Fiber reserve factor
7	DF- Fiber damage variable
8	Currently unused

<b>Extravar.</b>	<b>DESCRIPTION</b>
9	A11 - fiber orientation first principal value
10	A22 - fiber orientation first second value
11	q1/q11
12	q2/q12
13	-/q13
14	-/q31
15	-/q32
16	-/q33

17	FVF- Fiber-Volume-Fraction
18	FL- Fiber length

**References:**

- [1] Reithofer, P., et. al, \*MAT\_4A\_MICROMECH – micro mechanic based material model, 14<sup>th</sup> German LS-DYNA Conference (2016), Bamberg
- [2] Reithofer, P., et. al, \*MAT\_4A\_MICROMECH – Theory and application notes, 11<sup>th</sup> European LS-DYNA Conference (2017), Salzburg
- [3] Mori, T., Tanaka, K., Average Stress in Matrix and Average elastic Energy of Materials with misfitting Inclusions, Acta Metallurgica, Vol.21, pp.571-574, (1973).
- [4] Tucker Ch. L. III, Liang Erwin: Stiffness Predictions for Unidirectional Short-Fibre Composites: Review and Evaluation, Composites Science and Technology, 59, (1999)
- [5] Maewal A., Dandekar D.P.: Effective Thermoelastic Properties of Short-Fibre Composites, Acta Mechanica, 66, (1987)
- [6] Eshelby, J. D., The determination of the elastic field of an ellipsoidal inclusion, and related problems, Proceedings of the Royal Society, London, Vol.A, No241, pp.376-396, (1957).
- [7] Mlekusch, B., Kurzfaserverstärkte Thermoplaste, Dissertation, Montanuniversität Leoben (1997)
- [8] <http://micromec.4a.co.at>
- [9] Reithofer, P. et. al: *Material characterization of composites using micro mechanic models as key enabler*, NAFEMS DACH, Bamberg 2016
- [10] <http://impetus.4a.co.at>
- [11] Reithofer, P. et. al: *Short and long fiber reinforced thermoplastics material models in LS-DYNA*, 10th European LS-DYNA Conference, Würzburg 2015
- [12] <http://fibermap.4a.co.at>



<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
$RO_i$	Mass density for phase $i$ .
$E_i$	Young's modulus for phase $i$ .
$PR_i$	Poisson's ratio for phase $i$ .
X1, Y1, Z1	Coordinates of a point on the phase transition plane.
X2, Y2, Z2	Coordinates of a point that defines the exterior normal with the first point.
THKFAC	Scale factor applied to the shell thickness after the phase transformation.

**Phases:**

The material properties for each element are initialized using the data for the first phase. After the center of the element passes through the transition plane defined by the two points, the material properties are irreversibly changed to the second phase.

The plane is defined by two points. The first point, defined by the coordinates X1, Y1, and Z1, lies on the plane. The second point, defined by the coordinates X2, Y2, and Z2, define the exterior normal as a unit vector in the direction from the first point to the second point.

**Remarks:**

This hypoelastic material model may not be stable for finite (large) strains. If large strains are expected, a hyperelastic material model, e.g., \*MAT\_002 or \*MAT\_217, would be more appropriate.

**\*MAT\_OPTIONTROPIC\_ELASTIC\_PHASE\_CHANGE**

This is Material Type 217 a generalization of Material Type 2 for which material properties change on an element-by-element basis upon crossing a plane in space.

This material is valid *only* for shells. The stress update is incremental and the elastic constants are formulated in terms of Cauchy stress and true strain.

Available options include:

ORTHO

ANISO

such that the keyword cards appear:

\*MAT\_ORTHOTROPIC\_ELASTIC\_PHASE\_CHANGE or MAT\_217 (9 cards follow)

\*MAT\_ANISOTROPIC\_ELASTIC\_PHASE\_CHANGE or MAT\_217\_ANIS (11 cards follow)

**Orthotropic Card 1 (phase 1).** Card 1 for ORTHO keyword option for phase 1.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R01	EA	EB	EC	PRBA	PRCA	PRCB
Type	A8	F	F	F	F	F	F	F

**Orthotropic Card 2 (phase 1).** Card 2 for ORTHO keyword option for phase 1.

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT1	G	SIGF		
Type	F	F	F	F	F	F		

**Anisotropic Card 1 (phase 1).** Card 1 for ANISO keyword option for phase 1.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R02	C111	C121	C221	C131	C231	C331
Type	A8	F	F	F	F	F	F	F

**Anisotropic Card 2 (phase 1).** Card 2 for ANISO keyword option for phase 1.

Card 2	1	2	3	4	5	6	7	8
Variable	C141	C241	C341	C441	C151	C251	C351	C451
Type	F	F	F	F	F	F	F	F

**Anisotropic Card 3 (phase 1).** Card 3 for ANISO keyword option for phase 1.

Card 3	1	2	3	4	5	6	7	8
Variable	C551	C161	C261	C361	C461	C561	C661	AOPT1
Type	F	F	F	F	F	F	F	F

**Local Coordinate System Card 1 (phase 1).** Required for all keyword options

Card 4	1	2	3	4	5	6	7	8
Variable	XP1	YP1	ZP1	A11	A21	A31	MACF	IHIS
Type	F	F	F	F	F	F	I	F

**Local Coordinate System Card 2 (phase 1).** Required for all keyword options

Card 5	1	2	3	4	5	6	7	8
Variable	V11	V21	V31	D11	D21	D31	BETA1	REF
Type	F	F	F	F	F	F	F	F





**Anisotropic Card 6 (phase 2).** Card 3 for ANISO keyword option for phase 2.

Card 8	1	2	3	4	5	6	7	8
Variable	C552	C162	C262	C362	C462	C562	C662	
Type	F	F	F	F	F	F	F	

**Local Coordinate System Card 1 (phase 2).** Required for all keyword options

Card 9	1	2	3	4	5	6	7	8
Variable	XP2	YP2	ZP2	A12	A22	A32	MACF2	
Type	F	F	F	F	F	F	I	

**Local Coordinate System Card 2 (phase 2).** Required for all keyword options

Card 10	1	2	3	4	5	6	7	8
Variable	V12	V22	V32	D12	D22	D32	BETA2	
Type	F	F	F	F	F	F	F	

**Definition of transformation plane Card.**

Card 11	1	2	3	4	5	6	7	8
Variable	X1	Y1	Z1	X2	Y2	Z2	THKFAC	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	1.0	

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label not exceeding 8 characters must be specified.

RO*i*            Mass density for phase *i*.

**Define for the ORTHO option only:**

E*Ai*             $E_a$ , Young's modulus in *a*-direction for phase *i*.  
E*Bi*             $E_b$ , Young's modulus in *b*-direction for phase *i*.  
E*Ci*             $E_c$ , Young's modulus in *c*-direction phase *i* (nonzero value required but not used for shells).  
PR*BAi*         $\nu_{ba}$ , Poisson's ratio in the *ba* direction for phase *i*.  
PR*CAi*         $\nu_{ca}$ , Poisson's ratio in the *ca* direction for phase *i*.  
PR*CBi*         $\nu_{cb}$ , Poisson's ratio in the *cb* direction for phase *i*.  
G*ABi*         $G_{ab}$ , shear modulus in the *ab* direction for phase *i*.  
G*BCi*         $G_{bc}$ , shear modulus in the *bc* direction for phase *i*.  
G*CAi*         $G_{ca}$ , shear modulus in the *ca* direction for phase *i*.

**Due to symmetry define the upper triangular Cij's for the ANISO option only:**

C11*i*            The 1,1 term in the  $6 \times 6$  anisotropic constitutive matrix for phase *i*.  
Note that 1 corresponds to the *a* material direction  
C12*i*            The 1,2 term in the  $6 \times 6$  anisotropic constitutive matrix for phase *i*.  
Note that 2 corresponds to the *b* material direction  
:  
:  
C66*i*            The 6,6 term in the  $6 \times 6$  anisotropic constitutive matrix for phase *i*.

**Define AOPT for both options:**

AOPT*i*            Material axes option for phase *i*, see [Figure M2-1](#).

EQ.0.0: locally orthotropic with material axes determined by element nodes as shown in part (a) of [Figure M2-1](#). The *a*-direction is from node 1 to node 2 of the element. The *b*-direction is orthogonal to the *a*-direction and is in the plane formed by nodes 1, 2, and 4. When this option is used in two-dimensional planar and axisymmetric analysis, it is critical that the nodes in the element definition be numbered counterclockwise for this option to work cor-

rectly.

EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only.

EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with \*DEFINE\_COORDINATE\_VECTOR.

EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.

EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, **P**, which define the centerline axis. This option is for solid elements only.

LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on \*DEFINE\_COORDINATE\_NODES, \*DEFINE\_COORDINATE\_SYSTEM or \*DEFINE\_COORDINATE\_VECTOR). Available in R3 version of 971 and later.

G Shear modulus for frequency independent damping. Frequency independent damping is based of a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250-1000 times greater than SIGF. This option applies only to solid elements.

SIGF Limit stress for frequency independent, frictional, damping.

XPi, YPi, ZPi Define coordinates of the *i*<sup>th</sup> phase's point **p** for AOPT = 1 and 4.

A1i, A2i, A3i Define components of the *i*<sup>th</sup> phase's vector **a** for AOPT = 2.

MACFi Material axes change flag for brick elements in phase *i*:

EQ.1: No change, default,

EQ.2: switch material axes *a* and *b*,

EQ.3: switch material axes  $a$  and  $c$ ,  
EQ.4: switch material axes  $b$  and  $c$ .

IHIS	Flag for anisotropic stiffness terms initialization (for solid elements only). EQ.0: C11, C12, ... from Cards 1, 2, and 3 are used. EQ.1: C11, C12, ... are initialized by *INITIAL_STRESS_SOLID's history data.
V1i, V2i, V3i	Define components of the $i^{\text{th}}$ phase's vector $\mathbf{v}$ for AOPT = 3 and 4.
D1i, D2i, D3i	Define components of the $i^{\text{th}}$ phase's vector $\mathbf{d}$ for AOPT = 2.
BETAi	Material angle of $i^{\text{th}}$ phase in degrees for AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO.
REFi	Use reference geometry to initialize the stress tensor for the $i^{\text{th}}$ phase. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.
X1, Y1, Z1	Coordinates of a point on the phase transition page.
X2, Y2, Z2	Coordinates of a point that defines the exterior normal with the first point.
THKFAC	Scale factor applied to the shell thickness after the phase transformation.

**Phases:**

The material properties for each element are initialized using the data for the first phase. After the center of the element passes through the transition plane defined by the two points, the material properties are irreversibly changed to the second phase.

The plane is defined by two points. The first point, defined by the coordinates X1, Y1, and Z1, lies on the plane. The second point, defined by the coordinates X2, Y2, and Z2, define the exterior normal as a unit vector in the direction from the first point to the second point.

**Material Formulation:**

The material law that relates stresses to strains is defined as:

$$\mathbf{C} = \mathbf{T}^T \mathbf{C}_L \mathbf{T}$$

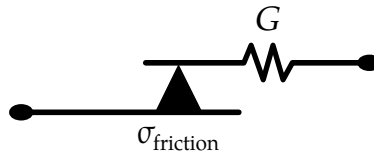
where  $\mathbf{T}$  is a transformation matrix, and  $\mathbf{C}_L$  is the constitutive matrix defined in terms of the material constants of the orthogonal material axes,  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ . The inverse of  $\mathbf{C}_L$  for the orthotropic case is defined as:

$$\mathbf{C}_L^{-1} = \begin{bmatrix} \frac{1}{E_a} & -\frac{\nu_{ba}}{E_b} & -\frac{\nu_{ca}}{E_c} & 0 & 0 & 0 \\ -\frac{\nu_{ab}}{E_a} & \frac{1}{E_b} & -\frac{\nu_{cb}}{E_c} & 0 & 0 & 0 \\ -\frac{\nu_{ac}}{E_a} & -\frac{\nu_{bc}}{E_b} & \frac{1}{E_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{ab}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{bc}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{ca}} \end{bmatrix}$$

Where,

$$\frac{\nu_{ab}}{E_a} = \frac{\nu_{ba}}{E_b}, \frac{\nu_{ca}}{E_c} = \frac{\nu_{ac}}{E_a}, \frac{\nu_{cb}}{E_c} = \frac{\nu_{bc}}{E_b}.$$

The frequency independent damping is obtained by having a spring and slider in series as shown in the following sketch:



This option applies only to orthotropic solid elements and affects only the deviatoric stresses.

The procedure for describing the principle material directions is explained for solid and shell elements for this material model and other anisotropic materials. We will call the material direction the  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$  coordinate system. The AOPT options illustrated in [Figure M2-1](#) can define the  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$  system for all elements of the parts that use the material, but this is not the final material direction. There  $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$  system defined by the AOPT options may be offset by a final rotation about the  $\mathbf{c}$ -axis. The offset angle we call BETA.

For solid elements, the BETA angle is specified in one of two ways. When using AOPT = 3, the BETA parameter defines the offset angle for all elements that use the material. The BETA parameter has no meaning for the other AOPT options. Alternatively, a BETA angle can be defined for individual solid elements as described in remark 5 for \*ELEMENT\_SOL-

ID\_ORTHO. The beta angle by the ORTHO option is available for all values of AOPT, and it overrides the BETA angle on the \*MAT card for AOPT = 3.

The directions determined by the material AOPT options may be overridden for individual elements as described in remark 3 for \*ELEMENT\_SOLID\_ORTHO. However, be aware that for materials with AOPT = 3, the final {**a**, **b**, **c**} system will be the system defined on the element card rotated about **c**-axis by the BETA angle specified on the \*MAT card.

There are two fundamental differences between shell and solid element orthotropic materials. First, the **c**-direction is always normal to a shell element such that the **a**-direction and **b**-directions are within the plane of the element. Second, for some anisotropic materials, shell elements may have unique fiber directions within each layer through the thickness of the element so that a layered composite can be modeled with a single element.

When AOPT = 0 is used in two-dimensional planar and axisymmetric analysis, it is critical that the nodes in the element definition be numbered counterclockwise for this option to work correctly.

Because shell elements have their **c**-axes defined by the element normal, AOPT = 1 and AOPT = 4 are not available for shells. Also, AOPT = 2 requires only the vector **a** be defined since **d** is not used. The shell procedure projects the inputted **a**-direction onto each element surface.

Similar to solid elements, the {**a**, **b**, **c**} coordinate system determined by AOPT is then modified by a rotation about the **c**-axis which we will call  $\phi$ . For those materials that allow a unique rotation angle for each integration point through the element thickness, the rotation angle is calculated by

$$\phi_i = \beta + \beta_i$$

where  $\beta$  is a rotation for the element, and  $\beta_i$  is the rotation for the  $i$ 'th layer of the element. The  $\beta$  angle can be input using the BETA parameter on the \*MAT data, or will be overridden for individual elements if the BETA keyword option for \*ELEMENT\_SHELL is used. The  $\beta_i$  angles are input using the ICOMP = 1 option of \*SECTION\_SHELL or with \*PART\_COMPOSITE. If  $\beta$  or  $\beta_i$  is omitted, they are assumed to be zero.

All anisotropic shell materials have the BETA option on the \*MAT card available for both AOPT = 0 and AOPT = 3, except for materials 91 and 92 which have it available for all values of AOPT, 0, 2, and 3.

All anisotropic shell materials allow an angle for each integration point through the thickness,  $\beta_i$ , except for materials 2, 86, 91, 92, 117, 130, 170, 172, and 194.

This discussion of material direction angles in shell elements also applies to thick shell elements which allow modeling of layered composites using \*INTEGRATION\_SHELL or \*PART\_COMPOSITE\_TSHELL.

# \*MAT\_218

# \*MAT\_MOONEY-RIVLIN\_PHASE\_CHANGE

## \*MAT\_MOONEY-RIVLIN\_PHASE\_CHANGE

This is Material Type 218, a generalization of Material Type 27, for which material properties change on an element-by-element basis upon crossing a plane in space.

### Phase 1 Card 1.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R01	PR1	A1	B1	REF		
Type	A8	F	F	F	F	F		

### Phase 1 Card 2.

Card 2	1	2	3	4	5	6	7	8
Variable	SGL1	SW1	ST1	LCID1				
Type	F	F	F	F				

### Phase 2 Card 1.

Card 3	1	2	3	4	5	6	7	8
Variable		R02	PR2	A2	B2			
Type		F	F	F	F			

### Phase 2 Card 2.

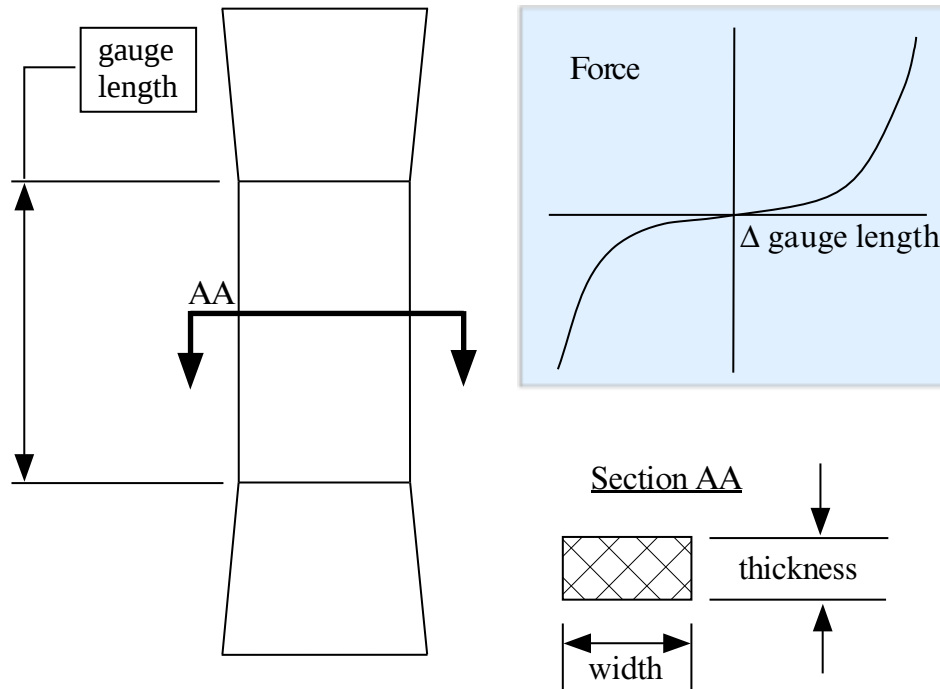
Card 4	1	2	3	4	5	6	7	8
Variable	SGL2	SW2	ST2	LCID2				
Type	F	F	F	F				



Transformation Plane Card.

Card 5	1	2	3	4	5	6	7	8
Variable	X1	Y1	Z1	X2	Y2	Z2	THKFAC	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	1.0	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO <sub><i>i</i></sub>	Mass density for phase <i>i</i> .
PR <sub><i>i</i></sub>	Poisson's ratio (value between 0.49 and 0.5 is recommended, smaller values may not work) where <i>i</i> indicates the phase.
A <sub><i>i</i></sub>	Constant for the <i>i</i> <sup>th</sup> phase, see literature and equations defined below.
B <sub><i>i</i></sub>	Constant for the <i>i</i> <sup>th</sup> phase, see literature and equations defined below.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword:*INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: off, EQ.1.0: on.



**Figure M218-1.** Uniaxial specimen for experimental data

If  $A = B = 0.0$ , then a least square fit is computed from tabulated uniaxial data via a load curve. The following information should be defined:

VARIABLE	DESCRIPTION
$SGL_i$	Specimen gauge length $l_0$ for the $i^{\text{th}}$ phase, see <a href="#">Figure M218-1</a> .
$SW_i$	Specimen width for the $i^{\text{th}}$ phase, see <a href="#">Figure M218-1</a> .
$ST_i$	Specimen thickness for the $i^{\text{th}}$ phase, see <a href="#">Figure M218-1</a> .
$LCID_i$	Curve ID for the $i^{\text{th}}$ phase, see *DEFINE_CURVE, giving the force versus actual change $\Delta L$ in the gauge length. See also <a href="#">Figure M218-2</a> for an alternative definition.
$X1, Y1, Z1$	Coordinates of a point on the phase transition plane.
$X2, Y2, Z2$	Coordinates of a point that defines the exterior normal with the first point.
THKFAC	Scale factor applied to the shell thickness after the phase transformation.

**Phases:**

The material properties for each element are initialized using the data for the first phase. After the center of the element passes through the transition plane defined by the two points, the material properties are irreversibly changed to the second phase.

The plane is defined by two points. The first point, defined by the coordinates  $X1$ ,  $Y1$ , and  $Z1$ , lies on the plane. The second point, defined by the coordinates  $X2$ ,  $Y2$ , and  $Z2$ , define the exterior normal as a unit vector in the direction from the first point to the second point.

**Material Formulation:**

The strain energy density function is defined as:

$$W = A(I - 3) + B(II - 3) + C(III^{-2} - 1) + D(III - 1)^2$$

where

$$C = 0.5 A + B$$

$$D = \frac{A(5\nu - 2) + B(11\nu - 5)}{2(1 - 2\nu)}$$

$$\nu = \text{Poisson's ratio}$$

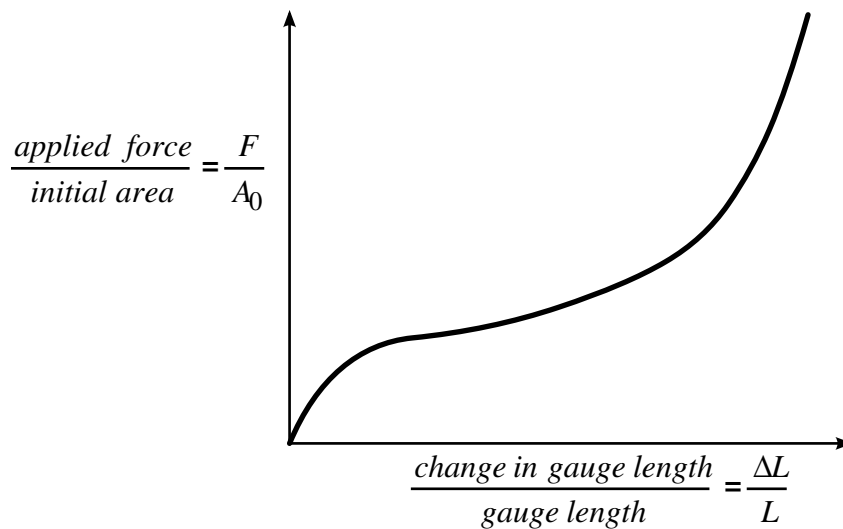
$$2(A + B) = \text{shear modulus of linear elasticity}$$

$$I, II, III = \text{invariants of right Cauchy-Green Tensor } C.$$

The load curve definition that provides the uniaxial data should give the change in gauge length,  $\Delta L$ , versus the corresponding force. In compression both the force and the change in gauge length must be specified as negative values. In tension the force and change in gauge length should be input as positive values. The principal stretch ratio in the uniaxial direction,  $\lambda_1$ , is then given by

$$\lambda_1 = \frac{L_0 + \Delta L}{L_0}$$

with  $L_0$  being the initial length and  $L$  being the actual length.



**Figure M218-2** The stress versus strain curve can be used instead of the force versus the change in the gauge length by setting the gauge length, thickness, and width to unity (1.0) and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force. \*MAT\_077\_O is a better alternative for fitting data resembling the curve above. \*MAT\_027 will provide a poor fit to a curve that exhibits a strong upturn in slope as strains become large.

Alternatively, the stress versus strain curve can also be input by setting the gauge length, thickness, and width to unity (1.0) and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force, see [Figure M218-1](#).

The least square fit to the experimental data is performed during the initialization phase and a comparison between the fit and the actual input is provided in the d3hsp file. It is a good idea to visually check to make sure it is acceptable. The coefficients *A* and *B* are also printed in the output file. It is also advised to use the material driver (see Appendix K) for checking out the material model.





Card 8	1	2	3	4	5	6	7	8
Variable	ERODE	ERPAR1	ERPAR2	RESIDS				
Type	I	F	F	F				
Default	0	none	none	0				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
EA	$E_a$ , Young's modulus in $a$ -direction. This is the modulus along the direction of fibers.
EB	$E_b$ , Young's modulus in $b$ -direction. This is the modulus transverse to fibers.
PRBA	$\nu_{ba}$ , Poisson's ratio, $ba$ (minor in-plane Poisson's ratio).
PRCB	$\nu_{cb}$ , Poisson's ratio, $cb$ (Poisson's ratio in the plane of isotropy).
GAB	$G_{ba}$ , Shear modulus, $ab$ (in-plane shear modulus).
NLAYER	Number of layers in the sub-laminate excluding symmetry. As an example, in a $[0/45/-45/90]_{3s}$ , NLAYER = 4.
R1	Non-local averaging radius.
R2	Currently not used.
NFREQ	Number of time steps between update of neighbor list for nonlocal smoothing. EQ.0: Do only one search at the start of the calculation
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <i>a</i>-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>\mathbf{p}</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</p>
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3 and 4.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA be overridden on the element card, see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO.



VARIABLE	DESCRIPTION
MACF	Material axes change flag for brick elements: EQ.1: No change, default, EQ.2: switch material axes $a$ and $b$ , EQ.3: switch material axes $a$ and $c$ , EQ.4: switch material axes $b$ and $c$ .
ANGLEi	Rotation angle in degrees of layers with respect to the material axes. Input one for each layer.
IMATT	Initiation strain for damage in matrix (transverse) under tensile condition.
IFIBT	Initiation strain for damage in the fiber (longitudinal) under tensile condition.
ILOCT	Initiation strain for the anti-locking mechanism. This parameter should be equal to the saturation strain for the fiber damage mechanism under tensile condition.
IDELT	Not working in the current version. Can be used for visualization purpose only.
SMATT	Saturation strain for damage in matrix (transverse) under tensile condition.
SFIBT	Saturation strain for damage in the fiber (longitudinal) under tensile condition.
SLOCT	Saturation strain for the anti-locking mechanism under tensile condition. The recommended value for this parameter is (ILOCT+0.02).
SDELT	Not working in the current version. Can be used for visualization purpose only.
IMATC	Initiation strain for damage in matrix (transverse) under compressive condition.
IFIBC	Initiation strain for damage in the fiber (longitudinal) under compressive condition.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ILOCC	Initiation strain for the anti-locking mechanism. This parameter should be equal to the saturation strain for the fiber damage mechanism under compressive condition.
IDELC	Initiation strain for delamination. Not working in the current version. Can be used for visualization purpose only.
SMATC	Saturation strain for damage in matrix (transverse) under compressive condition.
SFIBC	Saturation strain for damage in the fiber (longitudinal) under compressive condition.
SLOCC	Saturation strain for the anti-locking mechanism under compressive condition. The recommended value for this parameter is (ILOCC + 0.02).
SDELIC	Delamination strain. Not working in the current version. Can be used for visualization purpose only.
ERODE	Erosion Flag (see remarks) EQ.0: Erosion is turned off. EQ.1: Non-local strain based erosion criterion. EQ.2: Local strain based erosion criterion. EQ.3: Use both ERODE = 1 and ERODE = 2 criteria.
ERPAR1	The erosion parameter #1 used in ERODE types 1 and 3. ERPAR1 >= 1.0 and the recommended value is ERPAR1 = 1.2.
ERPAR2	The erosion parameter #2 used in ERODE types 2 and 3. The recommended value is five times SLOCC defined in cards 7 and 8.
RESIDS	Residual strength for layer damage

**Model Description:**

CODAM2 is developed for modeling the nonlinear, progressive damage behavior of laminated fiber-reinforced plastic materials. The model is based on the work by (Forghani, 2011; Forghani et al. 2011a; Forghani et al. 2011b) and is an extension of the original model, CODAM (Williams et al. 2003).

Briefly, the model uses a continuum damage mechanics approach and the following assumptions have been made in its development:

1. The material is an orthotropic medium consisting of a number of repeating units through the thickness of the laminate, called sub-laminates. e.g.  $[0/\pm 45/90]$  in a  $[0/\pm 45/90]_{8S}$  laminate.
2. The nonlinear behavior of the composite sub-laminate is only caused by damage evolution. Nonlinear elastic or plastic deformations are not considered.

### Formulation:

The in-plane secant stiffness of the damaged laminate is represented as the summation of the effective contributions of the layers in the laminate as shown.

$$\mathbf{A}^d = \sum t_k \mathbf{T}_k^T \mathbf{Q}_k^d \mathbf{T}_k$$

where  $\mathbf{T}_k$  is the transformation matrix for the strain vector, and  $\mathbf{Q}_k^d$  is the in-plane secant stiffness of  $k^{\text{th}}$  layer in the principal orthotropic plane, and  $t_k$  is the thickness of the  $k^{\text{th}}$  layer of an  $n$ -layered laminate.

A physically-based and yet simple approach has been employed here to derive the damaged stiffness matrix. Two reduction coefficients,  $R_f$  and  $R_m$ , that represent the reduction of stiffness in the longitudinal (fiber) and transverse (matrix) directions have been employed. The shear modulus has also been reduced by the matrix reduction parameter. The major and minor Poisson's ratios have been reduced by  $R_f$  and  $R_m$  respectively. A sub-laminate-level reduction,  $R_L$ , is incorporated to avoid spurious stress locking in the damaged zone. This would lead to an effective reduced stiffness matrix  $\mathbf{Q}_k^d$ . The reduction coefficients are equal to 1 in the undamaged condition and gradually decrease to 0 for a saturated damage condition.

$$\mathbf{Q}_k^d = R_L \begin{bmatrix} \frac{(R_f)_k E_1}{1 - (R_f)_k (R_m)_k \nu_{12} \nu_{21}} & \frac{(R_f)_k (R_m)_k \nu_{12} E_2}{1 - (R_f)_k (R_m)_k \nu_{12} \nu_{21}} & 0 \\ \frac{(R_f)_k (R_m)_k \nu_{12} E_2}{1 - (R_f)_k (R_m)_k \nu_{12} \nu_{21}} & \frac{(R_m)_k E_2}{1 - (R_f)_k (R_m)_k \nu_{12} \nu_{21}} & 0 \\ 0 & 0 & (R_m)_k G_{12} \end{bmatrix} = \mathbf{Q}_k^{dT}$$

where  $E_1$ ,  $E_2$ ,  $\nu_{12}$ ,  $\nu_{21}$ , and  $G_{12}$  are the elastic constants of the lamina.

**Damage Evolution:**

In CODAM2, the evolution of damage mechanisms are expressed in terms of equivalent strain parameters. The equivalent strain function that governs the fiber stiffness reduction parameter is written in terms of the longitudinal normal strains by

$$\varepsilon_{f,k}^{\text{eq}} = \varepsilon_{11,k}, \quad k = 1, \dots, n$$

The equivalent strain function that governs the matrix stiffness reduction parameter is written in an interactive form in terms of the transverse and shear components of the local strain.

$$\varepsilon_{m,k}^{\text{eq}} = \text{sign}(\varepsilon_{22,k}) \sqrt{(\varepsilon_{22,k})^2 + \left(\frac{\gamma_{12,k}}{2}\right)^2}, \quad k = 1, \dots, n$$

The sign of the transverse normal strain plays a very important role in the initiation and growth of damage since it indicates the compressive or tensile nature of the transverse stress. Therefore, the equivalent strain for the matrix damage carries the sign of the transverse normal strain.

Evolution of the overall damage mechanism (anti-locking) is written in terms of the maximum principal strains.

$$\varepsilon_L^{\text{eq}} = \max[\text{prn}(\varepsilon)]$$

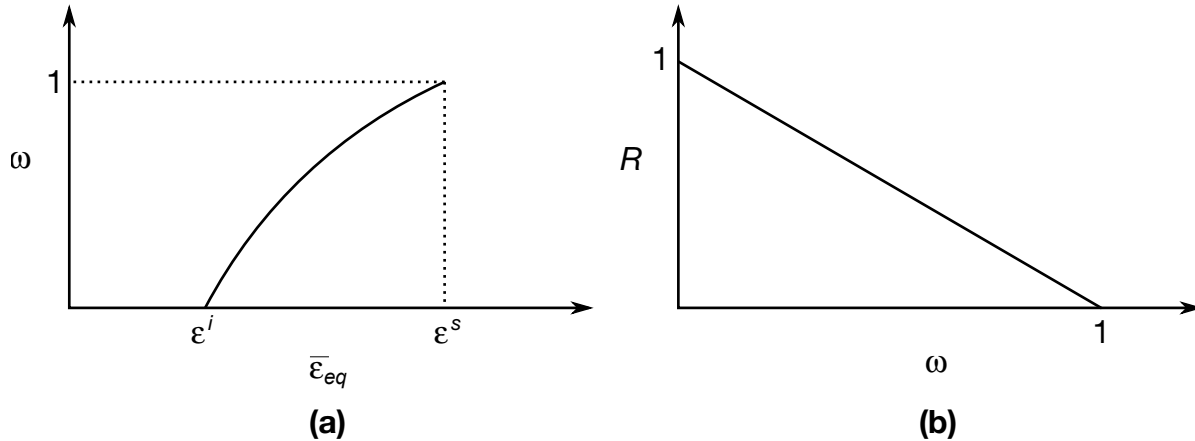
Within the framework of non-local strain-softening formulations adopted here, all damage modes, be it intra-laminar (i.e. fiber and matrix damage) or overall sub-laminate modes are considered to be a function of the non-local (averaged) equivalent strain defined as:

$$\bar{\varepsilon}_\alpha^{\text{eq}} = \int_{\Omega_x} \varepsilon_\alpha^{\text{eq}}(\mathbf{x}) w_\alpha(\mathbf{X} - \mathbf{x}) d\Omega$$

where the subscript  $\alpha$  denotes the mode of damage: fiber ( $\alpha = f$ ) and matrix ( $\alpha = m$ ) damage in each layer,  $k$ , within the sub-laminate or associated with the overall sub-laminate, namely, locking ( $\alpha = L$ ). Thus, for a given sub-laminate with  $n$  layers,  $\varepsilon_\alpha^{\text{eq}}$  and  $\bar{\varepsilon}_\alpha^{\text{eq}}$  are vectors of size  $2n + 1$ .  $\mathbf{X}$  represents the position vector of the original point of interest and  $\mathbf{x}$  denotes the position vector of all other points (Gauss points) in the averaging zone denoted by  $\Omega$ . In classical isotropic non-local averaging approach, this zone is taken to be spherical (or circular in 2D) with a radius of  $r$  (named R1 in the material input card). The parameter,  $r$ , which affects the size of the averaging zone, introduces a length scale into the model that is linked directly to the predicted size of the damage zone. Averaging is done with a bell-shaped weight function,  $w_\alpha$ , evaluated by

$$w_\alpha = \left[ 1 - \left(\frac{d}{r}\right)^2 \right]^2$$

where  $d$  is the distance from the integration point of interest to another integration point with the averaging zone.



**Figure M219-1.** illustrations of (a) damage parameter and (b) reduction parameter.

The damage parameters,  $\omega$ , are calculated as a function of the corresponding averaged equivalent strains. In CODAM2 the damage parameters are assumed to grow as a hyperbolic function of the damage potential (non-local equivalent strains) such that when used in conjunction with stiffness reduction factors that vary linearly with the damage parameters they result in a linear strain-softening response (or a bilinear stress-strain curve) for each mode of damage

$$\omega_{\alpha} = \frac{(|\bar{\varepsilon}_{\alpha}^{\text{eq}}| - \varepsilon_{\alpha}^i) \varepsilon_{\alpha}^s}{(\varepsilon_{\alpha}^s - \varepsilon_{\alpha}^i) |\bar{\varepsilon}_{\alpha}^{\text{eq}}|}, \quad |\bar{\varepsilon}_{\alpha}^{\text{eq}}| - \varepsilon_{\alpha}^i > 0$$

where superscripts  $i$  and  $s$  denote, respectively, the damage initiation and saturation values of the strain quantities to which they are assigned. The initiation and saturation parameters are defined in material cards #6 and #7. Damage is considered to be a monotonically increasing function of time,  $t$ , such that

$$\omega_{\alpha} = \max_{\tau < t} (\omega_{\alpha}^{\tau})$$

where  $\omega_{\alpha}^t$  is the value of  $\omega_{\alpha}$  for the current time (load state), and  $\omega_{\alpha}^{\tau}$  represents the state of damage at previous times  $\tau \leq t$ .

Damage is applied by scaling the layer stress by reduction parameters

$$R_{\alpha} = 1 - \omega_{\alpha}$$

where  $\alpha = f$  and  $\alpha = m$ . The layer stresses are summed and then then scaled by reduction parameter

$$R_L = 1 - \omega_L.$$

Figures M219-1 (a) and (b) show the relationship between the damage and reduction parameters

If the parameter RESIDS > 0, damage in the layers is limited such that

$$R_f = \max(\text{RESIDS}, 1 - \omega_f)$$

$$R_m = \max(\text{RESIDS}, 1 - \omega_m)$$

**Element Erosion:**

When  $\text{ERODE} > 0$ , an erosion criterion is checked at each integration point. Shell elements and thick shell elements will be deleted when the erosion criterion has been met at all integration points. Brick elements will be deleted when the erosion criterion is met at any of the integration points. For  $\text{ERODE} = 1$ , the erosion criterion is met when maximum principal strain exceeds either  $\text{SLOCT} \times \text{ERPAR1}$  for elements in tension, or  $\text{SLOCC} \times \text{ERPAR1}$  for elements in compression. Elements are in tension when the magnitude of the first principal strain is greater than the magnitude of the third principal strain and in compression when the third principal strain is larger. When  $R > 0$ , the  $\text{ERODE} = 1$  criterion is checked using the non-local (averaged) principal strain. For  $\text{ERODE} = 2$ , the erosion criterion is met when the local (non-averaged) maximum principal strain exceeds  $\text{ERPAR2}$ . For  $\text{ERODE} = 3$ , both of these erosion criteria are checked. For visualization purposes, the ratio of the maximum principal strain over the limit is stored in the location of plastic strain which is written by default to the `elout` and `d3plot` files.

**History Variables:**

History variables for CODAM2 are enumerated in the following tables. To include them in the D3PLOT database, use NEIPH (bricks) or NEIPS (shells) on `*DATABASE_EXTENT_BINARY`. For brick elements, add 4 to the variable numbers in the table because the first 6 history variables are reserved.

*Damage parameters*

<b>VARIABLE #</b>	<b>DESCRIPTION</b>
3	Overall (anti-locking) Damage.
4	Delamination Damage (for visualization only)
5	Fiber damage in the first layer
6	Matrix damage in the first layer
7	Fiber damage in the second layer
8	Matrix damage in the second layer
⋮	⋮
$3 + 2 \times \text{NLAYER}$	Fiber damage in the last layer
$4 + 2 \times \text{NLAYER}$	Matrix damage in the last layer

*Equivalent Strains used to evaluate damage (averaged if  $R1 > 0$ )*

<b>VARIABLE #</b>	<b>DESCRIPTION</b>
$5 + 2 \times \text{NLAYER}$	$\varepsilon_R^{\text{eq}}$
$6 + 2 \times \text{NLAYER}$	$\varepsilon_{f,1}^{\text{eq}}$
$7 + 2 \times \text{NLAYER}$	$\varepsilon_{m,1}^{\text{eq}}$
$8 + 2 \times \text{NLAYER}$	$\varepsilon_{f,2}^{\text{eq}}$
$9 + 2 \times \text{NLAYER}$	$\varepsilon_{m,2}^{\text{eq}}$
⋮	⋮
$4 + 4 \times \text{NLAYER}$	$\varepsilon_{f,n}^{\text{eq}}$
$5 + 4 \times \text{NLAYER}$	$\varepsilon_{m,n}^{\text{eq}}$

*Total Strain*

<b>VARIABLE #</b>	<b>DESCRIPTION</b>
$6 + 4 \times \text{NLAYER}$	$\varepsilon_x$
$7 + 4 \times \text{NLAYER}$	$\varepsilon_y$
$8 + 4 \times \text{NLAYER}$	$\varepsilon_z$
$9 + 4 \times \text{NLAYER}$	$\gamma_{xy}$
$10 + 4 \times \text{NLAYER}$	$\gamma_{yz}$
$11 + 4 \times \text{NLAYER}$	$\gamma_{zx}$



**\*MAT\_RIGID\_DISCRETE**

This is Material Type 220, a rigid material for shells or solids. Unlike \*MAT\_020, a \*MAT\_220 part can be discretized into multiple disjoint pieces and have each piece behave as an independent rigid body. The inertia properties for the disjoint pieces are determined directly from the finite element discretization.

Nodes of a \*MAT\_220 part cannot be shared by any other rigid part. A \*MAT\_220 part may share nodes with deformable structural and solid elements.

This material option can be used to model granular material where the grains interact through an automatic single surface contact definition. Another possible use includes modeling bolts as rigid bodies where the bolts belong to the same part ID. This model eliminates the need to represent each rigid piece with a unique part ID.

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A8	F	F	F				
Default	none	none	none	none				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.





**\*MAT\_221****\*MAT\_ORTHOTROPIC\_SIMPLIFIED\_DAMAGE**

Card 8	1	2	3	4	5	6	7	8
Variable	EPSC2C	CDAM2C	EPSD3C	EPSC3C	CDAM3C	EPSD12	EPSC12	CDAM12
Type	F	F	F	F	F	F	F	F
Default	0.	0.	0.	0.	0.	0.	0.	0.

Card 9	1	2	3	4	5	6	7	8
Variable	EPSD23	EPSC23	CDAM23	EPSD31	EPSC31	CDAM31		
Type	F	F	F	F	F	F		
Default	0.	0.	0.	0.	0.	0.		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
EA	$E_a$ , Young's modulus in $a$ -direction
EB	$E_b$ , Young's modulus in $b$ -direction
EC	$E_c$ , Young's modulus in $c$ -direction
PRBA	$\nu_{ba}$ , Poisson ratio
PRCA	$\nu_{ca}$ , Poisson ratio
PRCB	$\nu_{cb}$ , Poisson ratio
GAB	$G_{ab}$ , Shear modulus
GBC	$G_{bc}$ , Shear modulus
GCA	$G_{ca}$ , Shear modulus

VARIABLE	DESCRIPTION
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <i>a</i>-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <b>v</b>, and an originating point, <b>p</b>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</p>
MACF	<p>Material axes change flag for brick elements:</p> <p>EQ.1: No change, default,</p> <p>EQ.2: switch material axes <b>a</b> and <b>b</b>,</p> <p>EQ.3: switch material axes <b>a</b> and <b>c</b>,</p> <p>EQ.4: switch material axes <b>b</b> and <b>c</b>.</p>
XP, YP, ZP	Coordinates of point <b>p</b> for AOPT = 1 and 4
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3 and 4
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	Material angle in degrees for AOPT = 3, may be overridden on the element card, see *ELEMENT_SOLID_ORTHO.
NERODE	<p>Element erosion flag. For multi-integration point elements, each of the failure strains mentioned below for NERODE 2 and higher need only occur in one integration point to trigger element erosion, and for NERODE values 6 to 11, which require more than one failure strain be reached, those failure strains need not occur in the same integration point.</p> <p>EQ.0: No erosion (default).</p> <p>EQ.1: Erosion occurs when one failure strain is reached in all integration points.</p> <p>EQ.2: Erosion occurs when one failure strain is reached.</p> <p>EQ.3: Erosion occurs when a tension or compression failure strain in the <i>a</i>-direction is reached.</p> <p>EQ.4: Erosion occurs when as a tension or compression failure strain in the <i>b</i>-direction is reached.</p> <p>EQ.5: Erosion occurs when a tension or compression failure strain in the <i>c</i>-direction is reached.</p> <p>EQ.6: Erosion occurs when tension or compression failure strain in both the <i>a</i>- and <i>b</i>-directions are reached.</p> <p>EQ.7: Erosion occurs when tension or compression failure strain in both the <i>b</i>- and <i>c</i>-directions are reached.</p> <p>EQ.8: Erosion occurs when tension or compression failure strain in both the <i>a</i>- and <i>c</i>-directions are reached.</p> <p>EQ.9: Erosion occurs when tension or compression failure strain in all 3 directions are reached.</p> <p>EQ.10: Erosion occurs when tension or compression failure strain in both the <i>a</i>- and <i>b</i>-directions are reached and either of the out-of-plane failure shear strains (<i>bc</i> or <i>ac</i>) is reached. .</p> <p>EQ.11: Erosion occurs when tension failure strain in either the <i>a</i>- or <i>b</i>-directions is reached and either of the out-of-plane failure shear strains (<i>bc</i> or <i>ac</i>) is reached.</p>

VARIABLE	DESCRIPTION
NDAM	Damage flag: EQ.0: No damage (default) EQ.1: Damage in tension only (null for compression) EQ.2: Damage in tension and compression
EPS1TF	Failure strain in tension along the $a$ -direction
EPS2TF	Failure strain in tension along the $b$ -direction
EPS3TF	Failure strain in tension along the $c$ -direction
EPS1CF	Failure strain in compression along the $a$ -direction
EPS2CF	Failure strain in compression along the $b$ -direction
EPS3CF	Failure strain in compression along the $c$ -direction
EPS12F	Failure shear strain in the $ab$ -plane
EPS23F	Failure shear strain in the $bc$ -plane
EPS13F	Failure shear strain in the $ac$ -plane
EPD1T	Damage threshold in tension along the $a$ -direction, $\epsilon_{1t}^s$
EPSC1T	Critical damage threshold in tension along the $a$ -direction, $\epsilon_{1t}^c$
CDAM1T	Critical damage in tension along the $a$ -direction, $D_{1t}^c$
EPD2T	Damage threshold in tension along the $b$ -direction, $\epsilon_{2t}^s$
EPSC2T	Critical damage threshold in tension along the $b$ -direction, $\epsilon_{2t}^c$
CDAM2T	Critical damage in tension along the $b$ -direction, $D_{2t}^c$
EPD3T	Damage threshold in tension along the $c$ -direction, $\epsilon_{3t}^s$
EPSC3T	Critical damage threshold in tension along the $c$ -direction, $\epsilon_{3t}^c$
CDAM3T	Critical damage in tension along the $c$ -direction, $D_{3t}^c$
EPD1C	Damage threshold in compression along the $a$ -direction, $\epsilon_{1c}^s$
EPSC1C	Critical damage threshold in compression along the $a$ -direction, $\epsilon_{1c}^c$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
CDAM1C	Critical damage in compression along the $a$ -direction, $D_{1c}^c$
EPSD2C	Damage threshold in compression along the $b$ -direction, $\varepsilon_{2c}^s$
EPSC2C	Critical damage threshold in compression along the $b$ -direction, $\varepsilon_{2c}^c$
CDAM2C	Critical damage in compression along the $b$ -direction, $D_{2c}^c$
EPSD3C	Damage threshold in compression along the $c$ -direction, $\varepsilon_{3c}^s$
EPSC3C	Critical damage threshold in compression along the $c$ -direction, $\varepsilon_{3c}^c$
CDAM3C	Critical damage in compression along the $c$ -direction, $D_{3c}^c$
EPSD12	Damage threshold for shear in the $ab$ -plane, $\varepsilon_{12}^s$
EPSC12	Critical damage threshold for shear in the $ab$ -plane, $\varepsilon_{12}^c$
CDAM12	Critical damage for shear in the $ab$ -plane, $D_{12}^c$
EPSD23	Damage threshold for shear in the $bc$ -plane, $\varepsilon_{23}^s$
EPSC23	Critical damage threshold for shear in the $bc$ -plane, $\varepsilon_{23}^c$
CDAM23	Critical damage for shear in the $bc$ -plane, $D_{23}^c$
EPSD31	Damage threshold for shear in the $ac$ -plane, $\varepsilon_{31}^s$
EPSC31	Critical damage threshold for shear in the $ac$ -plane, $\varepsilon_{31}^c$
CDAM31	Critical damage for shear in the $ac$ -plane, $D_{31}^c$

**Remarks:**

If  $\varepsilon_k^c < \varepsilon_k^s$ , no damage is considered. Failure occurs only when failure strain is reached.

Failure can occur along the 3 orthotropic directions, in tension, in compression and for shear behavior. Nine failure strains drive the failure. When failure occurs, elements are deleted (erosion). Under the control of the NERODE flag, failure may occur either when only one integration point has failed, when several integration points have failed or when all integrations points have failed.

Damage applies to the 3 Young's moduli and the 3 shear moduli. Damage is different for tension and compression. Nine damage variables are used:  $d_{1t}, d_{2t}, d_{3t}, d_{1c}, d_{2c}, d_{3c}, d_{12}, d_{23}, d_{13}$ . The damaged flexibility matrix is:



$$-S^{\text{dam}} = \begin{pmatrix} \frac{1}{E_a(1-d_{1[t,c]})} & \frac{-v_{ba}}{E_b} & \frac{-v_{ca}}{E_c} & 0 & 0 & 0 \\ \frac{-v_{ba}}{E_b} & \frac{1}{E_b(1-d_{2[t,c]})} & \frac{-v_{cb}}{E_c} & 0 & 0 & 0 \\ \frac{-v_{ca}}{E_c} & \frac{-v_{cb}}{E_c} & \frac{1}{E_c(1-d_{3[t,c]})} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{ab}(1-d_{12})} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{bc}(1-d_{23})} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{ca}(1-d_{31})} \end{pmatrix}$$

The nine damage variables are calculated as follows:

$$d_k = \max \left( d_k; D_k^c \left\langle \frac{\varepsilon_k - \varepsilon_k^s}{\varepsilon_k^c - \varepsilon_k^s} \right\rangle_+ \right)$$

with  $k = 1t, 2t, 3t, 1c, 2c, 3c, 12, 23, 31$ .

$$\langle \cdot \rangle_+ \text{ is the positive part: } \langle x \rangle_+ = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Damage in compression may be deactivated with the NDAM flag. In this case, damage in compression is null, and only damage in tension and for shear behavior are taken into account.

The nine damage variables may be post-processed through additional variables. The number of additional variables for solids written to the d3plot and d3thdt databases is input by the optional \*DATABASE\_EXTENT\_BINARY card as variable NEIPH. These additional variables are tabulated below:

History Variable	Description	Value	LS-PrePost History Variable
$d_{1t}$	damage in traction along $a$	0 - no damage  $0 < d_k < D_k^c$ - damage	plastic strain
$d_{2t}$	damage in traction along $b$		1
$d_{3t}$	damage in traction along $c$		2
$d_{1c}$	damage in compression along $a$		3
$d_{2c}$	damage in compression along $b$		4
$d_{3c}$	damage in compression along $c$		5
$d_{12}$	shear damage in $ab$ -plane		6
$d_{23}$	shear damage in $bc$ -plane		7
$d_{13}$	shear damage in $ac$ -plane		8

## **\*MAT\_221**

## **\*MAT\_ORTHOTROPIC\_SIMPLIFIED\_DAMAGE**

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The first damage variable is stored as in the place of effective plastic strain. The eight other damage variables may be plotted in LS-PrePost as element history variables.



**Optional Card.**

Card 3	1	2	3	4	5	6	7	8
Variable	FAILOPT	NUMAVG	NCYFAIL	ERODE	LCPS			
Type	F	F	F	F	F			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus: GT.0.0: constant value is used LT.0.0: -E gives curve ID for temperature dependence
PR	Poisson's ratio.
CP	Specific heat (superseded by heat capacity in *MAT_THERMAL_OPTION if a coupled thermal/structural analysis).
TR	Room temperature.
BETA	Fraction of plastic work converted into heat: GT.0.0: constant value is used LT.0.0: -BETA gives either a curve ID for strain rate dependence or a table ID for strain rate <u>and</u> temperature dependence.
NUMINT	GT.0.0: Number of integration points which must fail before the element is deleted. Available for shells and solids. LT.0.0: -NUMINT is percentage of integration points/layers which must fail before shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells except as noted below EQ.-200: Turns off erosion for shells and solids. Not recommended unless used in conjunction with *CONSTRAINED_TIED_NODES_FAILURE.

VARIABLE	DESCRIPTION
LCK1	Load curve ID or Table ID. The load curve ID defines effective stress as a function of effective plastic strain. The table ID defines for each plastic strain rate value a load curve ID giving the (isothermal) effective stress versus effective plastic strain for that rate. As in *MAT_024, natural logarithmic strain rates can be used by setting the <i>first</i> strain rate to a negative value.
LCKT	Table ID defining for each temperature value a load curve ID giving the (quasi-static) effective stress versus effective plastic strain for that temperature.
LCF	Load curve ID or Table ID. The load curve ID defines plastic failure strain (or scale factor – see Remarks) as a function of triaxiality. The table ID defines for each Lode parameter a load curve ID giving the plastic failure strain versus triaxiality for that Lode parameter. (Table option only for solids and not yet generally supported). See Remarks for a description of the combination of LCF, LCG, LCH, and LCI.
LCG	Load curve ID defining plastic failure strain (or scale factor – see Remarks) as a function of plastic strain rate. If the <i>first</i> abscissa value in the curve corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for all abscissa values. See Remarks for a description of the combination of LCF, LCG, LCH, and LCI.
LCH	Load curve ID defining plastic failure strain (or scale factor – see Remarks) as a function of temperature. See Remarks for a description of the combination of LCF, LCG, LCH, and LCI.
LCI	Load curve ID, Table ID, or Table_3D ID. The load curve ID defines plastic failure strain (or scale factor – see Remarks) as a function of element size. The table ID defines for each triaxiality a load curve ID giving the plastic failure strain versus element size for that triaxiality. If a three dimensional table ID is referred, plastic failure strain can be a function of Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). See Remarks for a description of the combination of LCF, LCG, LCH, and LCI.
FAILOPT	Flag for additional failure criterion $F_2$ , see Remarks. EQ.0.0: off (default) EQ.1.0: on

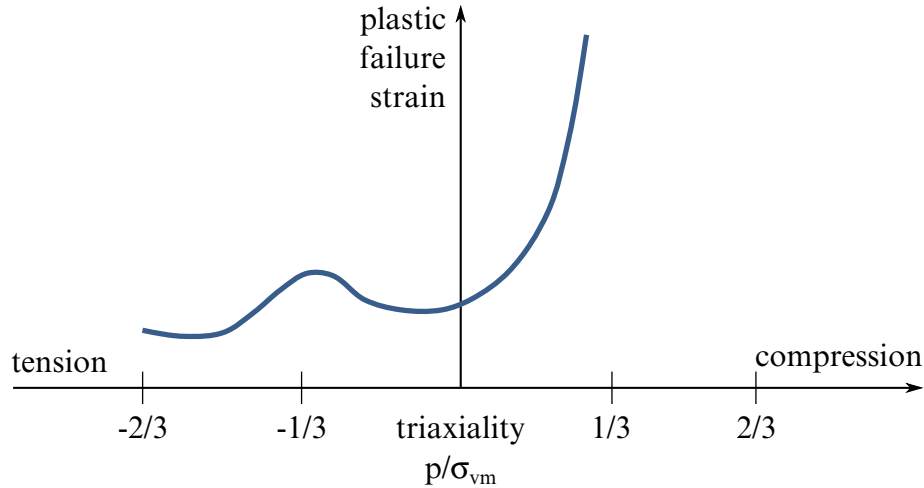
<b>VARIABLE</b>	<b>DESCRIPTION</b>
NUMAVG	Number of time steps for running average of plastic failure strain in the additional failure criterion. Default is 1 (no averaging).
NCYFAIL	Number of time steps that the additional failure criterion must be met before element deletion. Default is 1.
ERODE	Erosion flag (only for solid elements): EQ.0.0: default, element erosion is allowed. EQ.1.0: element does not erode; deviatoric stresses set to zero when element fails.
LCPS	Table ID with first principal stress limit as function of plastic strain (curves) and plastic strain rate (table). This option is for post-processing purposes only and gives an indication of areas in the structure where failure is likely to occur. History variable #17 shows a value of 1.0 for integration points that exceeded the limit, else a value of 0.0.

**Remarks:**

The flow stress  $\sigma_y$  is expressed as a function of plastic strain  $\varepsilon_p$ , plastic strain rate  $\dot{\varepsilon}_p$  and temperature  $T$  via the following formula (using load curves/tables LCK1 and LCKT):

$$s_y = k_1(\varepsilon_p, \dot{\varepsilon}_p) \frac{k_t(\varepsilon_p, T)}{k_t(\varepsilon_p, T_R)}$$

Note that  $T_R$  is a material parameter and should correspond to the temperature used when performing the room temperature tensile tests. If simulations are to be performed with an initial temperature  $T_i$  deviating from  $T_R$  then this temperature should be set using \*INITIAL\_STRESS\_SOLID/SHELL by setting history variable #14 for solid elements or history variable #10 for shell elements.



**Figure M224-1.** Typical failure curve for metal sheet, modeled with shell elements.

Optional plastic failure strain is defined as a function of triaxiality  $p/\sigma_{vm}$ , Lode parameter, plastic strain rate  $\dot{\epsilon}_p$ , temperature  $T$  and initial element size  $l_c$  (square root of element area for shells and volume over maximum area for solids) by

$$\epsilon_{pf} = f\left(\frac{p}{\sigma_{vm}}, \frac{27J_3}{2\sigma_{vm}^3}\right) g(\dot{\epsilon}_p) h(T) i\left(l_c, \frac{p}{\sigma_{vm}}\right)$$

using load curves/tables LCF, LCG, LCH and LCI. If more than one of these four variables LCF, LCG, LCH and LCI are defined, be aware that the net plastic failure strain is essentially the product of multiple functions as shown in the above equation. This means that one and only one of the variables LCF, LCG, LCH, and LCI can point to curve(s) that have plastic strain along the curve ordinate. The remaining nonzero variable(s) LCF, LCG, LCH, and LCI should point to curve(s) that have a unitless scaling factor along the curve ordinate.

A typical failure curve LCF for metal sheet, modeled with shell elements is shown in [Figure M224-1](#). Triaxiality should be monotonically increasing in this curve. A reasonable range for triaxiality is  $-2/3$  to  $2/3$  if shell elements are used (plane stress). For 3-dimensional stress states (solid elements), the possible range of triaxiality goes from  $-\infty$  to  $+\infty$ , but to get a good resolution in the internal load curve discretization (depending on parameter LCINT of \*CONTROL\_SOLUTION) one should define lower limits, e.g.  $-1$  to  $1$  if LCINT = 100 (default).

The default failure criterion of this material model depends on plastic strain evolution  $\dot{\epsilon}_p$  and on plastic failure strain  $\epsilon_{pf}$  and is obtained by accumulation over time:

$$F = \int \frac{\dot{\epsilon}_p}{\epsilon_{pf}} dt$$

where element erosion takes place when  $F \geq 1$ . This accumulation provides load-path dependent treatment of failure. The value of  $F$  is stored as history variable #8 for shells and #12 for solids.

An additional, load-path independent, failure criterion can be invoked by setting  $FAILOPT = 1$ , where the current state of plastic strain is used:

$$F_2 = \frac{\varepsilon_p}{\varepsilon_{pf}}$$

Two additional parameters can be used as countermeasures against stress oscillations for this failure criterion. With NUMAVG active, plastic failure strain is averaged over NUMAVG time steps for the  $F_2$  criterion. The value of  $F_2$ , taking into account any averaging per NUMAVG, is stored as history variable #14 for shells and #16 for solids. NUMAVG cannot exceed 30. NCYFAIL defines the number of time steps that  $F_2 \geq 1$  must be met before element deletion takes place. The number of time steps that  $F_2 \geq 1$  is stored as history variable #15 for shells and #19 for solids.

Temperature increase is caused by plastic work

$$T = T_R + \frac{\beta}{C_p \rho} \int \sigma_y \dot{\varepsilon}_p dt$$

with room temperature  $T_R$ , dissipation factor  $\beta$ , specific heat  $C_p$ , and density  $\rho$ .

For \*CONSTRAINED\_TIED\_NODES\_WITH\_FAILURE, the failure is based on the damage instead to the plastic strain.

History variables may be post-processed through additional variables. The number of additional variables for shells/solids written to the d3plot and d3thdt databases is input by the optional \*DATABASE\_EXTENT\_BINARY card as variable NEIPS/NEIPH. The relevant additional variables of this material model are tabulated below:



LS-PrePost history variable #	Shell elements	LS-PrePost history variable #	Solid elements
1	<i>plastic strain rate</i>	5	<i>plastic strain rate</i>
7	<i>plastic work</i>	8	<i>plastic failure strain</i>
8	<i>ratio of plastic strain to plastic failure strain</i>	9	<i>triaxiality</i>
9	<i>element size</i>	10	<i>Lode parameter</i>
10	<i>temperature</i>	11	<i>plastic work</i>
11	<i>plastic failure strain</i>	12	<i>ratio of plastic strain to plastic failure strain</i>
12	<i>triaxiality</i>	13	<i>element size</i>
17	<i>LCPS: critical value</i>	14	<i>temperature</i>
		17	<i>LCPS: critical value</i>

**\*MAT\_224\_GYS****\*MAT\_TABULATED\_JOHNSON\_COOK\_GYS****\*MAT\_TABULATED\_JOHNSON\_COOK\_GYS**

This is Material Type 224\_GYS. This is an isotropic elastic plastic material law with J3 dependent yield surface. This material considers tensile/compressive asymmetry in the material response, which is important for HCP metals like Titanium. The model is available for solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	CP	TR	BETA	NUMINT
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	1.0	1.0

Card 2	1	2	3	4	5	6	7	8
Variable	LCK1	LCKT	LCF	LCG	LCH	LCI		
Type	I	I	I	I	I	I		
Default	0	0	0	0	0	0		

Card 3	1	2	3	4	5	6	7	8
Variable	LCCR	LCCT	LCSR	LCST	IFLAG	SFIEPM	NITER	
Type	I	I	I	I	I	F	I	
Default	0	0	0	0	0	1	100	

**VARIABLE****DESCRIPTION**

MID Material identification. A unique number or label not exceeding 8 characters must be specified.

RO Mass density.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
E	Young's modulus: GT.0.0: constant value is used LT.0.0: temperature dependent Young's modulus given by load curve ID = -E
PR	Poisson's ratio.
CP	Specific heat.
TR	Room temperature.
BETA	Fraction of plastic work converted into heat.
NUMINT	Number of integration points which must fail before the element is deleted. EQ.-200: Turns off erosion for solids. Not recommended unless used in conjunction with *CONSTRAINED_TIED_NODES_FAILURE.
LCK1	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) effective stress versus effective plastic strain for that rate.
LCKT	Table ID defining for each temperature value a load curve ID giving the (quasi-static) effective stress versus effective plastic strain for that temperature.
LCF	Load curve ID or Table ID. The load curve ID defines plastic failure strain as a function of triaxiality. The table ID defines for each Lode parameter a load curve ID giving the plastic failure strain versus triaxiality for that Lode parameter. (Table option only for solids and not yet generally supported).
LCG	Load curve ID defining plastic failure strain as a function of plastic strain rate.
LCH	Load curve ID defining plastic failure strain as a function of temperature
LCI	Load curve ID or Table ID. The load curve ID defines plastic failure strain as a function of element size. The table ID defines for each triaxiality a load curve ID giving the plastic failure strain versus element size for that triaxiality.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCCR	Table ID. The curves in this table define compressive yield stress as a function of plastic strain or effective plastic strain (see IFLAG). The table ID defines for each plastic strain rate value or effective plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress versus plastic strain or effective plastic strain for that rate.
LCCT	Table ID defining for each temperature value a load curve ID giving the (quasi-static) compressive yield stress versus strain for that temperature. The curves in this table define compressive yield stress as a function of plastic strain or effective plastic strain (see IFLAG).
LCSR	Table ID. The load curves define shear yield stress in function of plastic strain or effective plastic strain (see IFLAG). The table ID defines for each plastic strain rate value or effective plastic strain rate value a load curve ID giving the (isothermal) shear yield stress versus plastic strain or effective plastic strain for that rate.
LCST	Table ID defining for each temperature value a load curve ID giving the (quasi-static) shear yield stress versus strain for that temperature. The load curves define shear yield stress as a function of plastic strain or effective plastic strain (see IFLAG).
IFLAG	Flag to specify abscissa for LCCR, LCCT, LCSR, LCST: EQ.0: Compressive and shear yields are given in a function of plastic strain as defined in the remarks (default). EQ.1: Compressive and shear yields are given in function of effective plastic strain.
SFIEPM	Scale factor on the initial estimate of the plastic multiplier.
NITER	Number of secant iterations to be performed.

**Remarks:**

If IFLAG = 0 the compressive and shear curves are defined as follows:

$$\sigma_c(\varepsilon_{pc}, \dot{\varepsilon}_{pc}), \quad \varepsilon_{pc} = \varepsilon_c - \frac{\sigma_c}{E}, \quad \dot{\varepsilon}_{pc} = \frac{\partial \varepsilon_{pc}}{\partial t}$$

$$\sigma_s(\gamma_{ps}, \dot{\gamma}_{ps}), \quad \gamma_{ps} = \gamma_s - \frac{\sigma_s}{G}, \quad \dot{\gamma}_{ps} = \frac{\partial \gamma_{ps}}{\partial t}$$

and two new history variables (#16 plastic strain in compression and #17 plastic strain in shear) are stored in addition to those history variables already stored in MAT\_224.

If IFLAG = 1 the compressive and shear curves are defined as follows:

$$\sigma_c(\dot{\lambda}, \lambda), \quad \sigma_s(\dot{\lambda}, \lambda), \quad \dot{W}_p = \sigma_{\text{eff}} \dot{\lambda}$$

History variables may be post-processed through additional variables. The number of additional variables for solids written to the d3plot and d3thdt databases is input by the optional \*DATABASE\_EXTENT\_BINARY card as variable NEIPH. The relevant additional variables of this material model are tabulated below:

<b>LS-PrePost history variable #</b>	<b>Solid elements</b>
5	<i>plastic strain rate</i>
8	<i>plastic failure strain</i>
9	<i>triaxiality</i>
10	<i>Lode parameter</i>
11	<i>plastic work</i>
12	<i>damage</i>
13	<i>element size</i>
14	<i>temperature</i>
16	<i>plastic strain in compression</i>
17	<i>plastic strain in shear</i>

**\*MAT\_225****\*MAT\_VISCOPLASTIC\_MIXED\_HARDENING****\*MAT\_VISCOPLASTIC\_MIXED\_HARDENING**

This is Material Type 225. An elasto-viscoplastic material with an arbitrary stress versus strain curve and arbitrary strain rate dependency can be defined. Kinematic, isotropic, or a combination of kinematic and isotropic hardening can be specified. Also, failure based on plastic strain can be defined.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	LCSS	BETA		
Type	A8	F	F	F	I	F		
Default	none	none	none	none	none	0.0		

Card 2	1	2	3	4	5	6	7	8
Variable	FAIL							
Type	F							
Default	1.0E+20							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.

VARIABLE	DESCRIPTION
LCSS	<p>Load curve ID or Table ID. Load curve ID defining effective stress versus effective plastic strain The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See <a href="#">Figure M24-1</a>. The stress versus effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress versus effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. NOTE: The strain rate values defined in the table may be given as the natural logarithm of the strain rate. If the <i>first</i> stress-strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used. Since the tables are internally discretized to equally space the points, natural logarithms are necessary, for example, if the curves correspond to rates from 10.e-04 to 10.e+04.</p>
BETA	<p>Hardening parameter, <math>0 &lt; \text{BETA} &lt; 1</math>.</p> <p>EQ.0.0: Pure kinematic hardening</p> <p>EQ.1.0: Pure isotropic hardening</p> <p>0.0 &lt; BETA &lt; 1.0: Mixed hardening</p>
FAIL	<p>Failure flag.</p> <p>LT.0.0: User defined failure subroutine is called to determine failure</p> <p>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.</p> <p>GT.0.0: Plastic strain to failure. When the plastic strain reachesthis value, the element is deleted from the calculation..</p>





Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	IOPT	C1	C2	IFLD			
Type	F	I	F	F	I			
Default	none	none	0.0	0.0	none			

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number must be specified.
RO	Mass density.
E	Young's modulus, E.
PR	Poisson's ratio, $\nu$ .
M	$m$ , the exponent in Barlat's yield criterion.
R00	$R_{00}$ , Lankford parameter in 0 degree direction.
R45	$R_{45}$ , Lankford parameter in 45 degree direction.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
R90	$R_{90}$ , Lankford parameter in 90 degree direction.
CB	The uppercase $B$ defined in the Yoshida's equations.
Y	Hardening parameter as defined in the Yoshida's equations.
SC	The lowercase $c$ defined in the Yoshida's equations.
K	Hardening parameter as defined in the Yoshida's equations.
RSAT	Hardening parameter as defined in the Yoshida's equations.
SB	The lowercase $b$ as defined in the Yoshida's equations.
H	Anisotropic parameter associated with work-hardening stagnation, defined in the Yoshida's equations.
HLCID	Load curve ID in keyword *DEFINE_CURVE, where true strain and true stress relationship is characterized. The load curve is optional, and is used for error calculation only.
IOPT	Kinematic hardening rule flag: EQ.0: Original Yoshida formulation, EQ.1: Modified formulation: define C1, C2 as below.
C1, C2	Constants used to modify $R$ : $R = RSAT \times [(C_1 + \bar{\epsilon}^p)^{c_2} - C_1^{c_2}]$
IFLD	ID of a load curve of the traditional Forming Limit Diagram (FLD) for the linear strain paths. In the load curve, abscissas represent minor strains while ordinates represent major strains. Define only when the NLP option is used. See the <a href="#">example</a> in the remarks section.
AOPT	Material axes option (see MAT_OPTION TROPIC_ELASTIC for a more complete description): EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA. EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDI-

VARIABLE	DESCRIPTION
	NATE_VECTOR:
	EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector $\mathbf{v}$ with the element normal:
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR): Available with the R3 release of Version 971 and later.
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

### On Barlat and Lian's yield criterion:

The  $R$ -values are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width  $W$  and thickness  $T$  are measured as function of strain. Then the corresponding  $R$ -value is given by:

$$R = \frac{\frac{dW}{d\varepsilon}/W}{\frac{dT}{d\varepsilon}/T}$$

Input R00, R45 and R90 to define sheet anisotropy in the rolling, 45 degree and 90 degree direction.

Barlat and Lian's [1989] anisotropic yield criterion  $\Phi$  for plane stress is defined as:

$$\Phi = a|K_1 + K_2|^m + a|K_1 - K_2|^m + c|2K_2|^m = 2\sigma_Y^m$$

for face centered cubic (FCC) materials exponent  $m = 8$  is recommended and for body centered cubic (BCC) materials  $m = 6$  may be used. Detailed description on the criterion can be found in \*MAT\_036 manual pages.

**On Yoshida nonlinear kinematic hardening model:****Background.**

The Yoshida's model accounts for cyclic plasticity including Bauschinger effect and cyclic hardening behavior. For detailed Yoshida's theory of nonlinear kinematic hardening rule and definitions of material constants CB, Y, SC, K, RSAT, SB, and H, refer to **Remarks** in \*MAT\_125 manual pages and in the original paper, "A model of large-strain cyclic plasticity describing the Bauschinger effect and workhardening stagnation", by Yoshida, F. and Uemori, T., *Int. J. Plasticity*, vol. 18, 661-689, 2002.

Further improvements in the original Yoshida's model, as described in a paper "Determination of Nonlinear Isotropic/Kinematic Hardening Constitutive Parameter for AHSS using Tension and Compression Tests", by Shi, M.F., Zhu, X.H., Xia, C., and Stoughton, T., in *NUMISHEET 2008 proceedings*, 137-142, 2008, included modifications to allow work hardening in large strain deformation region, avoiding the problem of earlier saturation, especially for Advanced High Strength Steel (AHSS). These types of steels exhibit continuous strain hardening behavior and a non-saturated isotropic hardening function. As described in the paper, the evolution equation for  $R$  (a part of the current radius of the bounding surface in deviatoric stress space), as is with the saturation type of isotropic hardening rule proposed in the original Yoshida model,

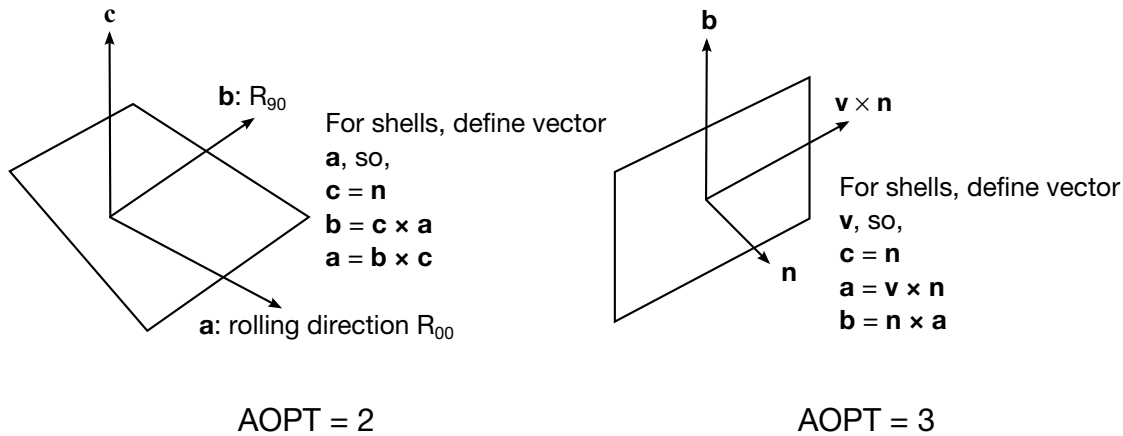
$$\dot{R} = m(R_{\text{sat}} - R)\dot{p}$$

is modified as,

$$R = \text{RSAT} \times [(C_1 + \bar{\epsilon}^p)^{c_2} - C_1^{c_2}]$$

For saturation type of isotropic hardening rule, set IOPT = 0, applicable to most of Aluminum sheet materials. In addition, the paper provides detailed variables used for this material model for DDQ, HSLA, DP600, DP780 and DP980 materials. Since the symbols used in the paper are different from what are used here, the following table provides a reference between symbols used in the paper and variables here in this keyword:

B	Y	C	m	K	b	h	$e^0$	N
CB	Y	SC	K	Rsat	SB	H	C1	C2



**Figure M226-1.** Defining sheet metal rolling direction.

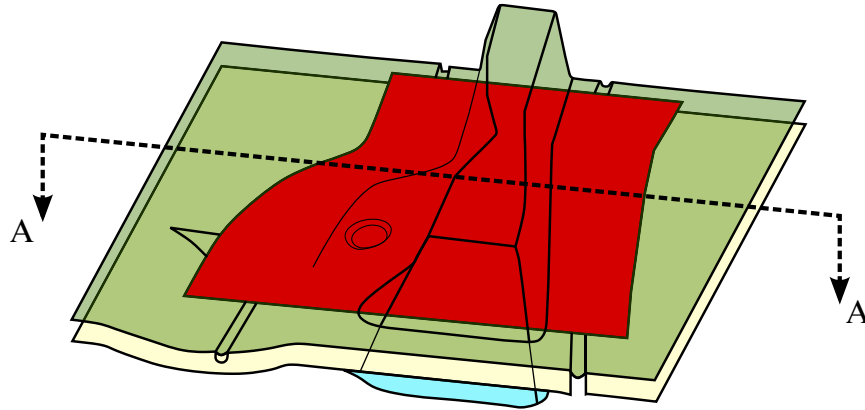
Using the modified formulation and the material properties provided by the paper, the predicted and tested results compare very well both in a full cycle tension and compression test and in a pre-strained tension and compression test, according to the paper. A set of experiments are required to fit (optimize) the Yoshida material constants, and these experiments include a uniaxial tension test (used for HLCID) to a sufficiently large strain range, a full cycle tension and compression test and a multiple cycle tension and compression test.

### Defining the rolling direction of a sheet metal.

The variable AOPT is used to define the rolling direction of the sheet metals. For shells, AOPT of 2 or 3 are relevant. When AOPT = 2, define vector components of  $\mathbf{a}$  in the direction of the rolling ( $R_{00}$ ); when AOPT = 3, define vector components of  $\mathbf{v}$  perpendicular to the rolling direction, as shown in [Figure M226-1](#).

### Application.

Application of the modified Yoshida's hardening rule in the metal forming industry has shown significant improvement in springback prediction accuracy, which is a pre-requisite for a successful stamping tool compensation, especially for AHSS type of sheet materials.



**Figure M226-2.** The NUMISHEET 2005 cross member and section definition.

In an example shown in [Figure M226-2](#), springback simulation was performed following drawing and trimming on the NUMISHEET 2005 cross member for aluminum alloy AL5182-O, using \*MAT\_226. In [Figure M226-3](#), springback shape was recovered from section A-A ([Figure M226-2](#)), and compared with those results from simulation using \*MAT\_037 and \*MAT\_125. Though all are remarkably close, results with \*MAT\_226 on the cross section ( $Y = -370$  mm) show better springback correlation to the measured test data than those with \*MAT\_125 and \*MAT\_37.

To improve convergence, it is recommended that \*CONTROL\_IMPLICIT\_FORMING type '1' be used when conducting springback simulation.

### **A Failure Criterion for Nonlinear Strain Paths (NLP):**

The NLP failure criterion and corresponding post processing procedures are described in the entries for \*MAT\_036 and \*MAT\_037. The history variables for every element stored in d3plot files include:

1. Formability Index (F.I.): #1
2. Strain ratio  $\beta$  (in-plane minor strain/major strain): #2
3. Effective strain from the planar isotropic assumption: #3

The entire time history can be plotted using *Post/History* menu in LS-PrePost v4.0. To enable the output of these history variables to the d3plot files, NEIPS on the \*DATABASE\_-EXTENT\_BINARY card must be set to at least 3. When plotting the formability index, first select the *history var #1* from the *Misc* in the *FriComp* menu. The pull-down menu under *FriComp* can be used to select minimum value 'Min' for necking failure determination (refer to *Tharrett and Stoughton's paper in 2003 SAE 2003-01-1157*). In *FriRang*, the option *None* is to be selected in the pull-down menu next to *Avg*. Lastly, set the simulation result to the last state in the *animation tool bar*. The index value ranges from 0.0 to 1.5. The non-linear forming limit is reached when the index reaches 1.0.

A partial keyword example is listed below when the option NLP is used. In this example, the traditional Forming Limit Diagram (FLD) which handles only the linear strain paths is defined by load curve ID 213.

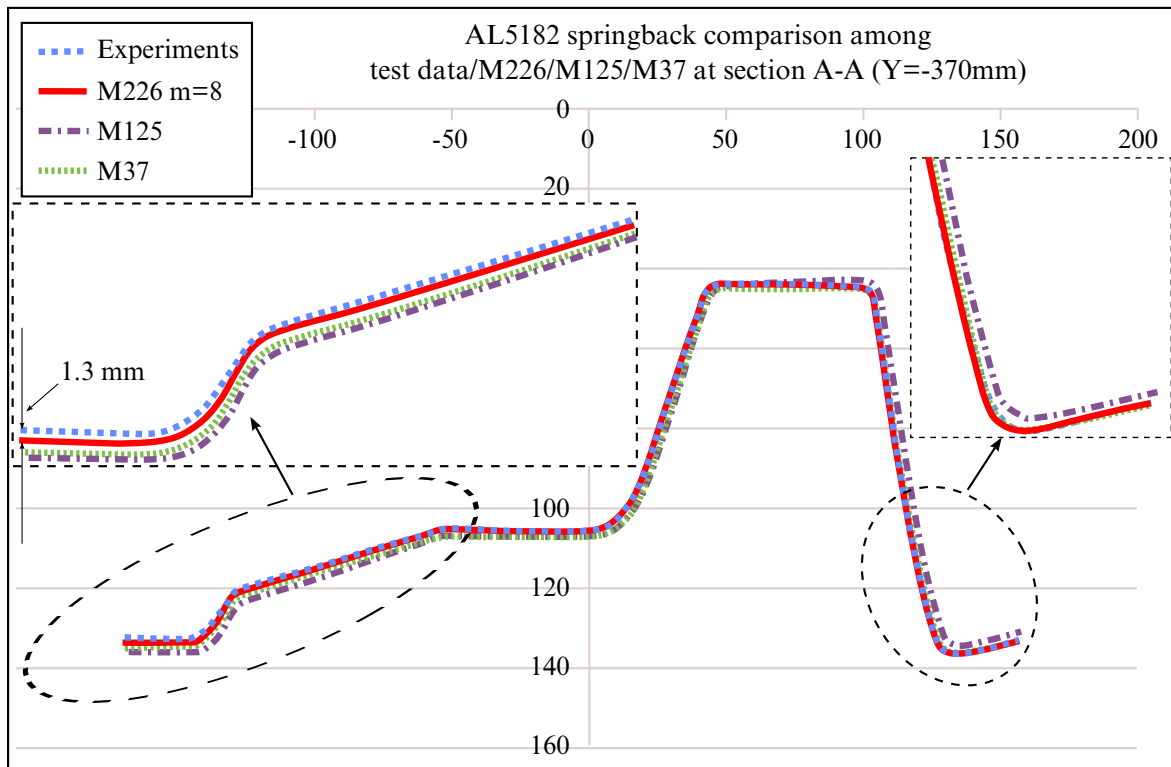
```
*MAT_KINEMATIC_HARDENING_BARLAT89_NLP
$#      mid      ro      e      pr      m      r00      r45      r90
        1 2.8900E-9  7.0E+4  0.333  8.0    0.699  0.776  0.775
$#      cb      y      sc      k      rsat      sb      h      hlcid
        122.3  110.2  577.5  12.0  201.7  16.5  0.16  0
$#      aopt      iopt      c1      c2      IFLD
        2      0      213
$#      xp      yp      zp      a1      a2      a3
        0.000  0.000  0.000  1.000000  0.000  0.000
$#      v1      v2      v3      d1      d2      d3      beta
        0.000  0.000  0.000  0.000  0.000  0.000  0.000
*DEFINE_CURVE
213
-0.300,0.36
-0.200,0.32
-0.114,0.266
-0.058,0.223
0.026,0.181
0.036,0.181
0.111,0.211
0.147,0.23
0.215,0.27
0.263,0.278
```

**Smoothing of the Strain Ratio  $\beta$ :**

Details related to the smoothing of  $\beta$  can be found in manual pages under \*CONTROL\_FORMING\_TOLERANC.

**Revision information:**

This material model is available starting in Revision 57717. The **NLP** option is available starting in Revision 95599. Smoothing of  $\beta$  is available starting in Revision 115432.



**Figure M226-3.** Springback prediction with \*MAT\_226 (Material properties courtesy of Ford Motor Company Research and Innovation Laboratory).



**\*MAT\_PML\_ELASTIC**

This is Material Type 230. This is a perfectly-matched layer (PML) material — an absorbing layer material used to simulate wave propagation in an unbounded isotropic elastic medium — and is available only for solid 8-node bricks (element type 2). This material implements the 3D version of the Basu-Chopra PML [Basu and Chopra (2003,2004), Basu (2009)].

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A8	F	F	F				
Default	none	none	none	none				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.

**Remarks:**

1. A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any static displacement.
2. It is assumed the material in the bounded domain near the layer is, or behaves like, an isotropic linear elastic material. The material properties of the layer should be set to the corresponding properties of this material.
3. The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem, e.g., for a half-space problem, the "top" of the box should be open.

4. Internally, LS-DYNA will partition the entire PML into regions which form the "faces", "edges" and "corners" of the above cuboid box, and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.
5. The layer should have 5-10 elements through its depth. Typically, 5-6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8-10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer, and should be small enough to sufficiently discretize all significant wavelengths in the problem.
6. The nodes on the outer boundary of the layer should be fully constrained.
7. The stress and strain values reported by this material do not have any physical significance.

**\*MAT\_PML\_ELASTIC\_FLUID**

This is Material Type 230\_FLUID. This is a perfectly-matched layer (PML) material with a pressure fluid constitutive law, to be used in a wave-absorbing layer adjacent to a fluid material (\*MAT\_ELASTIC\_FLUID) in order to simulate wave propagation in an unbounded fluid medium. See the Remarks sections of \*MAT\_PML\_ELASTIC (\*MAT\_230) and \*MAT\_ELASTIC\_FLUID (\*MAT\_001\_FLUID) for further details.

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	K	VC				
Type	A8	F	F	F				
Default	none	none	none	none				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
K	Bulk modulus
VC	Tensor viscosity coefficient

**\*MAT\_PML\_ACOUSTIC**

This is Material Type 231. This is a perfectly-matched layer (PML) material — an absorbing layer material used to simulate wave propagation in an unbounded acoustic medium — and can be used only with the acoustic pressure element formulation (element type 14). This material implements the 3D version of the Basu-Chopra PML for anti-plane motion [Basu and Chopra (2003,2004), Basu (2009)].

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	C					
Type	A8	F	F					
Default	none	none	none					

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
C	Sound speed

**Remarks:**

1. A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any hydrostatic pressure.
2. It is assumed the material in the bounded domain near the layer is an acoustic material. The material properties of the layer should be set to the corresponding properties of this material.
3. The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem, e.g., for a half-space problem, the “top” of the box should be open.

4. Internally, LS-DYNA will partition the entire PML into regions which form the "faces", "edges" and "corners" of the above cuboid box, and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.
5. The layer should have 5-10 elements through its depth. Typically, 5-6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8-10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer, and should be small enough to sufficiently discretize all significant wavelengths in the problem.
6. The nodes on the outer boundary of the layer should be fully constrained.
7. The pressure values reported by this material do not have any physical significance.

**\*MAT\_BIOT\_HYSTERETIC**

This is Material Type 232. This is a Biot linear hysteretic material, to be used for modeling the nearly-frequency-independent viscoelastic behaviour of soils subjected to cyclic loading, e.g. in soil-structure interaction analysis [Spanos and Tsavachidis (2001), Makris and Zhang (2000), Muscolini, Palmeri and Ricciardelli (2005)]. The hysteretic damping coefficient for the model is computed from a prescribed damping ratio by calibrating with an equivalent viscous damping model for a single-degree-of-freedom system. The damping increases the stiffness of the model and thus reduces the computed time-step size.

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	ZT	FD		
Type	A8	F	F	F	F	F		
Default	none	none	none	none	0.0	3.25		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
ZT	Damping ratio
FD	Dominant excitation frequency in Hz

**Remarks:**

1. The stress is computed as a function of the strain rate as

$$\sigma(t) = \int_0^t C_R(t - \tau) \dot{\epsilon}(\tau) d\tau$$

where

$$C_R(t) = C \left[ 1 + \frac{2\eta}{\pi} E_1(\beta t) \right]$$

with  $C$  being the elastic isotropic constitutive tensor,  $\eta$  the hysteretic damping factor, and  $\beta = 2\pi f_d/10$ , where  $f_d$  is the dominant excitation frequency in Hz. The function  $E_1$  is given by

$$E_1(s) = \int_s^{\infty} \frac{e^{-\zeta}}{\zeta} d\zeta$$

For efficient implementation, this function is approximated by a 5-term Prony series as

$$E_1(s) \approx \sum_{k=1}^5 b_k e^{a_k s}$$

such that  $b_k > 0$ .

2. The hysteretic damping factor  $\eta$  is obtained from the prescribed damping ratio  $\zeta$  as

$$\eta = \pi\zeta/\text{atan}(10) = 2.14\zeta$$

by assuming that, for a single degree-of-freedom system, the energy dissipated per cycle by the hysteretic material is the same as that by a viscous damper, if the excitation frequency matches the natural frequency of the system.

3. The consistent Young's modulus for this model is given by

$$E_c = E \left[ 1 + \frac{2\eta}{\pi} g \right]$$

where

$$g = \sum_{k=1}^5 b_k \frac{1}{a_k \beta \Delta t_n} [\exp(a_k \beta \Delta t_n) - 1]$$

Because  $g > 0$ , the computed element time-step size is smaller than that for the corresponding elastic element. Furthermore, the time-step size computed at any time depends on the previous time-step size. It can be demonstrated that the new computed time-step size stays within a narrow range of the previous time-step size, and for a uniform mesh, converges to a constant value. For  $f_d = 3.25\text{Hz}$  and  $\zeta = 0.05$ , the percentage decrease in time-step size can be expected to be about 12-15% for initial time-step sizes of less than 0.02 secs, and about 7-10% for initial time-step sizes larger than 0.02 secs.

4. The default value of the dominant frequency is chosen to be valid for earthquake excitation.

**\*MAT\_CAZACU\_BARLAT**

This is Material Type 233. This material model is for Hexagonal Closed Packet (HCP) metals and is based on the work by Cazacu et al. (2006). This model is capable of describing the yielding asymmetry between tension and compression for such materials. Moreover, a parameter fit is optional and can be used to find the material parameters that describe the experimental yield stresses. The experimental data that the user should supply consists of yield stresses for tension and compression in the 00 direction, tension in the 45 and the 90 directions, and a biaxial tension test.

Available options include:

<BLANK>

MAGNESIUM

Including MAGNESIUM invokes a material model developed by the USAMP consortium to simulate cast Magnesium under impact loading. The model includes rate effects having a tabulated failure model including equivalent plastic strain to failure as a function of stress triaxiality and effective plastic strain rate. Element erosion will occur when the number of integration points where the damage variable has reached unity reaches some specified threshold (NUMINT). Alternatively a Gurson type failure model can be activated, which requires less experimental data.

The input of the hardening curve for MAT\_233 requires the user to provide the evolution of the Cazacu-Barlat effective stress as a function of the energy conjugate plastic strain. With the MAGNESIUM option an alternative option for the hardening curve is available: von Mises effective stress as a function of equivalent plastic strain, which is energy conjugate to the von Mises stress.

Finally the MAGNESIUM option allows for distortional hardening by providing hardening curves as measured in tension and compression tests. This option is however incompatible with the activation of rate effects (visco-plasticity).

With the MAGNESIUM option this material model is also available for solid elements.

<p><b>NOTE:</b> Activating the MAGNESIUM options <i>requires</i> setting HR = 3 and FIT = 0.0. (Also see below)</p>
---



Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	HR	P1	P2	ITER
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	A	C11	C22	C33	LCID	E0	K	P3
Type	F	F	F	F	I	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT				C12	C13	C23	C44
Type	F				F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	FIT
Type	F	F	F	F	F	F	F	I

**Magnesium Card.** Additional card for MAGNESIUM keyword option.

Card 6	1	2	3	4	5	6	7	8
Variable	LC1ID	LC2ID	NUMINT	LCCID	ICFLAG	IDFLAG	LC3ID	EPSFG
Type	I	I	F	I	I	I	I	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material Identification number.
RO	Constant Mass density.
E	Young's modulus E.GT.0.0: constant value E.LT.0.0: load curve ID (-E) which defines the Young's modulus as a function of plastic strain.
PR	Poisson's ratio
HR	Hardening rules: HR.EQ.1.0: linear hardening (default) HR.EQ.2.0: exponential hardening (Swift) HR.EQ.3.0: load curve HR.EQ.4.0: exponential hardening (Voce) HR.EQ.5.0: exponential hardening (Gosh) HR.EQ.6.0: exponential hardening (Hocken-Sherby) HR must be set to 3 if the MAGNESIUM option is active
P1	Material parameter: HR.EQ.1.0: tangent modulus HR.EQ.2.0: $q$ , coefficient for exponential hardening law (Swift) HR.EQ.4.0: $a$ , coefficient for exponential hardening law (Voce) HR.EQ.5.0: $q$ , coefficient for exponential hardening law (Gosh) HR.EQ.6.0: $a$ , coefficient for exponential hardening law (Hockett-Sherby)

VARIABLE	DESCRIPTION
P2	Material parameter: HR.EQ.1.0: yield stress for the linear hardening law HR.EQ.2.0: $n$ , coefficient for (Swift) exponential hardening HR.EQ.4.0: $c$ , coefficient for exponential hardening law (Voce) HR.EQ.5.0: $n$ , coefficient for exponential hardening law (Gosh) HR.EQ.6.0: $c$ , coefficient for exponential hardening law (Hockett-Sherby)
ITER	Iteration flag for speed: ITER.EQ.0.0: fully iterative ITER.EQ.1.0: fixed at three iterations. Generally, ITER = 0.0 is recommended. However, ITER = 1.0 is faster and may give acceptable results in most problems.
A	Exponent in Cazacu-Barlat's orthotropic yield surface ( $A > 1$ )
C11	Material parameter (see card 5 pos. 8): FIT.EQ.1.0 or EQ.2.0: yield stress for tension in the 00 direction FIT.EQ.0.0: material parameter $c_{11}$
C22	Material parameter (see card 5 pos.8): FIT.EQ.1.0 or EQ.2.0: yield stress for tension in the 45 direction FIT.EQ.0.0: material parameter $c_{22}$
C33	Material parameter (see card 5 pos.8): FIT.EQ.1.0 or EQ.2.0: yield stress for tension in the 90 direction FIT.EQ.0.0: material parameter $c_{33}$
LCID	Load curve ID for the hardening law (HR.EQ.3.0), Table ID for rate dependent hardening if the MAGNESIUM option is active

<b>VARIABLE</b>	<b>DESCRIPTION</b>
E0	Material parameter: HR.EQ.2.0: $\epsilon_0$ , initial yield strain for exponential hardening law (Swift) (default = 0.0) HR.EQ.4.0: $b$ , coefficient for exponential hardening (Voce) HR.EQ.5.0: $\epsilon_0$ , initial yield strain for exponential hardening (Gosh), Default = 0.0 HR.EQ.6.0: $b$ , coefficient for exponential hardening law (Hockett-Sherby)
K	Material parameter (see card 5 pos.8): FIT.EQ.1.0 or EQ.2.0: yield stress for compression in the 00 direction FIT.EQ.0.0: material parameter (-1 < k<1)
P3	Material parameter: HR.EQ.5.0: $p$ , coefficient for exponential hardening (Gosh) HR.EQ.6.0: $n$ , exponent for exponential hardening law (Hockett-Sherby)

VARIABLE	DESCRIPTION
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for more complete description).</p> <p>AOPT.EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2 and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.</p> <p>AOPT.EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINED_COORDINATE_VECTOR.</p> <p>AOPT.EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle BETA, from a line in the plane of the element defined by the cross product of the vector V with the element normal.</p> <p>AOPT.LT.0.0: the absolute value of AOPT is coordinate system ID (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINED_COORDINATE_VECTOR). Available with the R3 release of 971 and later.</p>
C12	Material parameter. If parameter identification (FIT = 1.0) is turned on C12 is not used.
C13	Material parameter. If parameter identification (FIT = 1.0) is turned on C13 = 0.0
C23	Material parameter. If parameter identification (FIT = 1.0) is turned on C23 = 0.0
C44	<p>Material parameter (see card 5 pos.8)</p> <p>FIT.EQ.1.0 or EQ.2.0: yield stress for the balanced biaxial tension test.</p> <p>FIT.EQ.0.0: material parameter c44</p>
A1 - A3	Components of vector <b>a</b> for AOPT = 2.0
V1 - V3	Components of vector <b>v</b> for AOPT = 3.0
D1 - D3	Components of vector <b>d</b> for AOPT = 2.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	Material angle in degrees for AOPT = 0 and 3. NOTE, may be overridden on the element card, see *ELEMENT_SHELL_BETA
FIT	Flag for parameter identification algorithm:  FIT.EQ.0.0: No parameter identification routine is used. The variables K, C11, C22, C33, C44, C12, C13 and C23 are interpreted as material parameters. FIT MUST be set to zero if MAGNESIUM option is active  FIT.EQ.1.0: Parameter fit is used. The variables C11, C22, C33, C44 and K are interpreted as yield stresses in the 00, 45, 90 degree directions, the balanced biaxial tension and the 00 degree compression, respectively.  <i>It is recommended to always check the d3hsp file to see the fitted parameters before complex jobs are submitted.</i>  FIT.EQ.2.0: Same as EQ.1.0 but also produce contour plots of the yield surface. For each material three LS-PrePost ready xy-data files are created; Contour1_n, Contour2_n and Contour3_n where n equal the material number.
LC1ID	Load curve ID giving equivalent plastic strain to failure as a function of stress triaxiality or a table ID giving plastic strain to failure as a function of Lode parameter and stress triaxiality (solids)
LC2ID	Load curve ID giving equivalent plastic strain to failure as a function of equivalent plastic strain rate, the failure strain will be computed as the product of the values on LC1ID and LC2ID
NUMINT	Number of through thickness integration points which must fail before the element is deleted (inactive for solid elements)
LCCID	Load curve ID giving effective stress in function of plastic strain obtained from a compression stress, input of this load curve will activate distortional hardening and is NOT compatible with the use of strain rate effects
ICFLAG	Automated input conversion flag. If ICFLAG = 0 then the load curves provided under LCID and LCCID contain Cazacu-Barlat effective stress as a function of energy conjugate plastic strain. If ICFLAG = 1 then both load curves are given in terms of von Mises stress versus equivalent plastic strain

VARIABLE	DESCRIPTION
IDFLAG	Damage flag. If IDFLAG = 0 the failure model is of the Johnson Cook type and requires LC1ID and LC2ID as additional input. If IDFLAG = 1 the failure model is of the Gurson type and requires LC3ID and EPSFG as additional input
LC3ID	Load curve giving the critical void fraction of the Gurson model as a function of the plastic strain to failure measured in the uniaxial tensile test
EPSFG	Plastic strain to failure measured in the uniaxial tensile test, this value is used by the Gurson type failure model only.

### Remarks:

The material model #233 (MAT\_CAZACU\_BARLAT) is aimed for modeling materials with strength differential and orthotropic behavior under plane stress. The yield condition includes a parameter  $k$  that describes the asymmetry between yield in tension and compression. Moreover, to include the anisotropic behavior the stress deviator  $\mathbf{S}$  undergoes a linear transformation. The principal values of the Cauchy stress deviator are substituted with the principal values of the transformed tensor  $\mathbf{Z}$ , which is represented as a vector field, defined as:

$$\mathbf{Z} = \mathbf{C}\mathbf{S} \quad (233.1)$$

where  $\mathbf{S}$  is the field comprised of the four stresses deviator components  $S_I = (s_{11}, s_{22}, s_{33}, s_{12})$ ,

$$\mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3} \text{tr}(\boldsymbol{\sigma})\boldsymbol{\delta},$$

where  $\text{tr}(\boldsymbol{\sigma})$  is the trace of the Cauchy stress tensor and  $\boldsymbol{\delta}$  is the Kronecker delta. For the 2D plane stress condition, the orthotropic condition gives 7 independent coefficients. The tensor  $\mathbf{C}$  is represented by the 4x4 matrix

$$C_{IJ} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & \\ c_{12} & c_{22} & c_{23} & \\ c_{13} & c_{23} & c_{33} & \\ & & & c_{44} \end{pmatrix}.$$

The principal values of  $\mathbf{Z}$  are denoted  $\Sigma_1, \Sigma_2, \Sigma_3$  and are given as the eigenvalues to the matrix composed by the components  $\Sigma_{xx}, \Sigma_{yy}, \Sigma_{zz}, \Sigma_{xy}$  through

$$\Sigma_1 = \frac{1}{2} \left( \Sigma_{xx} + \Sigma_{yy} + \sqrt{(\Sigma_{xx} - \Sigma_{yy})^2 + 4\Sigma_{xy}^2} \right),$$

$$\Sigma_2 = \frac{1}{2} \left( \Sigma_{xx} + \Sigma_{yy} - \sqrt{(\Sigma_{xx} - \Sigma_{yy})^2 + 4\Sigma_{xy}^2} \right),$$

$$\Sigma_3 = \Sigma_{zz}$$

where

$$\begin{aligned} 3\Sigma_{xx} &= (2c_{11} - c_{12} - c_{13})\sigma_{xx} + (-c_{11} + 2c_{12} - c_{13})\sigma_{yy}, \\ 3\Sigma_{yy} &= (2c_{12} - c_{22} - c_{23})\sigma_{xx} + (-c_{12} + 2c_{22} - c_{23})\sigma_{yy}, \\ 3\Sigma_{zz} &= (2c_{13} - c_{23} - c_{33})\sigma_{xx} + (-c_{13} + 2c_{23} - c_{33})\sigma_{yy}, \\ \Sigma_{xy} &= c_{44}\sigma_{12} \end{aligned}$$

Note that the symmetry of  $\Sigma_{xy}$  follows from the symmetry of the Cauchy stress tensor.

The yield condition is written on the following form:

$$f(\Sigma, k, \varepsilon_{ep}) = \sigma_{\text{eff}}(\Sigma_1, \Sigma_2, \Sigma_3, k) - \sigma_y(\varepsilon_{ep}) \leq 0 \quad (233.2)$$

where  $\sigma_y(\varepsilon_{ep})$  is a function representing the current yield stress dependent on current effective plastic strain and  $k$  is the asymmetric parameter for yield in compression and tension. The effective stress  $\sigma_{\text{eff}}$  is given by

$$\sigma_{\text{eff}} = [(|\Sigma_1| - k\Sigma_1)^a + (|\Sigma_2| - k\Sigma_2)^a + (|\Sigma_3| - k\Sigma_3)^a]^{1/a} \quad (233.3)$$

where  $k \in [-1, 1]$ ,  $a \geq 1$ . Now, let  $\sigma_{00}^T$  and  $\sigma_{00}^C$  represent the yield stress along the rolling (00 degree) direction in tension and compression, respectively. Furthermore let  $\sigma_{45}^T$  and  $\sigma_{90}^T$  represent the yield stresses in the 45 and the 90 degree directions, and last let  $\sigma_B^T$  be the balanced biaxial yield stress in tension. Following Cazacu et al. (2006) the yield stresses can easily be derived.

To simplify the equations it is preferable to make the following definitions:

$$\begin{aligned} \Phi_1 &= \frac{1}{3}(2c_{11} - c_{12} - c_{13}) & \Psi_1 &= \frac{1}{3}(-c_{11} + 2c_{12} - c_{13}) \\ \Phi_2 &= \frac{1}{3}(2c_{12} - c_{22} - c_{23}) \quad \text{and} & \Psi_2 &= \frac{1}{3}(-c_{12} + 2c_{22} - c_{23}) \\ \Phi_3 &= \frac{1}{3}(2c_{13} - c_{23} - c_{33}) & \Psi_3 &= \frac{1}{3}(-c_{13} + 2c_{23} - c_{33}) \end{aligned}$$

The yield stresses can now be written as:

1. In the 00 degree direction:

$$\begin{aligned} \sigma_{00}^T &= \left[ \frac{(\sigma_{\text{eff}})^a}{(|\Phi_1| - k\Phi_1)^a + (|\Phi_2| - k\Phi_2)^a + (|\Phi_3| - k\Phi_3)^a} \right]^{1/a}, \\ \sigma_{00}^C &= \left[ \frac{(\sigma_{\text{eff}})^a}{(|\Phi_1| + k\Phi_1)^a + (|\Phi_2| + k\Phi_2)^a + (|\Phi_3| + k\Phi_3)^a} \right]^{1/a} \end{aligned} \quad (233.4)$$



2. In the 45 degree direction:

$$\sigma_{45}^T = \left[ \frac{(\sigma_{\text{eff}})^a}{(|\Lambda_1| - k\Lambda_1)^a + (|\Lambda_2| - k\Lambda_2)^a + (|\Lambda_3| - k\Lambda_3)^a} \right]^{1/a} \quad (233.5)$$

where

$$\Lambda_1 = \frac{1}{4} \left[ \Phi_1 + \Phi_2 + \Psi_1 + \Psi_2 + \sqrt{(\Phi_1 + \Psi_1 - \Phi_2 - \Psi_2)^2 + 4c_{44}^2} \right],$$

$$\Lambda_2 = \frac{1}{4} \left[ \Phi_1 + \Phi_2 + \Psi_1 + \Psi_2 - \sqrt{(\Phi_1 + \Psi_1 - \Phi_2 - \Psi_2)^2 + 4c_{44}^2} \right],$$

$$\Lambda_3 = \frac{1}{2} [\Phi_3 + \Psi_3].$$

3. In the 90 degree direction:

$$\sigma_{90}^T = \left[ \frac{(\sigma_{\text{eff}})^a}{(|\Psi_1| - k\Psi_1)^a + (|\Psi_2| - k\Psi_2)^a + (|\Psi_3| - k\Psi_3)^a} \right]^{1/a} \quad (233.6)$$

4. In the balanced biaxial yield occurs when both  $\sigma_{xx}$  and  $\sigma_{yy}$  are equal to:

$$\sigma_B^T = \left[ \frac{(\sigma_{\text{eff}})^a}{(|\Omega_1| - k\Omega_1)^a + (|\Omega_2| - k\Omega_2)^a + (|\Omega_3| - k\Omega_3)^a} \right]^{1/a} \quad (233.7)$$

where

$$\Omega_1 = \frac{1}{3} (c_{11} + c_{12} - 2c_{13})$$

$$\Omega_2 = \frac{1}{3} (c_{12} + c_{22} - 2c_{23})$$

$$\Omega_3 = \frac{1}{3} (c_{13} + c_{23} - 2c_{33})$$

### Hardening laws:

The implemented hardening laws are the following:

1. The Swift hardening law
2. The Voce hardening law
3. The Gosh hardening law
4. The Hockett-Sherby hardening law

5. A loading curve, where the yield stress is given as a function of the effective plastic strain

The Swift's hardening law can be written

$$\sigma_y(\varepsilon_{ep}) = q(\varepsilon_0 + \varepsilon_{ep})^n$$

where  $q$  and  $n$  are material parameters.

The Voce's equation says that the yield stress can be written in the following form

$$\sigma_y(\varepsilon_{ep}) = a - be^{-c\varepsilon_{ep}}$$

where  $a$ ,  $b$  and  $c$  are material parameters. The Gosh's equation is similar to Swift's equation. They only differ by a constant

$$\sigma_y(\varepsilon_{ep}) = q(\varepsilon_0 + \varepsilon_{ep})^n - p$$

where  $q$ ,  $\varepsilon_0$ ,  $n$  and  $p$  are material constants. The Hocket-Sherby equation resemblance the Voce's equation, but with an additional parameter added

$$\sigma_y(\varepsilon_{ep}) = a - be^{-c\varepsilon_{ep}^n}$$

where  $a$ ,  $b$ ,  $c$  and  $n$  are material parameters.

**Constitutive relation and material stiffness:**

The classical elastic constitutive equation for linear deformations is the well-known Hooke's law. This relation written in a rate formulation is given by

$$\dot{\sigma} = \mathbf{D}\dot{\varepsilon}_e \tag{233.8}$$

where  $\varepsilon_e$  is the elastic strain and  $\mathbf{D}$  is the constitutive matrix. An over imposed dot indicates differentiation respect to time. Introducing the total strain  $\varepsilon$  and the plastic strain  $\varepsilon_p$ , Eq. (233.8) is classically rewritten as

$$\dot{\sigma} = \mathbf{D}(\dot{\varepsilon} - \dot{\varepsilon}_p) \tag{233.9}$$

where

$$\mathbf{D} = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & & & \\ \nu & 1 & & & \\ & & \frac{1-\nu}{2} & & \\ & & & \frac{1-\nu}{2} & \\ & & & & \frac{1-\nu}{2} \end{pmatrix} \text{ and } (\dot{\varepsilon} - \dot{\varepsilon}_p) = \begin{pmatrix} \dot{\varepsilon}_{11} - (\dot{\varepsilon}_p)_{11} \\ \dot{\varepsilon}_{22} - (\dot{\varepsilon}_p)_{22} \\ 2[\dot{\varepsilon}_{12} - (\dot{\varepsilon}_p)_{12}] \\ 2[\dot{\varepsilon}_{13} - (\dot{\varepsilon}_p)_{13}] \\ 2[\dot{\varepsilon}_{23} - (\dot{\varepsilon}_p)_{23}] \end{pmatrix}.$$

The parameters  $E$  and  $\nu$  are the Young's modulus and Poisson's ratio, respectively.

The material stiffness  $\mathbf{D}_p$  that is needed for e.g., implicit analysis can be calculated from (233.9) as

$$\mathbf{D}_p = \frac{\partial \dot{\sigma}}{\partial \dot{\epsilon}}. \quad (233.10)$$

The associative flow rule for the plastic strain is usually written

$$\dot{\epsilon}_p = \lambda \frac{\partial f}{\partial \sigma} \quad (233.10)$$

and the consistency condition reads

$$\frac{df}{d\sigma} \dot{\sigma} + \frac{df}{d\epsilon_{ep}} \dot{\epsilon}_{ep} = 0. \quad (233.11)$$

Note that the centralized “dot” means scalar product between two vectors. Using standard calculus one easily derives from (1.9), (1.10) and (1.11) an expression for the stress rate

$$\dot{\sigma} = \left[ \mathbf{D} - \frac{\left( \mathbf{D} \frac{df}{d\sigma} \right) \cdot \left( \mathbf{D} \frac{df}{d\sigma} \right)}{\frac{df}{d\sigma} \cdot \left( \mathbf{D} \frac{df}{d\sigma} \right) - \frac{df}{d\epsilon_{ep}}} \right] \dot{\epsilon} \quad (233.12)$$

That means that the material stiffness used for implicit analysis is given by

$$\mathbf{D}_p = \mathbf{D} - \frac{\left( \mathbf{D} \frac{df}{d\sigma} \right) \cdot \left( \mathbf{D} \frac{df}{d\sigma} \right)}{\frac{df}{d\sigma} \cdot \left( \mathbf{D} \frac{df}{d\sigma} \right) - \frac{df}{d\epsilon_{ep}}}. \quad (233.13)$$

To be able to do a stress update we need to calculate the tangent stiffness and the derivative with respect to the corresponding hardening law.

When a suitable hardening law has been chosen the corresponding derivative is simple and will be left out from this document. However, the stress gradient of the yield surface is more complicated and will be outlined here.

$$\begin{aligned} \frac{\partial f}{\partial \sigma_{11}} = \frac{\partial f}{\partial \Sigma_3} \frac{1}{2} \frac{\partial f}{\partial \Sigma_1} & \left[ \left( 1 + \frac{\Sigma_{xx} - \Sigma_{yy}}{\sqrt{\Sigma_T}} \right) \Phi_1 + \left( 1 - \frac{\Sigma_{xx} - \Sigma_{yy}}{\sqrt{\Sigma_T}} \right) \Phi_2 \right] \\ & + \frac{1}{2} \frac{\partial f}{\partial \Sigma_2} \left[ \left( 1 - \frac{\Sigma_{xx} - \Sigma_{yy}}{\sqrt{\Sigma_T}} \right) \Phi_1 + \left( 1 + \frac{\Sigma_{xx} - \Sigma_{yy}}{\sqrt{\Sigma_T}} \right) \Phi_2 \right] + \Phi_3 \end{aligned} \quad (233.14)$$

$$\begin{aligned} \frac{\partial f}{\partial \sigma_{22}} = \frac{1}{2} \frac{\partial f}{\partial \Sigma_1} & \left[ \left( 1 + \frac{\Sigma_{xx} - \Sigma_{yy}}{\sqrt{\Sigma_T}} \right) \Psi_1 + \left( 1 - \frac{\Sigma_{xx} - \Sigma_{yy}}{\sqrt{\Sigma_T}} \right) \Psi_2 \right] \\ & + \frac{1}{2} \frac{\partial f}{\partial \Sigma_2} \left[ \left( 1 - \frac{\Sigma_{xx} - \Sigma_{yy}}{\sqrt{\Sigma_T}} \right) \Psi_1 + \left( 1 + \frac{\Sigma_{xx} - \Sigma_{yy}}{\sqrt{\Sigma_T}} \right) \Psi_2 \right] + \frac{\partial f}{\partial \Sigma_3} \Psi_3 \end{aligned} \quad (233.15)$$

and the derivative with respect to the shear stress component is

$$\frac{\partial f}{\partial \sigma_{12}} = c_{44} \frac{2\Sigma_{xy}}{\sqrt{\Sigma_T}} \left( \frac{\partial f}{\partial \Sigma_1} - \frac{\partial f}{\partial \Sigma_2} \right) \quad (233.16)$$

where

$$\Sigma_T = (\Sigma_{xx} - \Sigma_{yy})^2 + 4\Sigma_{xy}^2 \quad (233.17)$$

and

$$\frac{\partial f}{\partial \Sigma_i} = f(\Sigma, k, \varepsilon_{ep})^{\frac{1}{a}-1} (|\Sigma_i| - k\Sigma_i)^{a-1} (\text{sgn}(\Sigma_i) - k) \text{ for } i = 1,2,3 \quad (233.18)$$

### Implementation:

Assume that the stress and strain is known at time  $t^n$ . A trial stress  $\tilde{\sigma}^{n+1}$  at time  $t^{n+1}$  is calculated by assuming a pure elastic deformation, i.e.,

$$\tilde{\sigma}^{n+1} = \sigma^n + \mathbf{D}(\varepsilon^{n+1} - \varepsilon^n) \quad (233.19)$$

Now, if  $f(\Sigma, k, \varepsilon_{ep}) \leq 0$  the deformation is pure elastic and the new stress and plastic strain are determined as

$$\begin{aligned} \sigma^{n+1} &= \tilde{\sigma}^{n+1} \\ \varepsilon_{ep}^{n+1} &= \varepsilon_{ep}^n \end{aligned} \quad (233.20)$$

and the thickness strain increment is given by

$$\Delta\varepsilon_{33} = \varepsilon_{33}^{n+1} - \varepsilon_{33}^n = -\frac{\nu}{1-\nu} (\Delta\varepsilon_{11} + \Delta\varepsilon_{22}) \quad (233.21)$$

If the deformation is not pure elastic the stress is not inside the yield surface and a plastic iterative procedure must take place.

1. Set  $m = 0$ ,  $\sigma_{(0)}^{n+1} = \tilde{\sigma}^{n+1}$ ,  $\varepsilon_{ep(0)}^{n+1} = \varepsilon_{ep}^n$  and  $\Delta\varepsilon_{11}^{p(0)} = \Delta\varepsilon_{22}^{p(0)} = 0$
2. Determine the plastic multiplier as

$$\Delta\lambda = \frac{f(\sigma_{(m)}^{n+1}, \varepsilon_{ep(m)}^{n+1})}{\frac{df}{d\sigma}(\sigma_{(m)}^{n+1}) \cdot \mathbf{D} \frac{df}{d\sigma}(\sigma_{(m)}^{n+1}) - \frac{df}{d\varepsilon_{ep}}(\varepsilon_{ep(m)}^{n+1})} \quad (233.22)$$

3. Perform a plastic corrector step:  $\sigma_{(m+1)}^{n+1} = \sigma_{(m)}^{n+1} - \Delta\lambda \mathbf{D} \frac{df}{d\sigma}(\sigma_{(m)}^{n+1})$  and find the increments in plastic strain according to

$$\begin{aligned}
\varepsilon_{\text{ep}(m+1)}^{n+1} &= \varepsilon_{\text{ep}(m)}^{n+1} + \Delta\lambda \\
\Delta\varepsilon_{11}^{p(n+1)} &= \Delta\varepsilon_{11}^{p(n)} + \Delta\lambda \frac{\partial f}{\partial \sigma_{11}}(\sigma^{n+1}) \\
\Delta\varepsilon_{22}^{p(n+1)} &= \Delta\varepsilon_{22}^{p(n)} + \Delta\lambda \frac{\partial f}{\partial \sigma_{22}}(\sigma^{n+1})
\end{aligned} \tag{233.23}$$

4. If  $|f(\boldsymbol{\sigma}_{(m+1)}^{n+1}, \varepsilon_{\text{ep}}^n)| < \text{tol}$  or  $m = m_{\text{max}}$ ; stop and set

$$\begin{aligned}
\boldsymbol{\sigma}^{n+1} &= \boldsymbol{\sigma}_{(m+1)}^{n+1}, \\
\varepsilon_{\text{ep}}^{n+1} &= \varepsilon_{\text{ep}(m+1)}^{n+1}, \\
\Delta\varepsilon_{11}^p &= \Delta\varepsilon_{11}^{p(m+1)}, \\
\Delta\varepsilon_{22}^p &= \Delta\varepsilon_{22}^{p(m+1)},
\end{aligned} \tag{233.24}$$

otherwise set  $m = m + 1$  and return to 2.

The thickness strain increment is for plastic yield calculated as

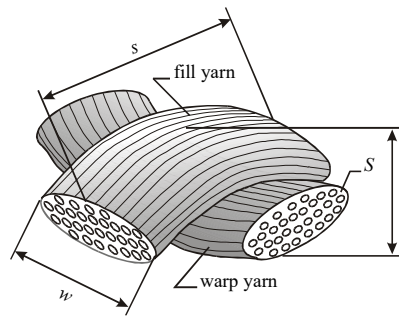
$$\Delta\varepsilon_{33} = -\frac{1}{1-\nu}(\Delta\varepsilon_{11} + \Delta\varepsilon_{22}) - \left(1 - \frac{\nu}{1-\nu}\right)(\Delta\varepsilon_{11}^p + \Delta\varepsilon_{22}^p) \tag{233.25}$$

The following history variables will be stored for the MAGNESIUM option:

HV1	equivalent plastic strain (energy conjugate to Cazacu-Barlat effective stress)
HV6	damage
HV7	plastic strain to failure
HV8	number of IP that failed
HV9	equivalent plastic strain (energy conjugate to von Mises stress)
HV10	effective stress (Cazacu-Barlat)
HV11	Gurson damage
HV12	void fraction
HV13	void fraction star
HV14	equivalent plastic strain (energy conjugate to von Mises stress)

**\*MAT\_VISCOELASTIC\_LOOSE\_FABRIC**

This is Material Type 234 developed and implemented by Tabiei et al [2004]. The model is a mechanism incorporating the crimping of the fibers as well as the trellising with reorientation of the yarns and the locking phenomenon observed in loose fabric. The equilibrium of the mechanism allows the straightening of the fibers depending on the fiber tension. The contact force at the fiber cross over point determines the rotational friction dissipating a part of the impact energy. The stress-strain relationship is viscoelastic based on a three-element model. The failure of the fibers is strain rate dependent. \*DAMPING\_PART\_MASS is recommended to be used in conjunction with this material model. This material is valid for modeling the elastic and viscoelastic response of loose fabric used in body armor, blade containments, and airbags.



**Figure M234-1.** Representative Volume Cell (RVC) of the model

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E1	E2	G12	EU	THL	THI
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	TA	W	s	T	H	S	EKA	EUA
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	VMB	C	G23	EKB	AOPT			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable	Not used	Not used	Not used	A1	A2	A3		
Type				F	F	F		

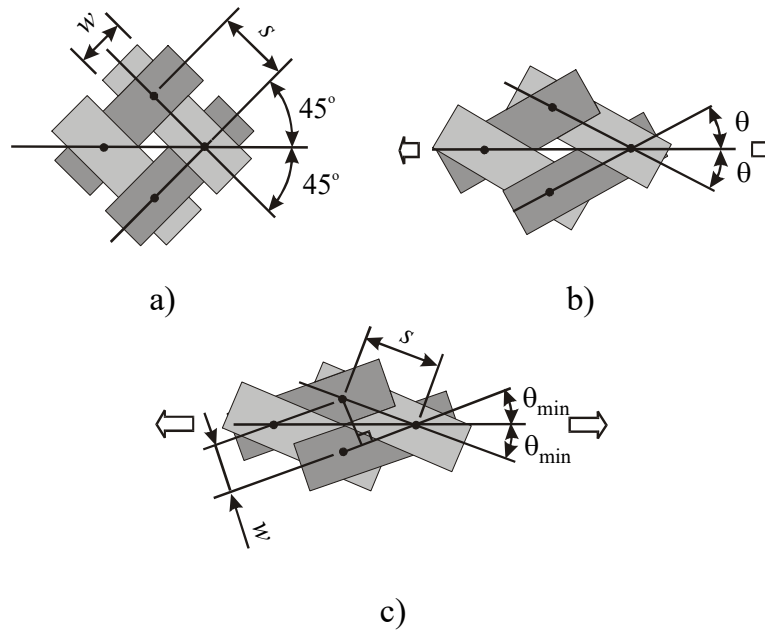
Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E1	$E_1$ , Young's modulus in the yarn axial-direction.
E2	$E_2$ , Young's modulus in the yarn transverse-direction.
G12	$G_{12}$ , Shear modulus of the yarns.
EU	Ultimate strain at failure.
THL	Yarn locking angle.
THI	Initial braid angle.
TA	Transition angle to locking.
W	Fiber width.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
S	Span between the fibers.
T	Real fiber thickness.
H	Effective fiber thickness.
S	Fiber cross-sectional area.
EKA	Elastic constant of element "a".
EUA	Ultimate strain of element "a".
VMB	Damping coefficient of element "b".
C	Coefficient of friction between the fibers.
G23	transverse shear modulus.
Ekb	Elastic constant of element "b"
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for more complete description).  AOPT.EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2 and 4, as with *DEFINE_COORDINATE_NODES.  AOPT.EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINED_COORDINATE_VECTOR.  AOPT.EQ.3.0: locally orthotropic material axes defined by the cross product of the vector V with the element normal.  AOPT.LT.0.0: the absolute value of AOPT is coordinate system ID (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINED_COORDINATE_VECTOR). Available with the R3 release of 971 and later.





**Figure M234-2.** Plain woven fabric as trellis mechanism: a) initial state; b) slightly stretched in bias direction; c) stretched to locking.

**Remarks:**

The parameters of the Representative Volume Cell (RVC) are: the yarn span,  $s$ , the fabric thickness,  $t$ , the yarn width,  $w$ , and the yarn cross-sectional area,  $A$ . The initially orthogonal yarns (see Fig. M234-2a) are free to rotate (see Fig. M234-2b) up to some angle and after that the lateral contact between the yarns causes the locking of the trellis mechanism and the packing of the yarns (see Fig. M234-2c). The minimum braid angle,  $\theta_{min}$ , can be calculated from the geometry and the architecture of the fabric material having the yarn width,  $w$ , and the span between the yarns,  $s$ :

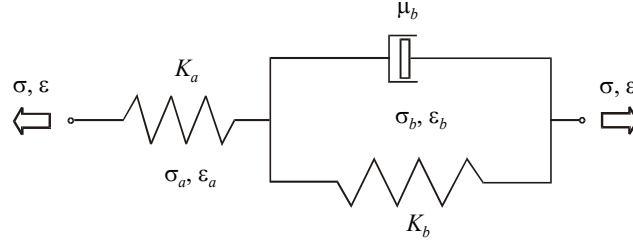
$$\sin(2\theta_{min}) = \frac{w}{s}$$

The other constrain angles as the locking range angle,  $\theta_{lock}$ , and the maximum braid angle,  $\theta_{max}$ , (see Fig) are easy to be determined then:

$$\theta_{lock} = 45^\circ - \theta_{min} , \quad \theta_{max} = 45^\circ + \theta_{lock}$$

The material behavior of the yarn can be simply described by a combination of one Maxwell element without the dashpot and one Kelvin-Voigt element. The 1-D model of viscoelasticity is shown in the following figure. The differential equation of viscoelasticity of the yarns can be derived from the model equilibrium as in the following equation:

$$(K_a + K_b)\sigma + \mu_b\dot{\sigma} = K_aK_b\varepsilon + \mu_bK_a\dot{\varepsilon}$$



**Figure M234-3.** Three-element viscoelasticity model

The input parameters for the viscoelasticity model of the material are only the static Young's modulus  $E_1$ , the Hookian spring coefficient (EKA)  $K_a$ , the viscosity coefficient (VMB)  $\mu_b$ , the static ultimate strain (EU)  $\varepsilon_{\max}$ , and the Hookian spring ultimate strain (EUA)  $\varepsilon_{a\max}$ . The other parameters can be obtained as follows:

$$K_b = \frac{K_a E_1}{K_a - E_1}$$

$$\varepsilon_{b\max} = \frac{K_a - E_1}{K_a} \varepsilon_{\max}$$

Applying the Eq. (18) for the fill and the warp yarns, we obtain the stress increments in the yarns,  $\Delta\sigma_f$  and  $\Delta\sigma_w$ . The stress in the yarns is updated for the next time step:

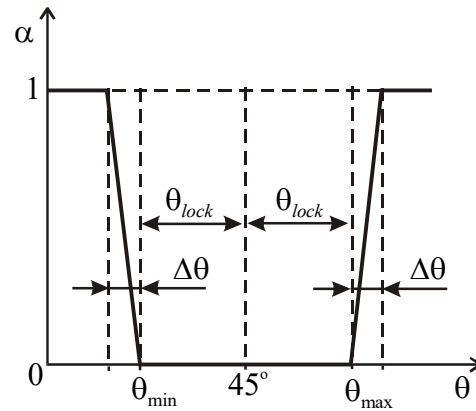
$$\sigma_f^{(n+1)} = \sigma_f^{(n)} + \Delta\sigma_f^{(n)}, \quad \sigma_w^{(n+1)} = \sigma_w^{(n)} + \Delta\sigma_w^{(n)}$$

We can imagine that the RVC is smeared to the parallelepiped in order to transform the stress acting on the yarn cross-section to the stress acting on the element wall. The thickness of the membrane shell element used should be equal to the effective thickness,  $t_e$ , that can be found by dividing the areal density of the fabric by its mass density. The in-plane stress components acting on the RVC walls in the material direction of the yarns are calculated as follows for the fill and warp directions:

$$\sigma_{f11}^{(n+1)} = \frac{2\sigma_f^{(n+1)} S}{st_e} \quad \sigma_{w11}^{(n+1)} = \frac{2\sigma_w^{(n+1)} S}{st_e}$$

$$\sigma_{f22}^{(n+1)} = \sigma_{f22}^{(n)} + \alpha E_2 \Delta\varepsilon_{f22}^{(n)} \quad \sigma_{w22}^{(n+1)} = \sigma_{w22}^{(n)} + \alpha E_2 \Delta\varepsilon_{w22}^{(n)}$$

$$\sigma_{f12}^{(n+1)} = \sigma_{f12}^{(n)} + \alpha G_{12} \Delta\varepsilon_{f12}^{(n)} \quad \sigma_{w12}^{(n+1)} = \sigma_{w12}^{(n)} + \alpha G_{12} \Delta\varepsilon_{w12}^{(n)}$$



**Figure M234-4.** The lateral contact factor as a function of average braid angle  $\theta$ .

where  $E_2$  is the transverse Young's modulus of the yarns,  $G_{12}$  is the longitudinal shear modulus, and  $\alpha$  is the lateral contact factor. The lateral contact factor is zero when the trellis mechanism is open and unity if the mechanism is locked with full lateral contact between the yarns. There is a transition range,  $\Delta\theta \times TA$ , of the average braid angle  $\theta$  in which the lateral contact factor,  $\alpha$ , is a linear function of the average braid angle. The graph of the function  $\alpha(\theta)$  is shown in Fig. [M234-4](#).

**\*MAT\_MICROMECHANICS\_DRY\_FABRIC**

This is Material Type 235 developed and implemented by Tabiei et al [2001]. The material model derivation utilizes the micro-mechanical approach and the homogenization technique usually used in composite material models. The model accounts for reorientation of the yarns and the fabric architecture. The behavior of the flexible fabric material is achieved by discounting the shear moduli of the material in free state, which allows the simulation of the trellis mechanism before packing the yarns. This material is valid for modeling the elastic response of loose fabric used in inflatable structures, parachutes, body armor, blade containments, and airbags.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E1	E2	G12	G23	V12	V23
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	XT	THI	THL	BFI	BWI	DSCF	CNST	ATLR
Type	F	F	F	F	F	F	F	F

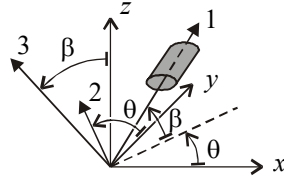
Card 3	1	2	3	4	5	6	7	8
Variable	VMB	VME	TRS	FFLG	AOPT			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable	Not used	Not used	Not used	A1	A2	A3		
Type				F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E1	$E_1$ , Young's modulus of the yarn in axial-direction.
E2	$E_2$ , Young's modulus of the yarn in transverse-direction.
G12	$G_{12}$ , shear modulus of the yarns.
G23	$G_{23}$ , transverse shear modulus of the yarns.
V12	Poisson's ratio.
V23	Transverse Poisson's ratio.
XT	Stress or strain to failure (see FFLG).
THI	Initial brade angle.
THL	Yarn locking angle.
BFI	Initial undulation angle in fill direction.
BWI	Initial undulation angle in warp direction.
DSCF	Discount factor
CNST	Reorientation damping constant
ATLR	Angle tolerance for locking
VME	Viscous modulus for normal strain rate
VMS	Viscous modulus for shear strain rate
TRS	Transverse shear modulus of the fabric layer



**Figure M235-1.** Yarn orientation schematic.

VARIABLE	DESCRIPTION
FFLG	Flag for stress-based or strain-based failure EQ.0: XT is a stress to failure NE.0: XT is a strain to failure
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for more complete description). AOPT.EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2 and 4, as with *DEFINE_COORDINATE_NODES. AOPT.EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINED_COORDINATE_VECTOR. AOPT.EQ.3.0: locally orthotropic material axes defined by the cross product of the vector V with the element normal. AOPT.LT.0.0: the absolute value of AOPT is coordinate system ID (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINED_COORDINATE_VECTOR). Available with the R3 release of 971 and later.
A1 - A3	Components of vector <b>a</b> for AOPT = 2.0
V1 - V3	Components of vector <b>v</b> for AOPT = 3.0
D1 - D3	Components of vector <b>d</b> for AOPT = 2.0

**Remarks:**

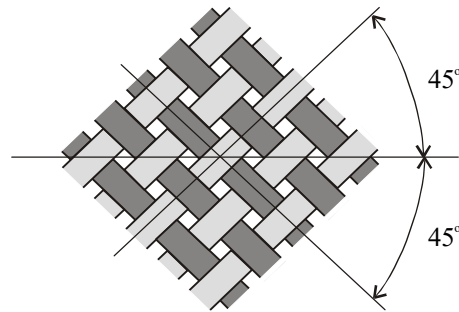
The Representative Volume Cell (RVC) approach is utilized in the micro-mechanical model development. The direction of the yarn in each sub-cell is determined by two angles – the braid angle,  $\theta$  (*the initial braid angle is 45 degrees*), and the undulation angle of the yarn, which is different for the fill and warp-yarns,  $\beta_f$  and  $\beta_w$  (*the initial undulations are normal*

few degrees), respectively. The starting point for the homogenization of the material properties is the determination of the yarn stiffness matrices.

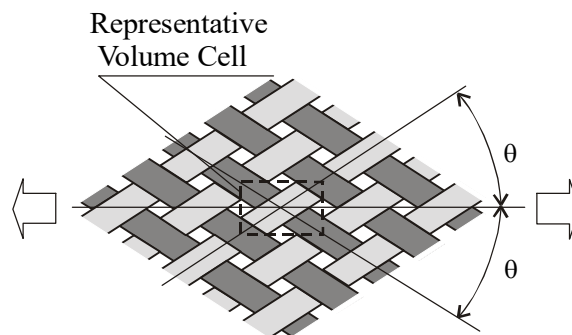
The elasticity tensor is given by

$$[C'] = [S']^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\mu G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\mu G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\mu G_{12}} \end{bmatrix}^{-1}$$

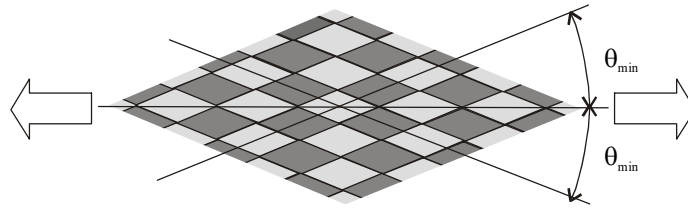
where  $E_1$ ,  $E_2$ ,  $\nu_{12}$ ,  $\nu_{23}$ ,  $G_{12}$  and  $G_{23}$  are Young's moduli, Poisson's ratios, and the shear moduli of the yarn material, respectively.  $\mu$  is a discount factor, which is function of the braid angle,  $\theta$ , and has value between  $\mu_0$  and 1 as shown in the next figure. Initially, in free stress state, the discount factor is a small value ( $DSCF = \mu_0 \ll 1$ ) and the material has very small resistance to shear deformation if any.



**Plain Woven Fabric: Free State**

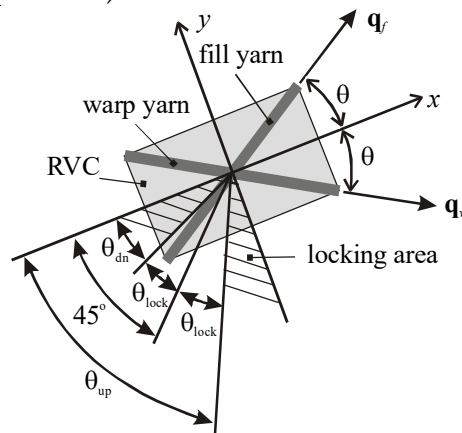


**Plain Woven Fabric: Stretched**



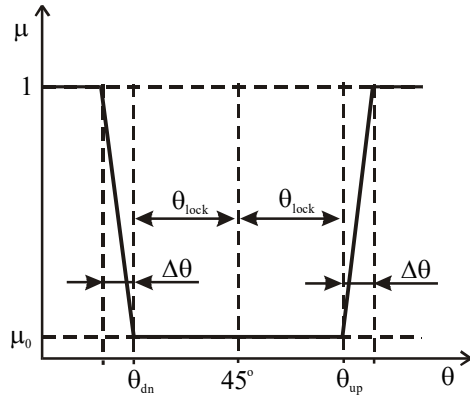
**Plain Woven Fabric: Compacted**

When the locking occurs, the fabric yarns are packed and they behave like elastic media. The discount factor is unity as shown in the next figure. The micro-mechanical model is developed to account for the reorientation of the yarns up to the locking angle. The locking angle,  $\theta_{lock}$ , can be obtained from the yarn width and the spacing parameter of the fabric using simple geometrical relationship. The transition range,  $\Delta\theta$  (angle tolerance for locking), can be chosen to be as small as possible, but big enough to prevent high frequency oscillations in transition to compacted state and depends on the range to the locking angle and the dynamics of the simulated problem. Reorientation damping constant is defined to damp some of the high frequency oscillations. A simple rate effect is added by defining the viscous modulus for normal or shear strain rate ( $VMB*\epsilon_{11}$  or  $22$  for normal components and  $VMS*\epsilon_{12}$  for the shear components).



**Locking Angles**





Discount factor as a function of braid angle,  $\theta$

**\*MAT\_SCC\_ON\_RCC**

This is Material Type 236 developed by Carney, Lee, Goldberg, and Santhanam [2007]. This model simulates silicon carbide coating on Reinforced Carbon-Carbon (RCC), a ceramic matrix and is based upon a quasi-orthotropic, linear-elastic, plane-stress model. Additional constitutive model attributes include a simple (i.e. non-damage model based) option that can model the tension crack requirement: a “stress-cutoff” in tension. This option satisfies the tension crack requirements by limiting the stress in tension but not compression, and having the tensile “yielding” (i.e. the stress-cutoff) be fully recoverable – not plasticity or damage based.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E0	E1	E2	E3	E4	E5
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	PR	G	G_SCL	TSL	EPS_TAN			
Type	F	F	F	F	F			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E0	E <sub>0</sub> , See Remarks below.
E1	E <sub>1</sub> , See Remarks below.
E2	E <sub>2</sub> , See Remarks below.
E3	E <sub>3</sub> , See Remarks below.
E4	E <sub>4</sub> , See Remarks below.
E5	E <sub>5</sub> , Young’s modulus of the yarn in transverse-direction.
PR	Poisson’s ratio.

VARIABLE	DESCRIPTION
G	Shear modulus
G_SCL	Shear modulus multiplier (default = 1.0).
TSL	Tensile limit stress
EPS_TAN	Strain at which E = tangent to the polynomial curve.

### Remarks:

This model for the silicon carbide coating on RCC is based upon a quasi-orthotropic, linear-elastic, plane-stress model, given by:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

Additional constitutive model requirements include a simple (i.e. non-damage model based) option that can model the tension crack requirement: a “stress-cutoff” in tension. This option satisfies the tension crack requirements by limiting the stress in tension but not compression, and having the tensile “yielding” (i.e. the stress-cutoff) be fully recoverable – not plasticity or damage based.

The tension stress-cutoff separately resets the stress to a limit value when it is exceeded in each of the two principal directions. There is also a strain-based memory criterion that ensures unloading follows the same path as loading: the “memory criterion” is the tension stress assuming that no stress cutoffs were in effect. In this way, when the memory criterion exceeds the user-specified cutoff stress, the actual stress will be set to that value. When the element unloads and the memory criterion falls back below the stress cutoff, normal behavior resumes. Using this criterion is a simple way to ensure that unloading does not result in any hysteresis. The cutoff criterion cannot be based on an effective stress value because effective stress does not discriminate between tension and compression, and also includes shear. This means that the in plane, 1- and 2- directions must be modeled as independent to use the stress cutoff. Because the Poisson’s ratio is not zero, this assumption is not true for cracks that may arbitrarily lie along any direction. However, careful examination of damaged RCC shows that generally, the surface cracks do tend to lie in the fabric directions, meaning that cracks tend to open in the 1- or the 2- direction independently. So the assumption of directional independence for tension cracks may be appropriate for the coating because of this observed orthotropy.

The quasi-orthotropic, linear-elastic, plane-stress model with tension stress cutoff (to simulate tension cracks) can model the as-fabricated coating properties, which do not show

nonlinearities, but not the non-linear response of the flight-degraded material. Explicit finite element analysis (FEA) lends itself to *nonlinear-elastic* stress-strain relation instead of linear-elastic. Thus, instead of  $\sigma = \mathbf{E}\varepsilon$ , the modulus will be defined as a function of some effective strain quantity, or  $\sigma = \mathbf{E}(\varepsilon_{\text{eff}}) \cdot \varepsilon$ , even though it is uncertain, from the available data, whether or not the coating response is completely nonlinear-elastic, and does not include some damage mechanism.

This nonlinear-elastic model cannot be implemented into a closed form solution or into an implicit solver; however, for explicit FEA such as is used for LS-DYNA impact analysis, the modulus can be adjusted at each time step to a higher or lower value as desired. In order to model the desired S-shape response curve of flight-degraded RCC coating, a function of strain that replicates the desired response must be found. It is assumed that the nonlinearities in the material are recoverable (elastic) and that the modulus is communicative between the 1- and 2- directions (going against the tension-crack assumption that the two directions do not interact). Sometimes stability can be a problem for this type of nonlinearity modeling, however, stability was not found to be a problem with the material constants used for the coating.

The von Mises strain is selected for the effective strain definition as it couples the 3-dimensional loading but reduces to uniaxial data, so that the desired uniaxial compressive response can be reproduced. So,

$$\varepsilon_{\text{eff}} = \frac{1}{\sqrt{2}} \frac{1}{1 + \nu} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_1 - \varepsilon_3)^2 + 3\gamma_{12}^2}$$

where for a 2-D, isotropic shell element case, the z-direction strain is given by:

$$\varepsilon_3 = \frac{-\nu}{1 - \nu} (\varepsilon_1 + \varepsilon_2)$$

The function for modulus is implemented as an arbitrary 5<sup>th</sup> order polynomial:

$$E(\varepsilon_{\text{eff}}) = A_0\varepsilon_{\text{eff}}^0 + A_1\varepsilon_{\text{eff}}^1 + \dots + A_5\varepsilon_{\text{eff}}^5$$

In the case of as-fabricated material the first coefficient ( $A_0$ ) is simply the modulus E, and the other coefficients ( $A_{n>0}$ ) are zero, reducing to a 0<sup>th</sup> order polynomial, or linear. To match the degraded stress-strain compression curve, a higher order polynomial is needed. Six conditions on stress were used (stress and its derivative at beginning, middle, and end of the curve) to obtain a 5<sup>th</sup> order polynomial, and then the derivative of that equation was taken to obtain modulus as a function of strain, yielding a 4<sup>th</sup> order polynomial that represents the degraded coating modulus vs. strain curve.

For values of strain which exceed the failure strain observed in the laminate compression tests, the higher order polynomial will no longer match the test data. Therefore, after a specified effective-strain, representing failure, the modulus is defined to be the tangent of the polynomial curve. As a result, the stress/strain response has a continuous derivative, which aids in avoiding numerical instabilities. The test data does not clearly define the

failure strain of the coating, but in the impact test it appears that the coating has a higher compressive failure strain in bending than the laminate failure strain.

The two dominant modes of loading which cause coating loss on the impact side of the RCC (the front-side) are in-plane compression and transverse shear. The in-plane compression is measured by the peak out of plane tensile strain,  $\varepsilon_3$ . As there is no direct loading of a shell element in this direction,  $\varepsilon_3$  is computed through Poisson's relation  $\varepsilon_3 = \frac{-\nu}{1-\nu}(\varepsilon_1 + \varepsilon_2)$ . When  $\varepsilon_3$  is tensile, it implies that the average of  $\varepsilon_1$  and  $\varepsilon_2$  is compressive. This failure mode will likely dominate when the RCC undergoes large bending, putting the front-side coating in high compressive strains. It is expected that a transverse shear failure mode will dominate when the debris source is very hard or very fast. By definition, the shell element cannot give a precise account of the transverse shear throughout the RCC's thickness. However, the Belytschko-Tsay shell element formulation in LS-DYNA has a first-order approximation of transverse shear that is based on the out-of-plane nodal displacements and rotations that should suffice to give a qualitative evaluation of the transverse shear. By this formulation, the transverse shear is constant through the entire shell thickness and thus violates surface-traction conditions. The constitutive model implementation records the peak value of the tensile out-of-plane strain ( $\varepsilon_3$ ) and peak root-mean-sum transverse-shear:  $\sqrt{\varepsilon_{13}^2 + \varepsilon_{23}^2}$ .

**\*MAT\_PML\_HYSTERETIC**

This is Material Type 237. This is a perfectly-matched layer (PML) material with a Biot linear hysteretic constitutive law, to be used in a wave-absorbing layer adjacent to a Biot hysteretic material (\*MAT\_BIOT\_HYSTERETIC) in order to simulate wave propagation in an unbounded medium with material damping. This material is the visco-elastic counterpart of the elastic PML material (\*MAT\_PML\_ELASTIC). See the Remarks sections of \*MAT\_PML\_ELASTIC (\*MAT\_230) and \*MAT\_BIOT\_HYSTERETIC (\*MAT\_232) for further details.

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	ZT	FD		
Type	A8	F	F	F	F	F		
Default	none	none	none	none	0.0	3.25		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
ZT	Damping ratio
FD	Dominant excitation frequency in Hz

**\*MAT\_PERT\_PIECEWISE\_LINEAR\_PLASTICITY**

This is Material Type 238. It is a duplicate of Material Type 24 (\*MAT\_PIECEWISE\_LINEAR\_PLASTICITY) modified for use with \*PERTURBATION\_MATERIAL and solid elements in an explicit analysis. It should give exactly the same values as the original material, if used exactly the same. It exists as a separate material type because of the speed penalty (an approximately 10% increase in the overall execution time) associated with the use of a material perturbation.

See Material Type 24 (\*MAT\_PIECEWISE\_LINEAR\_PLASTICITY) for a description of the material parameters. All of the documentation for Material Type 24 applies. Recommend practice is to first create the input deck using Material Type 24. Additionally, the CMP variable in the \*PERTURBATION\_MATERIAL must be set to affect a specific variables in the MAT\_238 definition as defined in the following table; for example, CMP = 5 will perturb the yield stress.

<b>CMP value</b>	<b>Material variable</b>
3	E
5	SIGY
6	ETAN
7	FAIL

# \*MAT\_240

# \*MAT\_COHESIVE\_MIXED\_MODE\_ELASTOPLASTIC\_RATE

## \*MAT\_COHESIVE\_MIXED\_MODE\_ELASTOPLASTIC\_RATE

This is Material Type 240. This model is a rate-dependent, elastic-ideally plastic cohesive zone model. It includes a tri-linear traction-separation law with a quadratic yield and damage initiation criterion in mixed-mode loading, while the damage evolution is governed by a power-law formulation. It can be used only with cohesive element formulations; see the variable ELFORM in \*SECTION\_SOLID and \*SECTION\_SHELL.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	EMOD	GMOD	THICK	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	G1C_0	G1C_INF	EDOT_G1	T0	T1	EDOT_T	FG1	LCG1C
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	G2C_0	G2C_INF	EDOT_G2	S0	S1	EDOT_S	FG2	LCG2C
Type	F	F	F	F	F	F	F	F

### VARIABLE

### DESCRIPTION

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
ROFLG	Flag for whether density is specified per unit area or volume. ROFLG = 0 specified density per unit volume (default), and ROFLG = 1 specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
INTFAIL	The number of integration points required for the cohesive element to be deleted. If it is zero, the element will not be deleted even if it satisfies the failure criterion. The value of INTFAIL may range from 1 to 4, with 1 the recommended value.
EMOD	The Young's modulus of the material
GMOD	The shear modulus of the material
THICK	GT.0.0: Cohesive thickness LE.0.0: Initial thickness is calculated from nodal coordinates
G1C_0	GT.0.0: Energy release rate G <sub>IC</sub> in Mode I LE.0.0: Lower bound value of rate-dependent G <sub>IC</sub>
G1C_INF	Upper bound value of rate-dependent G <sub>IC</sub> (only considered if G1C_0 < 0)
EDOT_G1	Equivalent strain rate at yield initiation to describe the rate dependency of G <sub>IC</sub> (only considered if G1C_0 < 0)
T0	GT.0.0: Yield stress in Mode I LT.0.0: Rate-dependency is considered, Parameter T0
T1	Parameter T1, only considered if T0 < 0: GT.0.0: Quadratic logarithmic model LT.0.0: Linear logarithmic model
EDOT_T	Equivalent strain rate at yield initiation to describe the rate dependency of the yield stress in Mode I (only considered if T0 < 0)
FG1	Parameter f <sub>G1</sub> to describe the tri-linear shape of the traction-separation law in Mode I, see remarks. GT.0.0: FG1 is ratio of fracture energies G <sub>I,P</sub> /G <sub>IC</sub> LT.0.0:  FG1  is ratio of displacements $(\delta_{n2} - \delta_{n1}) / (\delta_{nf} - \delta_{n1})$
LCG1C	Load curve ID which defines fracture energy G <sub>IC</sub> as a function of cohesive element thickness. G1C_0 and G1C_INF are ignored in that case.

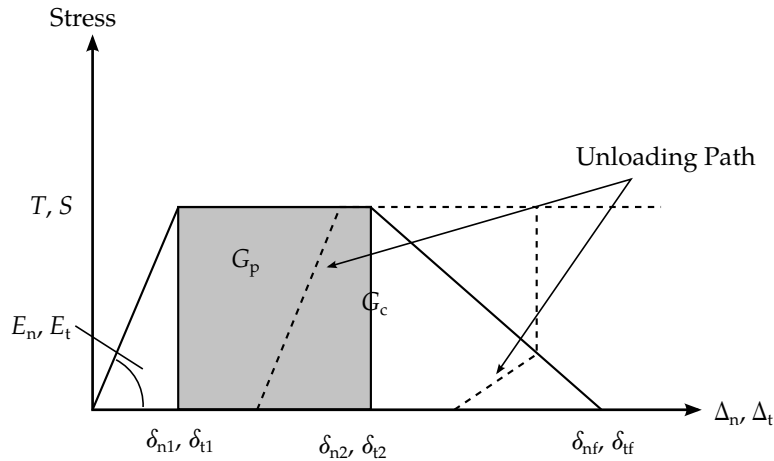


Figure M240-1. Trilinear traction separation law

VARIABLE	DESCRIPTION
G2C_0	GT.0.0: Energy release rate $G_{IIC}$ in Mode II LE.0.0: Lower bound value of rate-dependent $G_{IIC}$
G2C_INF	Upper bound value of $G_{IIC}$ (only considered if $G2C_0 < 0$ )
EDOT_G2	Equivalent strain rate at yield initiation to describe the rate dependency of $G_{IIC}$ (only considered if $G2C_0 < 0$ )
S0	GT.0.0: Yield stress in Mode II LT.0.0: Rate-dependency is considered, Parameter S0
S1	Parameter S1, only considered if $S0 < 0$ : GT.0.0: Quadratic logarithmic model is applied LT.0.0: Linear logarithmic model is applied
EDOT_S	Equivalent strain rate at yield initiation to describe the rate dependency of the yield stress in Mode II (only considered if $S0 < 0$ )
FG2	Parameter $f_{G2}$ to describe the tri-linear shape of the traction-separation law in Mode II, see remarks. GT.0.0: FG2 is ratio of fracture energies $G_{II,P}/G_{IIC}$ LT.0.0: $ FG2 $ is ratio of displacements $(\delta_{t2} - \delta_{t1})/(\delta_{tf} - \delta_{t1})$
LCG2C	Load curve ID which defines fracture energy $G_{IIC}$ as a function of cohesive element thickness. $G2C_0$ and $G2C\_INF$ are ignored in that case.

**Remarks:**

The model is a tri-linear elastic-ideally plastic Cohesive Zone Model, which was developed by Marzi et al. [2009]. It looks similar to \*MAT\_185, but considers effects of plasticity and rate-dependency. Since the entire separation at failure is plastic, no brittle fracture behavior can be modeled with this material type.

The separations  $\Delta_n$  in normal (peel) and  $\Delta_t$  in tangential (shear) direction are calculated from the element's separations in the integration points,

$$\Delta_n = \max(u_n, 0)$$

and

$$\Delta_t = \sqrt{u_{t1}^2 + u_{t2}^2},$$

$u_n$ ,  $u_{t1}$  and  $u_{t2}$  are the separations in normal and in the both tangential directions of the element coordinate system. The total (mixed-mode) separation  $\Delta_m$  is determined by

$$\Delta_m = \sqrt{\Delta_n^2 + \Delta_t^2}.$$

The initial stiffnesses in both modes are calculated from the elastic Young's and shear moduli and are respectively,

$$E_n = \frac{\text{EMOD}}{\text{THICK}}$$

$$E_t = \frac{\text{GMOD}}{\text{THICK}},$$

where THICK, the element's thickness, is an input parameter. Unless the input THICK > 0 it is calculated from the distance between the initial positions of the element's corner nodes (Nodes 1-5, 2-6, 3-7 and 4-8, respectively).

While the total energy under the traction-separation law is given by  $G_C$ , one further parameter is needed to describe the exact shape of the tri-linear material model. If the area (energy) under the constant stress (plateau) region is denoted  $G_P$  (see M240-1), a parameter  $f_G$  defines the shape of the traction-separation law,

for mode I loading:

$$0 \leq f_{G1} = \frac{G_{I,P}}{G_{IC}} < 1 - \frac{T^2}{2G_{IC}E_n} < 1$$

for mode II loading:

$$0 \leq f_{G2} = \frac{G_{II,P}}{G_{IIC}} < 1 - \frac{S^2}{2G_{IIC}E_t} < 1$$

As a recommended alternative, the shape of the tri-linear model can be described by the following displacement ratios (triggered by negative input values for  $f_G$ ):

for mode I loading:

$$0 < |f_{G1}| = \left| \frac{\delta_{n2} - \delta_{n1}}{\delta_{nf} - \delta_{n1}} \right| < 1$$

for mode II loading:

$$0 < |f_{G2}| = \left| \frac{\delta_{t2} - \delta_{t1}}{\delta_{tf} - \delta_{t1}} \right| < 1$$

While  $f_{G1}$  and  $f_{G2}$  are always constant values,  $T$ ,  $S$ ,  $G_{IC}$  and  $G_{IIC}$  may be chosen as functions of an equivalent strain rate  $\dot{\epsilon}_{eq}$ , which is evaluated by

$$\dot{\epsilon}_{eq} = \frac{\sqrt{\dot{u}_n^2 + \dot{u}_{t1}^2 + \dot{u}_{t2}^2}}{\text{THICK}},$$

where  $\dot{u}_n$ ,  $\dot{u}_{t1}$  and  $\dot{u}_{t2}$  are the velocities corresponding to the separations  $u_n$ ,  $u_{t1}$  and  $u_{t2}$ .

For the yield stresses, two rate dependent formulations are implemented:

1. A quadratic logarithmic function:

for mode I if  $T0 < 0$  and  $T1 > 0$ :

$$T(\dot{\epsilon}_{eq}) = |T0| + |T1| \left[ \max \left( 0, \ln \frac{\dot{\epsilon}_{eq}}{\text{EDOT}_T} \right) \right]^2$$

for mode II if  $S0 < 0$  and  $S1 > 0$ :

$$S(\dot{\epsilon}_{eq}) = |S0| + |S1| \left[ \max \left( 0, \ln \frac{\dot{\epsilon}_{eq}}{\text{EDOT}_S} \right) \right]^2$$

2. A linear logarithmic function:

for mode I if  $T0 < 0$  and  $T1 < 0$ :

$$T(\dot{\epsilon}_{eq}) = |T0| + |T1| \max \left( 0, \ln \frac{\dot{\epsilon}_{eq}}{\text{EDOT}_T} \right)$$

for mode II if  $S0 < 0$  and  $S1 < 0$ :

$$S(\dot{\epsilon}_{eq}) = |S0| + |S1| \max \left( 0, \ln \frac{\dot{\epsilon}_{eq}}{\text{EDOT}_S} \right)$$

Alternatively,  $T$  and  $S$  can be set to constant values:

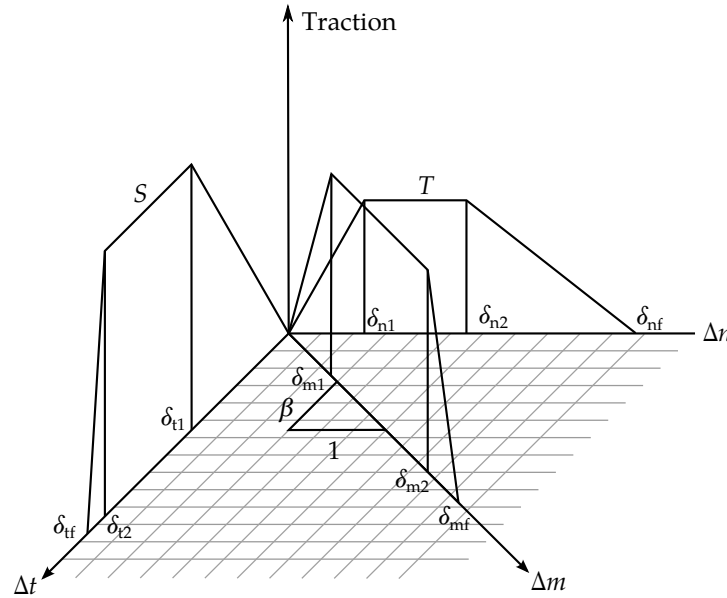
for mode I if  $T0 > 0$ :

$$T(\dot{\epsilon}_{eq}) = T0$$

for mode II if  $S0 > 0$ :

$$S(\dot{\epsilon}_{eq}) = S0$$

The rate-dependency of the fracture energies are given by



**Figure M240-2.** Trilinear mixed mode traction-separation law

if  $G1C\_0 < 0$ :

$$G_{IC}(\dot{\epsilon}_{eq}) = |G1C\_0| + (G1C\_INF - |G1C\_0|) \exp\left(-\frac{EDOT\_G1}{\dot{\epsilon}_{eq}}\right)$$

if  $G2C\_0 < 0$ :

$$G_{IIC}(\dot{\epsilon}_{eq}) = |G2C\_0| + (G2C\_INF - |G2C\_0|) \exp\left(-\frac{EDOT\_G2}{\dot{\epsilon}_{eq}}\right)$$

If positive values are chosen for  $G1C\_0$  or  $G2C\_0$ , no rate-dependency is considered for this parameter and its value remains constant as specified by the user.

As an alternative, fracture energies  $GIC$  and  $GIIC$  can be defined as functions of cohesive element thickness by using load curves  $LCG1C$  and  $LCG2C$ . In that case, parameters  $G1C\_0$ ,  $G1C\_INF$ ,  $G2C\_0$ , and  $G2C\_INF$  will be ignored and no rate dependence is considered.

It should be noticed, that the equivalent strain rate  $\dot{\epsilon}_{eq}$  is updated until  $\Delta_m > \delta_{m1}$ , then the model behavior depends on the equivalent strain rate at yield initiation.

Having defined the parameters describing the single modes, the mixed-mode behavior is formulated by quadratic initiation criteria for both yield stress and damage initiation, while the damage evolution follows a Power-Law.

Due to reasons of readability, the following simplifications are made,

$$T = T(\dot{\epsilon}_{eq}), S = S(\dot{\epsilon}_{eq}), G_{IC} = G_{IC}(\dot{\epsilon}_{eq}) \text{ and } G_{IIC} = G_{IIC}(\dot{\epsilon}_{eq}).$$

The mixed-mode yield initiation displacement  $\delta_{m1}$  is defined as

$$\delta_{m1} = \delta_{n1} \delta_{t1} \sqrt{\frac{1 + \beta^2}{\delta_{t1}^2 + (\beta \delta_{n1})^2}}$$

where  $\delta_{n1} = \frac{T}{E_n}$  and  $\delta_{t1} = \frac{S}{E_t}$  are the single-mode yield initiation displacements and  $\beta = \frac{\Delta_t}{\Delta_n}$  is the mixed-mode ratio. Analog to the yield initiation, the damage initiation displacement  $\delta_{m2}$  is defined:

$$\delta_{m2} = \delta_{n2} \delta_{t2} \sqrt{\frac{1 + \beta^2}{\delta_{t2}^2 + (\beta \delta_{n2})^2}}$$

where

$$\delta_{n2} = \delta_{n1} + \frac{f_{G1} G_{IC}}{T}$$

$$\delta_{t2} = \delta_{t1} + \frac{f_{G2} G_{IIC}}{S}$$

With  $\gamma = \arccos\left(\frac{\langle u_n \rangle}{\Delta_m}\right)$ , the ultimate (failure) displacement  $\delta_{mf}$  can be written,

$$\delta_{mf} = \frac{\delta_{m1}(\delta_{m1} - \delta_{m2}) E_n G_{IIC} \cos^2 \gamma + G_{IC} (2G_{IIC} + \delta_{m1}(\delta_{m1} - \delta_{m2}) E_t \sin^2 \gamma)}{\delta_{m1} (E_n G_{IIC} \cos^2 \gamma + E_t G_{IC} \sin^2 \gamma)}$$

This formulation describes a power-law damage evolution with an exponent  $\eta = 1.0$  (see \*MAT\_138).

After the shape of the mixed-mode traction-separation law has been determined by  $\delta_{m1}$ ,  $\delta_{m2}$  and  $\delta_{mf}$ , the plastic separation in each element direction,  $u_{n,P}$ ,  $u_{t1,P}$  and  $u_{t2,P}$  can be calculated. The plastic separation in peel direction is given by

$$u_{n,P} = \max(u_{n,P,\Delta t-1}, u_n - \delta_{m1} \cos(\gamma), 0).$$

In shear direction, a shear yield separation  $\delta_{t,y}$ ,

$$\delta_{t,y} = \sqrt{(u_{t1} - u_{t1,P,\Delta t-1})^2 + (u_{t2} - u_{t2,P,\Delta t-1})^2},$$

is defined. If  $\delta_{t,y} > \delta_{m1} \sin \gamma$ , the plastic shear separations in the element coordinate system are updated,

$$u_{t1,P} = u_{t1,P,\Delta t-1} + u_{t1} - u_{t1,\Delta t-1}$$

$$u_{t2,P} = u_{t2,P,\Delta t-1} + u_{t2} - u_{t2,\Delta t-1}.$$

In the formulas above,  $\Delta t - 1$  indicates the individual value from the last time increment. In case  $\Delta_m > \delta_{m2}$ , the damage initiation criterion is satisfied and a damage variable  $D$  increases monotonically,

$$D = \max\left(\frac{\Delta_m - \delta_{m2}}{\delta_{mf} - \delta_{m2}}, D_{\Delta t-1}, 0\right).$$

When  $\Delta_m > \delta_{mf}$ , complete damage ( $D = 1$ ) is reached and the element fails in the corresponding integration point.

Finally, the peel and the shear stresses in element directions are calculated,

$$\sigma_{t1} = E_t(1 - D)(u_{t1} - u_{t1,P})$$

$$\sigma_{t2} = E_t(1 - D)(u_{t2} - u_{t2,P}).$$

In peel direction, no damage under pressure loads is considered if  $u_n - u_{n,P} > 0$

$$\sigma_n = E_n(u_n - u_{n,P})$$

otherwise,

$$\sigma_n = E_n(1 - D)(u_n - u_{n,P})$$

**Reference:**

S. Marzi, O. Hesebeck, M. Brede and F. Kleiner (2009), A Rate-Dependent, Elasto-Plastic Cohesive Zone Mixed-Mode Model for Crash Analysis of Adhesively Bonded Joints, In Proceeding: 7<sup>th</sup> European LS-DYNA Conference, Salzburg

**\*MAT\_JOHNSON\_HOLMQUIST\_JH1**

This is Material Type 241. This Johnson-Holmquist Plasticity Damage Model is useful for modeling ceramics, glass and other brittle materials. This version corresponds to the original version of the model, JH1, and Material Type 110 corresponds to JH2, the updated model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G	P1	S1	P2	S2	C
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	EPSI	T		ALPHA	SFMAX	BETA	DP1	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	EPFMIN	EPFMAX	K1	K2	K3	FS		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Density.
G	Shear modulus.
P1	Pressure point 1 for intact material.
S1	Effective stress at P1.
P2	Pressure point 2 for intact material.
S2	Effective stress at P2.



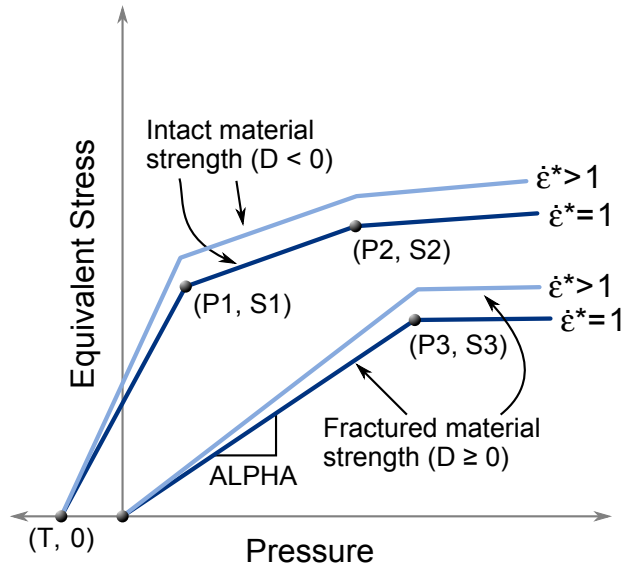
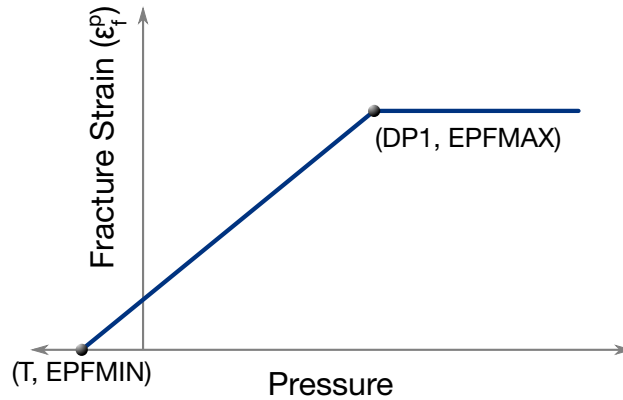


Figure M241-1. Strength: equivalent stress versus pressure.

VARIABLE	DESCRIPTION
C	Strain rate sensitivity factor.
EPSI	Quasi-static threshold strain rate. See *MAT_015.
T	Maximum tensile pressure strength. This value is positive in tension.
ALPHA	Initial slope of the fractured material strength curve. See <a href="#">Figure M241-1</a> .
SFMAX	Maximum strength of the fractured material.
BETA	Fraction of elastic energy loss converted to hydrostatic energy (affects bulking pressure (history variable 1) that accompanies damage).
DP1	Maximum compressive pressure strength. This value is positive in compression.
EPFMIN	Plastic strain for fracture at tensile pressure $T$ . See <a href="#">Figure M241-2</a> .
EPFMAX	Plastic strain for fracture at compressive pressure $DP1$ . See <a href="#">Figure M241-1</a> .
K1	First pressure coefficient (equivalent to the bulk modulus).
K2	Second pressure coefficient.



**Figure M241-2.** Fracture strain versus pressure.

VARIABLE	DESCRIPTION
K3	Third pressure coefficient.
FS	Element deletion criteria. LT.0: delete if $P < FS$ (tensile failure). EQ.0: no element deletion (default). GT.0: delete element if the $\bar{\epsilon}^p > FS$ .

**Remarks:**

The equivalent stress for both intact and fractured ceramic-type materials is given by

$$\sigma_y = (1 + c \ln \bar{\epsilon}^*) \sigma(P)$$

where  $\sigma(P)$  is evaluated according to [Figure M241-1](#).

$$D = \sum \Delta \epsilon^p / \epsilon_f^p(P)$$

represents the accumulated damage (history variable 2) based upon the increase in plastic strain per computational cycle and the plastic strain to fracture is evaluated according to [Figure M241-2](#).

In undamaged material, the hydrostatic pressure is given by

$$P = k_1 \mu + k_2 \mu^2 + k_3 \mu^3 + \Delta P$$

in compression and by

$$P = k_1 \mu + \Delta P$$

in tension where  $\mu = \rho / \rho_0 - 1$ . A fraction, between 0 and 1, of the elastic energy loss,  $\beta$ , is converted into hydrostatic potential energy (pressure). The pressure increment,  $\Delta P$ , associated with the increment in the hydrostatic potential energy is calculated at fracture,

where  $\sigma_y$  and  $\sigma_y^f$  are the intact and failed yield stresses respectively. This pressure increment is applied both in compression and tension, which is not true for JH2 where the increment is added only in compression.

$$\Delta P = -k_1\mu_f + \sqrt{(k_1\mu_f)^2 + 2\beta k_1\Delta U}$$

$$\Delta U = \frac{\sigma_y - \sigma_y^f}{6G}$$





Card 8	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number must be specified.
RO	Mass density.
E	Young's modulus, E.
PR	Poisson's ratio, $\nu$ .
M	Flow potential exponent. For face centered cubic (FCC) materials $m = 8$ is recommended and for body centered cubic (BCC) materials $m = 6$ may be used.
ALPHA1	$\alpha_1$ , material constant in Barlat's yield equation.
ALPHA2	$\alpha_2$ , material constant in Barlat's yield equation.
ALPHA3	$\alpha_3$ , material constant in Barlat's yield equation.
ALPHA4	$\alpha_4$ , material constant in Barlat's yield equation.
ALPHA5	$\alpha_5$ , material constant in Barlat's yield equation.
ALPHA6	$\alpha_6$ , material constant in Barlat's yield equation.
ALPHA7	$\alpha_7$ , material constant in Barlat's yield equation.
ALPHA8	$\alpha_8$ , material constant in Barlat's yield equation.
CB	The uppercase B defined in the Yoshida's equations.
Y	Anisotropic parameter associated with work-hardening stagnation, defined in the Yoshida's equations.
SC	The lowercase c defined in the Yoshida's equations.
K	Hardening parameter as defined in the Yoshida's equations.
RSAT	Hardening parameter as defined in the Yoshida's equations.
SB	The lowercase b as defined in the Yoshida's equations.

VARIABLE	DESCRIPTION
H	Anisotropic parameter associated with work-hardening stagnation, defined in the following Yoshida's equations.
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description): EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector $\mathbf{v}$ with the element normal. LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available with the R3 release of Version 971 and later.
IOPT	Kinematic hardening rule flag: EQ.0: Original Yoshida formulation. EQ.1: Modified formulation. Define C1, C2 below.
C1, C2	Constants used to modify R: $R = \text{RSAT} \times [(C_1 + \bar{\epsilon}^p)^{c_2} - C_1^{c_2}]$
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.

**Remarks:**

1. A total of eight parameters ( $\alpha_1$  to  $\alpha_8$ ) are needed to describe the yield surface. The parameters can be determined with tensile tests in three directions and an equal

biaxial tension test. For detailed theoretical background and material parameters of some typical FCC materials, please see remarks in \*MAT\_133 and Barlat's 2003 paper.

2. NUMISHEET 2005 provided a complete set of the parameters of AL5182-O for Benchmark #2, the cross member, as below (flow potential exponent  $M = 8$ ):

$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	$\alpha_8$
0.94	1.08	0.97	1.0	1.0	1.02	1.03	1.11

3. For a more detailed description on the Yoshida model and parameters, please see Remarks in \*MAT\_226 and \*MAT\_125.
4. For information on variable AOPT please see remarks in \*MAT\_226.
5. To improve convergence, it is recommended that \*CONTROL\_IMPLICIT\_FORMING type '1' be used when conducting springback simulation.
6. This material model is available in LS-DYNA R5 Revision 58432 or later releases.



\*MAT\_HILL\_90

This is Material Type 243. This model was developed by Hill [1990] for modeling sheets with anisotropic materials under plane stress conditions. This material allows the use of the Lankford parameters for the definition of the anisotropy. All features of this model are the same as in \*MAT\_036, only the yield condition and associated flow rules are replaced by the Hill90 equations.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	HR	P1	P2	ITER
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	M	R00 / AH	R45 / BH	R90 / CH	LCID	E0	SPI	P3
Type	F	F	F	F	I	F	F	F

**Hardening Card.** Additional Card for  $M < 0$ .

Card 3	1	2	3	4	5	6	7	8
Variable	CRC1	CRA1	CRC2	CRA2	CRC3	CRA3	CRC4	CRA4
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT	C	P	VLCID		FLAG		
Type	F	F	F	I		F		

Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

This card is optional.

Card 6	1	2	3	4	5	6	7	8
Variable	USRFAIL							
Type	F							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus, E GT.0.0: Constant value, LT.0.0: Load curve ID = (-E) which defines Young's Modulus as a function of plastic strain. See Remark 1.
PR	Poisson's ratio, $\nu$

VARIABLE	DESCRIPTION
HR	Hardening rule: EQ.1.0: linear (default), EQ.2.0: exponential (Swift) EQ.3.0: load curve or table with strain rate effects EQ.4.0: exponential (Voce) EQ.5.0: exponential (Gosh) EQ.6.0: exponential (Hocket-Sherby) EQ.7.0: load curves in three directions EQ.8.0: table with temperature dependence EQ.9.0: 3d table with temperature and strain rate dependence
P1	Material parameter: HR.EQ.1.0: Tangent modulus, HR.EQ.2.0: k, strength coefficient for Swift exponential hardening HR.EQ.4.0: a, coefficient for Voce exponential hardening HR.EQ.5.0: k, strength coefficient for Gosh exponential hardening HR.EQ.6.0: a, coefficient for Hocket-Sherby exponential hardening HR.EQ.7.0: load curve ID for hardening in 45 degree direction. See Remark 2.
P2	Material parameter: HR.EQ.1.0: Yield stress HR.EQ.2.0: n, exponent for Swift exponential hardening HR.EQ.4.0: c, coefficient for Voce exponential hardening HR.EQ.5.0: n, exponent for Gosh exponential hardening HR.EQ.6.0: c, coefficient for Hocket-Sherby exponential hardening HR.EQ.7.0: load curve ID for hardening in 90 degree direction. See Remark 2.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ITER	Iteration flag for speed: ITER.EQ.0.0: fully iterative ITER.EQ.1.0: fixed at three iterations Generally, ITER = 0 is recommended. However, ITER = 1 is somewhat faster and may give acceptable results in most problems.
M	m, exponent in Hill's yield surface, absolute value is used if negative. Typically, m ranges between 1 and 2 for low-r materials, such as aluminum (AA6111: $m \approx 1.5$ ), and is greater than 2 for high r-values, as in steel (DP600: $m \approx 4$ ).
CRC $n$	Chaboche-Rousselier hardening parameters, see remarks.
CRA $n$	Chaboche-Rousselier hardening parameters, see remarks.
R00	R <sub>00</sub> , Lankford parameter in 0 degree direction GT.0.0: Constant value, LT.0.0: Load curve or Table ID = (-R00) which defines R value as a function of plastic strain (Curve) or as a function of temperature and plastic strain (Table). See Remark 3.
R45	R <sub>45</sub> , Lankford parameter in 45 degree direction GT.0.0: Constant value, LT.0.0: Load curve or Table ID = (-R45) which defines R value as a function of plastic strain (Curve) or as a function of temperature and plastic strain (Table). See Remarks 2 and 3.
R90	R <sub>90</sub> , Lankford parameter in 90 degree direction GT.0.0: Constant value, LT.0.0: Load curve or Table ID = (-R90) which defines R value as a function of plastic strain (Curve) or as a function of temperature and plastic strain (Table). See Remarks 2 and 3.
AH	a, Hill90 parameter, which is read instead of R00 if FLAG = 1.
BH	b, Hill90 parameter, which is read instead of R45 if FLAG = 1.
CH	c, Hill90 parameter, which is read instead of R90 if FLAG = 1.

VARIABLE	DESCRIPTION
LCID	Load curve/table ID for hardening in the 0 degree direction. See Remark 1.
E0	<p>Material parameter</p> <p>HR.EQ.2.0: <math>\epsilon_0</math> for determining initial yield stress for Swift exponential hardening. (Default = 0.0)</p> <p>HR.EQ.4.0: b, coefficient for Voce exponential hardening</p> <p>HR.EQ.5.0: <math>\epsilon_0</math> for determining initial yield stress for Gosh exponential hardening. (Default = 0.0)</p> <p>HR.EQ.6.0: b, coefficient for Hocket-Sherby exponential hardening</p>
SPI	<p>if <math>\epsilon_0</math> is zero above and HR = 2.0. (Default = 0.0)</p> <p>EQ.0.0: <math>\epsilon_0 = (E/k)^{1/(n-1)}</math></p> <p>LE.0.02: <math>\epsilon_0 = \text{SPI}</math></p> <p>GT.0.02: <math>\epsilon_0 = (\text{SPI}/k)^{1/n}</math></p> <p>If HR = 5.0 the strain at plastic yield is determined by an iterative procedure based on the same principles as for HR.EQ.2.0.</p>
P3	<p>Material parameter:</p> <p>HR.EQ.5.0: p, parameter for Gosh exponential hardening</p> <p>HR.EQ.6.0: n, exponent for Hocket-Sherby exponential hardening</p>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element nor-</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	mal.
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available with the R3 release of Version 971 and later.
C	C in Cowper-Symonds strain rate model
P	p in Cowper-Symonds strain rate model, p = 0.0 for no strain rate effects
VLCID	Volume correction curve ID defining the relative volume change (change in volume relative to the initial volume) as a function of the effective plastic strain. This is only used when nonzero. See Remark 1.
FLAG	Flag for interpretation of parameters. If FLAG = 1, parameters AH, BH, and CH are read instead of R00, R45, and R90. See Remark 4.
XP, YP, ZP	Coordinates of point <b>p</b> for AOPT = 1.
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3.
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.
USRFAIL	User defined failure flag USRFAIL.EQ.0: no user subroutine is called USRFAIL.EQ.1: user subroutine matusr_24 in dyn21.f is called

**Remarks:**

1. The effective plastic strain used in this model is defined to be plastic work equivalent. A consequence of this is that for parameters defined as functions of effective plastic strain, the rolling (00) direction should be used as reference direction. For instance, the hardening curve for HR = 3 is the stress as function of strain for uniaxial tension in the rolling direction, VLCID curve should give the relative volume change as function of strain for uniaxial tension in the rolling direction and

load curve given by -E should give the Young's modulus as function of strain for uniaxial tension in the rolling direction. Optionally the curve can be substituted for a table defining hardening as function of plastic strain rate (HR = 3) or temperature (HR = 8).

2. Exceptions from the rule above are curves defined as functions of plastic strain in the 45 and 90 directions, i.e., P1 and P2 for HR = 7 and negative R45 or R90. The hardening curves are here defined as measured stress as function of measured plastic strain for uniaxial tension in the direction of interest, i.e., as determined from experimental testing using a standard procedure. Moreover, the curves defining the R values are as function of the measured plastic strain for uniaxial tension in the direction of interest. These curves are transformed internally to be used with the effective stress and strain properties in the actual model. The effective plastic strain does not coincide with the plastic strain components in other directions than the rolling direction and may be somewhat confusing to the user. Therefore the von Mises work equivalent plastic strain is output as history variable #2 if HR = 7 or if any of the R-values is defined as function of the plastic strain.
3. The R-values in curves are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width W and thickness T are measured as function of strain. Then the corresponding R-value is given by:

$$R = \frac{\frac{dW}{d\varepsilon} / W}{\frac{dT}{d\varepsilon} / T}$$

4. The anisotropic yield criterion  $\Phi$  for plane stress is defined as:

$$\Phi = K_1^m + K_3 K_2^{(m/2)-1} + c^m K_4^{m/2} = (1 + c^m - 2a + b) \sigma_Y^m$$

where  $\sigma_Y$  is the yield stress and  $K_i = 1,4$  are given by:

$$K_1 = |\sigma_x + \sigma_y|$$

$$K_2 = |\sigma_x^2 + \sigma_y^2 + 2\sigma_{xy}^2|$$

$$K_3 = -2a(\sigma_x^2 - \sigma_y^2) + b(\sigma_x - \sigma_y)^2$$

$$K_4 = |(\sigma_x - \sigma_y)^2 + 4\sigma_{xy}^2|$$

If FLAG = 0, the anisotropic material constants a, b, and c are obtained through R<sub>00</sub>, R<sub>45</sub>, and R<sub>90</sub> using these 3 equations:

$$1 + 2R_{00} = \frac{c^m - a + \{(m+2)/2m\}b}{1 - a + \{(m-2)/2m\}b}$$

$$1 + 2R_{45} = c^m$$

$$1 + 2R_{90} = \frac{c^m + a + \{(m+2)/2m\}b}{1 + a + \{(m-2)/2m\}b}$$

If FLAG = 1, material parameters a (AH), b (BH), and c (CH) are used directly.

For material parameters a, b, c, and m, the following condition has to be fulfilled, otherwise an error termination occurs:

$$1 + c^m - 2a + b > 0$$

Two even more strict conditions should ensure convexity of the yield surface according to Hill (1990). A warning message will be dumped if at least one of them is violated:

$$b > -2\left(\frac{m}{2}\right)^{-1} c^m$$

$$b > a^2 - c^m$$

The yield strength of the material can be expressed in terms of k and n:

$$\sigma_Y = k\varepsilon^n = k(\varepsilon_{yp} + \bar{\varepsilon}^p)^n$$

where  $\varepsilon_{yp}$  is the elastic strain to yield and  $\bar{\varepsilon}^p$  is the effective plastic strain (logarithmic). If SIGY is set to zero, the strain to yield is found by solving for the intersection of the linearly elastic loading equation with the strain hardening equation:

$$\sigma = E\varepsilon$$

$$\sigma = k\varepsilon^n$$

which gives the elastic strain at yield as:

$$\varepsilon_{yp} = \left(\frac{E}{k}\right)^{\left[\frac{1}{n-1}\right]}$$

If SIGY yield is nonzero and greater than 0.02 then:

$$\varepsilon_{yp} = \left(\frac{\sigma_Y}{k}\right)^{\left[\frac{1}{n}\right]}$$

The other available hardening models include the Voce equation given by

$$\sigma_Y(\varepsilon_p) = a - be^{-c\varepsilon_p},$$

the Gosh equation given by

$$\sigma_Y(\varepsilon_p) = k(\varepsilon_0 + \varepsilon_p)^n - p,$$

and finally the Hockett-Sherby equation given by

$$\sigma_Y(\varepsilon_p) = a - be^{-c\varepsilon_p^n}.$$

For the Gosh hardening law, the interpretation of the variable SPI is the same, i.e., if set to zero the strain at yield is determined implicitly from the intersection of the strain hardening equation with the linear elastic equation.



To include strain rate effects in the model we multiply the yield stress by a factor depending on the effective plastic strain rate. We use the Cowper-Symonds' model, hence the yield stress can be written

$$\sigma_Y(\varepsilon_p, \dot{\varepsilon}_p) = \sigma_Y^s(\varepsilon_p) \left[ 1 + \left( \frac{\dot{\varepsilon}_p}{C} \right)^{1/p} \right]$$

where  $\sigma_Y^s$  denotes the static yield stress,  $C$  and  $p$  are material parameters,  $\dot{\varepsilon}_p$  is the effective plastic strain rate.

5. A kinematic hardening model is implemented following the works of Chaboche and Roussilier. A back stress  $\alpha$  is introduced such that the effective stress is computed as

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12})$$

The back stress is the sum of up to four terms according to

$$\alpha_{ij} = \sum_{k=1}^4 \alpha_{ij}^k$$

and the evolution of each back stress component is as follows

$$\delta \alpha_{ij}^k = C_k \left( a_k \frac{s_{ij}}{\sigma_{\text{eff}}} - \alpha_{ij}^k \right) \delta \varepsilon_p$$

where  $C_k$  and  $a_k$  are material parameters,  $s_{ij}$  is the deviatoric stress tensor,  $\sigma_{\text{eff}}$  is the effective stress and  $\varepsilon_p$  is the effective plastic strain.

**\*MAT\_UHS\_STEEL**

This material model is developed for both shell and solid models. It is mainly suited for hot stamping processes where phase transformations are crucial. It has five phases and it is assumed that the blank is fully austenitized before cooling. The basic constitutive model is based on the work done by P. Akerstrom [2, 7].

Automatic switching between cooling and heating of the blank is under development. To activate the heating algorithm, set HEAT = 1 or 2 and add the appropriate input Cards. See the description of the HEAT parameter below. HEAT = 0 as is the default activates only the cooling algorithm and no extra cards need to be read in. Also note that for HEAT = 0 you **must** check that the initial temperature of this material is above the start temperature for the ferrite transformation. The transformation temperatures are echoed in the message and in the d3hsp file.

If HEAT > 0 the temperature that instantaneous transform all ferrite back to austenite is also echoed in the message file. If you want to heat up to 100% austenite you must let the specimen's temperature exceed that temperature.

**Features Added in 2014:**

1. Young's modulus and Poisson ratio can now be given as temperature dependent load curves or by a table definition with a load curve for each phase. See [Remark 8](#).
2. Latent heat can now be given for each phase. See [Remark 9](#).
3. Thermal expansion can now be given for each phase [Remark 10](#).
4. Advanced reaction kinetic modifications include the ability to tailor the start temperatures and the activation energies. The martensite start temperature can be dependent on the plastic strain and triaxiality, and the activation energies can be scaled with the plastic strain as well.
5. Hardness calculation improved when tempering is active. Improvements are achieved in the bainite and martensite phases (experimental). See [Remark 11](#).

<p><b>NOTE:</b> For this material "weight%" means "ppm × 10<sup>-4</sup>".</p>
--





**Heat Card 2.** Additional Card for HEAT =1.

Card 9	1	2	3	4	5	6	7	8
Variable	GRA	GRB	EXPA	EXPB	GRCC	GRCM	HEATN	TAU2
Type	F	F	F	F	F	F	F	F
Default	3.11	7520.	1.0	1.0	none	none	1.0	4.806

**Reaction Card.** Addition card for REACT = 1.

Card 10	1	2	3	4	5	6	7	8
Variable	FS	PS	BS	MS	MSIG	LCEPS23	LCEPS4	LCEPS5
Type	F	F	F	F	F	I	I	I
Default	0.0	0.0	0.0	0.0	none	none	none	none

**Tempering Card.** Additional card for TEMPR = 1.

Card 11	1	2	3	4	5	6	7	8
Variable	LCH4	LCH5	DTCRIT	TSAMP				
Type	I	I	F	F				
Default	0	0	0.0	0.0				

**Computational Welding Mechanics Card.** Additional card for CWM = 1.

Card 11	1	2	3	4	5	6	7	8
Variable	TASTART	TAEND	TLSTART	TLEND	EGHOST	PGHOST	AGHOST	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

<u>VARIABLE</u>	<u>DESCRIPTION</u>	<u>BASELINE VALUE</u>
MID	Material ID, a unique number has to be chosen.	
RO	Material density	7830 Kg/m <sup>3</sup>
E	Youngs' modulus: GT.0.0: constant value is used LT.0.0: LCID or TABID. Temperature dependent Youngs' modulus given by load curve ID = -E or a Table ID = -E. When using a table to describe the Youngs modulus see <a href="#">Remark 8</a> for more information.	100 GPa [1]
PR	Poisson's ratio: GT.0.0: constant value LT.0.0: LCID or TABID: Temperature dependent Poisson ratio given by load curve or table ID = -PR. The table input is described in <a href="#">Remark 8</a> .	0.30 [1]
TUNIT	Number of time units per hour. Default is seconds, that is 3600 time units per hour. It is used only for hardness calculations.	3600.

<b><u>VARIABLE</u></b>	<b><u>DESCRIPTION</u></b>	<b><u>BASELINE VALUE</u></b>
CRSH	<p>Switch to use a simple and fast material model but with the actual phases active.</p> <p>EQ.0: The original model where phase transitions are active and trip is used.</p> <p>EQ.1: A simpler and faster version. This option is mainly when transferring the quenched blank into a crash analysis where all properties from the cooling are maintained. This option must be used with a <b>*INTERFACE_SPRINGBACK</b> keyword and should be used after a quenching analysis.</p> <p>EQ.2: Same as 0 but trip effect is not used.</p>	0
PHASE	<p>Switch to include or exclude middle phases from the simulation.</p> <p>EQ.0: All phases active (default)</p> <p>EQ.1: pearlite and bainite excluded</p> <p>EQ.2: bainite excluded</p> <p>EQ.3: ferrite and pearlite excluded</p> <p>EQ.4: ferrite and bainite excluded</p> <p>EQ.5: exclude middle phases (only austenite → martensite)</p>	0

<u>VARIABLE</u>	<u>DESCRIPTION</u>	<u>BASELINE VALUE</u>
HEAT	<p>Switch to activate the heating algorithms</p> <p>EQ.0: Heating is not activated. That means that no transformation to Austenite is possible.</p> <p>EQ.1: Heating is activated: That means that only transformation to Austenite is possible.</p> <p>EQ.2: Automatic switching between cooling and heating. LS-DYNA checks the temperature gradient and calls the appropriate algorithms. For example, this can be used to simulate the heat affected zone during welding.</p> <p>LT.0: Switch between cooling and heating is defined by a time dependent load curve with id ABS(HEAT). The ordinate should be 1.0 when heating is applied and 0.0 if cooling is preferable.</p>	
LCY1	<p>Load curve or Table ID for austenite hardening.</p> <p><u>IF LCID</u></p> <p>input yield stress versus effective plastic strain.</p> <p><u>IF TABID.GT.0:</u></p> <p>2D table. Input temperatures as table values and hardening curves as targets for those temperatures (see *DEFINE_TABLE)</p> <p><u>IF TABID.LT.0:</u></p> <p>3D table. Input temperatures as main table values and strain rates as values for the sub tables, and hardening curves as targets for those strain rates.</p>	[5]



<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
LCY2	Load curve ID for ferrite hardening (stress versus eff. pl. str.)	
LCY3	Load curve or Table ID for pearlite. See LCY1 for description.	
LCY4	Load curve or Table ID for bainite. See LCY1 for description.	
LCY5	Load curve or Table ID for martensite. See LCY1 for description.	
KFERR	Correction factor for boron in the ferrite reaction.	$1.9 \times 10^5$ [2]
KPEAR	Correction factor for boron in the pearlite reaction.	$3.1 \times 10^3$ [2]
B	Boron [weight %]	0.003 [2, 4]
C	Carbon [weight %]	0.23 [2, 4]
Co	Cobolt [weight %]	0.0 [2, 4]
Mo	Molybdenum [weight %]	0.0 [2, 4]
Cr	Chromium [weight %]	0.21 [2, 4]
Ni	Nickel [weight %]	0.0 [2, 4]
Mn	Manganese [weight %]	1.25 [2, 4]
Si	Silicon [weight %]	0.29 [2, 4]
V	Vanadium [weight %]	0.0 [2, 4]
W	Tungsten [weight %]	0.0
Cu	copper [weight %]	0.0
P	Phosphorous [weight %]	0.013
Al	Aluminium [weight %]	0.0
As	Arsenic [weight %]	0.0
Ti	Titanium [weight %]	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
CWM	Flag for computational welding mechanics input. One additional input card is read. EQ.1.0: Active EQ.0.0: Inactive	
LCTRE	Load curve for transformation induced strains. See <a href="#">Remark 13</a> for more information.	
THEXP1	Coefficient of thermal expansion in austenite	$25.1 \times 10^{-6}$ 1/K [7]
THEXP5	Coefficient of thermal expansion in martensite	$11.1 \times 10^{-6}$ 1/K [7]
LCTH1	Load curve for the thermal expansion coefficient for austenite: LT.0.0: curve ID = -LA and TREF is used as reference temperature GT.0.0: curve ID = LA	0
LCTH5	Load curve for the thermal expansion coefficient for martensite: LT.0.0: curve ID = -LA and TREF is used as reference temperature GT.0.0: curve ID = LA	0
TREF	Reference temperature for thermal expansion. Used if and only if LA.LT.0.0 or/and LM.LT.0.0	293.15
LAT1	Latent heat for the decomposition of austenite into ferrite, pearlite and bainite. GT.0.0: Constant value LT.0.0: Curve ID or Table ID. See <a href="#">Remark 9</a> for more information.	$590 \times 10^6$ J/m <sup>3</sup> [2]

VARIABLE	DESCRIPTION	BASELINE VALUE
LAT5	<p>Latent heat for the decomposition of austenite into martensite.</p> <p>GT.0.0: Constant value</p> <p>LT.0.0: Curve ID:Note that LAT 5 is ignored if a Table ID is used in LAT1.</p>	$640 \times 10^6 \text{ J/m}^3$ [2]
TABTH	<p>Table definition for thermal expansion coefficient. With this option active THEXP1, THEXP2, LCTH1 and LCTH5 are ignored. See <a href="#">Remark 10</a>.</p> <p>GT.0: A table for instantaneous thermal expansion (TREF is ignored).</p> <p>LT.0: A table with thermal expansion with reference to TREF.</p>	
QR2	<p>Activation energy divided by the universal gas constant for the diffusion reaction of the austenite-ferrite reaction: Q2/R. R = 8.314472 [J/mol K].</p>	$10324 \text{ K}$ [3] = $(23000 \text{ cal/mole}) \times$ $(4.184 \text{ J/cal}) /$ $(8.314 \text{ J/mole/K})$
QR3	<p>Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-pearlite reaction: Q3/R. R = 8.314472 [J/mol K].</p>	$13432. \text{ K}$ [3]
QR4	<p>Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-bainite reaction: Q4/R. R = 8.314472 [J/mol K].</p>	$15068. \text{ K}$ [3]
ALPHA	<p>Material constant for the martensite phase. A value of 0.011 means that 90% of the available austenite is transformed into martensite at 210 degrees below the martensite start temperature (see <code>messag</code> file for information), whereas a value of 0.033 means a 99.9% transformation.</p>	0.011
GRAIN	<p>ASTM grain size number for austenite, usually a number between 7 and 11.</p>	6.8
TOFFE	<p>Number of degrees that the ferrite is bleeding over into the pearlite reaction.</p>	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
TOFPE	Number of degrees that the pearlite is bleeding over into the bainite reaction	0.0
TOFBA	Number of degrees that the bainite is bleeding over into the martensite reaction.	0.0
PLMEM2	Memory coefficient for the plastic strain that is carried over from the austenite. A value of 1 means that all plastic strains from austenite is transferred to the <i>ferrite</i> phase and a value of 0 means that nothing is transferred.	0.0
PLMEM3	Same as PLMEM2 but between austenite and pearlite.	0.0
PLMEM4	Same as PLMEM2 but between austenite and bainite.	0.0
PLMEM5	Same as PLMEM3 but between austenite and martensite.	0.0
STRC	Effective strain rate parameter C. STRC.LT.0.0: load curve id = -STRC STRC.GT.0.0: constant value STRC.EQ.0.0: strain rate NOT active	0.0
STRP	Effective strain rate parameter P. STRP.LT.0.0: load curve id = -STRP STRP.GT.0.0: constant value STRP.EQ.0.0: strain rate NOT active	0.0
REACT	Flag for advanced reaction kinetics input. One additional input card is read. EQ.1.0: Active EQ.0.0: Inactive	0.0
TEMPER	Flag for tempering input. One additional input card is read. EQ.1.0: Active EQ.0.0: Inactive	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
AUST	If a heating process is initiated at $t = 0$ this parameters sets the initial amount of austenite in the blank. If heating is activated at $t > 0$ during a simulation this value is ignored. Note that, $\text{AUST} + \text{FERR} + \text{PEAR} + \text{BAIN} + \text{MART} = 1.0$	0.0
FERR	See AUST for description	0.0
PEAR	See AUST for description	0.0
BAIN	See AUST for description	0.0
MART	See AUST for description	0.0
GRK	Growth parameter $k$ ( $\mu\text{m}^2/\text{sec}$ )	$10^{11}$ [9]
GRQR	Grain growth activation energy (J/mol) divided by the universal gas constant. $Q/R$ where $R = 8.314472$ (J/mol K)	$3 \times 10^4$ [9]
TAU1	Empirical grain growth parameter $c_1$ describing the function $\tau(T)$	$2.08 \times 10^8$ [9]
GRA	Grain growth parameter A	[9]
GRB	Grain growth parameter B. A table of recommended values of GRA and GRB is included in <a href="#">Remark 7</a> .	[9]
EXPA	Grain growth parameter a	1.0 [9]
EXPB	Grain growth parameter b	1.0 [9]
GRCC	Grain growth parameter with the concentration of non-metals in the blank, weight% of C or N	[9]
GRCM	Grain growth parameter with the concentration of metals in the blank, lowest weight% of Cr, V, Nb, Ti, Al.	[9]
HEATN	Grain growth parameter n for the austenite formation	1.0 [9]
TAU2	Empirical grain growth parameter $c_2$ describing the function $\tau(T)$	4.806 [9]

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
FS	<p>Manual start temperature Ferrite</p> <p>GT.0.0: Same temperature is used for heating and cooling.</p> <p>LT.0.0: Curve ID: Different start temperatures for cooling and heating given by load curve ID = -FS. First ordinate value is used for cooling, last ordinate value for heating.</p>	
PS	Manual start temperature Pearlite. See FS for description.	
BS	Manual start temperature Bainite. See FS for description.	
MS	Manual start temperature Martensite. See FS for description.	
MSIG	<p>Describes the increase of martensite start temperature for cooling due to applied stress.</p> <p>LT.0: Load Curve ID describes MSIG as a function of triaxiality (pressure / effective stress).</p> $MS^* = MS + MSIG \times \sigma_{eff}$	
LCEPS23	<p>Load Curve ID dependent on plastic strain that scales the activation energy QR2 and QR3.</p> $QR_x = Q_x \times CEPS23(\epsilon_{pl}) / R$	
LCEPS4	<p>Load Curve ID dependent on plastic strain that scales the activation energy QR4.</p> $QR_4 = Q_4 \times LCEPS4(\epsilon_{pl}) / R$	
LCEPS5	<p>Load Curve ID which describe the increase of the martensite start temperature for cooling as a function of plastic strain.</p> $MS^* = MS + MSIG \times \sigma_{eff} + LCEPS5(\epsilon_{pl})$	

VARIABLE	DESCRIPTION	BASELINE VALUE
LCH4	Load curve ID of Vicker hardness vs. temperature for Bainite hardness calculation.	
LCH5	Load curve ID of Vicker hardness vs. temperature for Martensite hardness calculation.	
DTCRIT	Critical cooling rate to detect holding phase.	
TSAMP	Sampling interval for temperature rate monitoring to detect the holding phase	
TASTART	Annealing temperature start	
TAEND	Annealing temperature end	
TLSTART	Birth temperature start	
EGHOST	Young's modulus for ghost (quiet) material	
PGHOST	Poisson's ratio for ghost (quiet) material	
AGHOST	Thermal expansion coefficient for ghost (quiet) material	

### Discussion:

The phase distribution during cooling is calculated by solving the following rate equation for each phase transition

$$\dot{X}_k = g_k(G, C, T_k, Q_k) f_k(X_k), \quad k = 2,3,4$$

where  $g_k$  is a function, taken from Li et al., dependent on the grain number  $G$ , the chemical composition  $C$ , the temperature  $T$  and the activation energy  $Q$ . Moreover, the function  $f$  is dependent on the actual phase  $X_k = x_k/x_{eq}$

$$f_k(X_k) = X_k^{0.4(X_k-1)} (1 - X_k)^{0.4X_k}, \quad k = 2,3,4$$

The true amount of martensite, i.e.,  $k = 5$ , is modelled by using the true amount of the austenite left after the bainite phase:

$$x_5 = x_1 [1 - e^{-\alpha(MS-T)}],$$

where  $x_1$  is the true amount of austenite left for the reaction,  $\alpha$  is a material dependent constant and  $MS$  is the start temperature of the martensite reaction.

The start temperatures are automatically calculated based on the composition:

1. Ferrite,

$$\begin{aligned} FS = & 1185 - 203 \times \sqrt{C} - 15.2 \times Ni + 44.7 \times Si + 104 \times V + 31.5 \times Mo + 13.1 \times W \\ & - 30 \times Mn - 11 \times Cr - 20 \times Cu + 700 \times P + 400 \times Al + 120 \times As \\ & + 400 \times Ti \end{aligned}$$

2. Pearlite,

$$PS = 996 - 10.7 \times Mn - 16.9 \times Ni + 29 \times Si + 16.9 \times Cr + 290 \times As + 6.4 \times W$$

3. Bainite,

$$BS = 910 - 58 \times C - 35 \times Mn - 15 \times Ni - 34 \times Cr - 41 \times Mo$$

4. Martensite,

$$\begin{aligned} MS = & 812 - 423 \times C - 30.4 \times Mn - 17.7 \times Ni - 12.1 \times Cr - 7.5 \times Mo + 10 \times Co \\ & - 7.5 \times Si \end{aligned}$$

where the element weight values are input on Cards 2 through 4.

The automatic start temperatures are printed to the `messag` file and if they are not accurate enough you can manually set them in the input deck (must be set in absolute temperature, Kelvin). If `HEAT > 0`, the temperature `FSnc` (ferrite without C) is also echoed. If the specimen exceeds that temperature all ferrite that is left is instantaneous transformed to austenite.

**Remarks:**

1. **History Variables.** History variables 1 through 8 include the different phases, the Vickers hardness, the yield stress and the ASTM grain size number. Set `NEIPS = 8` (shells) or `NEIPH = 8` (solids) on `*DATABASE_EXTENT_BINARY`.



History Variable	Description
1	Amount austenite
2	Amount ferrite
3	Amount pearlite
4	Amount bainite
5	Amount martensite
6	Vickers hardness
7	Yield stress
8	ASTM grain size number (a low value means large grains and vice versa)

- Excluding Phases.** To exclude a phase from the simulation, set the PHASE parameter accordingly.
- STRC and STRP.** Note that both strain rate parameters must be set to include the effect. It is possible to use a temperature dependent load curve for both parameters simultaneously or for one parameter keeping the other constant.
- TUNIT.** TUNIT is time units per hour and is only used for calculating the Vicker Hardness, as default it is assumed that the time unit is seconds. If other time unit is used, for example milliseconds, then TUNIT must be changed to  $TUNIT = 3.6 \times 10^6$
- TSF.** The thermal speedup factor TSF of \*CONTROL\_THERMAL\_SOLVER is used to scale reaction kinetics and hardness calculations in this material model. On the other hand, strain rate dependent properties (see LCY1 to LCY5 or STRC/STRP) are not scaled by TSF.
- CRSH.** With the CRSH = 1 option it is now possible to transfer the material properties from a hot stamping simulation (CRSH = 0) into another simulation. The CRSH = 1 option reads a dynain file from a simulation with CRSH = 0 and keeps all the history variables (austenite, ferrite, pearlite, bainite, martensite, etc) constant. This will allow steels with inhomogeneous strength to be analysed in, for example, a crash simulation. The speed with the CRSH = 1 option is comparable with \*MAT\_024. Note that for keeping the speed the temperature used in the CRSH simulation should be constant and the thermal solver should be inactive.

7. **HEAT.** When HEAT is activated the re-austenitization and grain growth algorithms are also activated. The grain growth is activated when the temperature exceeds a threshold value that is given by

$$T = \frac{B}{A - \log_{10}[(\text{GRCM})^a(\text{GRCC})^b]}$$

and the rate equation for the grain growth is,

$$\dot{g} = \frac{k}{2g} e^{-\frac{Q}{RT}}$$

The rate equation for the phase re-austenitization is given in Oddy (1996) and is here mirrored

$$\dot{x}_a = n \left[ \ln \left( \frac{x_{eu}}{x_{eu} - x_a} \right) \right]^{\frac{n-1}{n}} \left[ \frac{x_{eu} - x_a}{\tau(T)} \right]$$

where n is the parameter HEATN. The temperature dependent function  $\tau(T)$  is given from Oddy as  $\tau(T) = c_1(T - T_s)^{c_2}$ . The empirical parameters  $c_1$  and  $c_2$  are calibrated in Oddy to  $2.06 \times 10^8$  and 4.806 respectively. Note that  $\tau$  above given in **seconds**.

Recommended values for GRA and GRB are given in the following table.

Compound	Metal	Non-metal	GRA	GRB
Cr <sub>23</sub> C <sub>6</sub>	Cr	C	5.90	7375
V <sub>4</sub> C <sub>3</sub>	V	C <sub>0.75</sub>	5.36	8000
TiC	Ti	C	2.75	7000
NbC	Nb	C <sub>0.7</sub>	3.11	7520
Mo <sub>2</sub> C	Mo	C	5.0	7375
Nb(CN)	Nb	(CN)	2.26	6770
VN	V	N	3.46 + 0.12%Mn	8330
AlN	Al	N	1.03	6770
NbN	Nb	N	4.04	10230
TiN	Ti	N	0.32	8000

8. **Using the Table Capability for Temperature Dependence of Young's Modulus.** Use \*DEFINE\_TABLE\_2D and set the abscissa value equal to 1 for the austenite YM-curve, equal to 2 for the ferrite YM-curve, equal to 3 for the pearlite YM curve, equal to 4 for the bainite YM-curve and finally equal to 5 for the martensite YM-curve. If you use the PHASE option you only need to define the curves

for the included phases, but you can define all five. LS-DYNA uses the number 1-5 to get the right curve for the right phase. The total YM is calculated by a linear mixture law:  $YM = YM1 \times PHASE1 + \dots + YM5 \times PHASE5$ . For example:

```

*DEFINE_TABLE_2D
$ The number before curve id:s define which phase the curve
$ will be applied to. 1 = Austenite, 2 = Ferrite, 3 = Pearlite,
$ 4 = Bainite and 5 = Martensite.
      1000      0.0      0.0
              1.0          100
              2.0          200
              3.0          300
              4.0          400
              5.0          500

$
$ Define curves 100 - 500
*DEFINE_CURVE
$ Austenite Temp (K) - YM-Curve (MPa)
      100      0      1.0      1.0
      1300.0      50.E+3
      223.0      210.E+3

```

9. **Using the Table Capability for Latent Heat.** When using a table ID for the latent heat (LAT1) you can describe all phase transition individually. Use \*DEFINE\_TABLE\_2D and set the abscissa values to the corresponding phase transition number. That is, 2 for the austenite to ferrite, 3 for the austenite to pearlite, 4 for the austenite to bainite and 5 for the austenite to martensite. Remark 8 demonstrates the form a correct table definition. If a curve is missing, the corresponding latent heat for that transition will be set to zero. Also, when a table is used the LAT2 is ignored. If HEAT > 0 you also have the option to include latent heat for the transition back to Austenite. This latent heat curve is marked as 1 in the table definition of LAT1.
  
10. **Using the Table Capability for Thermal Expansion.** When using a table ID for the thermal expansion you can specify the expansion characteristics for each phase. That is, you can have a curve for each of the 5 phases (austenite, ferrite, pearlite, bainite, and martensite). The input is identical to the above table definitions. The table must have the abscissa values between 1 and 5 where the number correspond to phase 1 to 5. To exclude one phase from influencing the thermal expansion you simply input a curve that is zero for that phase or even easier, exclude that phase number in the table definition. For example, to exclude the bainite phase you only define the table with curves for the indices 1, 2, 3 and 5.
  
11. **TEMPER.** Tempering is activated by setting TEMPER to 1. When active the default hardness calculation for bainite and martensite is altered to use an incremental update formula. The total hardness is given by  $\sum_{i=1}^5 HV_i \times x_i$ . When holding phases are detected the hardness for Bainite and Martensite is updated according to

$$HV_4^{n+1} = \frac{x_4^n}{x_4^{n+1}} HV_4^n + \frac{\Delta x_4}{x_4^{n+1}} h_4(T), \quad \Delta x_4 = x_4^{n+1} - x_4^n$$

$$\text{HV}_5^{n+1} = \frac{x_5^n}{x_5^{n+1}} \text{HV}_5^n + \frac{\Delta x_5}{x_5^{n+1}} h_5(T), \quad \Delta x_5 = x_5^{n+1} - x_5^n$$

We detect the holding phase for Bainite and Martensite when the temperature is in the appropriate range and if average temperature rate is below DTCRIT. The average temperature rate is calculated as  $\frac{T_{\text{tresh}}}{t_{\text{tresh}}}$  where the  $T_{\text{tresh}}^{n+1} = T_{\text{tresh}}^n + |\dot{T}|\Delta t$  and  $t_{\text{tresh}}^{n+1} = t_{\text{tresh}}^n + \Delta t$ . The average temperature and time are updated until  $t_{\text{tresh}} \geq t_{\text{samp}}$ .

12. **CWM (Welding).** When computational welding mechanics is activated with CWM = 1 the material can be defined to be initially in a quiet state. In this state the material (often referred to as ghost material) has thermo-mechanical properties defined by an additional card. The material is activated when the temperature reaches the birth temperature. See MAT\_CWM (MAT\_270) for a detailed description.
13. **LCTRE (Transformation Induced Strains).** Transformation induced strains can be included with a load curve LCTRE as a function of temperature. The load curve represents the difference between the hard phases and the austenite phase in the dilatometer curves. Therefore, positive curve values result in a negative transformation strain for austenitization and a positive transformation strain for the phase transformation from austenite to one of the hard phases.

#### References:

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2. P. Akerstrom and M. Oldenburg, "Austenite Decomposition During Press hardening of a Boron Steel – Computer Simulation and Test", Journal of Material processing technology, 174 (2006), pp399-406.
3. M.V Li, D.V Niebuhr, L.L Meekisho and D.G Atteridge, "A Computatinal model for te prediction of steel hardenability", Metallurgical and materials transactions B, 29B, 661-672, 1998.
4. D.F. Watt, "An Algorithm for Modelling Microstructural Development in Weld heat-Affected Zones (Part A) Reaction Kinetics", Acta metal. Vol. 36., No. 11, pp. 3029-3035, 1988.
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6. Malek Naderi, "Hot Stamping of Ultra High Strength Steels", Doctor of Engineering Dissertation, Technical University Aachen, Germany, 2007.

7. P. Akerstrom, "Numerical Implementation of a Constitutive model for Simulation of Hot Stamping", Division of Solid Mechanics, Lulea University of technology, Sweden.
8. Malek Naderi, "A numerical and Experimental Investigation into Hot Stamping of Boron Alloyed Heat treated Steels", Steel research Int. 79 (2008) No. 2.
9. A.S. Oddy, J.M.J. McDill and L. Karlsson, "Microstructural predictions including arbitrary thermal histories, reaustenitization and carbon segregation effects" (1996).

**Boron steel composition from the literature:**

Element	HAZ code	Akerstrom (2)	Naderi (8)	ThyssenKrupp(4) (max amount)
B		0.003	0.003	0.005
C	0.168	0.23	0.230	0.250
Co				
Mo	0.036			0.250
Cr	0.255	0.211	0.160	0.250
Ni	0.015			
Mn	1.497	1.25	1.18	1.40
Si	0.473	0.29	0.220	0.400
V	0.026			
W				
Cu	0.025			
P	0.012	0.013	0.015	0.025
Al	0.020			
As				
Ti			0.040	0.05
S		0.003	0.001	0.010



**Anisotropic Card 1.** Card 1 format used for ANISO keyword option.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	C11	C12	C22	C13	C23	C33
Type	A8	F	F	F	F	F	F	F

**Anisotropic Card 2.** Card 2 format used for ANISO keyword option.

Card 2	1	2	3	4	5	6	7	8
Variable	C14	C24	C34	C44	C15	C25	C35	C45
Type	F	F	F	F	F	F	F	F

**Anisotropic Card 3.** Card 3 format used for ANISO keyword option.

Card 3	1	2	3	4	5	6	7	8
Variable	C55	C16	C26	C36	C46	C56	C66	AOPT
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	I	

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	REF
Type	F	F	F	F	F	F	F	F

**Remarks:**

1. A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any static displacement.
2. It is assumed the material in the bounded domain near the layer is, or behaves like, a linear ortho/anisotropic material. The material properties of the layer should be set to the corresponding properties of this material.
3. The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem, e.g., for a half-space problem, the "top" of the box should be open.
4. Internally, LS-DYNA will partition the entire PML into regions which form the "faces", "edges" and "corners" of the above cuboid box, and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.
5. The layer should have 5 - 10 elements through its depth. Typically, 5 - 6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8 - 10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer, and should be small enough to sufficiently discretize all significant wavelengths in the problem.
6. The nodes on the outer boundary of the layer should be fully constrained.
7. The stress and strain values reported by this material do not have any physical significance.



**\*MAT\_PML\_NULL**

This is Material Type 246. This is a perfectly-matched layer (PML) material with a pressure fluid constitutive law computed using an equation of state, to be used in a wave-absorbing layer adjacent to a fluid material (\*MAT\_NULL with an EOS) in order to simulate wave propagation in an unbounded fluid medium. Only \*EOS\_LINEAR\_POLYNOMIAL and \*EOS\_GRUNEISEN are allowed with this material. See the Remarks section of \*MAT\_NULL (\*MAT\_009) for further details. Accurate results are to be expected only for the case where the EOS presents a linear relationship between the pressure and volumetric strain.

This material is a variant of MAT\_PML\_ELASTIC (MAT\_230) and is available only for solid 8-node bricks (element type 2).

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	MU					
Type	A8	F	F					
Default	none	none	0.0					

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
MU	Dynamic viscosity coefficient

**Remarks:**

1. A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any static displacement.
2. It is assumed the material in the bounded domain near the layer is, or behaves like, an linear fluid material. The material properties of the layer should be set to the corresponding properties of this material.

3. The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem, e.g., for a half-space problem, the “top” of the box should be open.
4. Internally, LS-DYNA will partition the entire PML into regions which form the “faces”, “edges” and “corners” of the above cuboid box, and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.
5. The layer should have 5-10 elements through its depth. Typically, 5-6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8-10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer, and should be small enough to sufficiently discretize all significant wavelengths in the problem.
6. The nodes on the outer boundary of the layer should be fully constrained.
7. The stress and strain values reported by this material do not have any physical significance.

\*MAT\_PHS\_BMW

This is Material Type 248. This model is intended for hot stamping processes with phase transformation effects. It is available for shell elements *only* and is based on [Material Type 244 \(\\*MAT\\_UHS\\_STEEL\)](#). As compared with Material Type 244 Material Type 248 features:

1. a more flexible choice of evolution parameters,
2. an approach for transformation induced strains,
3. and a more accurate density calculation of individual phases.

Thus the metal physical effects can be taken into account calculating the volume fractions of ferrite, pearlite, bainite and martensite for fast supercooling as well as for slow cooling conditions. Furthermore, this material model features cooling-rate dependence for several of its more crucial material parameters in order to accurately calculate the Time-Temperature-Transformation diagram dynamically. A detailed description can be found in Hippchen et al. [2013] and Hippchen [2014].

**NOTE 1:** For this material “weight%” means “ppm × 10<sup>-4</sup>”.

**NOTE 2:** For this material the phase fractions are calculated in volume percent (vol%).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	TUNIT	TRIP	PHASE	HEAT
Type	I	F	F	F	F	I	I	I
Defaults	none	none	none	none	3600	0	0	0

Card 2	1	2	3	4	5	6	7	8
Variable	LCY1	LCY2	LCY3	LCY4	LCY5	C_F	C_P	C_B
Type	I	I	I	I	I	F	F	F
Defaults	none	none	none	none	none	0.0	0.0	0.0

Card 3	1	2	3	4	5	6	7	8
Variable	C	Co	Mo	Cr	Ni	Mn	Si	V
Type	F	F	F	F	F	F	F	F
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Card 4	1	2	3	4	5	6	7	8
Variable	W	Cu	P	Al	As	Ti	B	
Type	F	F	F	F	F	F	F	
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

Card 5	1	2	3	4	5	6	7	8
Variable			TABRH0		TREF	LAT1	LAT5	TABTH
Type			I		F	F	F	I
Defaults			none		none	0.0	0.0	none

Card 6	1	2	3	4	5	6	7	8
Variable	QR2	QR3	QR4	ALPHA	GRAIN	TOFFE	TOFPE	TOFBA
Type	F	F	F	F	F	F	F	F
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Card 7	1	2	3	4	5	6	7	8
Variable	PLMEM2	PLMEM3	PLMEM4	PLMEM5	STRC	STRP		
Type	F	F	F	F	F	F		
Defaults	0.0	0.0	0.0	0.0	0.0	0.0		

Card 8	1	2	3	4	5	6	7	8
Variable	FS	PS	BS	MS	MSIG	LCEPS23	LCEPS4	LCEPS5
Type	F	F	F	F	F	I	I	I
Defaults	0.0	0.0	0.0	0.0	none	none	none	none

Card 9	1	2	3	4	5	6	7	8
Variable	LCH4	LCH5	DTCRIT	TSAMP	ISLC	IEXTRA		
Type	I	I	F	F	I	I		
Defaults	0	0	0.0	0.0	0	0		

Card 10	1	2	3	4	5	6	7	8
Variable	ALPH_M	N_M	PHI_M	PSI_M	OMG_F	PHI_F	PSI_F	CR_F
Type	F	F	F	F	F	F	F	F
Defaults	0.0428	0.191	0.382	2.421	0.41	0.4	0.4	0.0

Card 11	1	2	3	4	5	6	7	8
Variable	OMG_P	PHI_P	PSI_P	CR_P	OMG_B	PHI_B	PSI_B	CR_B
Type	F	F	F	F	F	F	F	F
Defaults	0.32	0.4	0.4	0.0	0.29	0.4	0.4	0.0

**Heat Card 1.** Additional Card for HEAT  $\neq$  0.

Card 12	1	2	3	4	5	6	7	8
Variable	AUST	FERR	PEAR	BAIN	MART	GRK	GRQR	TAU1
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.08E+8

**Heat Card 2.** Additional Card for HEAT  $\neq$  0.

Card 13	1	2	3	4	5	6	7	8
Variable	GRA	GRB	EXPA	EXPB	GRCC	GRCM	HEATN	TAU2
Type	F	F	F	F	F	F	F	F
Default	3.11	7520.	1.0	1.0	none	none	1.0	4.806

**Extra Card 1.** Additional Card for IEXTRA = 1.

Card 14	1	2	3	4	5	6	7	8
Variable	FUNCA	FUNCB	FUNCM	TCVUP	TCVLO	CVCRT	TCVSL	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

**Extra Card 2.** Additional Card for IEXTRA = 2.

Card 15	1	2	3	4	5	6	7	8
Variable	EPSP	EXPON						
Type	F	F						
Default	0.0	0.0						

<u>VARIABLE</u>	<u>DESCRIPTION</u>	<u>BASELINE VALUE</u>
MID	Material ID, a unique number has to be chosen.	
RO	Material density at room temperature (necessary for calculating transformation induced strains)	7830 Kg/m <sup>3</sup>
E	Youngs' modulus: GT.0.0: constant value is used LT.0.0: LCID or TABID. Temperature dependent Young's modulus given by load curve or table ID = -E. When using a table to describe the Young's modulus see Remark 10 for more information.	100.e+09 Pa [1]

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
PR	Poisson's ratio:  GT.0.0: constant value is used  LT.0.0: LCID or TABID. Temperature dependent Poisson's ratio given by load curve or table ID = -PR. The table input is described in Remark 10.	0.30 [1]
TUNIT	Number of time units per hour. Default is seconds, that is 3600 time units per hour. It is used only for hardness calculations.	3600.
TRIP	Flag to activate (0) or deactivate (1) trip effect calculation.	0
PHASE	Switch to exclude middle phases from the simulation.  EQ.0: all phases active (default) EQ.1: pearlite and bainite active EQ.2: bainite active EQ.3: ferrite and pearlite active EQ.4: ferrite and bainite active EQ.5: no active middle phases (only austenite → martensite)	0
HEAT	Heat flag as in MAT_244, see there for details.  EQ.0: Heating is not activated. EQ.1: Heating is activated. EQ.2: Automatic switching between cooling and heating.  LT.0: Switch between cooling and heating is defined by a time dependent load curve with id ABS(HEAT).	



VARIABLE	DESCRIPTION	BASELINE VALUE
LCY1	<p>Load curve or Table ID for austenite hardening.</p> <p><u>if LCID</u></p> <p>input yield stress versus effective plastic strain.</p> <p><u>if TABID.GT.0:</u></p> <p>2D table. Input temperatures as table values and hardening curves as targets for those temperatures (see *DEFINE_TABLE)</p> <p><u>if TABID.LT.0:</u></p> <p>3D table. Input temperatures as main table values and strain rates as values for the sub tables, and hardening curves as targets for those strain rates.</p>	[5]
LCY2	Load curve or Table ID for ferrite. See LCY1 for description.	
LCY3	Load curve or Table ID for pearlite. See LCY1 for description.	
LCY4	Load curve or Table ID for bainite. See LCY1 for description.	
LCY5	Load curve or Table ID for martensite. See LCY1 for description.	
C_F	Alloy dependent factor $C_f$ for ferrite (controls the alloying effects beside of Boron on the time-temperature-transformation start line of ferrite).	
C_P	Alloy dependent factor $C_p$ for pearlite (see C_F for description).	
C_B	Alloy dependent factor $C_b$ for bainite (see C_F for description).	
C	Carbon [weight %]	0.23 [2, 4]
Co	Cobolt [weight %]	0.0 [2, 4]
Mo	Molybdenum [weight %]	0.0 [2, 4]

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
Cr	Chromium [weight %]	0.21 [2, 4]
Ni	Nickel [weight %]	0.0 [2, 4]
Mn	Manganese [weight %]	1.25 [2, 4]
Si	Silicon [weight %]	0.29 [2, 4]
V	Vanadium [weight %]	0.0 [2, 4]
W	Tungsten [weight %]	0.0
Cu	Copper [weight %]	0.0
P	Phosphorous [weight %]	0.013
Al	Aluminium [weight %]	0.0
As	Arsenic [weight %]	0.0
Ti	Titanium [weight %]	0.0
B	Boron [weight %]	0.0
TABRHO	Table definition for phase and temperature dependent densities. Needed for calculation of transformation induced strains.	
TREF	Reference temperature for thermal expansion (only necessary for thermal expansion calculation with the secant method).	293.15
LAT1	Latent heat for the decomposition of austenite into ferrite, pearlite and bainite. GT.0.0: Constant value LT.0.0: Curve ID or Table ID: See remark 11 for more information.	590.e+06 J/m <sup>3</sup> [2]
LAT5	Latent heat for the decomposition of austenite into martensite. GT.0.0: Constant value LT.0.0: Curve ID:Note that LAT 5 is ignored if a Table ID is used in LAT1.	640.e+06 J/m <sup>3</sup> [2]

<u>VARIABLE</u>	<u>DESCRIPTION</u>	<u>BASELINE VALUE</u>
TABTH	Table definition for thermal expansion coefficient. See remarks for more information how to input this table.  GT.0: A table for instantaneous thermal expansion (TREF is ignored).  LT.0: A table with thermal expansion with reference to TREF.	
QR2	Activation energy divided by the universal gas constant for the diffusion reaction of the austenite-ferrite reaction: Q2/R. $R = 8.314472$ [J/mol K].	10324 K [3] = (23000 cal/mole) × (4.184 J/cal) / (8.314 J/mole/K)
QR3	Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-pearlite reaction: Q3/R. $R = 8.314472$ [J/mol K].	13432. K [3]
QR4	Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-bainite reaction: Q4/R. $R = 8.314472$ [J/mol K].	15068. K [3]
ALPHA	Material constant for the martensite phase. A value of 0.011 means that 90% of the available austenite is transformed into martensite at 210 degrees below the martensite start temperature (see message file for information), whereas a value of 0.033 means a 99.9% transformation.	0.011
GRAIN	ASTM grain size number $G$ for austenite, usually a number between 7 and 11.	6.8
TOFFE	Number of degrees that the ferrite is bleeding over into the pearlite reaction: $T_{off,f}$ .	0.0
TOFPE	Number of degrees that the pearlite is bleeding over into the bainite reaction: $T_{off,p}$ .	0.0
TOFBA	Number of degrees that the bainite is bleeding over into the martensite reaction: $T_{off,b}$ .	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
PLMEM2	Memory coefficient for the plastic strain that is carried over from the austenite. A value of 1 means that all plastic strains from austenite is transferred to the ferrite phase and a value of 0 means that nothing is transferred.	0.0
PLMEM3	Same as PLMEM2 but between austenite and pearlite.	0.0
PLMEM4	Same as PLMEM2 but between austenite and bainite.	0.0
PLMEM5	Same as PLMEM3 but between austenite and martensite.	0.0
STRC	Cowper and Symonds strain rate parameter $C$ . STRC.LT.0.0: load curve id = -STRC STRC.GT.0.0: constant value STRC.EQ.0.0: strain rate NOT active	0.0
STRP	Cowper and Symonds strain rate parameter $P$ . STRP.LT.0.0: load curve id = -STRP STRP.GT.0.0: constant value STRP.EQ.0.0: strain rate NOT active	0.0
FS	Manual start temperature ferrite, $F_S$ . GT.0.0: Same temperature is used for heating and cooling. LT.0.0: Curve ID:Different start temperatures for cooling and heating given by load curve ID = -FS. First ordinate value is used for cooling, last ordinate value for heating.	
PS	Manual start temperature pearlite, $P_S$ . See FS for description.	

VARIABLE	DESCRIPTION	BASELINE VALUE
BS	Manual start temperature bainite, $B_S$ . See FS for description.	
MS	Manual start temperature martensite, $M_S$ . See FS for description.	
MSIG	Describes the increase of martensite start temperature for cooling due to applied stress.  LT.0: Load Curve ID describes MSIG as a function of triaxiality (pressure / effective stress).  $MS^* = MS + MSIG \times \sigma_{eff}$	
LCEPS23	Load Curve ID dependent on plastic strain that scales the activation energy QR2 and QR3.  $QR_n = Q_n \times LCEPS23(\varepsilon_{pl}) / R$	
LCEPS4	Load Curve ID dependent on plastic strain that scales the activation energy QR4.  $QR4 = Q4 \times LCEPS4(\varepsilon_{pl}) / R$	
LCEPS5	Load Curve ID which describe the increase of the martensite start temperature for cooling as a function of plastic strain.  $MS^* = MS + MSIG \times \sigma_{eff} + LCEPS5(\varepsilon_{pl})$	
LCH4	Load curve ID of Vickers hardness vs. temperature for bainite hardness calculation.	
LCH5	Load curve ID of Vickers hardness vs. temperature for martensite hardness calculation.	
DTCRIT	Critical cooling rate to detect holding phase.	
TSAMP	Sampling interval for temperature rate monitoring to detect the holding phase	

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
ISLC	<p>Flag for definition of evolution parameters on Cards 10 and 11.</p> <p>EQ.0.0: All 16 parameters on Cards 10 and 11 are constant values.</p> <p>EQ.1.0: PHI_F, CR_F, PHI_P, CR_P, PHI_B, and CR_B are load curves defining values as functions of cooling rate. The remaining 10 parameters on Cards 10 and 11 are constant values.</p> <p>EQ.2.0: All 16 parameters on Cards 10 and 11 are load curves defining values as functions of cooling rate.</p>	
IEXTRA	Flag to read extra cards	
ALPH_M	Martensite evolution parameter $\alpha_m$	0.0428
N_M	Martensite evolution parameter $n_m$	0.191
PHI_M	Martensite evolution parameter $\varphi_m$	0.382
PSI_M	Martensite evolution exponent $\psi_m$ , if $\psi_m < 0$ then $\psi_m =  \psi_m (2 - \zeta_a)$	2.421
OMG_F	Ferrite grain size factor $\omega_f$ (mainly controls the alloying effect of Boron on the time-temperature-transformation start line of ferrite)	0.41
PHI_F	Ferrite evolution parameter $\varphi_f$ (controls the incubation time till 1vol% of ferrite is built)	0.4
PSI_F	Ferrite evolution parameter $\psi_f$ (controls the time till 99vol% of ferrite is built without effect on the incubation time)	0.4
CR_F	Ferrite evolution parameter $C_{r,f}$ (retardation coefficient to influence the kinetics of phase transformation of ferrite, should be determined at slow cooling conditions, can also be defined in dependency to the cooling rate)	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
OMG_P	Pearlite grain size factor $\omega_p$ (see OMG_F for description)	0.32
PHI_P	Pearlite evolution parameter $\varphi_p$ (see PHI_F for description)	0.4
PSI_P	Pearlite evolution parameter $\psi_p$ (see PSI_F for description)	0.4
CR_P	Pearlite evolution parameter $C_{r,p}$ (see CR_F for description)	0.0
OMG_B	Bainite grain size factor $\omega_b$ (see OMG_F for description)	0.32
PHI_B	Bainite evolution parameter $\varphi_b$ (see PHI_F for description)	0.4
PSI_B	Bainite evolution parameter $\psi_b$ (see PSI_F for description)	0.4
CR_B	Bainite evolution parameter $C_{r,b}$ (see CR_F for description)	0.0
AUST	If a heating process is initiated at $t = 0$ this parameters sets the initial amount of austenite in the blank. If heating is activated at $t > 0$ during a simulation this value is ignored. Note that, $\text{AUST} + \text{FERR} + \text{PEAR} + \text{BAIN} + \text{MART} = 1.0$	0.0
FERR	See AUST for description	0.0
PEAR	See AUST for description	0.0
BAIN	See AUST for description	0.0
MART	See AUST for description	0.0
GRK	Growth parameter $k$ ( $\mu\text{m}^2/\text{sec}$ )	1.0E+11[9]
GRQR	Grain growth activation energy (J/mol) divided by the universal gas constant. Q/R where $R = 8.314472$ (J/mol K)	3.0E+4[9]
TAU1	Empirical grain growth parameter $c_1$ describing the function $\tau(T)$	2.08E+8 [9]

<b>VARIABLE</b>	<b>DESCRIPTION</b>	<b>BASELINE VALUE</b>
GRA	Grain growth parameter A	[9]
GRB	Grain growth parameter B. A table of recommended values of GRA and GRB is included in Remark 7 of *MAT_244.	[9]
EXPA	Grain growth parameter <i>a</i>	1.0 [9]
EXPB	Grain growth parameter <i>b</i>	1.0 [9]
GRCC	Grain growth parameter with the concentration of non metals in the blank, weight% of C or N	[9]
GRCM	Grain growth parameter with the concentration of metals in the blank, lowest weight% of Cr, V, Nb, Ti, Al.	[9]
HEATN	Grain growth parameter <i>n</i> for the austenite formation	1.0[9]
TAU2	Empirical grain growth parameter $c_2$ describing the function $\tau(T)$	4.806[9]
FUNCA	ID of a *DEFINE_FUNCTION for saturation stress A (Hockett-Sherby approach)	
FUNCB	ID of a *DEFINE_FUNCTION for initial yield stress B (Hockett-Sherby approach)	
FUNCM	ID of a *DEFINE_FUNCTION for saturation rate M (Hockett-Sherby approach)	
TCVUP	Upper temperature for determination of average cooling velocity	
TCVLO	Lower temperature for determination of average cooling velocity	
CVCRIT	Critical cooling velocity. If the average cooling velocity is smaller or equal CVCRIT, the cooling rate at TCVSL is used.	
TCVSL	Temperature for determination of cooling velocity for small cooling velocities.	
EPSP	Plastic strain in Hockett-Sherby approach	
EXPON	Exponent in Hockett-Sherby approach	



**Remarks:**

1. **Start Temperatures.** Start temperatures for ferrite, pearlite, bainite, and martensite can be defined manually via FS, PS, BS, and MS. Or they are initially defined using the following composition equations:

$$F_S = 273.15 + 912 - 203 \times \sqrt{C} - 15.2 \times \text{Ni} + 44.7 \times \text{Si} + 104 \times \text{V} + 31.5 \times \text{Mo} \\ + 13.1 \times \text{W} - 30 \times \text{Mn} - 11 \times \text{Cr} - 20 \times \text{Cu} + 700 \times \text{P} + 400 \times \text{Al} \\ + 120 \times \text{As} + 400$$

$$P_S = 273.15 + 723 - 10.7 \times \text{Mn} - 16.9 \times \text{Ni} + 29 \times \text{Si} + 16.9 \times \text{Cr} + 290 \times \text{As} \\ + 6.4 \times \text{W}$$

$$B_S = 273.15 + 637 - 58 \times \text{C} - 35 \times \text{Mn} - 15 \times \text{Ni} - 34 \times \text{Cr} - 41 \times \text{Mo}$$

$$M_S = 273.15 + 539 - 423 \times \text{C} - 30.4 \times \text{Mn} - 17.7 \times \text{Ni} - 12.1 \times \text{Cr} - 7.5 \times \text{Mo} \\ + 10 \times \text{Co} - 7.5 \times \text{Si}$$

2. **Martensite Phase Evolution.** Martensite phase evolution according to Lee et al. [2008, 2010] if  $\text{PSI}_M > 0$ :

$$\frac{d\tilde{\zeta}_m}{dT} = \alpha_m (M_S - T)^n \tilde{\zeta}_m^{\varphi_m} (1 - \tilde{\zeta}_m)^{\psi_m}$$

Martensite phase evolution according to Lee et al. [2008, 2010] with extension by Hippchen et al. [2013] if  $\text{PSI}_M < 0$ :

$$\frac{d\tilde{\zeta}_m}{dT} = \alpha_m (M_S - T)^n \tilde{\zeta}_m^{\varphi_m} (1 - \tilde{\zeta}_m)^{\psi_m(2-\tilde{\zeta}_a)}$$

3. **Phase Change Kinetics for Ferrite, Pearlite and Bainite.**

$$\frac{d\tilde{\zeta}_f}{dt} = 2^{\omega_f G} \frac{\exp\left(-\frac{Q_f}{RT}\right)}{C_f} (F_S - T)^3 \frac{\tilde{\zeta}_f^{\varphi_f(1-\tilde{\zeta}_f)} (1 - \tilde{\zeta}_f)^{\psi_f \tilde{\zeta}_f}}{\exp(C_{r,f} \tilde{\zeta}_f^2)}$$

$$\text{for } F_S \geq T \geq (P_S - T_{\text{off},f})$$

$$\frac{d\tilde{\zeta}_p}{dt} = 2^{\omega_p G} \frac{\exp\left(-\frac{Q_p}{RT}\right)}{C_p} (P_S - T)^3 \frac{\tilde{\zeta}_p^{\varphi_p(1-\tilde{\zeta}_p)} (1 - \tilde{\zeta}_p)^{\psi_p \tilde{\zeta}_p}}{\exp(C_{r,p} \tilde{\zeta}_p^2)}$$

$$\text{for } P_S \geq T \geq (B_S - T_{\text{off},p})$$

$$\frac{d\tilde{\zeta}_b}{dt} = 2^{\omega_b G} \frac{\exp\left(-\frac{Q_b}{RT}\right)}{C_b} (B_S - T)^2 \frac{\tilde{\zeta}_b^{\varphi_b(1-\tilde{\zeta}_b)} (1 - \tilde{\zeta}_b)^{\psi_b \tilde{\zeta}_b}}{\exp(C_{r,b} \tilde{\zeta}_b^2)}$$

$$\text{for } M_S \geq T \geq (M_S - T_{\text{off},b})$$

4. **History Variables.** History variables of this material model are listed in the following table. To be able to post-process that data, parameters NEIPS (shells) or NEIPH (solids) have to be defined on \*DATABASE\_EXTENT\_BINARY.

History Variable	Description
1	Amount austenite
2	Amount ferrite
3	Amount pearlite
4	Amount bainite
5	Amount martensite
6	Vickers hardness
7	Yield stress
8	ASTM grain size number
9	Young's modulus
10	Saturation stress A (H-S approach)
11	Initial yield stress B (H-S approach)
12	Saturation rate M (H-S approach)
13	Yield stress of H-S approach $\sigma_y = A - (A - B) \bullet e^{-M \bullet EPSP^{EXPON}}$
17	Temperature rate
19	Current temperature
25	Plastic strain rate
26	Effective thermal expansion coefficient

5. **Choosing/Excluding Phases.** To exclude a phase from the simulation, set the PHASE parameter accordingly.
6. **Strain Rate Effects.** Note that both strain rate parameters (STRC and STRP) must be set to include the effect. It is possible to use a temperature dependent load curve for both parameters simultaneously or for one parameter keeping the other constant.
7. **Time Units.** TUNIT is time units per hour and is only used for calculating the Vicker Hardness, as default it is assumed that the time unit is seconds. If other

time unit is used, for example milliseconds, then TUNIT must be changed to TUNIT = 3.6 × 10<sup>6</sup>

- 8. **Thermal Speedup Factor.** The thermal speedup factor TSF of \*CONTROL\_THERMAL\_SOLVER is used to scale reaction kinetics and hardness calculations in this material model. On the other hand, strain rate dependent properties (see LCY1 to LCY5 or STRC/STRP) are not scaled by TSF.
- 9. **Re-austenization and Grain Growth with HEAT Option.** When HEAT is activated the re-austenitization and grain growth algorithms are also activated. See MAT\_244 for details.
- 10. **Phase Indexed Tables.** When using a Table ID for describing the Young’s modulus as dependent on the temperature Use \*DEFINE\_TABLE\_2D and set the abscissa value equal to 1 for the austenite YM-curve, equal to 2 for the ferrite YM-curve, equal to 3 for the pearlite YM curve, equal to 4 for the bainite YM-curve and finally equal to 5 for the martensite YM-curve. When using the PHASE option only the curves for the included phases are required, but all five phases may be included. The total YM is calculated by a linear mixture law:

$$YM = YM1 \times PHASE1 + \dots + YM5 \times PHASE5$$

For example:

```
*DEFINE_TABLE_2D
$ The number before curve id:s define which phase the curve
$ will be applied to. 1 = Austenite, 2 = Ferrite, 3 = Pearlite,
$ 4 = Bainite and 5 = Martensite.
      1000      0.0      0.0
              1.0              100
              2.0              200
              3.0              300
              4.0              400
              5.0              500

$
$ Define curves 100 - 500
*DEFINE_CURVE
$ Austenite Temp (K) - YM-Curve (MPa)
      100      0      1.0      1.0
      1300.0      50.E+3
      223.0      210.E+3
```

- 11. **Phase-indexed Latent Heat Table.** A Table ID may be specified for the Latent heat (LAT1) to describe each phase change individually. Use \*DEFINE\_TABLE\_2D and set the abscissa values to the corresponding phase transition number. That is, 2 for the Austenite – Ferrite, 3 for the Austenite – Pearlite, 4 for the Austenite – Bainite and 5 for the Austenite – Martensite. See Remark 7 for an example of a correct table definition. If a curve is missing, the corresponding latent heat for that transition will be set to zero. Also, when a table is used the LAT2 is ignored. If HEAT.GT.0 you also have the option to include latent heat for the transition back to Austenite. This latent heat curve is marked as 1 in the table definition of LAT1.

12. **Phase-indexed Thermal Expansion Table.** Tables are supported for defining different thermal expansion properties for each phase. The input is identical to the above table definitions. The Table must have the abscissa values between 1 and 5 where the number correspond to phase 1 to 5. To exclude one phase from influencing the thermal expansion you simply input a curve that is zero for that phase or even easier, exclude that phase number in the table definition. For example to exclude the bainite phase you only define the table with curves for the indices 1, 2, 3 and 5.
  
13. **Phase-indexed Transformation Induced Strain Properties.** Transformation induced strains can be define with a table TABRHO, where densities are defined as functions of phase (table abscissas) and temperature (load curves).

**\*MAT\_REINFORCED\_THERMOPLASTIC**

This is material type 249. This material model describes a reinforced thermoplastic composite material. The reinforcement is defined as an anisotropic hyper-elastic material with up to three distinguished fiber directions. It can be used to model unidirectional layers as well as woven and non-crimped fabrics. The matrix is modeled with a simple thermal elasto-plastic material formulation. For a composite an additive composition of fiber and matrix stresses is used.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EM	LCEM	PRM	LCPRM	LCSIGY	BETA
Type	A8	F	F	I	F	I	I	F

Card 2	1	2	3	4	5	6	7	8
Variable	NFIB	AOPT	XP	YP	ZP	A1	A2	A3
Type	I	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	MANGL	THICK
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	IDF1	ALPH1	EF1	LCEF1	G23_1	G31_1		
Type	I	F	F	I	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	G12	LCG12	ALOC12	GLOC12	METH12			
Type	F	I	F	F	I			

Card 6	1	2	3	4	5	6	7	8
Variable	IDF2	ALPH2	EF2	LCEF2	G23_2	G31_2		
Type	I	F	F	I	F	F		

Card 7	1	2	3	4	5	6	7	8
Variable	G23	LCG23	ALOC23	GLOC23	METH23			
Type	F	I	F	F	I			

Card 8	1	2	3	4	5	6	7	8
Variable	IDF3	ALPH3	EF3	LCEF3	G23_3	G31_3		
Type	I	F	F	I	F	F		

**The following card is optional**

Card 9	1	2	3	4	5	6	7	8
Variable	POSTV							
Type	I							

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label not exceeding 8 characters must be specified.

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RO	Density.
EM	Young's modulus of matrix material.
LCEM	Curve ID for Young's modulus of matrix material versus temperature. With this option active, EM is ignored.
PR	Poisson's ratio for matrix material
LCPR	Curve ID for Poisson's ratio of matrix material versus temperature. With this option active, PR is ignored.
LCSIGY	Load curve or table ID for strain hardening of the matrix. <u>IF LCSIGY refers to a curve</u> Input yield stress versus effective plastic strain. <u>IF LCSIGY refers to a table:</u> Input temperatures as table values and hardening curves as targets for those temperatures (see *DEFINE_TABLE)
BETA	Parameter for mixed hardening. Set $\beta = 0$ for pure kinematic hardening and $\beta = 1$ for pure isotropic hardening.
NFIB	Number of fiber families to be considered.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle MANGL.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, MANGL, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</p>
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.
MANGL	Material angle in degrees for AOPT = 0 and 3, may be overwritten on the element card, see *ELEMENT_SHELL_BETA.
THICK	<p>Balance thickness changes of the material due to the matrix description by scaling fiber stresses</p> <p>EQ.0: No scaling</p> <p>EQ.1: Scaling</p>
IDF $i$	ID for $i$ -th fiber family for post-processing
ALPH $i$	Orientation angle $\alpha_i$ for $i$ -th fiber with respect to overall material direction
EF $i$	Young's modulus for $i$ -th fiber family



VARIABLE	DESCRIPTION
LCEFi	Curve ID for stress versus fiber strain of $i$ -th fiber. With this option active, EFi is ignored.
G23 <sub><math>i</math></sub>	Transversal shear modulus orthogonal to direction of fiber $i$
G31 <sub><math>i</math></sub>	Transversal shear modulus in direction of fiber $i$
G <sub><math>ij</math></sub>	Linear shear modulus for shearing between fiber $i$ and $j$
LCG <sub><math>ij</math></sub>	Curve ID for shear stress versus shearing between of $i$ -th and $j$ -th fiber. With this option active, G <sub><math>ij</math></sub> is ignored. For details see parameter METH <sub><math>ij</math></sub> .
ALOC <sub><math>ij</math></sub>	Locking angle (in radians) for shear between fiber families $i$ and $j$
GLOC <sub><math>ij</math></sub>	Linear shear modulus for shear angles larger than ALOC <sub><math>ij</math></sub>
METH <sub><math>ij</math></sub>	Option for shear between fiber $i$ and $j$ (see remarks below): <ul style="list-style-type: none"> <li>EQ.0: Elastic shear response, curve LCG<sub><math>ij</math></sub> defines shear stress vs. scalar product of fibers directions.</li> <li>EQ.1 Elasto-plastic shear response, curve LCG<sub><math>ij</math></sub> defines yield shear stress vs. normalized scalar product of fiber directions.</li> <li>EQ.2: Elastic shear response, curve LCG<sub><math>ij</math></sub> defines shear stress vs. shear angle between fibers given in rad.</li> <li>EQ.3: Elasto-plastic shear response, curve LCG<sub><math>ij</math></sub> defines yield shear stress vs. normalized shear angle between fibers.</li> <li>EQ.4: Elastic shear response, curve LCG<sub><math>ij</math></sub> defines shear stress vs. shear angle between fibers given in rad. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching.</li> <li>EQ.5: Elasto-plastic shear response, curve LCG<sub><math>ij</math></sub> defines yield shear stress vs. normalized shear angle between fibers. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching.</li> <li>EQ.10: Elastic shear response, curve LCG<sub><math>ij</math></sub> defines shear stress vs. shear angle between fibers given in rad. This option is tailored for woven fabrics and guarantees a pure shear stress response.</li> </ul>

VARIABLE	DESCRIPTION
	EQ.11: Elasto-plastic shear response, curve LCG <sub>ij</sub> defines yield shear stress vs. normalized shear angle. This option is tailored for woven fabrics and guarantees a pure shear stress response
POSTV	Defines additional history variables that might be useful for post-processing. See remarks below for details.

### Stress calculation:

This material features an additive split of matrix and reinforcement contributions, i.e. the combined stress response  $\sigma$  equals the sum  $\sigma^m + \sigma^f$ . The matrix mechanics is described by an elasto-plastic material formulation with a von-Mises yield criterion.

The contribution of the reinforcement is formulated as a hyperelastic material. Based on the orientation angle  $\alpha_i$  of the  $i$ -th fiber family an initial fiber direction  $\mathbf{m}_i^0$  is computed. By using the deformation gradient  $\mathbf{F}$  the current fiber configuration is defined as  $\mathbf{m}_i = \mathbf{F} \mathbf{m}_i^0$  containing all necessary information on fiber strain and reorientation.

Following standard textbook mechanics for anisotropic and hyperelastic materials, the elastic stresses within the fibers due to tension or compression are given as

$$\sigma_T^f = \sum_{i=1}^n \frac{1}{J} f_i(\lambda_i) (\mathbf{m}_i \otimes \mathbf{m}_i),$$

where the function  $f_i$  of the fiber strain  $\lambda_i$  corresponds to the load curve LCEF<sub>i</sub>.

The shear behavior of the reinforcement can be controlled by METH<sub>ij</sub>. For values less than 10, the behavior is again standard textbook mechanics:

$$\sigma_S^f = \sum_{i=1}^2 \frac{1}{J} g_{i,i+1}(\kappa_{i,i+1}) (\mathbf{m}_i \otimes \mathbf{m}_{i+1})$$

Where  $\kappa_{i,i+1}$  represents the employed shear measure (scalar product or shear angle in rad). In general, the dyadic product  $\mathbf{m}_i \otimes \mathbf{m}_{i+1}$  does not define a shear stress tensor. This might result in unphysical shear behavior in case of woven fabrics. Therefore, METH<sub>ij</sub> = 10 or 11 have been devised such that a pure shear stress tensor  $\sigma_S^f$  is always obtained.

For even values of METH<sub>ij</sub>, an elastic shear response is assumed. If defined, the load curve LCG<sub>ij</sub> corresponds to function  $g_{i,j}$ . In this case the values of G<sub>ij</sub>, ALOC<sub>ij</sub> and GLOC<sub>ij</sub> are ignored.

For odd values of METH<sub>ij</sub> on the other hand, an elasto-plastic shear behavior is assumed and the load curve LCG<sub>ij</sub> defines the yield stress value as function of a **normalized** shear parameter. This implies that the load curve has to be defined for abscissa values between

0.0 and 1.0. A first elastic regime, which is controlled by the linear shear stiffness  $G_{ij}$ , is assumed until the yield stress given in the load curve for normalized shear value 0.0 is reached. A second linear elastic regime is defined for shear angles ( $\xi_{ij}$ ) / fiber angles ( $\eta_{ij}$ ) larger than the locking angle  $ALOC_{ij}$ . The corresponding stiffness in that regime is  $GLOC_{ij}$ . At the transition point to the second elastic regime, the shear stress corresponds to the load curve value for a normalized shear of 1.0.

### History data:

This material formulation outputs additional data for post-processing to the set of history variables if requested by the user. The parameter POSTV defines the data to be written. Its value is calculated as

$$POSTV = a_1 + 2 a_2 + 4 a_3 + 8 a_4 + 16 a_5 + 32 a_6.$$

Each flag  $a_i$  is a binary (can be either 1 or 0) and corresponds to one particular post-processing variable according to the following table.

Flag	Description	Variables	# hist
$a_1$	Fiber angle	$\eta_{12}, \eta_{23}$	2
$a_2$	Fiber ID	IDF1, IDF2, IDF3	3
$a_3$	Fiber strain	$\lambda_1, \lambda_2, \lambda_3$	3
$a_4$	Fiber direction	$\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3$	9
$a_5$	Individual fiber stresses	$f_1(\lambda_1), f_2(\lambda_2), f_3(\lambda_3)$	3
$a_6$	Fiber stress tensor	$\sigma_{11}^f, \sigma_{22}^f, \sigma_{33}^f, \sigma_{12}^f, \sigma_{23}^f, \sigma_{31}^f$	6

The above table also shows the order of output as well as the number of extra history variables associated with the particular flag. In total NXH extra variables are required depending on the choice of parameter POSTV. For example, the maximum number of additional variables is  $NXH = 26$  for  $POSTV = 63$ .

A more complete list of potentially helpful history variables are given in the following table. In this table, "Position" refers to the history variable number as listed by LS-PrePost when post-processing the d3plot database. The variable NEIPS in \*DATABASE\_EXTENT\_BINARY must to set to output these history variables.

Position	Description
3	Number of Fibers
4	NXH
5→NXH+4	Variables as described in preceding table
NXH+5, NXH+6	Shear angles $\zeta_{12}$ and $\zeta_{23}$
NXH+7 → NXH+12	Matrix stress tensor in element coordinate system
NXH+13 → NXH+21	Deformation gradient

## **\*MAT\_REINFORCED\_THERMOPLASTIC\_UDFIBER \*MAT\_249\_UDFIBER**

### **\*MAT\_REINFORCED\_THERMOPLASTIC\_UDFIBER**

This is material type 249. It describes a material with unidirectional fiber reinforcements and considers up to three distinguished fiber directions. Each fiber family is described by a spatially transversely isotropic neo-Hookean constitutive law. The implementation is based on an adapted version of the material described by Bonet and Burton (1998). The material is only available for thin shell elements and in explicit simulations.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EM	PRM	G	EZDEF		
Type	A8	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	NFIB	AOPT	XP	YP	ZP	A1	A2	A3
Type	I	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	MANGL	
Type	F	F	F	F	F	F	F	

Card 4	1	2	3	4	5	6	7	8
Variable	IDF1	ALPH1	EF1	KAP1				
Type	I	F	F	F				

**\*MAT\_249\_UDFIBER \*MAT\_REINFORCED\_THERMOPLASTIC\_UDFIBER**

Card 5	1	2	3	4	5	6	7	8
Variable	IDF2	ALPH2	EF2	KAP2				
Type	I	F	F	F				

Card 6	1	2	3	4	5	6	7	8
Variable	IDF3	ALPH3	EF3	KAP3				
Type	I	F	F	F				

**VARIABLE**

**DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Density.
EM	Isotropic young's modulus $E_{iso}$ .
PR	Poisson's ratio $\nu$ .
G	Linear shear modulus $G_{fib}$ .
EZDEF	Algorithmic parameter. If set to 1, last row of deformation gradient is not updated during the calculation.
NFIB	Number of fiber families to be considered.

## **\*MAT\_REINFORCED\_THERMOPLASTIC\_UDFIBER \*MAT\_249\_UDFIBER**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle MANGL.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0:: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, MANGL, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</p>
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2.
MANGL	Material angle in degrees for AOPT = 0 and 3, may be overwritten on the element card, see *ELEMENT_SHELL_BETA.
IDF $_i$	ID for $i$ -th fiber family for post-processing.
ALPH $_i$	Orientation angle $\alpha_i$ for $i$ -th fiber with respect to overall material direction
EF $_i$	Young's modulus $E_i$ for $i$ -th fiber family.
KAP $_i$	Fiber volume ratio $\kappa_i$ of $i$ -th fiber family.

**Stress calculation:**

In this model up to three distinguished fiber families are considered. It is assumed that there is no interaction between the families and, thus, that the resulting stress tensor is given by the sum of the single fiber responses, each to be calculated as the sum of an isotropic and a spatially transversely isotropic neo-Hookean stress contribution,  $\sigma_i^{\text{iso}}$  and  $\sigma_i^{\text{tr}}$ , respectively. The implementation is based on the work of Bonet and Burton (1998), adapted by BMW for simulation of unidirectional fabrics, see references below.

In order to determine the isotropic stress tensor  $\sigma_i^{\text{iso}}$ , an isotropic shear modulus  $\mu$  and an isotropic bulk modulus  $\lambda_i$  have to be defined from the input values as:

$$\mu = \frac{E_{\text{iso}}}{2(1 + \nu)} \quad \text{and} \quad \lambda_i = \frac{E_{\text{iso}}(\nu + n_i \nu^2)}{2(1 + \nu)}.$$

Here, the variable  $n_i$  denotes the ratio between stiffness orthogonally to the fibers and in fiber direction, i.e.  $n_i = E_{\text{iso}}/E_i$ . Using the left Cauchy-Green tensor  $\mathbf{b}$  the isotropic neo-Hookean model reads:

$$\sigma_i^{\text{iso}} = \frac{\mu}{J} (\mathbf{b} - \mathbf{I}) + \lambda_i (J - 1) \mathbf{I}.$$

Based on the orientation angel  $\alpha_i$  of the  $i$ -th fiber family an initial fiber direction  $\mathbf{m}_i^0$  is computed. The deformation gradient  $\mathbf{F}$  is used to define the current fiber configuration as  $\mathbf{m}_i = \mathbf{F} \mathbf{m}_i^0$ . This vector contains all necessary information on fiber elongation and reorientation.

The spatially transversely isotropic neo-Hookean formulation is given by:

$$J \sigma_i^{\text{tr}} = 2\beta_i (I_4 - 1) \mathbf{I} + 2(\alpha + 2\beta_i \ln J + 2\gamma_i (I_4 - 1)) \mathbf{m}_i \otimes \mathbf{m}_i - \alpha (\mathbf{b} \mathbf{m}_i \otimes \mathbf{m}_i + \mathbf{m}_i \otimes \mathbf{b} \mathbf{m}_i)$$

with material parameters

$$\alpha = \mu - G_{\text{fib}}, \quad \beta_i = \frac{E_{\text{iso}} \nu^2 (1 - n_i)}{4m_i (1 + \nu)}, \quad m_i = 1 - \nu - 2n_i \nu^2,$$

$$\gamma_i = \frac{E_i \kappa_i (1 - \nu)}{8m} - \frac{\lambda_i + 2\mu}{8} + \frac{\alpha}{2} - \beta_i.$$



## **\*MAT\_REINFORCED\_THERMOPLASTIC\_UDFIBER \*MAT\_249\_UDFIBER**

The parameter EZDEF activates a modification of the model. Instead of the standard deformation gradient  $\mathbf{F}$ , a modified tensor  $\tilde{\mathbf{F}}$  is employed to calculate current fiber directions  $\mathbf{m}_i$  and left Cauchy-Green tensor  $\mathbf{b}$ . In tensor  $\tilde{\mathbf{F}}$  only the first two rows of the deformation gradient are updated based on the deformation of the element. This simplification can in some cases increase the stability of the model especially if the structure undergoes large deformations.

### **References:**

- Bonet, J., and A. J. Burton. "A simple orthotropic, transversely isotropic hyperelastic constitutive equation for large strain computations." *Computer methods in applied mechanics and engineering* 162.1 (1998): 151-164.
- Senner, T., et al. "A modular modeling approach for describing the in-plane forming behavior of unidirectional non-crimp-fabrics." *Production Engineering* 8.5 (2014): 635-643.
- Senner, T., et al. "Bending of unidirectional non-crimp-fabrics: experimental characterization, constitutive modeling and application in finite element simulation." *Production Engineering* 9.1 (2015): 1-10.

### **History data:**

Position	Description
3	ID of 1 <sup>st</sup> fiber
4	ID of 2 <sup>nd</sup> fiber
5	ID of 3 <sup>rd</sup> fiber
6 → 8	Current direction of 1 <sup>st</sup> fiber
9 → 11	Current direction of 2 <sup>nd</sup> fiber
12 → 14	Current direction of 3 <sup>rd</sup> fiber
15	Number of fibers
16	Projected orthogonal fiber strain (1 <sup>st</sup> fiber)
17	Projected parallel fiber strain (1 <sup>st</sup> fiber)
18	Shear angle (1 <sup>st</sup> fiber) in rad
19	Euler-Almansi strain (1 <sup>st</sup> fiber)
20	Porosity (1 <sup>st</sup> fiber)
21	Fiber volume ratio (1 <sup>st</sup> fiber)
22	Projected orthogonal fiber strain (2 <sup>nd</sup> fiber)

## **\*MAT\_249\_UDFIBER \*MAT\_REINFORCED\_THERMOPLASTIC\_UDFIBER**

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23	Projected parallel fiber strain (2 <sup>nd</sup> fiber)
24	Shear angle (2 <sup>nd</sup> fiber) in rad
25	Euler-Almansi strain (2 <sup>nd</sup> fiber)
26	Porosity (2 <sup>nd</sup> fiber)
27	Fiber volume ratio (2 <sup>nd</sup> fiber)
28	Projected orthogonal fiber strain (3 <sup>rd</sup> fiber)
29	Projected parallel fiber strain (3 <sup>rd</sup> fiber)
30	Shear angle (3 <sup>rd</sup> fiber) in rad
31	Euler-Almansi strain (3 <sup>rd</sup> fiber)
32	Porosity (3 <sup>rd</sup> fiber)
33	Fiber volume ratio (3 <sup>rd</sup> fiber)

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**\*MAT\_251****\*MAT\_TAILORED\_PROPERTIES**

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's modulus.
PR	Poisson's ratio.
FAIL	<p>Failure flag.</p> <p>LT.0.0: User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure</p> <p>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.</p> <p>GT.0.0: Effective plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.</p>
TDEL	Minimum time step size for automatic element deletion.
LCSS	Load curve ID or Table ID (see full description of MAT_024). Load curve for stress vs. plastic strain. 2-D table for stress vs. plastic strain as a function of strain rates. 3-D table for stress vs. plastic strain as a function of strain rates as a function of history variable values (see HISVN).
VP	<p>Formulation for rate effects:</p> <p>EQ.0.0: Scale yield stress (default),</p> <p>EQ.1.0: Viscoplastic formulation.</p>

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
HISVN	Location of history variable in the history array of *INITIAL_STRESS_SHELL that is used to evaluate the 3-D table LCSS.
PHASE	Constant value to evaluate the 3-D table LCSS. Only used if HISVN = 0.
EPS1 - EPS8	Effective plastic strain values (optional). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8.

**Remarks:**

If the 3-D table is used for LCSS, interpolation is used to find the corresponding stress value for the current plastic strain, strain rate, and history variable. In addition, extrapolation is used for the history variable evaluation, which means that some upper and lower "limit curves" have to be used, if extrapolation is not desired.

If material history is written to dynain file using \*INTERFACE\_SPRINGBACK\_LSDYNA, the history variable of material 251 (e.g. hardness, temperature, ...) is written to position HISV6 of \*INITIAL\_STRESS\_SHELL.

It is recommended to set HISVN = 6 and to put the history variable on position HISV6 if \*MAT\_251 is used in combination with \*MAT\_ADD\_...

**\*MAT\_TOUGHENED\_ADHESIVE\_POLYMER**

This is Material Type 252, the Toughened Adhesive Polymer model (TAPO). It is based on non-associated  $I_1 - J_2$  plasticity constitutive equations and was specifically developed to represent the mechanical behaviour of crash optimized high-strength adhesives under combined shear and tensile loading. This model includes material softening due to damage, rate-dependency, and a constitutive description for the mechanical behaviour of bonded connections under compression.

A detailed description of this material can be found in Matzenmiller and Burbulla [2013]. This material model can be used with solid elements or with cohesive elements in combination with \*MAT\_ADD\_COHESIVE.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	FLG	JCFL	DOPT	
Type	A8	F	F	F	I	I	I	

Card 2	1	2	3	4	5	6	7	8
Variable	LCSS	TAU0	Q	B	H	C	GAM0	GAMM
Type	I	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	A10	A20	A1H	A2H	A2S	POW		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable			D1	D2	D3	D4	D1C	D2C
Type			F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density $\rho$ .
E	Young's modulus $E$ .
PR	Poisson's ratio $\nu$ .
FLG	Flag to choose between yield functions $f$ and $\hat{f}$ , see Remarks. EQ.0.0: Cap in tension. and <i>Drucker &amp; Prager</i> in compression, EQ.2.0: Cap in tension. and <i>von Mises</i> in compression.
JCFL	Johnson & Cook constitutive failure criterion flag, see Remarks. EQ.0.0: use triaxiality factor only in tension, EQ.1.0: use triaxiality factor in tension and compression.
DOPT	Damage criterion flag $\hat{D}$ or $\check{D}$ , see Remarks. EQ.0.0: damage model uses damage plastic strain $r$ , EQ.1.0: damage model uses plastic arc length $\gamma_v$ .
LCSS	Curve ID or Table ID.  <u>If LCSS is a curve ID:</u> The curve specifies yield stress $\tau_Y$ as a function of plastic strain $r$ .  <u>If LCSS is a Table ID:</u> For each strain rate value the table specifies a curve ID giving the yield stress versus plastic strain for that strain rate (see DEFINE_TABLE) or it defines for each temperature value a table ID which, in turn, maps strain rates to curves giving the yield stress as a function of plastic strain (see DEFINE_TABLE_3D).  The yield stress versus plastic strain curve for the lowest value of strain rate or temperature is used when the strain rate or temperature falls below the minimum value. Likewise, maximum values cannot be exceeded. Hardening variables are ignored with this option (TAU0, Q, B, H, C, GAM0, and GAMM).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TAU0	Initial shear yield stress $\tau_0$ .
Q	Isotropic nonlinear hardening modulus $q$ .
B	Isotropic exponential decay parameter $b$ .
H	Isotropic linear hardening modulus $H$ .
C	Strain rate coefficient $C$ .
GAM0	Quasi-static threshold strain rate $\gamma_0$ .
GAMM	Maximum threshold strain rate $\gamma_m$ .
A10	Yield function parameter: initial value $a_{10}$ of $a_1 = \hat{a}_1(r)$ .
A20	Yield function parameter: initial value $a_{20}$ of $a_2 = \hat{a}_2(r)$ .
A1H	Yield function parameter $a_1^H$ for formative hardening (ignored if FLG.EQ.2).
A2H	Yield function parameter $a_2^H$ for formative hardening (ignored if FLG.EQ.2).
A2S	Plastic potential parameter $a_2^*$ for hydrostatic stress term.
POW	Exponent $n$ of the phenomenological damage model.
D1	Johnson & Cook failure parameter $d_1$ .
D2	Johnson & Cook failure parameter $d_2$ .
D3	Johnson & Cook failure parameter $d_3$ .
D4	Johnson & Cook rate dependent failure parameter $d_4$ .
D1C	Johnson & Cook damage threshold parameter $d_{1c}$ .
D2C	Johnson & Cook damage threshold parameter $d_{2c}$ .

**Remarks:**

Two different  $I_1$ - $J_2$  yield criteria for isotropic plasticity can be defined by parameter FLG:

1. FLG = 0 is used for the yield criterion  $f$  which is changed at the case of hydrostatic pressure  $I_1 = 0$  into the *Drucker & Prager* model (DP)



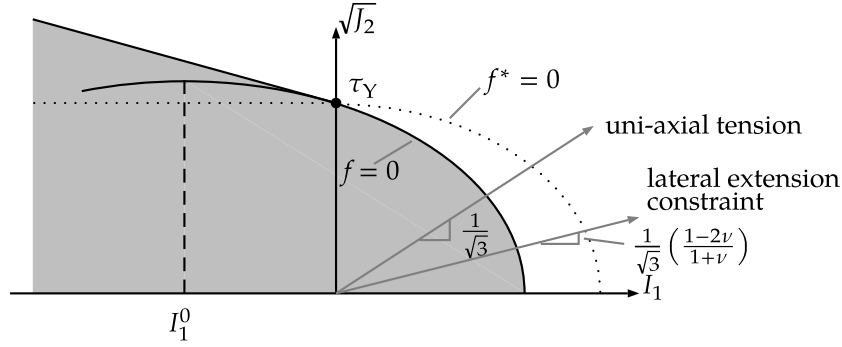


Figure M252-1. Yield function  $f$  and plastic flow potential  $f^*$

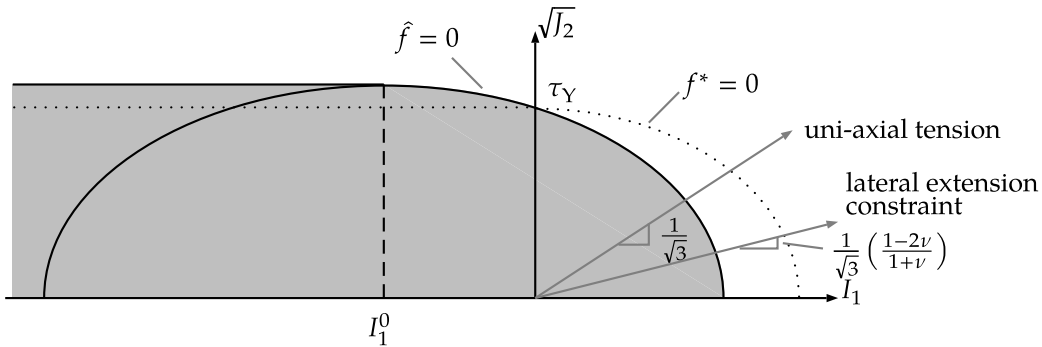


Figure M252-2. Yield function  $\hat{f}$  and plastic flow potential  $f^*$

$$f := \frac{J_2}{(1-D)^2} + \frac{1}{\sqrt{3}} a_1 \tau_0 \frac{I_1}{1-D} + \frac{a_2}{3} \left\langle \frac{I_1}{1-D} \right\rangle^2 - \tau_Y^2 = 0$$

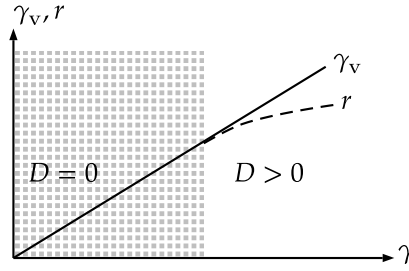
with the *Maccauley* bracket  $\langle \bullet \rangle$ , the first invariant of the stress tensor  $I_1 = \text{tr } \sigma$ , and the second invariant of the stress deviator  $J_2 = (1/2)\text{tr}(\mathbf{s})^2$ , see Figure M252-1.

2. FLG = 2 is used for the yield criterion  $\hat{f}$  which is changed at the vertex into the deviatoric *von Mises* yield function – see Figure M252-2 – and is used for conservative calculation in case of missing uniaxial compression or combined compression and shear experiments:

$$\hat{f} := \frac{J_2}{(1-D)^2} + \frac{a_2}{3} \left\langle \frac{I_1}{1-D} + \frac{\sqrt{3} a_1 \tau_0}{2 a_2} \right\rangle^2 - \left( \tau_Y^2 + \frac{a_1^2 \tau_0^2}{4 a_2} \right) = 0$$

The yield functions  $f$  and  $\hat{f}$  are formulated in terms of the effective stress tensor  $\tilde{\sigma} = \sigma / (1 - D)$  and the isotropic material damage  $D$  according to the continuum damage mechanics in Lemaitre [1992]. The stress tensor  $\sigma$  is defined in terms of the elastic strain  $\epsilon^e$  and the isotropic damage  $D$ :

$$\sigma = (1 - D) C \epsilon^e$$



**Figure M252-3.** Accumulated plastic strain  $\gamma_v$  and damage plastic strain  $r$  versus strain  $\gamma$

The continuity  $(1 - D)$  in the elastic constitutive equation above degrades the fourth order elastic stiffness tensor  $\mathbb{C}$ ,

$$\mathbb{C} = 2G \left( \mathbb{I} - \frac{1}{3} \mathbf{1} \otimes \mathbf{1} \right) + K \mathbf{1} \otimes \mathbf{1}$$

with shear modulus  $G$ , bulk modulus  $K$ , fourth order identity tensor  $\mathbb{I}$ , and second order identity tensor  $\mathbf{1}$ . The plastic strain rate  $\dot{\epsilon}^P$  is given by the non-associated flow rule

$$\dot{\epsilon}^P = \lambda \frac{\partial f^*}{\partial \boldsymbol{\sigma}} = \frac{\lambda}{(1-D)^2} \left( \mathbf{s} + \frac{2}{3} a_2^* \langle I_1 \rangle \mathbf{1} \right)$$

with the potential  $f^*$  and an additional parameter  $a_2^* < a_2$  to reduce plastic dilatancy.

$$f^* := \frac{J_2}{(1-D)^2} + \frac{a_2^*}{3} \left\langle \frac{I_1}{1-D} \right\rangle^2 - \tau_Y^2$$

The plastic arc length  $\dot{\gamma}_v$  characterizes the inelastic response of the material and is defined by the Euclidean norm:

$$\dot{\gamma}_v := \sqrt{2 \operatorname{tr}(\dot{\epsilon}^P)^2} = \frac{2\lambda}{(1-D)^2} \sqrt{J_2 + \frac{2}{3} (a_2^* \langle I_1 \rangle)^2}$$

In addition, the arc length of the damage plastic strain rate  $\dot{r}$  is introduced by means of the arc length  $\dot{\gamma}_v$  and the continuity  $(1 - D)$  as in Lemaitre [1992], where  $\tilde{I}_1 = I_1/(1 - D)$  and  $\tilde{J}_2 = J_2/(1 - D)^2$  are the effective stress invariants, see [Figure M252-3](#).

$$\dot{r} := (1 - D) \dot{\gamma}_v = 2\lambda \sqrt{\tilde{J}_2 + \frac{2}{3} (a_2^* \langle \tilde{I}_1 \rangle)^2}$$

The rate-dependent yield strength for shear  $\tau_Y$  can be defined by two alternative expressions. The first representation is an analytic expression for  $\tau_Y$ :

$$\tau_Y = (\tau_0 + R) \left[ 1 + C \left( \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_0} \right\rangle - \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_m} \right\rangle \right) \right], \text{ with } \dot{\gamma} = \sqrt{2 \operatorname{tr}(\dot{\epsilon})^2}$$

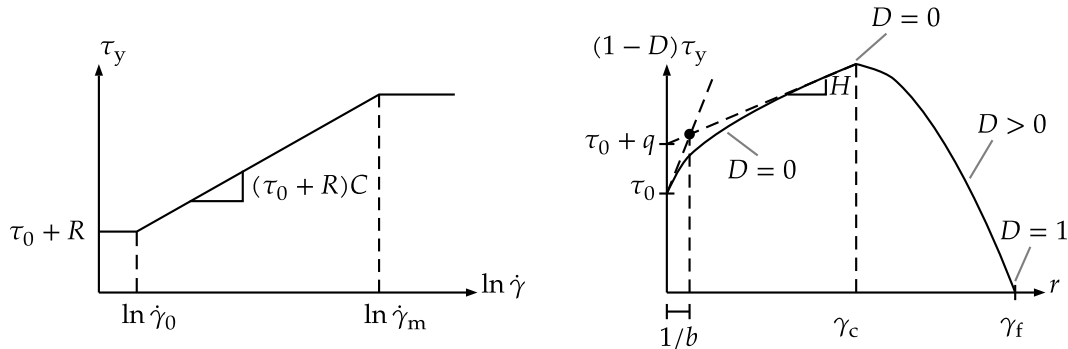
where the first factor  $(\tau_0 + R)$  in  $\tau_Y$  is given by the static yield strength with the initial yield  $\tau_0$  and the non-linear hardening contribution

$$R = q[1 - \exp(-br)] + Hr$$

The second factor [...] in  $\tau_Y$  describes the rate dependency of the yield strength by a modified Johnson & Cook approach with the reference strain rates  $\dot{\gamma}_0$  and  $\dot{\gamma}_m$  which limit the shear strength  $\tau_Y$ , see [Figure M252-4](#).

The second representation of the yield strength  $\tau_Y$  is the table definition LCSS, where hardening can be defined as a function of plastic strain, strain rate, and temperature.

Toughened structural adhesives show distortional hardening under plastic flow, i.e. the



**Figure M252-4.** Rate-dependent tensile strength  $\tau_Y$  versus effective strain rate  $\dot{\gamma}$  (left) and effective damage plastic strain  $r$  (right)

yield surface changes its shape. This formative hardening can be phenomenological described by simple evolution equations of parameters  $a_1 = \hat{a}_1(r) \wedge a_2 = \hat{a}_2(r)$  in the yield criterions  $f$  with the initial values  $a_{10}$  and  $a_{20}$ :

$$a_1 = \hat{a}_1(r) \wedge \dot{a}_1 = a_1^H \dot{r}$$

$$a_2 = \hat{a}_2(r) \wedge a_2 \geq 0 \wedge \dot{a}_2 = a_2^H \dot{r}$$

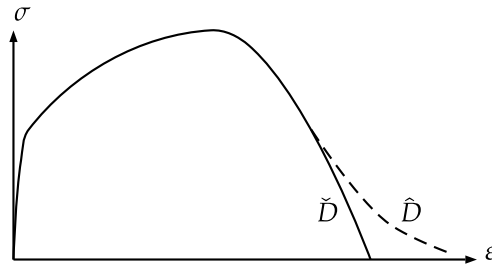
The parameters  $a_1^H$  and  $a_2^H$  can take positive or negative values as long as the inequality  $a_2 \geq 0$  is satisfied. The criterion  $a_2 \geq 0$  ensures an elliptic yield surface. The yield criterion  $\hat{f}$  uses only the initial values  $a_1 = a_{10}$  and  $a_2 = a_{20}$  without the distortional hardening.

The empirical isotropic damage model  $D$  is based on the approach in Lemaitre [1985]. Two different evolution equations  $\dot{D}(r, \dot{r})$  and  $\dot{D}(\dot{\gamma}_v, \dot{\gamma}_v)$  are available, [Figure M252-5](#) see. The damage variable  $D$  is formulated in terms of the damage plastic strain rate  $\dot{r}$  (DOPT = 0)

$$\dot{D} = \hat{D}(r, \dot{r}) = n \left\langle \frac{r - \gamma_c}{\gamma_f - \gamma_c} \right\rangle^{n-1} \frac{\dot{r}}{\gamma_f - \gamma_c}$$

or of the plastic arc length  $\dot{\gamma}_v$  (DOPT = 1)

$$\dot{D} = \check{D}(\dot{\gamma}_v, \dot{\gamma}_v) = n \left\langle \frac{\dot{\gamma}_v - \gamma_c}{\gamma_f - \gamma_c} \right\rangle^{n-1} \frac{\dot{\gamma}_v}{\gamma_f - \gamma_c}$$



**Figure M252-5.** Influence of DOPT on damage softening

where  $r$  in contrast to  $\gamma_v$  increases non-proportionally slowly, see [Figure M252-5](#). The strains at the thresholds  $\gamma_c$  and  $\gamma_f$  for damage initiation and rupture are functions of the triaxiality  $T = \sigma_m / \sigma_{eq}$  with the hydrostatic stress  $\sigma_m = I_1/3$  and the von Mises equivalent stress  $\sigma_{eq} = \sqrt{3}J_2$  as in Johnson and Cook [1985].

$$\gamma_c = [d_{1c} + d_{2c} \exp(-d_3 \langle T \rangle)] \left( 1 + d_4 \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_0} \right\rangle \right)$$

$$\gamma_f = [d_1 + d_2 \exp(-d_3 \langle T \rangle)] \left( 1 + d_4 \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_0} \right\rangle \right)$$

The option JCFL controls the influence of triaxiality  $T = \sigma_m / \sigma_{eq}$  in the pressure range for the thresholds  $\gamma_c$  and  $\gamma_f$ . JCFL = 0 makes use of the Macauley bracket  $\langle T \rangle$  for the triaxiality  $T = \sigma_m / \sigma_{eq}$  and JCFL = 1 omits the Macauley bracket  $\langle T \rangle$ .

**History Variables:**

VARIABLE	DESCRIPTION
1	damage variable $D$
2	plastic arc length $\gamma_v$
3	effective strain rate
4	temperature
5	yield stress
6	damaged yield stress
7	triaxiality



Card 4	1	2	3	4	5	6	7	8
Variable	PTTAB1	PTTAB2	PTTAB3	PTTAB4	PTTAB5			
Type	I	I	I	I	I			
Card 5	1	2	3	4	5	6	7	8
Variable	PTEPS	PTRIP				GRAI		
Type	I	F				F		

**Phase Yield Stress Cards.** For each of the N phases, one parameter SIGY<sub>i</sub> has to be specified. A keyword card (with a "\*" in column 1) terminates this input if less than 10 cards are used.

Optional	1	2	3	4	5	6	7	8
Variable	SIGY1	SIGY2	SIGY3	SIGY4	SIGY5	SIGY6	SIGY7	SIGY8
Type	I	I	I	I	I	I	I	I

**VARIABLE**

**DESCRIPTION**

- MID Material identification. A unique number or label not exceeding 8 characters must be specified.
- RO Mass density  $\rho$ .
- N Number of phases
- YM Youngs' modulus:
  - GT.0.0: constant value is used
  - LT.0.0: LCID or TABID. Temperature dependent Youngs' modulus given by load curve ID = -E or a Table ID = -E. Use TABID to describe temperature dependent modulus for each phase individually.

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
PR	Poisson's modulus: GT.0.0: constant value is used LT.0.0: LCID or TABID. Temperature dependent Poisson's ratio given by load curve ID = -E or a Table ID = -E. Use TABID to describe temperature dependent parameter for each phase individually.
MIX	Load curve ID with initial phase concentrations
MIXR	LCID or TABID for mixture rule. Use a TABID to define a temperature dependency
TASTART	Annealing temperature start
TAEND	Annealing temperature end
CTE	Coefficient of thermal expansion: GT.0.0: constant value is used LT.0.0: LCID or TABID. Temperature dependent CTE given by load curve ID = -CTE or a Table ID = -CTE. Use Table ID to describe temperature dependent CTE for each phase individually.
DTEMP	Maximum temperature variation within a time step. If exceeded during the analysis a local sub-cycling is used
TIME	Number of time units per hour. Default is seconds, that is 3600 time units per hour.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PTLAW	<p>Table ID to define phase transformation model as a function of source phase and target phase. The values in *DEFINE_TABLE are the phase numbers before transformation (source phase). The curves referenced by the table specify transformation model (ordinate) versus phase number after transformation (abscissa).</p> <p>LT.0: Transformation model used in cooling</p> <p>EQ.0: No transformation</p> <p>GT.0: Transformation model is used in heating</p> <p>There are four possible transformation models which can be specified as ordinate values of the curves:</p> <p>EQ.1: Koinstinen-Marburger</p> <p>EQ.2: JMAK</p> <p>EQ.3: Akerstrom (only for cooling)</p> <p>EQ.4: Oddy (only for heating)</p>
PTSTR	<p>Table ID to define start temperatures for the transformations as function of source phase and target phase. The values in *DEFINE_TABLE are the phase numbers before transformation (source phase). The curves referenced by the table specify start temperature (ordinate) versus phase number after transformation (abscissa).</p>
PTEND	<p>Table ID to define end temperatures for the transformations as function of source phase and target phase. The values in *DEFINE_TABLE are the phase numbers before transformation (source phase). The curves referenced by the table specify end temperature (ordinate) versus phase number after transformation (abscissa).</p>
PTXi	<p>Table ID defining the <math>i</math>-th scalar-valued phase transformation parameter as function of source phase and target phase (see description of transformation models below to see which parameters apply). The values in *DEFINE_TABLE are the phase numbers before transformation (source phase). The curves referenced by the table specify scalar parameter (ordinate) versus phase number after transformation (abscissa).</p>



VARIABLE	DESCRIPTION
PTTAB $i$	Table ID of 3D table defining the $i$ -th tabulated phase transformation parameter as function of source phase and target phase (see description of transformation models below to see which parameters apply). The values in *DEFINE_TABLE_3D are the phase numbers before transformation (source phase). The values in the 2D tables referenced by *DEFINE_TABLE_3D are the phase number after transformation. The curves referenced by the 2D tables specify tabulated parameter (ordinate) versus either temperature or temperature rate (abscissa).
PTEPS	Table ID containing transformation induced strains as function of source phase and target phase.
PTRIP	Flag for transformation induced plasticity (TRIP). Algorithm active for positive value of PTRIP.
GRAIN	Initial grain size.

### Remarks:

This material features temperature and phase composition dependent elastic plastic behavior. The phase composition is determined using a list of generic phase transformation mechanisms the user can choose from for each of the possible phase transformations. So far, four different transformation models have been implemented to describe the transition from source phase  $x_a$  to target phase  $x_b$ :

1. Koistinen-Marburger:

This formulation is tailored for non-diffusive transformations. The temperature dependent amount of the target phase is computed as

$$x_b = x_a(1.0 - e^{-\alpha(T_{start}-T)})$$

The factor  $\alpha$  is to be defined in table PTX1.

2. Generalized Johnson-Mehl-Avrami-Kolmogorov (JMAK):

This widely used model employs the evolution equation

$$\frac{dx_b}{dt} = n(T)(k_{ab}x_a - k'_{ab}x_b) \left( \ln \left( \frac{k_{ab}(x_a + x_b)}{k_{ab}x_a - k'_{ab}x_b} \right) \right)^{\frac{n(T)-1.0}{n(T)}}$$

for which the factors

$$k_{ab} = \frac{x_{eq}(T)}{\tau(T)} f(\dot{T}), k'_{ab} = \frac{1.0 - x_{eq}(T)}{\tau(T)} f'(\dot{T})$$

have to be defined.

As user input, load curve data for the exponent  $n(T)$  in PTTAB1, the equilibrium concentration  $x_{eq}(T)$  in PTTAB2, the relaxation time  $\tau(T)$  in PTTAB3, and the

temperature rate correction factors  $f(\dot{T})$  and  $f'(\dot{T})$  in PTTAB4 and PTTAB5, respectively, are expected.

3. Kirkaldy:

Similar to the implementation of \*MAT\_244, the transformation for cooling phases can be computed by the evolution equation

$$\frac{dX_b}{dt} = 2^{0.5(G-1)} f(C) (T_{start} - T)^{n_T} D(T) \frac{X_b^{n_1(1.0-X_b)} (1.0 - X_b)^{n_2 X_b}}{Y(X_b)}$$

formulated in the normalized phase concentration  $X_b = \frac{x_b}{x_{eq}(T)}$ .

In contrast to \*MAT\_244, the parameters for the evolution equation are not determined from the chemical composition of the material but defined directly as user input. The scalar data in PTX1 to PTX4 are interpreted as  $f(C)$ ,  $n_T$ ,  $n_1$ , and  $n_2$ . Tabulated data for  $D(T)$ ,  $Y(X_b)$ , and  $x_{eq}(T)$  are given in PTTAB1 to PTTAB3.

4. Oddy:

For phase transformation in heating, the equation of Oddy can be used, which can be interpreted as a simplified JAMK relation and reads

$$\frac{dx_b}{dt} = n \frac{x_a}{c_1(T - T_{start})^{-c_2}} \left( \ln \left( \frac{(x_a + x_b)}{x_a} \right) \right)^{\frac{n-1.0}{n}}$$

Its application requires the input of three scalar parameters  $n$ ,  $c_1$ ,  $c_2$  that are read from the respective positions in the tables in PTX1 to PTX3.

**\*MAT\_PIECEWISE\_LINEAR\_PLASTIC\_THERMAL**

This is material type 255, an isotropic elastoplastic material with thermal properties. It can be used for both explicit and implicit analyses. Young’s modulus and Poisson’s ratio can depend on the temperature by defining two load curves. Moreover, the yield stress in tension and compression are given as load curves for different temperatures by using two tables. The thermal coefficient of expansion can be given as a constant ALPHA or as a load curve, see LALPHA at position 3 on card 2. A positive curve ID for LALPHA models the instantaneous thermal coefficient, whereas a negatives curve ID models the thermal coefficient relative to a reference temperature, TREF. The strain rate effects are modelled with the Cowper-Symonds rate model with the parameters C and P on card 1. Failure can be based on effective plastic strain or using the \*MAT\_ADD\_EROSION keyword.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	C	P	FAIL	TDEL
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	TABIDC	TABIDT	LALPHA		VP			
Type	I	I	I		F			

Card 3	1	2	3	4	5	6	7	8
Variable	ALPHA	TREF						
Type	F	F						

**VARIABLE**

**DESCRIPTION**

MID Material identification. A unique number or label not exceeding 8 characters must be specified.

RO Mass density.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
E	<p>Young's modulus:</p> <p>LT.0.0: E is given as a function of temperature, T. The curve consists of (T,E) data pairs. Enter  E  on the DEFINE_CURVE keyword.</p> <p>GT.0.0: E is constant.</p>
PR	<p>Poisson's ratio.</p> <p>LT.0.0:  PR  is the LCID for Poisson's ratio versus temperature.</p> <p>GT.0.0: PR is constant</p>
C	Strain rate parameter. See <a href="#">Remark 1</a> .
P	Strain rate parameter. See <a href="#">Remark 1</a> .
FAIL	Effective plastic strain when the material fails. User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure when FAIL < 0. Note that for solids the *MAT_ADD_EROSION can be used for additional failure criteria.
TDEL	A time step less than TDEL is not allowed. A step size less than TDEL trigger automatic element deletion. This option is ignored for implicit analyses.
TABIDC	Table ID for yield stress in compression, see <a href="#">Remark 2</a> .
TABIDT	Table ID for yield stress in tension, see <a href="#">Remark 2</a> .
LALPHA	<p>Load curve ID for thermal expansion coefficient as a function of temperature.</p> <p>GT.0.0: the instantaneous thermal expansion coefficient based on the following formula:</p> $d\varepsilon_{ij}^{\text{thermal}} = \alpha(T)dT\delta_{ij}$ <p>LT.0.0: the thermal coefficient is defined relative a reference temperature TREF, such that the total thermal strain is given by:</p> $\varepsilon_{ij}^{\text{thermal}} = \alpha(T)(T - T_{\text{ref}})\delta_{ij}$ <p>With this option active, ALPHA is ignored.</p>

VARIABLE	DESCRIPTION
VP	Formulation for rate effects, see <a href="#">Remarks 1</a> and <a href="#">2</a> . EQ.0.0: effective total strain rate (default) NE.0.0: effective plastic strain rate
ALPHA	Coefficient of thermal expansion
TREF	Reference temperature, which is required if and only if LALPHA is given with a negative load curve ID.

**Remarks:**

- Strain Rate Effects.** The strain rate effect is modelled by using the Cowper and Symonds model which scales the yield stress according to the factor

$$1 + \left( \frac{\dot{\epsilon}_{\text{eff}}}{C} \right)^{1/P}$$

where  $\dot{\epsilon}_{\text{eff}} = \sqrt{\text{tr}(\dot{\epsilon}\dot{\epsilon}^T)}$  is the Euclidean norm of the total strain rate tensor if VP = 0 (default), otherwise  $\dot{\epsilon}_{\text{eff}} = \dot{\epsilon}_{\text{eff}}^p$ .

- Yield Stress Tables.** The dependence of the yield stresses on the effective plastic strains is given in two tables.
  - TABIDC gives the behaviour of the yield stresses in compression
  - TABIDT gives the behaviour of the yield stresses in tension.

The table indices consist of temperatures, and at each temperature a yield stress curve must be defined.

Both TABIDC and TABIDT can be 3D tables, in which temperatures indexes the main table and strain rates are defined as values for the sub tables with hardening curves as targets for those strain rates. If the same yield stress should be used in both tension and compression, only one table needs to be defined and the same TABID is put in position 1 and 2 on card 2. If VP = 0, effective total strain rates are used in the 3D tables, otherwise plastic strain rates.

- History Variables.** Two history variables are added to the d3plot file, the Young's modulus and the Poisson's ratio, respectively. They can be requested through the \*DATABASE\_EXTENT\_BINARY keyword.

4. **Nodal Temperatures.** Nodal temperatures must be defined by using a coupled analysis or some other way to define the temperatures, such as \*LOAD\_THERMAL\_VARIABLE or \*LOAD\_THERMAL\_LOAD\_CURVE.

**\*MAT\_AMORPHOUS\_SOLIDS\_FINITE\_STRAIN**

This is material type 256, an isotropic elastic-viscoplastic material model intended to describe the behaviour of amorphous solids such as polymeric glasses. The model accurately captures the hardening-softening-hardening sequence and the Bauschinger effect experimentally observed at tensile loading and unloading respectively. The formulation is based on hyperelasticity and uses the multiplicative split of the deformation gradient  $F$  which makes it naturally suitable for both large rotations and large strains. Stress computations are performed in an intermediate configuration and are therefore preceded by a pull-back and followed by a push-forward. The model was originally developed by Anand and Gurtin [2003] and implemented for solid elements by Bonnaud and Faleskog [2008]

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	G	MR	LL	NU0	M
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA	H0	SCV	B	ECV	G0	S0	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
K	Bulk modulus
G	Shear modulus
MR	Kinematic hardening parameter: $\mu_R$ (see Eq.1)
LL	Kinematic hardening parameter: $\lambda_L$ (see Eq.1)
NU0	Creep parameter: $v_0$ (see Eq.2)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
M	Creep parameter: $m$ (see Eq.2)
ALPHA	Creep parameter: $\alpha$ (see Eq.2)
H0	Isotropic hardening parameter: $h_0$ (see Eq.3-5)
SCV	Isotropic hardening parameter: $s_{cv}$ (see Eq.3-5)
B	Isotropic hardening parameter: $b$ (see Eq.3-5)
ECV	Isotropic hardening parameter: $\eta_{cv}$ (see Eq.3-5)
G0	Isotropic hardening parameter: $g_0$ (see Eq.3-5)
S0	Isotropic hardening parameter: $s_0$ (see Eq.3-5)

**Remarks:**

1. Kinematic hardening gives rise to the second hardening occurrence in the hardening-softening-hardening sequence. The constants  $\mu_R$  and  $\lambda_L$  enter the back stress  $\mu B$  (where  $B$  is the left Cauchy-Green deformation tensor) through the function  $\mu$  according to:

$$\mu = \mu_r \left( \frac{\lambda_L}{3\lambda^p} \right) L^{-1} \left( \frac{\lambda^p}{\lambda_L} \right) \quad (256.1)$$

Where  $\lambda^p = \frac{1}{\sqrt{3}} \sqrt{\text{tr}(B^p)}$  and  $B^p$  is the plastic part of the left Cauchy-Green deformation tensor and where  $L$  is the Langevin function defined by,

$$L(X) = \coth(X) - X^{-1}$$

2. This material model assumes plastic incompressibility. Nevertheless in order to account for the different behaviours in tension and compression a Drucker-Prager law is included in the creep law according to:

$$\nu^p = \nu_0 \left( \frac{\bar{\tau}}{s + \alpha\pi} \right)^{1/m} \quad (256.2)$$

Where  $\nu^p$  is the equivalent plastic shear strain rate,  $\bar{\tau}$  the equivalent shear stress,  $s$  the internal variable defined below and  $-\pi$  the hydrostatic stress.

3. Isotropic hardening gives rise to the first hardening occurrence in the hardening-softening-hardening sequence. Two coupled internal variables are defined:  $s$  the



resistance to plastic flow and  $\eta$  the local free volume. Their evolution equations read:

$$\dot{s} = h_0 \left[ 1 - \frac{s}{\tilde{s}(\eta)} \right] v^p \quad (256.3)$$

$$\dot{\eta} = g_0 \left( \frac{s}{s_{cv}} - 1 \right) v^p \quad (256.4)$$

$$\tilde{s}(\eta) = s_{cv} [1 + b(\eta_{cv} - \eta)] \quad (256.5)$$

4. Typical material parameters values are given in Ref.1 for Polycarbonate:

PolyC 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	G	MR	LL	NU0	M
Value			2.24GPa	0.857GPa	11.0MPa	1.45	0.0017s <sup>-1</sup>	0.011

PolyC 2	1	2	3	4	5	6	7	8
Variable	ALPHA	H0	SCV	B	ECV	G0	S0	
Value	0.08	2.75GPa	24.0MPa	825	0.001	0.006	20.0MPa	

[1] Anand, L., Gurtin, M.E., 2003, "A theory of amorphous solids undergoing large deformations, with application to polymeric glasses," *International Journal of Solids and Structures*, 40, pp. 1465-1487.



Card 3	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							
Default	none							

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

**VARIABLE**

**DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's Modulus
PR	Poisson's ratio

<b>VARIABLE</b>	<b>DESCRIPTION</b>
R00, R45, R90	Lankford parameters in rolling (0°), diagonal (45°) and transverse (90°) directions, respectively; determined from experiments. Note if R00, R45 and R90 are not defined or set as 0.0, then $R00 = R45 = R90 = 1.0$ , which degenerates to the Von-Mises yield.
SIG00, SIG45, SIG90, SIG_B	SIG00: the initial yield stress from uniaxial tension tests in rolling (0°) direction; SIG45: the initial yield stress from uniaxial tension tests in diagonal (45°) direction; SIG90: the initial yield stress from uniaxial tension tests in transverse(90°) directions; SIG_B: the initial yield stress from equi-biaxial stretching tests. Note if SIG00, SIG45, SIG90 and SIG_B are not defined, they will be set to the first stress point defined by LCIDS, and it degenerates to the associated flow rule.
LCIDS	ID of a load curve defining stress vs. strain hardening behavior from a uniaxial tension test along the rolling direction.
LCIDV	ID of a load curve defining stress scale factors vs. strain rates, determined from experiments. An example of the curve can be found in <a href="#">Figure M260A-2</a> . In addition, strain rates are stored in history variable #5 and strain rate scale factors are stored in history variable #6. To turn on the variables for viewing in <i>LS-PrePost</i> , set NEIPS to at least "6" in *DATABASE_EXTENT_BINARY. It is very useful to know what levels of strain rates, and strain rate scale factors in a particular simulation. Once d3plot files are opened in <i>LS-PrePost</i> , individual element time history can be plotted via menu option <i>Post</i> → <i>History</i> , or a color contour of the entire part can be viewed with the menu option <i>Post</i> → <i>FriComp</i> → <i>Misc</i> .
SCALE	This variable can be used to speed up the simulation while equalizing the strain rate effect, useful especially in cases where the pulling speed or punch speed is slow. For example, if the pulling speed is at 15 mm/s but running the simulation at this speed will take a long time, the pulling speed can be increased to 500 mm/s while SCALE can be set to 0.03, giving the same results as those from 15 mm/s, but with the benefit of greatly reduced computational time, see <a href="#">Figures M260A-3</a> and <a href="#">M260A-4</a> . Note the increased absolute value (within a reasonable range) of mass scaling $-1.0 \times dt2ms$ frequently used in forming simulation does not affect the strain rates, as shown in the <a href="#">Figure M260A-5</a> .

VARIABLE	DESCRIPTION
EF0, PLIM, Q, GAMA, M	Material parameters for the option XUE. The parameter $k$ in the original paper is assumed to be 1.0. For details, refer to Xue, L., Wierzbicki, T.'s 2009 paper "Numerical simulation of fracture mode transition in ductile plates" in the <i>International Journal of Solids and Structures</i> .
AOPT	<p>Material axes option (see MAT_OPTION TROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.</p> <p>EQ.1.0: locally orthotropic with material axes determined by the point <math>\mathbf{p}</math> in space and the global location of the element center; this is the <math>\mathbf{a}</math>-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by the vector <math>\mathbf{a}</math> for shells and by both vectors <math>\mathbf{a}</math> and <math>\mathbf{d}</math> for solids, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal. The plane of a solid element is the mid-surface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</p>
XP, YP, ZP	Coordinates of point $\mathbf{p}$ for AOPT = 1.
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2, for shells and solids.
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3.
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2, for solids.

**The Stoughton Non-associated Flow Rule:**

In a non-associated flow rule, material yield function does not equal to the plastic flow potential. According to Thomas B. Stoughton's paper titled "A non-associated flow rule for sheet metal forming" in 2002 International Journal of Plasticity 18, 687-714, and "A pressure-sensitive yield criterion under a non-associated flow rule for sheet metal forming" in 2004 International Journal of Plasticity 20, 705-731, plastic potential is defined by:

$$\bar{\sigma}_p = \sqrt[2]{\sigma_{11}^2 + \lambda_p \sigma_{22}^2 - 2\nu_p \sigma_{11} \sigma_{22} + 2\rho_p \sigma_{12}^2}$$

where  $\sigma_{ij}$  is the stress tensor component; where also,

$$\lambda_p = \frac{1 + \frac{1}{r_{90}}}{1 + \frac{1}{r_0}},$$

$$\nu_p = \frac{r_0}{1 + r_0},$$

$$\rho_p = \frac{\frac{1}{r_0} + \frac{1}{r_{90}}}{1 + \frac{1}{r_0}} \left( \frac{1}{2} + r_{45} \right).$$

where  $r_0, r_{45}, r_{90}$  are Lankford parameters in the rolling (0°), the diagonal (45°) and the transverse (90°) directions, respectively.

Yield function is defined by:

$$\bar{\sigma}_y = \sqrt[2]{\sigma_{11}^2 + \lambda_y \sigma_{22}^2 - 2\nu_y \sigma_{11} \sigma_{22} + 2\rho_y \sigma_{12}^2}$$

where,

$$\lambda_y = \left( \frac{\sigma_0}{\sigma_{90}} \right)^2,$$

$$\nu_y = \frac{1}{2} \left[ 1 + \lambda_y - \left( \frac{\sigma_0}{\sigma_b} \right)^2 \right],$$

$$\rho_y = \frac{1}{2} \left[ \left( \frac{2\sigma_0}{\sigma_{45}} \right)^2 - \left( \frac{\sigma_0}{\sigma_b} \right)^2 \right].$$

where  $\sigma_0, \sigma_{45}, \sigma_{90}$  are the initial yield stresses from uniaxial tension tests in the rolling (0°), diagonal (45°) and transverse (90°) directions, respectively.  $\sigma_b$  is the initial yield stress from an equi-biaxial stretching test.

The required stress-strain hardening curve must be for uniaxial tension along the rolling direction. Strain rate sensitivity is implemented as an option, by defining a curve (LCIDV)

of strain rates vs. stress scale factors, see [Figure M260A-2](#).

The variable SCALE is very useful in speeding up the simulation while equalizing the strain rate effect. For example, if the real, physical pulling speed is at 15 mm/s but running at this speed will take a long time, one could increase the pulling speed to 500 mm/s while setting the SCALE to 0.03, resulting in the same results as those from 15 mm/s with the

benefit of greatly reduced computational time. See examples in [Verification](#).

**History Variables:**

- 1. Strain rates: history variable #5.
- 2. Strain rate scale factors: history variable #6.

**Verification:**

Uniaxial tension tests were done on a single shell element as shown in [Figure M260A-1](#). Strain rate effect LCIDV is input as shown in [Figure M260A-2](#). In [Figure M260A-3](#), pulling stress vs. strain from various test conditions are compared with input stress-strain curve A. In summary, using the parameter SCALE, the element can be pulled much faster (500 mm/s vs. 15 mm/s) but achieve the same stress vs. strain results, the same strain rates (history variable #5), and the same strain rate scale factor (history variable #6 in [Figure M260A-4](#)). Simulation speed can be improved further with increased mass scaling ( $-1.0 \times dt^2ms$ ) without affecting the results, see [Figure M260A-5](#).

A partial keyword input is provided below, for the case with pulling speed of 500 mm/s, strain hardening curve ID of 100, LCIDV curve ID of 105, and strain rate scale factor of 0.03.

```

*KEYWORD
*parameter_expression
R endtime      0.012
R v            500.0
*CONTROL_TERMINATION
$  ENDTIM      ENDCYC      DTMIN      ENDNEG      ENDMAS
&endtime
*MAT_STOUGHTON_NON_ASSOCIATED_FLOW
$#  mid      Ro      E      PR      R00      R45      R90      SIG00
    1  7.8000E-9  2.10E05  0.300000  1.1      1.2      1.3      311.0
$  SIG45      SIG90      SIG_B      LCIDS      LCIDV      SCALE
    305.4      321.1      290.3      100      105      0.03
$  AOPT
    3
$  XP      YP      ZP      A1      A2      A3

```

```
$      V1      V2      V3      D1      D2      D3      BETA
      1.0
*DEFINE_CURVE
  100
    0.00000E+00      0.30130E+03
    0.10000E-01      0.42295E+03
    0.20000E-01      0.47991E+03
    0.30000E-01      0.52022E+03
    0.40000E-01      0.55126E+03
    0.50000E-01      0.57615E+03
                                     ⋮
*DEFINE_CURVE
  105
    0.00000E+00      0.10000E+01
    0.10000E+00      0.10608E+01
    0.50000E+00      0.10828E+01
    0.10000E+01      0.10923E+01
                                     ⋮
*END
```

**Corresponding Parameter Summary:**

The following table lists variable names used in this material model and corresponding symbols in Xue’s paper, for the option XUE:

EF0	PLIM	Q	GAMA	M
$\epsilon_{f0}$	$P_{lim}$	$q$	$\gamma$	$m$

**Revision Information:**

This material model is available starting in Revision 101821 in explicit, SMP only.

- 3. The option XUE: Revision 112711.
- 4. Default values set for R00, R45 and R90, SIG00, SIG45, SIG90 and SIG\_B: Revision 116093.



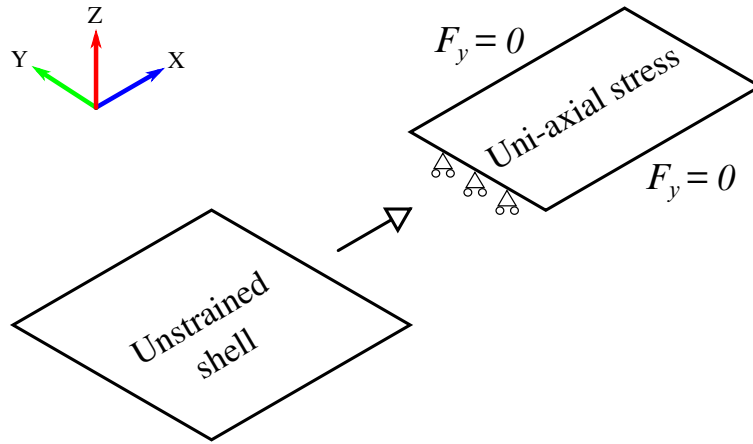


Figure M260A-1. Uniaxial tension tests on a single shell element.

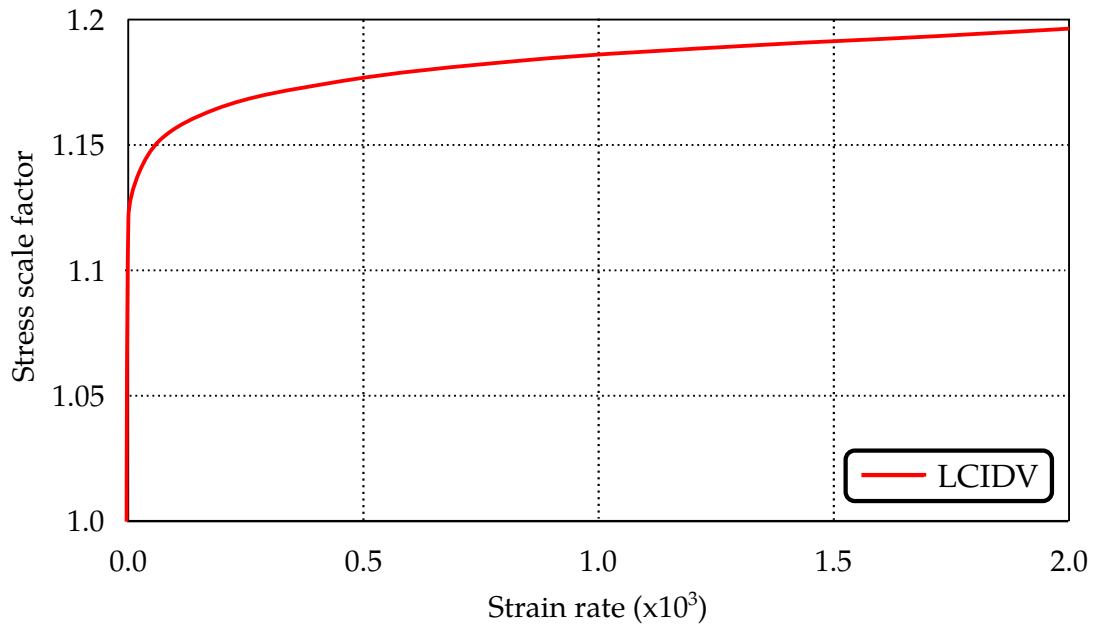
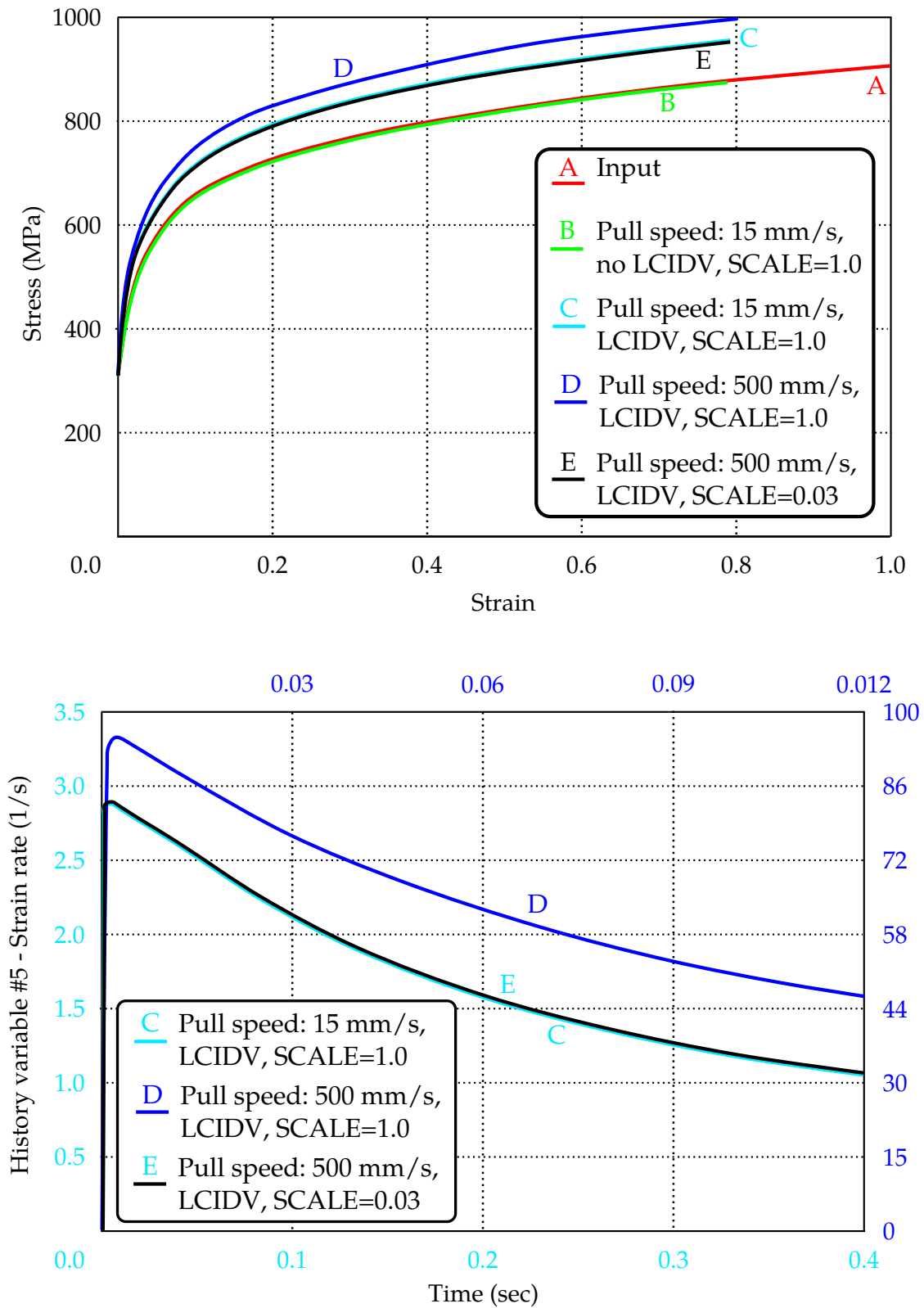


Figure M260A-2. Input LCIDV.



**Figure M260A-3.** Recovered stress-strain curve (top) and strain rates (bottom) under various conditions shown.

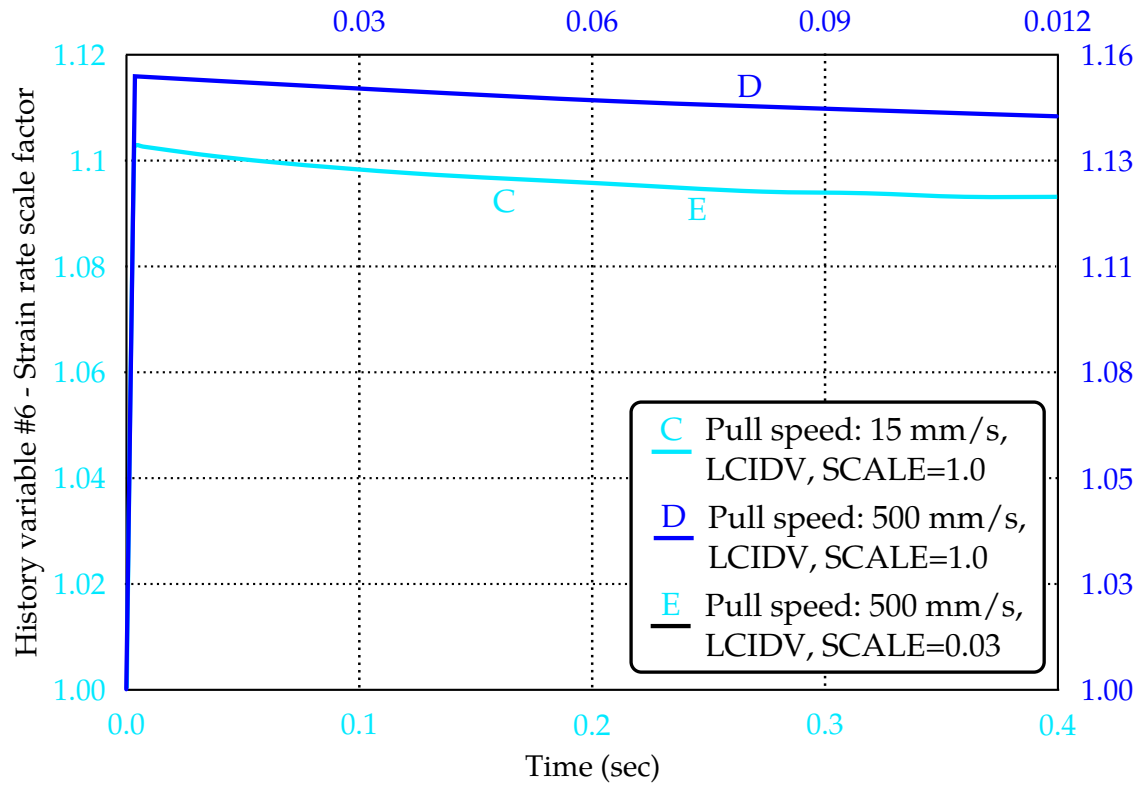


Figure M260A-4. Recovered strain rate scale factors under various conditions shown.

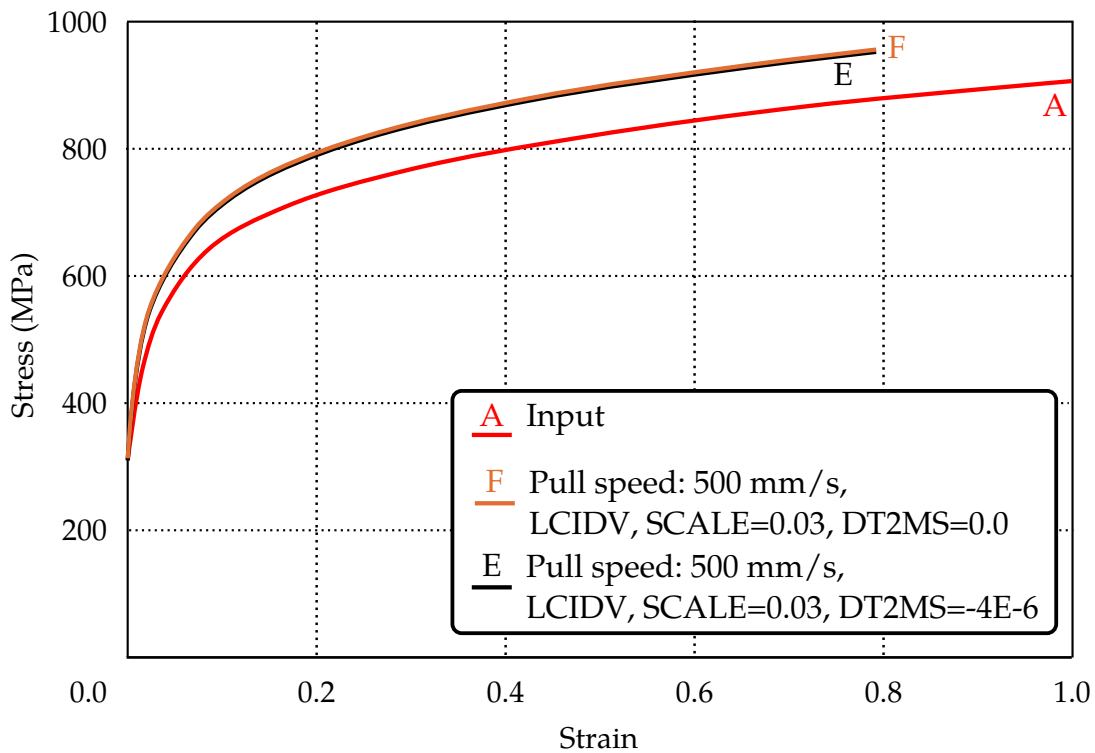


Figure M260A-5. Effect of mass scaling (-1.0\*dt2ms).

# \*MAT\_260B

# \*MAT\_MOHR\_NON\_ASSOCIATED\_FLOW

## \*MAT\_MOHR\_NON\_ASSOCIATED\_FLOW\_{OPTION}

This is Material Type 260B. This material model is implemented based on the papers by Mohr, D., et al. (2010) and Roth, C.C., Mohr, D. (2014). The Johnson-Cook plasticity model of strain hardening, strain rate hardening, and temperature soften effect is modified with a mixed Swift-Voce strain hardening function, coupled with a non-associated flow rule which accounts for the difference between directional dependency of the  $r$ -values (planar anisotropic), and planar isotropic material response of certain Advanced High Strength Steels (AHSS). A ductile fracture model is included based on Hosford-Coulomb fracture initiation model. This model applies to shell elements only.

Available options include:

<BLANK>

XUE

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	P12	P22	P33	G12
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	-0.5	1.0	3.0	-0.5

Card 2	1	2	3	4	5	6	7	8
Variable	G22	G33	LCIDS	LCIDV	LCIDT	LFLD	LFRAC	W0
Type	F	F	I	I	I	I	I	F
Default	1.0	3.0	none	none	none	0	none	none

Card 3	1	2	3	4	5	6	7	8
Variable	A	B0	GAMMA	C	N	SCALE	SIZE0	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	1.0	none	

Card 4	1	2	3	4	5	6	7	8
Variable	TREF	TMELT	M	ETA	CP	TINI	DEPS0	DEPSAD
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

**Define the following card only for the option XUE:**

Card 5	1	2	3	4	5	6	7	8
Variable	EFO	PLIM	Q	GAMA	M			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

Card 5	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							
Default	none							

**\*MAT\_260B****\*MAT\_MOHR\_NON\_ASSOCIATED\_FLOW**

Card 6	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		
Default				none	none	none		

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3					
Type	F	F	F					
Default	none	none	none					

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
E	Young's Modulus
PR	Poisson's ratio
P12, P22, P33	Yield function parameters, defined by Lankford parameters in rolling ( $0^\circ$ ), diagonal ( $45^\circ$ ) and transverse ( $90^\circ$ ) directions, respectively; see <a href="#">Non-associated Flow Rule</a> .
G12, G22, G33	Plastic flow potential parameters, defined by Lankford parameters in rolling ( $0^\circ$ ), diagonal ( $45^\circ$ ) and transverse ( $90^\circ$ ) directions, respectively; see <a href="#">Non-associated Flow Rule</a> .
LCIDS	Load curve ID defining stress vs. strain hardening behavior from a uniaxial tension test; must be along the rolling direction. Also see <a href="#">A Modified Johnson-Cook</a> .

VARIABLE	DESCRIPTION
LCIDV	Load curve ID defining stress scale factors vs. strain rates ( <a href="#">Figure M260B-1</a> middle); determined from experiments. Strain rates are stored in history variable #5. Strain rate scale factors are stored in history variable #6. To turn on the variables for viewing in <i>LS-PrePost</i> , set NEIPS to at least "6" in *DATABASE_EXTENT_BINARY. It is very useful to know what levels of strain rates, and strain rate scale factors in a particular simulation. Once d3plot files are opened in <i>LS-PrePost</i> , individual element time history can be plotted via menu option <i>Post</i> → <i>History</i> , or a color contour of the entire part can be viewed with the menu option <i>Post</i> → <i>FriComp</i> → <i>Misc</i> . Also see <a href="#">A Modified Johnson-Cook</a> .
LCIDT	Load curve ID defining stress scale factors vs. temperature in Kelvin ( <a href="#">Figure M260B-1</a> bottom), determined from experiments. Temperatures are stored in history variable #4. Temperature scale factors are stored in history variable #7. To turn on this variable for viewing in <i>LS-PrePost</i> , set NEIPS to at least "7" in *DATABASE_EXTENT_BINARY. It is very useful to know what levels of temperatures and temperature scale factors in a particular simulation. Once d3plot files are opened in <i>LS-PrePost</i> , individual element time history can be plotted via menu option <i>Post</i> → <i>History</i> , or a color contour of the entire part can be viewed with the menu option <i>Post</i> → <i>FriComp</i> → <i>Misc</i> . Also see <a href="#">A Modified Johnson-Cook</a> .
LFLD	Load curve ID defining traditional Forming Limit Diagram for linear strain paths.
LFRAC	Load curve ID defining a fracture limit curve. Leave this field empty if parameters A, B0, GAMMA, C, N are defined. However, if this field is defined, parameters A, B0, GAMMA, C, N will be ignored even if they are defined.
W0	Neck (FLD failure) width, typically is the blank thickness.
A, B0, GAMMA, C, N	Material parameters ( $a$ , $b_0$ , $\gamma$ , $c$ , $n$ ) for the rate-dependent Hosford-Coulomb fracture initiation model, see <a href="#">Rate-dependent Hosford-Coulomb</a> . Ignored if LFRAC is defined.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SCALE	This variable can be used to speed up the simulation while equalizing the strain rate effect, useful especially in cases where the pulling speed or punch speed is slow. For example, if the pulling speed is at 15 mm/s but running the simulation at this speed will take a long time, the pulling speed can be increased to 500 mm/s while "SCALE" can be set to 0.03, giving the same results as those from 15 mm/s, but with the benefit of greatly reduced computational time, see examples and Figures in <b>*MAT_260A</b> for details. Furthermore, the increased absolute value (within a reasonable range) of mass scaling $-1.0*dt2ms$ frequently used in forming simulation does not affect the strain rates, as shown in the examples and Figures in <b>*MAT_260A</b> .
SIZE0	Fracture gage length used in an experimental measurement, typically between 0.2~0.5mm.
TREF, TMELT, M, ETA, CP, TINI, DEPS0, DEPSAD	Material parameters for strain softening effect due to temperature. TINI is the initial temperature. See <a href="#">A Modified Johnson-Cook</a> for other parameters' definitions.
EF0, PLIM, Q, GAMA, M	Material parameters for the option XUE. The parameter $k$ in the original paper is assumed to be 1.0. For details, refer to Xue, L., Wierzbicki, T.'s 2009 paper " <i>Numerical simulation of fracture mode transition in ductile plates</i> " in the <i>International Journal of Solids and Structures</i> .



VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_OPTION TROPIC_ELASTIC for a more complete description): <ul style="list-style-type: none"> <li>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.</li> <li>EQ.2.0: globally orthotropic with material axes determined by the vector <b>a</b> for shells, as with *DEFINE_COORDINATE_VECTOR.</li> <li>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal.</li> <li>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</li> </ul>
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3.

### Non-associated Flow Rule:

Referring to Mohr, D., Dunand, M., and Kim, K-H.'s 2010 and 2014 papers in the *International Journal of Plasticity*, Hill's 1948 quadratic yield function is written as:

$$f(\sigma, k) = \bar{\sigma} - k = 0$$

where  $\sigma$  is the Cauchy stress tensor and  $\bar{\sigma}$  the equivalent stress is defined by:

$$\bar{\sigma} = \sqrt{(\mathbf{P}\sigma) \bullet \sigma}$$

Where  $\mathbf{P}$  is a symmetric positive-definite matrix defined through three independent parameters  $P_{12}$ ,  $P_{22}$ ,  $P_{33}$ :

$$\mathbf{P} = \begin{bmatrix} 1 & P_{12} & 0 \\ P_{12} & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix}$$

Flow rule, which defines the incremental plastic strain tensor, is written as follows:

$$d\epsilon_p = d\delta \frac{\partial g(\sigma)}{\partial \sigma}$$

where  $d\delta$  is a scalar plastic multiplier. The plastic potential function  $g(\sigma)$  can be defined as a quadratic function in stress space:

$$g(\sigma) = \sqrt{(\mathbf{G}\sigma) \bullet \sigma}$$

with,

$$\mathbf{G} = \begin{bmatrix} 1 & G_{12} & 0 \\ G_{12} & G_{22} & 0 \\ 0 & 0 & G_{33} \end{bmatrix}$$

When  $\mathbf{P} \neq \mathbf{G}$ , it leads to non-associated flow rule. For example,  $\mathbf{P}$  can represent isotropic von-Mises yield surface by setting  $P_{11} = P_{22} = 1.0, P_{12} = -0.5, P_{33} = 3.0$ .  $\mathbf{G}$  can represent an orthotropic plastic flow potential by setting:

$$G_{12} = \frac{r_0}{1 + r_0},$$

$$G_{22} = \frac{r_0(1 + r_{90})}{r_{90}(1 + r_0)},$$

$$G_{33} = \frac{(1 + 2r_{45})(r_0 + r_{90})}{r_{90}(1 + r_0)}.$$

where  $r_0, r_{45}, r_{90}$  are Lankford coefficients in the rolling, diagonal and transverse direction. Experiments have shown on the stress level, some AHSS (Advanced High Strength Steel), e.g., DP590, and TRIP780 show strong directional dependency of  $r$ -values, while nearly the same stress-strain curves have been measured in all directions. The directional dependency of  $r$ -values suggests planar anisotropy while the material response on the stress level is planar isotropic, which is the main reason to employ the non-associated flow rule.

The associated flow rule is recovered if  $\mathbf{P} = \mathbf{G}$ .

### **A Modified Johnson-Cook Plasticity Model with Mixed Swift-Voce Hardening:**

The Johnson-Cook plasticity model (1983) multiplicatively decomposes the deformation resistance into three functions representing the effect of strain hardening, strain rate and temperature. The Johnson-Cook model is modified to include hardening saturation with a mixed Swift-Voce hardening law (Sung et al, *A plastic constitutive equation incorporating strain, strain-rate, and temperature, International Journal of Plasticity, 2010*), which gives a better description of the hardening at large strain levels, thus improving the prediction of the necking and post-necking response of metal sheet:

$$\sigma_y = \left( \alpha(A(\bar{\epsilon}_{pl} + \epsilon_0)^n) + (1 - \alpha) \left( k_0 + Q(1 - e^{-\beta \bar{\epsilon}_{pl}}) \right) \right) \left( 1 + C \ln \left( \frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\epsilon}_0} \right) \right) \left( 1 - \left( \frac{T - T_r}{T_m - T_r} \right)^m \right)$$

where  $\bar{\epsilon}_{pl}$  and  $\dot{\bar{\epsilon}}_{pl}$  are effective plastic strain and strain rate, respectively;  $T_m$  (TMELT),  $T_r$  (TREF) and  $T$  are the melting temperature, reference temperature (ambient temperature 293 kelvin) and current temperature, respectively;  $m$  (M) is an exponent coefficient. For other symbols' definitions refer to the aforementioned paper.

To make this material model more general and flexible, three load curves are used to define the three components of the deformation resistance. A load curve (LCIDS) is used to describe the strain hardening:

$$\text{LCIDS: } \left( \alpha(A(\bar{\epsilon}_{pl} + \epsilon_0)^n) + (1 - \alpha) \left( k_0 + Q(1 - e^{-\beta\bar{\epsilon}_{pl}}) \right) \right)$$

Strain rate is described by a load curve LCIDV (stress scale factor vs. strain rates, [Figure](#)

[M260B-1](#) middle), which scales the stresses based on the strain rates during a simulation:

$$\text{LCIDV: } \left( 1 + C \ln \left( \frac{\dot{\bar{\epsilon}}_{pl}}{\dot{\epsilon}_0} \right) \right)$$

The temperature softening effect is defined by another load curve LCIDT (stress scale factor vs. temperature, [Figure M260B-1](#) bottom), which scales the stresses based on the temperatures during the simulation:

$$\text{LCIDT: } \left( 1 - \left( \frac{T - T_r}{T_m - T_r} \right)^m \right)$$

The temperature effect is a self-contained model, in other words, it does not require thermal exchange with the environment, and it calculates temperatures based on plastic strain and strain rate.

The temperature evolution is determined with:

$$dT = \omega[\dot{\bar{\epsilon}}_{pl}] \frac{\eta_k}{\rho C_p} \bar{\sigma} d\bar{\epsilon}_{pl}$$

Where  $\eta_k$  (ETA) is Taylor-Quinney coefficient,  $\rho$  (R0) is the mass density and  $C_p$  (CP) is the heat capacity; also where,

$$\omega[\dot{\bar{\epsilon}}_{pl}] = \begin{cases} 0 & \text{for } \dot{\bar{\epsilon}}_{pl} < \dot{\epsilon}_{it} \\ \frac{(\dot{\bar{\epsilon}}_{pl} - \dot{\epsilon}_{it})^2 (3\dot{\epsilon}_a - 2\dot{\bar{\epsilon}}_{pl} - \dot{\epsilon}_{it})}{(\dot{\epsilon}_a - \dot{\epsilon}_{it})^3} & \text{for } \dot{\epsilon}_{it} \leq \dot{\bar{\epsilon}}_{pl} \leq \dot{\epsilon}_a \\ 1 & \text{for } \dot{\epsilon}_a < \dot{\bar{\epsilon}}_{pl} \end{cases}$$

where  $\dot{\epsilon}_{it} > 0$  and  $\dot{\epsilon}_a > \dot{\epsilon}_{it}$  define the limits of the respective domains of isothermal and adiabatic conditions ( $\dot{\epsilon}_a = \text{DEPSAD}$ ). For simplification,  $\dot{\epsilon}_{it} = \dot{\epsilon}_0$  (DEPS0).

As shown in a single shell element uniaxial stretching (Figure M260B-1), the general effect of the LCIDV is to elevate the strain hardening behavior as the strain rate increases (curve “D” in Figure M260B-2 top), while the effect of the LCIDT is strain softening as temperature rises (curve “C” in Figure M260B-2 top). A combined effect of both LCIDV and LCIDT may result in strain hardening initially before temperature rise enough to cause the strain softening in the model (curve “E” in Figure M260B-2 top). The temperature and strain rates calculated for each element can be viewed with history variables #4 and #5 (curves “C” and “D” in Figure M260B-2 bottom), respectively, while the strain rate scale factors and temperature scale factors can be viewed with history variable #6 and #7, respectively.

### **Rate-dependent Hosford-Coulomb Fracture Initiation Model:**

An extension of the Hosford-Coulomb fracture initiation model is used to account for the effect of strain rate on ductile fracture. The damage accumulation is calculated through history variable #3, and fracture occurs at an equivalent plastic strain  $\bar{\epsilon}_f$  when the variable reaches 1.0:

$$\int_0^{\epsilon_f} \frac{d\bar{\epsilon}_{pl}}{\bar{\epsilon}_f^{pr}[\eta, \bar{\theta}]} = 1$$

Where  $\bar{\epsilon}_f^{pr}$ ,  $\eta$ ,  $\bar{\theta}$  are strain to fracture, stress triaxiality and the Lode parameter, respectively.

The fracture parameters A, B0, GAMMA, C, N ( $a, b_0, \gamma, c, n$ ) are indicated in the following equations. Strain to fracture for proportional load:

$$\bar{\epsilon}_f^{pr}[\eta, \bar{\theta}] = b(1 + c)^{\frac{1}{n}} \left( \left\{ \frac{1}{2} ((f_1 - f_2)^a + (f_2 - f_3)^a + (f_1 - f_3)^a) \right\}^{\frac{1}{a}} + c(2\eta + f_1 + f_3) \right)^{-\frac{1}{n}}$$

where  $a$  is the Hosford exponent,  $c$  is the friction coefficient controlling the effect of triaxiality,  $n$  is the stress state sensitivity.

The Lode angle parameter dependent trigonometric functions:

$$f_1[\bar{\theta}] = \frac{2}{3} \cos \left[ \frac{\pi}{6} (1 - \bar{\theta}) \right], f_2[\bar{\theta}] = \frac{2}{3} \cos \left[ \frac{\pi}{6} (3 + \bar{\theta}) \right], f_3[\bar{\theta}] = -\frac{2}{3} \cos \left[ \frac{\pi}{6} (1 + \bar{\theta}) \right]$$

and coefficient  $b$  (strain to fracture for uniaxial or equi-biaxial stretching):

$$b = \begin{cases} b_0 & \text{for } \dot{\epsilon}_p < \dot{\epsilon}_0 \\ b_0 \left( 1 + \gamma \ln \left[ \frac{\dot{\epsilon}_p}{\dot{\epsilon}_0} \right] \right) & \text{for } \dot{\epsilon}_p > \dot{\epsilon}_0 \end{cases}$$

where  $\gamma$  is the strain rate sensitivity.

### Corresponding Parameters Summary:

The following table lists variable names used in this material model and corresponding symbols employed in the papers:

P12	P22	P33	G12	G22	G33	A	B0	GAMMA	C	N
$P_{12}$	$P_{22}$	$P_{33}$	$G_{12}$	$G_{22}$	$G_{33}$	$a$	$b_0$	$\gamma$	$c$	$n$

TREF	TMELT	M	ETA	CP	DEPSO	DEPSAD	R0
$T_r$	$T_m$	$m$	$\eta_k$	$C_p$	$\dot{\epsilon}_{it}/\dot{\epsilon}_0$	$\dot{\epsilon}_a$	$\rho$

The following table lists variable names used in this material model and corresponding symbols in Xue's paper, for the option XUE:

EFO	PLIM	Q	GAMA	M
$\epsilon_{f0}$	$P_{lim}$	$q$	$\gamma$	$m$

### History Variables Summary:

1. Damage accumulation: history variable #3. Elements will be deleted if this variable reaches 1.0 for more than half of the through-thickness integration points (Revision 109792).

- 2. Temperatures: history variable #4.
- 3. Strain rates: history variable #5.
- 4. Strain rate scale factors: history variable #6.
- 5. Temperature scale factor: history variable #7.

**Keyword Example Input:**

A sample material input card can be found below, with parameters from Mohr, D., et al.(2010) and Roth, C.C., Mohr, D. (2014).

```

*MAT_MOHR_NON_ASSOCIATED_FLOW
$#      mid      R0      E      PR      P12      P22      P33      G12
      1 7.8000E-9  2.10E05  0.300000  -0.5      1.0      3.0     -0.4946
$      G22      G33      LCIDS      LCIDV      LCIDT      LFLD      LFRAC      W0
      0.9318      2.4653      100      105      102
$      A      B0      GAMMA      C      N      SCALE
      1.97      0.82      0.025      0.00      0.199      3.132E-3
$      TREF      TMELT      M      ETA      CP      TINI      DEPSO      DEPSAD
      293.0      1673.70      0.921      0.9      420.0      293.0      0.001164      1.379
$      AOPT
      3
$      XP      YP      ZP      A1      A2      A3
$      V1      V2      V3      D1      D2      D3      BETA
      1.0
*DEFINE_CURVE
100
      0.00000E+00      0.30130E+03
      0.10000E-01      0.42295E+03
      0.20000E-01      0.47991E+03
      0.30000E-01      0.52022E+03
      0.40000E-01      0.55126E+03
      ::
*DEFINE_CURVE
105
      0.00000E+00      0.10000E+01
      0.10000E+00      0.10608E+01
      0.50000E+00      0.10828E+01
      0.10000E+01      0.10923E+01
      ::
*DEFINE_CURVE
102
      0.29300E+03      0.10000E+01
      0.33300E+03      0.96168E+00
      0.37300E+03      0.92744E+00
      0.41300E+03      0.89459E+00
      0.45300E+03      0.86261E+00
      ::

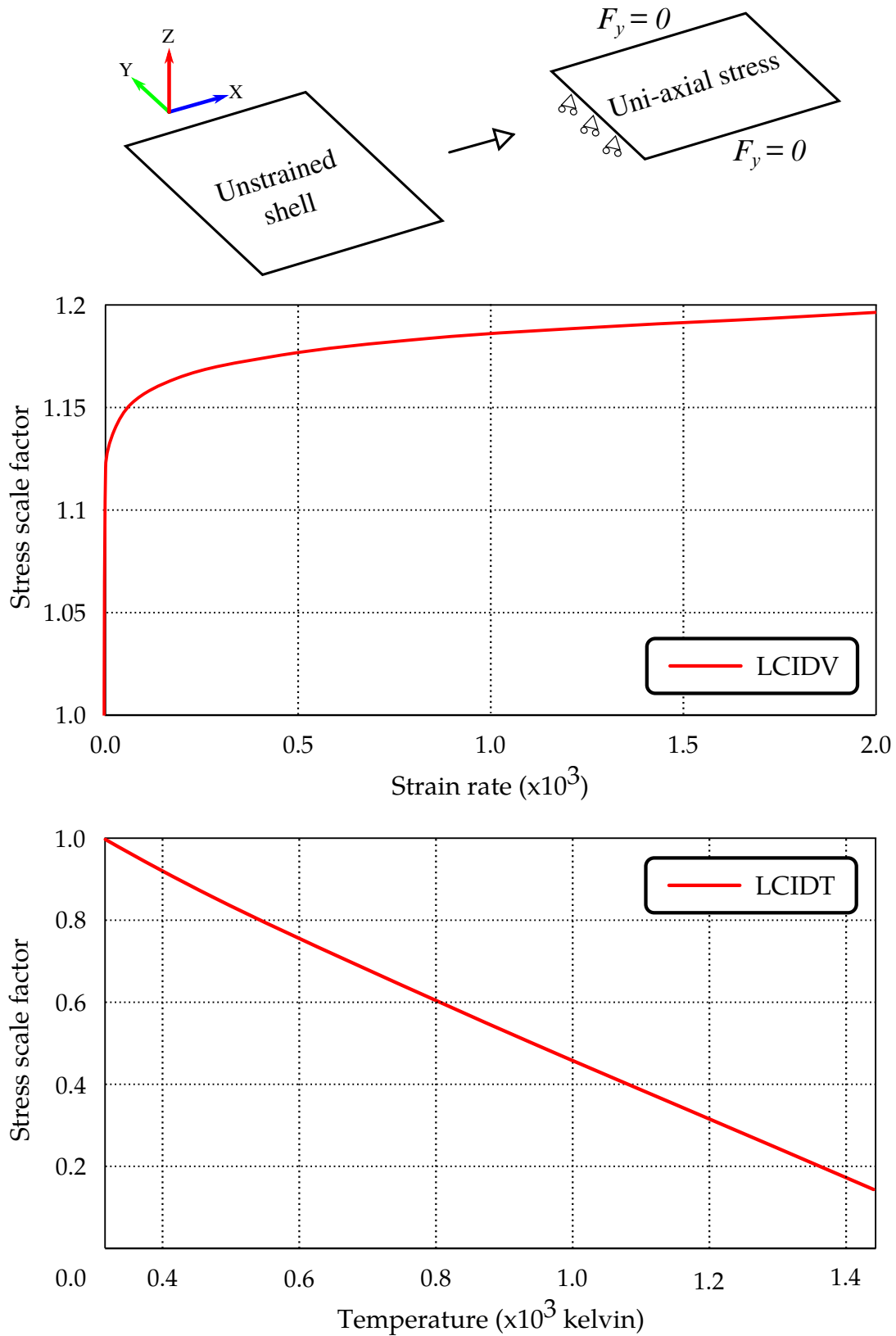
```

**Revision Information:**

This material model is available in SMP starting in Revision 102375. Revision history is listed below:

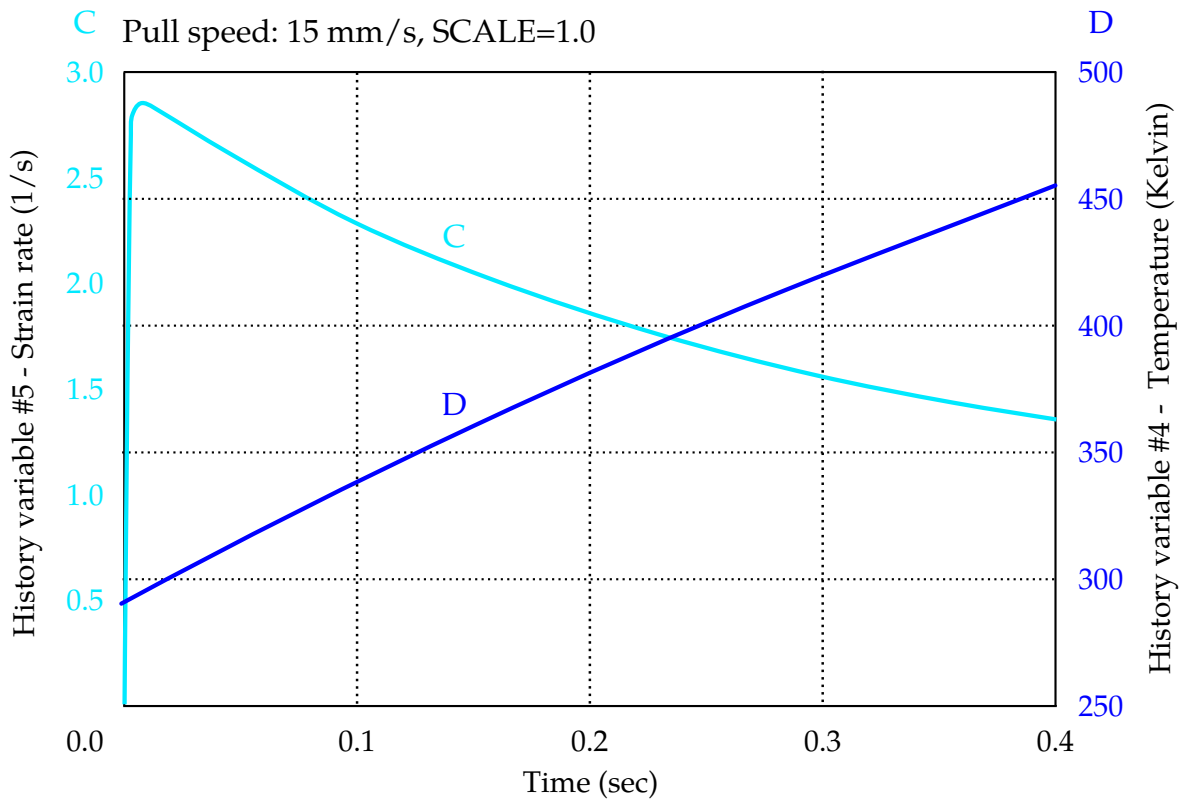
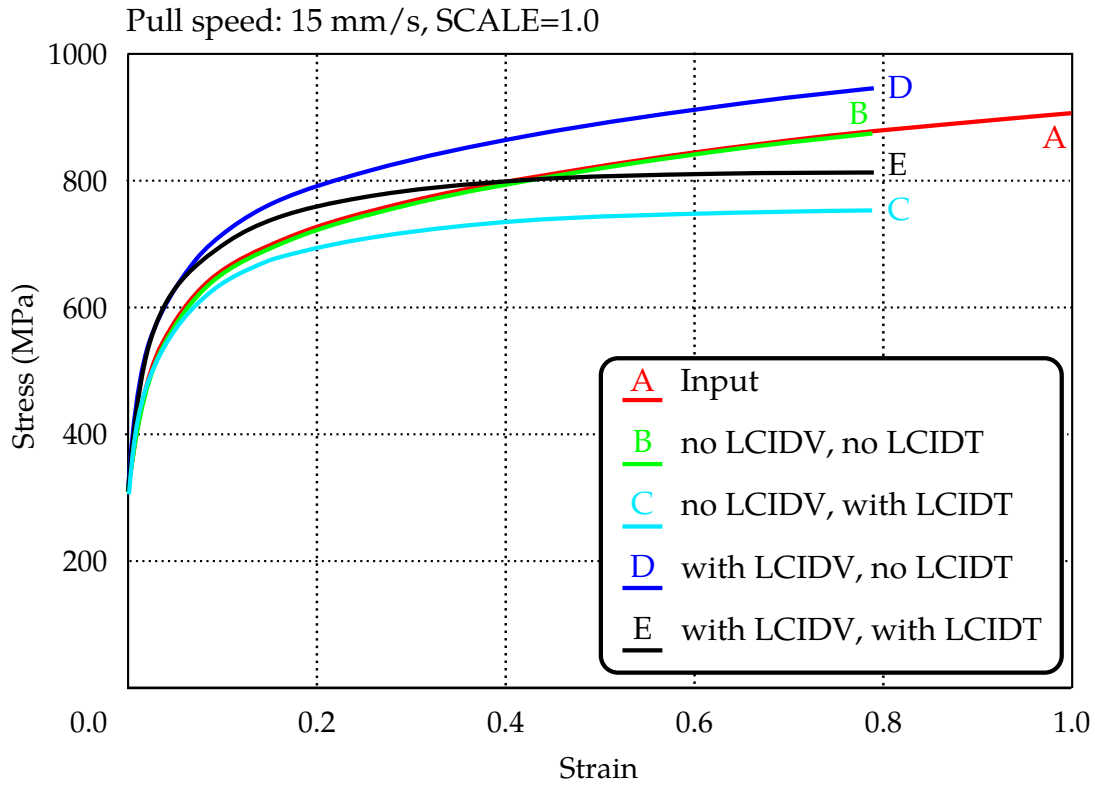
- 1) Element deletion feature based on damage accumulation: Revision 109792.
- 2) The option XUE is available starting on Revision 111531.

- 3) Set default values for P12, P22, P33, G12, G22 and G33: Revision 116262.



**Figure M260B-1.** Uniaxial stretching on a single shell element; Input curves LCIDV and LCIDT.





**Figure M260B-2.** Results of a single element uniaxial stretching - stress-strain curves (top), strain rates and temperature history under various conditions.



Card 5	1	2	3	4	5	6	7	8
Variable	ENKINK	ENA	ENB	ENT	ENL			
Type	F	F	F	F	F			

Card 6	1	2	3	4	5	6	7	8
Variable	XC	XT	YC	YT	SL			
Type	F	F	F	F	F			

Card 7	1	2	3	4	5	6	7	8
Variable	FIO	SIGY	LCSS	BETA	PFL	PUCK	SOFT	DT
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
EA	$E_a$ , Young's modulus in $a$ -direction (longitudinal)
EB	$E_b$ , Young's modulus in $b$ -direction (transverse)
EC	$E_c$ , Young's modulus in $c$ -direction
PRBA	$\nu_{ba}$ , Poisson's ratio $ba$
PRCA	$\nu_{ca}$ , Poisson's ratio $ca$
PRCB	$\nu_{cb}$ , Poisson's ratio $cb$
GAB	$G_{ab}$ , shear modulus $ab$
GBC	$G_{bc}$ , shear modulus $bc$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GCA	$G_{ca}$ , shear modulus $ca$
AOPT	<p>Material axes option (see MAT_{OPTION}TROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle MANGLE.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <math>a</math>-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle (MANGLE) from a line in the plane of the element defined by the cross product of the vector <math>v</math> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>v</math>, and an originating point, <math>p</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
DAF	<p>Flag to control failure of an integration point based on longitudinal (fiber) tensile failure:</p> <p>EQ.0.0: IP fails if any damage variable reaches 1.0.</p> <p>EQ.1.0: no failure of IP due to fiber tensile failure. This condition corresponds to history variable “<math>da(i)</math>” reaching 1.0</p>

VARIABLE	DESCRIPTION
DKF	Flag to control failure of an integration point based on longitudinal (fiber) compressive failure: EQ.0.0: IP fails if any damage variable reaches 1.0. EQ.1.0: no failure of IP due to fiber compressive failure. This condition corresponds to history variable "dkink(i)" reaching 1.0.
DMF	Flag to control failure of an integration point based on transverse (matrix) failure: EQ.0.0: IP fails if any damage variable reaches 1.0. EQ.1.0: no failure of IP due to matrix failure. This condition corresponds to history variable "dmat(i)" reaching 1.0.
EFS	Maximum effective strain for element layer failure. A value of unity would equal 100% strain. GT.0.0: fails when effective strain calculated assuming material is vol-ume preserving exceeds EFS. LT.0.0: fails when effective strain calculated from the full strain tensor exceeds  EFS .
XP, YP, ZP	Coordinates of point <b>p</b> for AOPT = 1 and 4.
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2.
V1, V2, V3	Define components of vector <b>v</b> for AOPT = 3.
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2.
MANGLE	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. MANGLE may be overridden on the element card, see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO.
ENKINK	Fracture toughness for longitudinal (fiber) compressive failure mode. GT.0.0: The given value will be regularized with the characteristic element length. LT.0.0: Load curve ID = (-ENKINK) which defines the fracture toughness for fiber compressive failure mode as a function of characteristic element length. No further regularization.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ENA	<p>Fracture toughness for longitudinal (fiber) tensile failure mode.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-ENA) which defines the fracture toughness for fiber tensile failure mode as a function of characteristic element length. No further regularization.</p>
ENB	<p>Fracture toughness for intralaminar matrix tensile failure.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-ENB) which defines the fracture toughness for intralaminar matrix tensile failure as a function of characteristic element length. No further regularization.</p>
ENT	<p>Fracture toughness for intralaminar matrix transverse shear failure.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-ENT) which defines the fracture toughness for intralaminar matrix transverse shear failure as a function of characteristic element length. No further regularization.</p>
ENL	<p>Fracture toughness for intralaminar matrix longitudinal shear failure.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-ENL) which defines the fracture toughness for intralaminar matrix longitudinal shear failure as a function of characteristic element length. No further regularization.</p>
XC	<p>Longitudinal compressive strength, <math>a</math>-axis (positive value).</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-XC) which defines the longitudinal compressive strength vs. longitudinal strain rate (<math>\dot{\epsilon}_{aa}</math>).</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XT	Longitudinal tensile strength, <i>a</i> -axis. GT.0.0: constant value LT.0.0: Load curve ID = (-XT) which defines the longitudinal tensile strength vs. longitudinal strain rate ( $\dot{\epsilon}_{aa}$ ).
YC	Transverse compressive strength, <i>b</i> -axis (positive value). GT.0.0: constant value LT.0.0: Load curve ID = (-YC) which defines the transverse compressive strength vs. transverse strain rate ( $\dot{\epsilon}_{bb}$ ).
YT	Transverse tensile strength, <i>b</i> -axis. GT.0.0: constant value LT.0.0: Load curve ID = (-YT) which defines the transverse tensile strength vs. transverse strain rate ( $\dot{\epsilon}_{bb}$ ).
SL	Longitudinal shear strength. GT.0.0: constant value LT.0.0: Load curve ID = (-SL) which defines the longitudinal shear strength vs. in-plane shear strain rate ( $\dot{\epsilon}_{ab}$ ).
FIO	Fracture angle in pure transverse compression (in degrees, default = 53.0).
SIGY	In-plane shear yield stress.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCSS	<p>Load curve ID or Table ID.</p> <p><b>Load Curve.</b> When LCSS is a Load curve ID, it defines the non-linear in-plane shear-stress as a function of in-plane shear-strain.</p> <p><b>Tabular Data.</b> The table maps in-plane strain rate values (<math>\dot{\epsilon}_{ab}</math>) to a load curve giving the in-plane shear-stress as a function of in-plane shear-strain. For strain rates below the minimum value, the curve for the lowest defined value of strain rate is used. Likewise, when the strain rate exceeds the maximum value, the curve for the highest defined value of strain rate is used.</p> <p><b>Logarithmically Defined Table.</b> If the <i>first</i> curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for <i>all</i> stress-strain curves. Since the tables are internally discretized to equally spaced points, natural logarithms are necessary, for example, if the curves correspond to rates from <math>10^{-4}</math> to <math>10^4</math>. Computing natural logarithms can substantially increase the computational time on certain computer architectures</p>
BETA	<p>Hardening parameter for in-plane shear plasticity (<math>0.0 \leq \text{BETA} \leq 1.0</math>).</p> <p>EQ.0.0: Pure kinematic hardening</p> <p>EQ.1.0: Pure isotropic hardening</p> <p><math>0.0 &lt; \text{BETA} &lt; 1.0</math>: mixed hardening.</p>
PFL	<p>Percentage of layers which must fail until crashfront is initiated. E.g. <math> \text{PFL}  = 80.0</math>, then 80% of layers must fail until strengths are reduced in neighboring elements. Default: all layers must fail. A single layer fails if 1 in-plane IP fails (<math>\text{PFL} &gt; 0</math>) or if 4 in-plane IPs fail (<math>\text{PFL} &lt; 0</math>).</p>
PUCK	<p>Flag for evaluation and post-processing of Puck's inter-fiber-failure criterion (IFF, see Puck, Kopp and Knops [2002]).</p> <p>EQ.0.0: no evaluation of Puck's IFF-criterion.</p> <p>EQ.1.0: Puck's IFF-criterion will be evaluated.</p>
SOFT	<p>Softening reduction factor for material strength in crashfront elements (default = 1.0).</p>



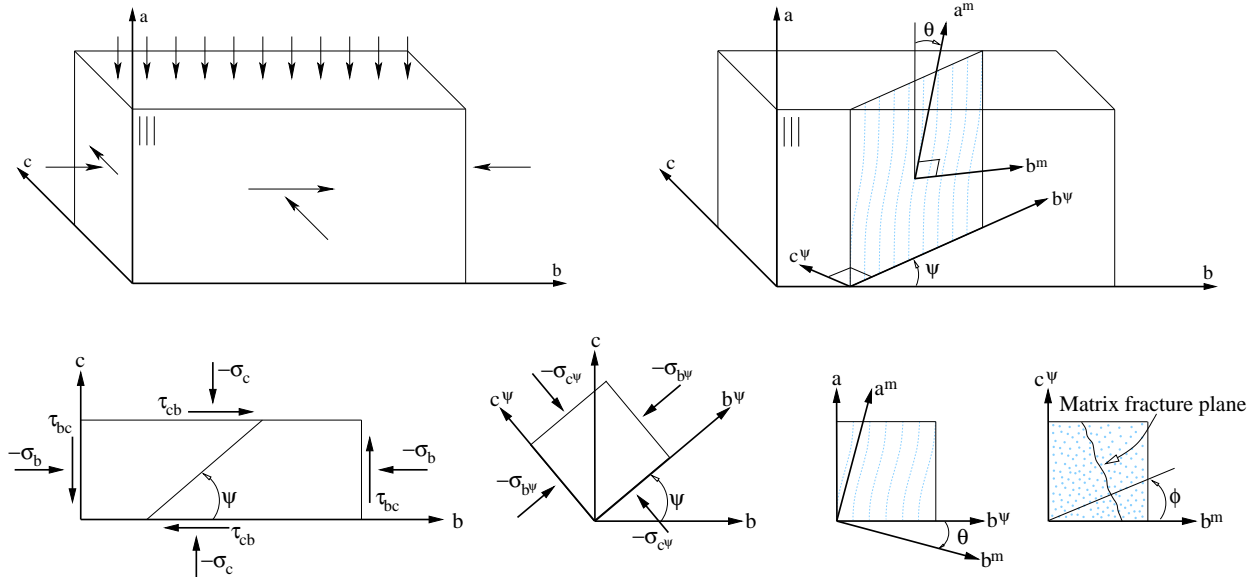


Figure M261-1. Definition of angles and stresses in fracture plane

VARIABLE	DESCRIPTION
DT	Strain rate averaging option. EQ.0.0: Strain rate is evaluated using a running average. LT.0.0: Strain rate is evaluated using average of last 11 time steps. GT.0.0: Strain rate is averaged over the last DT time units.

**Remarks: Failure Surfaces**

The failure surface to limit the elastic domain is assembled by *four* sub-surfaces, representing different failure mechanisms. See Figure M261-1 for definition of angles. They are defined as follows:

1. longitudinal (fiber) tension,

$$f_a = \frac{\sigma_a}{X_T} = 1$$

2. longitudinal (fiber) compression (3D-kinking model) – (transformation to fracture plane),

$$f_{kink} = \begin{cases} \left( \frac{\tau_T}{S_T - \mu_T \sigma_n} \right)^2 + \left( \frac{\tau_L}{S_L - \mu_L \sigma_n} \right)^2 = 1 & \text{if } \sigma_b^m \leq 0 \\ \left( \frac{\sigma_n}{Y_T} \right)^2 + \left( \frac{\tau_T}{S_T} \right)^2 + \left( \frac{\tau_L}{S_L} \right)^2 = 1 & \text{if } \sigma_b^m > 0 \end{cases}$$

with

$$S_T = \frac{Y_C}{2 \tan(\phi_0)} ; \quad \mu_T = -\frac{1}{\tan(2\phi_0)} ; \quad \mu_L = S_L \frac{\mu_T}{S_T}$$

$$\sigma_n = \frac{\sigma_{b^m} + \sigma_{c^\psi}}{2} + \frac{\sigma_{b^m} - \sigma_{c^\psi}}{2} \cos(2\phi) + \tau_{b^m c^\psi} \sin(2\phi)$$

$$\tau_T = -\frac{\sigma_{b^m} - \sigma_{c^\psi}}{2} \sin(2\phi) + \tau_{b^m c^\psi} \cos(2\phi)$$

$$\tau_L = \tau_{a^m b^m} \cos(\phi) + \tau_{c^\psi a^m} \sin(\phi)$$

3. transverse (matrix) failure: transverse tension,

$$f_{mat} = \left(\frac{\sigma_n}{Y_T}\right)^2 + \left(\frac{\tau_T}{S_T}\right)^2 + \left(\frac{\tau_L}{S_L}\right)^2 = 1 \quad \text{if } \sigma_n \geq 0$$

with

$$\sigma_n = \frac{\sigma_b + \sigma_c}{2} + \frac{\sigma_b - \sigma_c}{2} \cos(2\phi) + \tau_{bc} \sin(2\phi)$$

$$\tau_T = -\frac{\sigma_b - \sigma_c}{2} \sin(2\phi) + \tau_{bc} \cos(2\phi)$$

$$\tau_L = \tau_{ab} \cos(\phi) + \tau_{ca} \sin(\phi)$$

4. transverse (matrix) failure: transverse compression/shear,

$$f_{mat} = \left(\frac{\tau_T}{S_T - \mu_T \sigma_n}\right)^2 + \left(\frac{\tau_L}{S_L - \mu_L \sigma_n}\right)^2 = 1 \quad \text{if } \sigma_n < 0$$

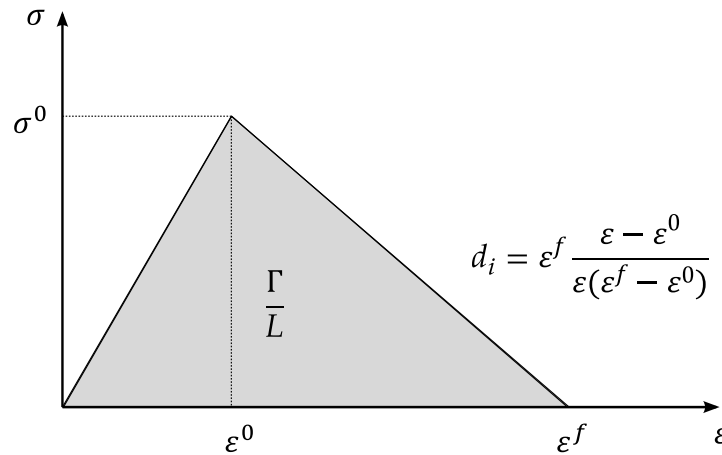


Figure M261-2. Damage evolution law

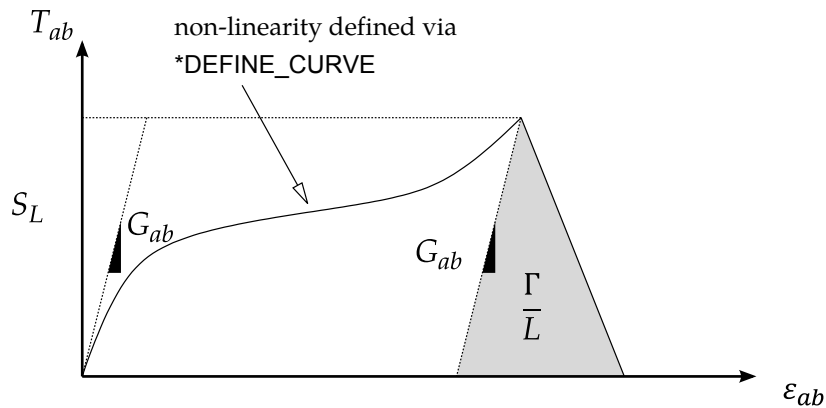


Figure M261-3. Definition of non-linear in-plane shear behavior

**Remarks: Damage Evolution:**

As long as the stress state is located within the failure surface the model behaves orthotropic elastic. When reaching the failure criteria the effective (undamaged) stresses will be reduced by a factor of  $(1 - d)$ , where the damage variable  $d$  represents one of the damage variables defined for the different failure mechanisms ( $d_{da}, d_{kink}, d_{mat}$ ). The growth of these damage variables is driven by a linear damage evolution law based on fracture toughnesses ( $\Gamma \rightarrow ENKINK, ENA, ENB, ENT, ENL$ ) and a characteristic internal element length,  $L$ , to account for objectivity. See [Figure M261-2](#).

**Remarks: Nonlinear In-Plane Shear:**

To account for the characteristic non-linear in-plane shear behavior of laminated fiber-reinforced composites a 1D elasto-plastic formulation is coupled to a linear damage behavior once the maximum allowable stress state for shear failure is reached. The non-linearity of the shear behavior can be introduced via the definition of an explicit shear

stress vs. engineering shear strain curve (LCSS) with \*DEFINE\_CURVE. See [Figure M261-3](#) (in which epsilon designates engineering shear strain rather than tensorial shear strain).

**Remarks: References:**

More detailed information about this material model can be found in Pinho, Iannucci and Robinson [2006].

**Remarks: Element Deletion:**

When failure has occurred in all the composite layers (through-thickness integration points), the element is deleted. Elements which share nodes with the deleted element become “crashfront” elements and can have their strengths reduced by using the SOFT parameter. An earlier initiation of crashfront elements is possible by using the parameter PFL.

**Remarks: History Variables:**

The number of additional integration point variables written to the LS-DYNA database is input by the \*DATABASE\_EXTENT\_BINARY definition with the variable NEIPS (shells) and NEIPH (solids). These additional variables are tabulated below ( $i$  = integration point):

History Variable	Description	Value	LS-PrePost history variable
$fa(i)$	fiber tensile mode	0 → 1: elastic 1: failure criterion reached	1
$fkink(i)$	fiber compressive mode		2
$fmat(i)$	matrix mode		3
			4
$da(i)$	damage fiber tension	0: elastic 1: fully damaged	5
$dkink(i)$	damage fiber compression		6
$dmat(i)$	damage transverse		7
			8
$dam(i)$	crashfront	-1: element intact 10 - 8: element in crashfront +1: element failed	9
$fmt\_p(i)$	tensile matrix mod (Puck criteria)	0 → 1: elastic 1: failure criterion reached	10
$fmc\_p(i)$	compressive matrix mode (Puck criteria)		11
$theta\_p(i)$	angle of fracture plane (radians, Puck criteria)		12
$e1dot(i)$	Averaged strain rate in longitudinal direction		54
$e2dot(i)$	Averaged strain rate in transverse direction		55
$e4dot(i)$	Averaged engineering shear strain rate in in-plane direction		56



Card 5	1	2	3	4	5	6	7	8
Variable	GXC	GXT	GYC	GYT	GSL	GXCO	GXT0	
Type	F	F	F	F	F	F	F	

Card 6	1	2	3	4	5	6	7	8
Variable	XC	XT	YC	YT	SL	XCO	XT0	
Type	F	F	F	F	F	F	F	

Card 7	1	2	3	4	5	6	7	8
Variable	FIO	SIGY	ETAN	BETA	PFL	PUCK	SOFT	DT
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density
EA	$E_a$ , Young's modulus in $a$ -direction (longitudinal)
EB	$E_b$ , Young's modulus in $b$ -direction (transverse)
EC	$E_c$ , Young's modulus in $c$ -direction
PRBA	$\nu_{ba}$ , Poisson's ratio $ba$
PRCA	$\nu_{ca}$ , Poisson's ratio $ca$
PRCB	$\nu_{cb}$ , Poisson's ratio $cb$
GAB	$G_{ab}$ , shear modulus $ab$
GBC	$G_{bc}$ , shear modulus $bc$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GCA	$G_{ca}$ , shear modulus $ca$
AOPT	<p>Material axes option (see MAT_OPTION TROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle MANGLE.</p> <p>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the <math>a</math>-direction. This option is for solid elements only.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle (MANGLE) from a line in the plane of the element defined by the cross product of the vector <math>v</math> with the element normal.</p> <p>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>v</math>, and an originating point, <math>p</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
DAF	<p>Flag to control failure of an integration point based on longitudinal (fiber) tensile failure:</p> <p>EQ.0.0: IP fails if any damage variable reaches 1.0.</p> <p>EQ.1.0: no failure of IP due to fiber tensile failure, <math>da(i)=1.0</math></p>



VARIABLE	DESCRIPTION
DKF	Flag to control failure of an integration point based on longitudinal (fiber) compressive failure: EQ.0.0: IP fails if any damage variable reaches 1.0. EQ.1.0: no failure of IP due to fiber compressive failure, $dkink(i) = 1.0$ .
DMF	Flag to control failure of an integration point based on transverse (matrix) failure: EQ.0.0: IP fails if any damage variable reaches 1.0. EQ.1.0: no failure of IP due to matrix failure, $dmat(i)=1.0$
EFS	Maximum effective strain for element layer failure. A value of unity would equal 100% strain. GT.0.0: fails when effective strain calculated assuming material is vol-ume preserving exceeds EFS. LT.0.0: fails when effective strain calculated from the full strain tensor exceeds  EFS .
XP YP ZP	Coordinates of point <b>p</b> for AOPT = 1 and 4.
A1 A2 A3	Define components of vector <b>a</b> for AOPT = 2.
V1 V2 V3	Define components of vector <b>v</b> for AOPT = 3.
D1 D2 D3	Define components of vector <b>d</b> for AOPT = 2.
MANGLE	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. MANGLE may be overridden on the element card, see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO.
GXC	Fracture toughness for longitudinal (fiber) compressive failure mode. GT.0.0: The given value will be regularized with the characteristic element length. LT.0.0: Load curve ID = (-GXC) which defines the fracture toughness for fiber compressive failure mode as a function of characteristic element length. No further regularization.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GXT	<p>Fracture toughness for longitudinal (fiber) tensile failure mode.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-GXT) which defines the fracture toughness for fiber tensile failure mode as a function of characteristic element length. No further regularization.</p>
GYC	<p>Fracture toughness for transverse compressive failure mode.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-GYC) which defines the fracture toughness for transverse compressive failure mode as a function of characteristic element length. No further regularization.</p>
GYT	<p>Fracture toughness for transverse tensile failure mode.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-GYT) which defines the fracture toughness for transverse tensile failure mode as a function of characteristic element length. No further regularization.</p>
GSL	<p>Fracture toughness for in-plane shear failure mode.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-GSL) which defines the fracture toughness for in-plane shear failure mode as a function of characteristic element length. No further regularization.</p>
GXCO	<p>Fracture toughness for longitudinal (fiber) compressive failure mode to define bi-linear damage evolution.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-GXCO) which defines the fracture toughness for fiber compressive failure mode to define bi-linear damage evolution as a function of characteristic element length. No further regularization.</p>

VARIABLE	DESCRIPTION
GXTO	<p>Fracture toughness for longitudinal (fiber) tensile failure mode to define bi-linear damage evolution.</p> <p>GT.0.0: The given value will be regularized with the characteristic element length.</p> <p>LT.0.0: Load curve ID = (-GXTO) which defines the fracture toughness for fiber tensile failure mode to define bi-linear damage evolution as a function of characteristic element length. No further regularization.</p>
XC	<p>Longitudinal compressive strength, <math>a</math>-axis (positive value).</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-XC) which defines the longitudinal compressive strength vs. longitudinal strain rate (<math>\dot{\epsilon}_{aa}</math>).</p>
XT	<p>Longitudinal tensile strength, <math>a</math>-axis.</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-XT) which defines the longitudinal tensile strength vs. longitudinal strain rate (<math>\dot{\epsilon}_{aa}</math>).</p>
YC	<p>Transverse compressive strength, <math>b</math>-axis (positive value).</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-YC) which defines the transverse compressive strength vs. transverse strain rate (<math>\dot{\epsilon}_{bb}</math>).</p>
YT	<p>Transverse tensile strength, <math>b</math>-axis.</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-YT) which defines the transverse tensile strength vs. transverse strain rate (<math>\dot{\epsilon}_{bb}</math>).</p>
SL	<p>Shear strength, <math>ab</math> plane.</p> <p>GT.0.0: constant value</p> <p>LT.0.0: Load curve ID = (-SL) which defines the longitudinal shear strength vs. in-plane shear strain rate (<math>\dot{\epsilon}_{ab}</math>).</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XCO	Longitudinal compressive strength at inflection point (positive value). GT.0.0: constant value LT.0.0: Load curve ID = (-XCO) which defines the longitudinal compressive strength at inflection point vs. longitudinal strain rate ( $\dot{\epsilon}_{aa}$ ).
XTO	Longitudinal tensile strength at inflection point. GT.0.0: constant value LT.0.0: Load curve ID = (-XTO) which defines the longitudinal tensile strength at inflection point vs. longitudinal strain rate ( $\dot{\epsilon}_{aa}$ ).
FIO	Fracture angle in pure transverse compression (in degrees, default = 53.0).
SIGY	In-plane shear yield stress. GT.0.0: constant value LT.0.0: Load curve ID = (-SIGY) which defines the in-plane shear yield stress vs. in-plane shear strain rate ( $\dot{\epsilon}_{ab}$ ).
ETAN	Tangent modulus for in-plane shear plasticity. GT.0.0: constant value LT.0.0: Load curve ID = (-ETAN) which defines the tangent modulus for in-plane shear plasticity vs. in-plane shear strain rate ( $\dot{\epsilon}_{ab}$ ).
BETA	Hardening parameter for in-plane shear plasticity ( $0.0 \leq \text{BETA} \leq 1.0$ ). EQ.0.0: Pure kinematic hardening EQ.1.0: Pure isotropic hardening $0.0 < \text{BETA} < 1.0$ : mixed hardening.
PFL	Percentage of layers which must fail until crashfront is initiated. E.g. $ \text{PFL}  = 80.0$ , then 80% of layers must fail until strengths are reduced in neighboring elements. Default: all layers must fail. A single layer fails if 1 in-plane IP fails ( $\text{PFL} > 0$ ) or if 4 in-plane IPs fail ( $\text{PFL} < 0$ ).

VARIABLE	DESCRIPTION
PUCK	Flag for evaluation and post-processing of Puck's inter-fiber-failure criterion (IFF, see Puck, Kopp and Knops [2002]). EQ.0.0: no evaluation of Puck's IFF-criterion. EQ.1.0: Puck's IFF-criterion will be evaluated.
SOFT	Softening reduction factor for material strength in crashfront elements (default = 1.0).
DT	Strain rate averaging option. EQ.0.0: Strain rate is evaluated using a running average. LT.0.0: Strain rate is evaluated using average of last 11 time steps. GT.0.0: Strain rate is averaged over the last DT time units.

**Remarks:**

The failure surface to limit the elastic domain is assembled by *four* sub-surfaces, representing different failure mechanisms. They are defined as follows:

1. longitudinal (fiber) tension,

$$\phi_{1+} = \frac{\sigma_{11} - \nu_{12}\sigma_{22}}{X_T} = 1$$

2. longitudinal (fiber) compression – (transformation to fracture plane),

$$\phi_{1-} = \frac{\langle |\sigma_{12}^m| + \mu_L \sigma_{22}^m \rangle}{S_L} = 1$$

with

$$\mu_L = -\frac{S_L \cos(2\phi_0)}{Y_C \cos^2(\phi_0)}$$

$$\sigma_{22}^m = \sigma_{11} \sin^2(\varphi^c) + \sigma_{22} \cos^2(\varphi^c) - 2|\sigma_{12}| \sin(\varphi^c) \cos(\varphi^c)$$

$$\sigma_{12}^m = (\sigma_{22} - \sigma_{11}) \sin(\varphi^c) \cos(\varphi^c) + |\sigma_{12}| (\cos^2(\varphi^c) - \sin^2(\varphi^c))$$

and

$$\varphi^c = \arctan \left[ \frac{1 - \sqrt{1 - 4 \left( \frac{S_L}{X_C} + \mu_L \right) \frac{S_L}{X_C}}}{2 \left( \frac{S_L}{X_C} + \mu_L \right)} \right]$$

3. transverse (matrix) failure: perpendicular to the laminate mid-plane,

$$\phi_{2+} = \begin{cases} \sqrt{(1-g)\frac{\sigma_{22}}{Y_T} + g\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\sigma_{12}}{S_L}\right)^2} = 1 & \sigma_{22} \geq 0 \\ \frac{\langle |\sigma_{12}| + \mu_L \sigma_{22} \rangle}{S_L} = 1 & \sigma_{22} < 0 \end{cases}$$

4. transverse (matrix) failure: transverse compression/shear,

$$\phi_{2-} = \sqrt{\left(\frac{\tau_T}{S_T}\right)^2 + \left(\frac{\tau_L}{S_L}\right)^2} = 1 \quad \text{if} \quad \sigma_{22} < 0$$

with

$$\mu_T = -\frac{1}{\tan(2\phi_0)}$$

$$S_T = Y_C \cos(\phi_0) \left[ \sin(\phi_0) + \frac{\cos(\phi_0)}{\tan(2\phi_0)} \right]$$

$$\theta = \arctan\left(\frac{-|\sigma_{12}|}{\sigma_{22} \sin(\phi_0)}\right)$$

$$\tau_T = \langle -\sigma_{22} \cos(\phi_0) [\sin(\phi_0) - \mu_T \cos(\phi_0) \cos(\theta)] \rangle$$

$$\tau_L = \langle \cos(\phi_0) [|\sigma_{12}| + \mu_L \sigma_{22} \cos(\phi_0) \sin(\theta)] \rangle$$

So long as the stress state is located within the failure surface the model behaves orthotropic elastic. The constitutive law is derived on basis of a proper definition for the ply complementary free energy density  $G$ , whose second derivative with respect to the stress tensor leads to the compliance tensor  $\mathbf{H}$

$$\mathbf{H} = \frac{\partial^2 G}{\partial \sigma^2} = \begin{bmatrix} \frac{1}{(1-d_1)E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{(1-d_2)E_2} & 0 \\ 0 & 0 & \frac{1}{(1-d_6)G_{12}} \end{bmatrix}, \quad \begin{aligned} d_1 &= d_{1+} \frac{\langle \sigma_{11} \rangle}{|\sigma_{11}|} + d_{1-} \frac{\langle -\sigma_{11} \rangle}{|\sigma_{11}|} \\ d_2 &= d_{2+} \frac{\langle \sigma_{22} \rangle}{|\sigma_{22}|} + d_{2-} \frac{\langle -\sigma_{22} \rangle}{|\sigma_{22}|} \end{aligned}$$

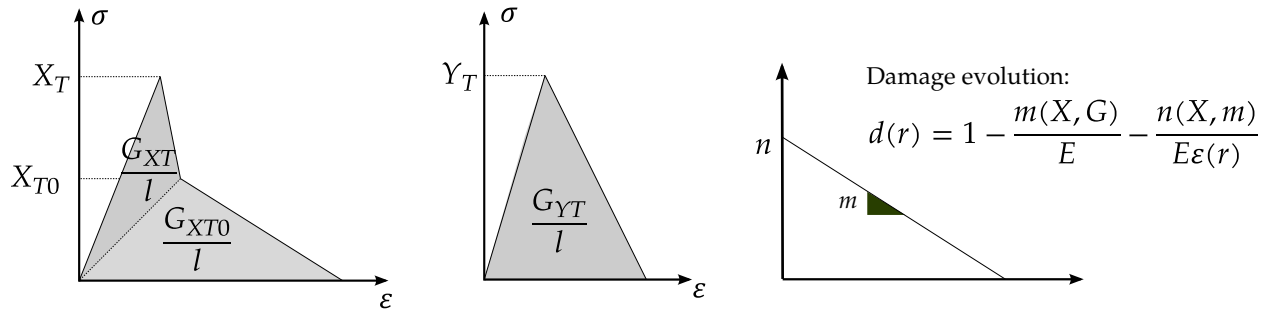


Figure M262-1. Damage evolution law

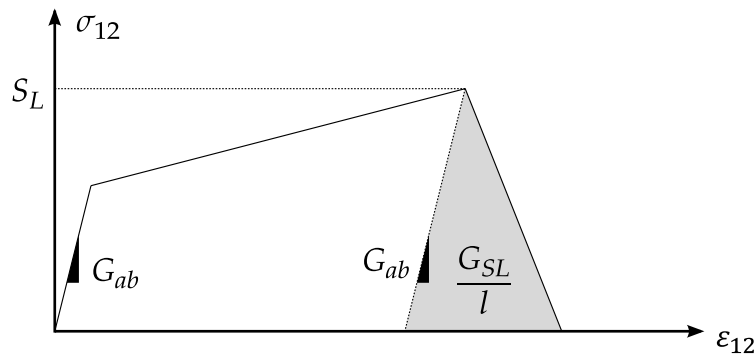


Figure M262-2. In-plane shear behavior

Once the stress state reaches the failure criterion a set of scalar damage variables ( $d_{1-}$ ,  $d_{1+}$ ,  $d_{2-}$ ,  $d_{2+}$ ,  $d_6$ ) is introduced associated with the different failure mechanisms. A bi-linear (longitudinal direction) and a linear (transverse direction) damage evolution law is utilized to define the development of the damage variables driven by the fracture toughness and a characteristic internal element length to account for objectivity. See [Figure M262-1](#).

To account for the characteristic non-linear in-plane shear behavior of laminated fiber-reinforced composites a 1D elasto-plastic formulation with linear hardening is coupled to a linear damage behavior once the maximum allowable stress state for shear failure is reached. See [Figure M262-2](#).

More detailed information about this material model can be found in Maimí, Camanho, Mayugo and Dávila [2007].

When failure has occurred in all the composite layers (through-thickness integration points), the element is deleted. Elements which share nodes with the deleted element become “crashfront” elements and can have their strengths reduced by using the SOFT parameter. An earlier initiation of crashfront elements is possible by using the parameter PFL.

The number of additional integration point variables written to the LS-DYNA database is input by the \*DATABASE\_EXTENT\_BINARY definition with the variable NEIPS (shells) and NEIPH (solids). These additional variables are tabulated below ( $i$  = integration point):

History Variable	Description	Value	LS-PrePost history variable
$\phi_{1+}(i)$	fiber tensile mode	0 → 1: elastic 1: failure criterion reached	1
$\phi_{1-}(i)$	fiber compressive		2
$\phi_{2+}(i)$	tensile matrix mode		3
$\phi_{2-}(i)$	compressive matrix mode		4
$d_{1+}(i)$	damage fiber tension	0: elastic 1: fully damaged	5
$d_{1-}(i)$	damage fiber compression		6
$d_2(i)$	damage transverse		7
$d_6(i)$	damage in-plane shear		8
dam( <i>i</i> )	crashfront	-1: element intact 10 - 8: element in crashfront +1: element failed	9
fmt_p( <i>i</i> )	tensile matrix mod (Puck criteria)	0 → 1: elastic 1: failure criterion reached	10
fmc_p( <i>i</i> )	compressive matrix mode (Puck criteria)		11
theta_p( <i>i</i> )	angle of fracture plane (radians, Puck criteria)		12



**\*MAT\_TABULATED\_JOHNSON\_COOK\_ORTHO\_PLASTICITY**

This is Material Type 264. This is an orthotropic elastic plastic material law with J3 dependent yield surface. This material considers tensile/compressive asymmetry in the material response, which is important for HCP metals like Titanium. The model is available for solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	CP	TR	BETA	NUMINT
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	1.0	1.0

Card 2	1	2	3	4	5	6	7	8
Variable	LCT00R	LCT00T	LCF	LCG	LCH	LCI		
Type	F	F	F	F	F	F		
Default	0	0	0	0	0	0		

Card 3	1	2	3	4	5	6	7	8
Variable	LCC00R	LCC00T	LCS45R	LCS45T		SFIEPM	NITER	AOPT
Type	F	F	F	F	F	F		F
Default	0	0	0	0	0	1	100	

Card 4	1	2	3	4	5	6	7	8
Variable	LCT90R	LCT45R	LCTTHR	LCC90R	LCC45R	LCCTHR		
Type	F	F	F	F	F	F		
Default	0	0	0	0	0	0		

Card 5	1	2	3	4	5	6	7	8
Variable	LCT90T	LCT45T	LCTTHT	LCC90T	LCC45T	LCCTHT		
Type	F	F	F	F	F	F		
Default	0	0	0	0	0	0		

Card 6	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	F	

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID                      Material identification. A unique number or label not exceeding 8 characters must be specified.

RO                        Mass density.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
E	Young's modulus: GT.0.0: constant value is used LT.0.0: temperature dependent Young's modulus given by load curve ID = -E
PR	Poisson's ratio.
CP	Specific heat.
TR	Room temperature.
BETA	Fraction of plastic work converted into heat.
NUMINT	Number of integration points which must fail before the element is deleted. EQ.-200: Turns off erosion for solids. Not recommended unless used in conjunction with *CONSTRAINED_TIED_NODES_FAILURE.
LCT00R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress versus plastic strain for that rate in the 00 degree direction.
LCT00T	Table ID defining for each temperature value a load curve ID giving the (quasi-static) tensile yield stress versus plastic strain for that temperature in the 00 degree direction.
LCF	Load curve ID or Table ID. The load curve ID defines plastic failure strain as a function of triaxiality. The table ID defines for each Lode parameter a load curve ID giving the plastic failure strain versus triaxiality for that Lode parameter. (Table option only for solids and not yet generally supported).
LCG	Load curve ID defining plastic failure strain as a function of plastic strain rate.
LCH	Load curve ID defining plastic failure strain as a function of temperature
LCI	Load curve ID or Table ID. The load curve ID defines plastic failure strain as a function of element size. The table ID defines for each triaxiality a load curve ID giving the plastic failure strain versus element size for that triaxiality.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCC00R	Table ID. The curves in this table define compressive yield stress as a function of plastic strain. The table ID defines for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress versus plastic strain for that rate in the 00 direction.
LCC00T	Table ID defining for each temperature value a load curve ID giving the (quasi-static) compressive yield stress versus strain for that temperature. The curves in this table define compressive yield stress as a function of plastic strain in the 00 direction.
LCS45R	Table ID. The load curves define shear yield stress in function of plastic strain. The table ID defines for each plastic strain rate value a load curve ID giving the (isothermal) shear yield stress versus plastic strain for that rate in the 45 degree direction.
LCS45T	Table ID defining for each temperature value a load curve ID giving the (quasi-static) shear yield stress versus strain for that temperature. The load curves define shear yield stress as a function of plastic strain or effective plastic strain (see IFLAG) in the 45 degree direction.
SFIEPM	Scale factor on the initial estimate of the plastic multiplier.
NITER	Maximum number of iterations for the plasticity algorithm

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see <i>MAT_OPTION TROPIC_ELASTIC</i> for a more complete description):</p> <p>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <i>*DEFINE_COORDINATE_NODES</i>, and then rotated about the shell element normal by an angle BETA.</p> <p>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with <i>*DEFINE_COORDINATE_VECTOR</i>.</p> <p>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <i>v</i> with the element normal.</p> <p>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on <i>*DEFINE_COORDINATE_NODES</i>, <i>*DEFINE_COORDINATE_SYSTEM</i> or <i>*DEFINE_COORDINATE_VECTOR</i>). Available with the R3 release of Version 971 and later.</p>
LCT90R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress versus plastic strain for that rate in the 90 degree direction.
LCT45R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress versus plastic strain for that rate in the 45 degree direction.
LCTTHR	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress versus plastic strain for that rate in the thickness degree direction.
LCC90R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress versus plastic strain for that rate in the 90 degree direction.
LCC45R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress versus plastic strain for that rate in the 45 degree direction.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCCTHR	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress versus plastic strain for that rate in the thickness degree direction.
LCT90T	Table ID defining for each temperature value a load curve ID giving the (quasistatic) tensile yield stress versus plastic strain for that rate in the 90 degree direction.
LCT45T	Table ID defining for each temperature value a load curve ID giving the (quasistatic) tensile yield stress versus plastic strain for that rate in the 45 degree direction.
LCTTHT	Table ID defining for each temperature value a load curve ID giving the (quasistatic) tensile yield stress versus plastic strain for that rate in the thickness degree direction.
LCC90T	Table ID defining for each temperature value a load curve ID giving the (quasistatic) compressive yield stress versus plastic strain for that rate in the 90 degree direction.
LCC45T	Table ID defining for each temperature value a load curve ID giving the (quasistatic) compressive yield stress versus plastic strain for that rate in the 45 degree direction.
LCCTHT	Table ID defining for each temperature value a load curve ID giving the (quasistatic) compressive yield stress versus plastic strain for that rate in the thickness degree direction.
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.
MACF	
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3.
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

**Remarks:**

If IFLAG = 0 the compressive and shear curves are defined as follows:

$$\sigma_c(\varepsilon_{pc}, \dot{\varepsilon}_{pc}), \quad \varepsilon_{pc} = \varepsilon_c - \frac{\sigma_c}{E}, \quad \dot{\varepsilon}_{pc} = \frac{\partial \varepsilon_{pc}}{\partial t}$$

$$\sigma_s(\gamma_{ps}, \dot{\gamma}_{ps}), \quad \gamma_{ps} = \gamma_s - \frac{\sigma_s}{G}, \quad \dot{\gamma}_{ps} = \frac{\partial \gamma_{ps}}{\partial t}$$

and two new history variables (#15 plastic strain in compression and #16 plastic strain in shear) are stored in addition to those history variables already stored in MAT\_224.

If IFLAG = 1 the compressive and shear curves are defined as follows:

$$\sigma_c(\dot{\lambda}, \lambda), \quad \sigma_s(\dot{\lambda}, \lambda), \quad \dot{W}_p = \sigma_{\text{eff}} \dot{\lambda}$$

History variables may be post-processed through additional variables. The number of additional variables for solids written to the d3plot and d3thdt databases is input by the optional \*DATABASE\_EXTENT\_BINARY card as variable NEIPH. The relevant additional variables of this material model are tabulated below:

<b>LS-PrePost history variable #</b>	<b>Solid elements</b>
5	<i>plastic strain rate</i>
6	<i>Compressive plastic strain</i>
7	<i>Shear plastic strain</i>
8	<i>plastic failure strain</i>
9	<i>triaxiality</i>
10	<i>Lode parameter</i>
11	<i>plastic work</i>
12	<i>ratio of plastic strain to plastic failure strain</i>
13	<i>element size</i>
14	<i>temperature</i>
15	<i>plastic strain in compression</i>
16	<i>plastic strain in shear</i>

**\*MAT\_TISSUE\_DISPERSSED**

This is Material Type 266. This material is an invariant formulation for dispersed orthotropy in soft tissues, e.g., heart valves, arterial walls or other tissues where one or two collagen fibers are used. The passive contribution is composed of an isotropic and two anisotropic parts. The isotropic part is a simple neo-Hookean model. The first anisotropic part is passive, with two collagen fibers to choose from: (1) a simple exponential model and (2) a more advanced crimped fiber model from Freed et al. [2005]. The second anisotropic part is active described in Guccione et al. [1993] and is used for active contraction.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	F	SIGMA	MU	KAPPA	ACT	INIT
Type	I	F	F	F	F	F	I	I

Card 2	1	2	3	4	5	6	7	8
Variable	FID	ORTH	C1	C2	C3	THETA	NHMOD	
Type	I	I	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	ACT1	ACT2	ACT3	ACT4	ACT5	ACT6	ACT7	ACT8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	ACT9	ACT10						
Type	F	F						

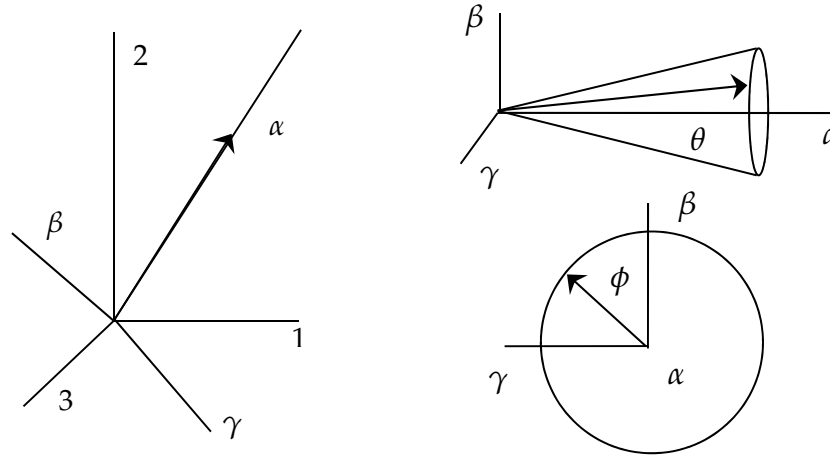


Card 5	1	2	3	4	5	6	7	8
Variable	AOPT	BETA	XP	YP	ZP	A1	A2	A3
Type	I	F	F	F	F	F	F	F

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number must be specified.
RO	Mass density.
F	Fiber dispersion parameter governs the extent to which the fiber dispersion extends to the third dimension. $F = 0$ and $F = 1$ apply to 2D splay with the normal to the membrane being in the $\beta$ and the $\gamma$ -directions, respectively (see <a href="#">Figure M266-1</a> ). $F = 0.5$ applies to 3D splay with transverse isotropy. Splay will be orthotropic whenever $F \neq 0.5$ . This parameter is ignored if $INIT = 1$ .
SIGMA	The parameter SIGMA governs the extent of dispersion, such that as SIGMA goes to zero, the material symmetry reduces to pure transverse isotropy. Conversely, as SIGMA becomes large, the material symmetry becomes isotropic in the plane. This parameter is ignored if $INIT = 1$ .
MU	MU is the isotropic shear modulus that models elastin. MU should be chosen such that the following relation is satisfied: $0.5 (3KAPPA - 2MU) / (3KAPPA + MU) < 0.5.$ Instability can occur for implicit simulations if this quotient is close to 0.5. A modest approach is a quotient between 0.495 and 0.497.
KAPPA	Bulk modulus for the hydrostatic pressure.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ACT	ACT = 1 indicates that an active model will be used that acts in the mean fiber-direction. The active model, like the passive model, will be dispersed by SIGMA and F, or if INIT = 1, with the *INITIAL_FIELD_SOLID keyword.
INIT	INIT = 1 indicates that the anisotropy eigenvalues will be given by *INITIAL_FIELD_SOLID variables in the global coordinate system (see Remark 1).
FID	The passive fiber model number. There are two passive models available: FID = 1 or FID = 2. They are described in Remark 2.
ORTH	ORTH specifies the number (1 or 2) of fibers used. When ORTH = 2 two fiber families are used and arranged symmetrically THETA degrees from the mean fiber direction and lying in the tissue plane.
C1-C3	Passive fiber model parameters.
THETA	The angle between the mean fiber direction and the fiber families. The parameter is active only if ORTH = 2 and is particularly important in vascular tissues (e.g. arteries)
NHMOD	Neo-Hooke model flag EQ.0.0: original implementation (modified Neo-Hooke) EQ.1.0: standard Neo-Hooke model (as in umat45 of dyn21.f)
ACT1 - ACT10	Active fiber model parameters. Note that ACT10 is an input for a time dependent load curve that overrides some of the ACTx values. See section 2 below.
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description): EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element nor-



**Figure M266-1.** The plot on the left relates the global coordinates (1, 2, 3) to the local coordinates  $(\alpha, \beta, \gamma)$ , selected so the mean fiber direction in the reference configuration is align with the  $\alpha$ -axis. The plots on the right show how the unit vector for a specific fiber within the fiber distribution of a 3D tissue is oriented with respect to the mean fiber direction via angles  $\theta$  and  $\phi$ .

VARIABLE	DESCRIPTION
	mal.
	LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available with the R3 release of Version 971 and later.
BETA	Material angle in degrees for AOPT = 3, may be overridden on the element card *ELEMANT_SOLID_ORTHO.
P1 - P3	P1, P2 and P3 define the coordinates of point P for AOPT = 1 and AOPT = 4.
A1 - A3	A1, A2 and A3 define the components of vector A for AOPT = 2.
D1 - D3	D1, D2 and D3 define components of vector D for AOPT = 2.
V1 - V3	V1, V2 and V3 define components of vector V for AOPT = 3 and AOPT = 4.

Details of the passive model can be found in Freed et al. (2005) and Einstein et al. (2005). The stress in the reference configuration consists of a deviatoric matrix term, a hydrostatic pressure term, and either one (ORTHO = 1) or two (ORTH = 2) fiber terms:

$$\mathbf{S} = \kappa J(J - 1)\mathbf{C}^{-1} + \mu J^{-2/3} \mathbf{DEV} \left[ \frac{1}{4} (\mathbf{I} - \bar{\mathbf{C}}^{-2}) \right] + J^{-2/3} \sum_{i=1}^n [\sigma_i(\lambda_i) + \varepsilon_i(\lambda_i)] \mathbf{DEV}[\mathbf{K}_i]$$

where  $\mathbf{S}$  is the second Piola-Kirchhoff stress tensor,  $J$  is the Jacobian of the deformation gradient,  $\kappa$  is the bulk modulus,  $\sigma_i$  is the passive fiber stress model used, and  $\varepsilon_i$  is the corresponding active fiber model used. The operator  $\mathbf{DEV}$  is the deviatoric projection:

$$\mathbf{DEV}[\bullet] = (\bullet) - \frac{1}{3} \text{tr}[(\bullet)\mathbf{C}]\mathbf{C}^{-1}$$

where  $\mathbf{C}$  is the right Cauchy-Green deformation tensor. The dispersed fourth invariant  $\lambda = \sqrt{\text{tr}[\mathbf{K}\bar{\mathbf{C}}]}$ , where  $\bar{\mathbf{C}}$  is the isochoric part of the Cauchy-Green deformation. Note that  $\lambda$  is not a stretch in the classical way, since  $\mathbf{K}$  embeds the concept of dispersion.  $\mathbf{K}$  is called the dispersion tensor or anisotropy tensor and is given in global coordinates. The passive and active fiber models are defined in the fiber coordinate system. In effect the dispersion tensor rotates and weights these one dimensional models, such that they are both three-dimensional and in the Cartesian framework.

In the case where, the splay parameters SIGMA and F are specified,  $\mathbf{K}$  is given by:

$$\mathbf{K}_i = \frac{1}{2} \mathbf{Q}_i \begin{bmatrix} 1 + e^{-2\text{SIGMA}^2} & 0 & 0 \\ 0 & F(1 - e^{-2\text{SIGMA}^2}) & 0 \\ 0 & 0 & (1 - F)(1 - e^{-2\text{SIGMA}^2}) \end{bmatrix} \mathbf{Q}_i^T$$

where  $\mathbf{Q}$  is the transformation tensor that rotates from the local to the global Cartesian system. In the case when INIT = 1, the dispersion tensor is given by

$$\mathbf{K}_i = \mathbf{Q}_i \begin{pmatrix} \chi_i^1 & 0 & 0 \\ 0 & \chi_i^2 & 0 \\ 0 & 0 & \chi_i^3 \end{pmatrix} \mathbf{Q}_i^T$$

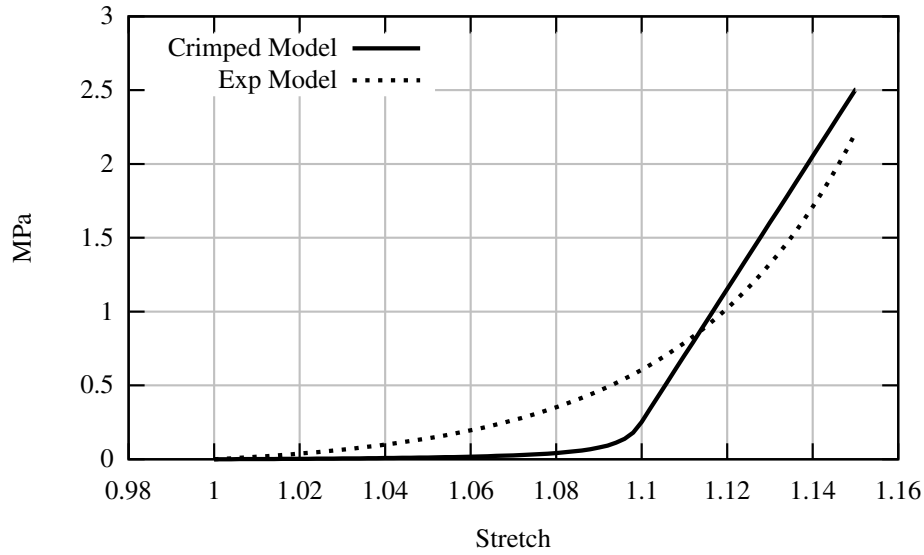
where the  $\chi$ :s are given on the \*INITIAL\_FIELD\_SOLID card. For the values to be physically meaningful  $\chi_i^1 + \chi_i^2 + \chi_i^3 = 1$ . It is the responsibility of the user to assure that this condition is met, no internal checking for this is done. These values typically come from diffusion tensor data taken from the myocardium.

**Remarks:**

1. Passive fiber models. Currently there are two models available.
  - a) If FID = 1 a crimped fiber model is used. It is solely developed for collagen fibers. Given  $H_0$  and  $R_0$  compute:

$$L_0 = \sqrt{(2\pi)^2 + (H_0)^2}, \Lambda = \frac{L_0}{H_0}$$

and



**Figure M266-2.** both the Crimped and the Exponential fiber models visualized. Here  $\Lambda = 1.1$  is the transition point in the crimped model.

$$E_s = \frac{E_f H_0}{H_0 + \left(1 + \frac{37}{6\pi^2} + 2\frac{L_0^2}{\pi^2}\right)(L_0 - H_0)}$$

Now if the fiber stretches  $\lambda < \Lambda$  the fiber stress is given by:

$$\sigma = \zeta E_s (\lambda - 1)$$

where

$$\zeta = \frac{6\pi^2(\Lambda^2 + (4\pi^2 - 1)\lambda^2)\lambda}{\Lambda(3H_0^2(\Lambda^2 - \lambda^2)(3\Lambda^2 + (8\pi^2 - 3)\lambda^2) + 8\pi^2(10\Lambda^2 + (3\pi^2 - 10)\lambda^2))}$$

and if  $\lambda > \Lambda$  the fiber stress equals:

$$\sigma = E_s(\lambda - 1) + E_f(\lambda - \Lambda).$$

In [Figure M266-1](#) the fiber stress is rendered with  $H_0 = 27.5$ ,  $R_0 = 2$  and the transition point becomes  $\Lambda = 1.1$ .

- b) The second fiber model available (FID = 2) is a simpler but more useful model for the general fiber reinforced rubber. The fiber stress is simply given by:

$$\sigma = C_1 \left[ e^{\frac{C_2}{2}(\lambda^2 - 1)} - 1 \right].$$

The difference between the two fiber models is given in [Figure M266-2](#).

The active model for myofibers (ACT = 1) is defined in Guccione et al. (1993) and is given by:

$$\sigma = T_{\max} \frac{Ca_0^2}{Ca_0^2 + ECa_{50}^2} C(t)$$

where

$$ECa_{50}^2 = \frac{(Ca_0)_{\max}}{\sqrt{e^{B(l_r\sqrt{2(\lambda-1)+1}-l_0)-1}}}$$

and  $B$  is a constant,  $(Ca_0)_{\max}$  is the maximum peak intracellular calcium concentration,  $l_0$  is the sarcomere length at which no active tension develops and  $l_r$  is the stress free sarcomere length. The function  $C(t)$  is defined in one of two ways. First it can be given as:

$$C(t) = \frac{1}{2} (1 - \cos\omega(t))$$

where

$$\omega = \begin{cases} \pi \frac{t}{t_0} & 0 \leq t < t_0 \\ \pi \frac{t - t_0 + t_r}{t_r} & t_0 \leq t < t_0 + t_r \\ 0 & t_0 + t_r \leq t \end{cases}$$

and  $t_r = ml_R\lambda + b$ . Secondly, it can also be given as a load curve. If a load curve should be used its index must be given in ACT10. Note that all variables that correspond to  $\omega$  are neglected if a load curve is used. The active parameters on Card 3 and 4 are interpreted as:

ACT1	ACT2	ACT3	ACT4	ACT5	ACT6	ACT7	ACT8	ACT9	ACT10
$T_{\max}$	$Ca_0$	$(Ca_0)_{\max}$	$B$	$l_0$	$t_0$	$m$	$b$	$l_R$	LCID

**References:**

1. Freed AD., Einstein DR. and Vesely I., Invariant formulation for dispersed transverse isotropy in aortic heart valves – An efficient means for modeling fiber splay, Biomechan model Mechanobiol, 4, 100-117, 2005.
2. Guccione JM., Waldman LK., McCulloch AD., Mechanics of Active Contraction in Cardiac Muscle: Part II – Cylindrical Models of the Systolic Left Ventricle, J. Bio Mech, 115, 82-90, 1993.

**\*MAT\_EIGHT\_CHAIN\_RUBBER**

This is Material Type 267. This is an advanced rubber-like model that is tailored for glassy polymers and similar materials. It is based on Arruda's eight chain model but enhanced with non elastic properties.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	MU	N	MULL	VISPL	VISEL
Type	I	F	F	F	I	I	I	I
Default	none	none	0.0	0.0	0	0	0	0

Card 2	1	2	3	4	5	6	7	8
Variable	YLD0	FP	GP	HP	LP	MP	NP	PMU
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Card 3	1	2	3	4	5	6	7	8
Variable	M1	M2	M3	M4	M5	TIME	VCON	
Type	F	F	F	F	F	F	F	
Default	See MULL	See MULL	See MULL	See MULL	See MULL	0.0	9.0	





Card 8-14	1	2	3	4	5	6	7	8
Variable	TAUi	BETAI						
Type	F	F						
Default	0.0	0.0						

**VARIABLE**

**DESCRIPTION**

MID	Material identification. A unique number must be specified
RO	Mass density.
K	<p>Bulk modulus. To get almost incompressible behavior set this to one or two orders of magnitude higher than MU. Note that the Poisson's ratio should be kept at a realistic value.</p> $v = \frac{3K - 2MU}{2(3K + MU)}$
MU	<p>Shear modulus. MU is the product of the number of molecular chains per unit volume (n), Boltzmann's constant (k) and the absolute temperature (T). Thus MU = nkT.</p>
MULL	<p>Parameter describing which softening algorithm that shall be used.</p> <p>EQ.1: Strain based Mullins effect from Qi and Boyce, see theory section below for details</p> <p>M1 = A (Qi recommends 3.5)  M2 = B (Qi recommends 18.0)  M3 = Z (Qi recommends 0.7)  M4 = vs (between 0 and 1 and less than vss)  M5 = vss (between 0 and 1 and greater than vs)</p> <p>EQ.2: Energy based Mullins, a modified version of Roxburgh and Ogden model. M1 &gt; 0, M2 &gt; 0 and M3 &gt; 0 must be set. See Theory section for details.</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
VISPL	<p>Parameter describing which viscoplastic formulation that should be used, see the theory section for details.</p> <p>EQ.0: No viscoplasticity.</p> <p>EQ.1: 2 parameters standard model, K1 and S1 must be set.</p> <p>EQ.2: 6 parameters G'Sells model, K1,K2,K3,S1,S2 and S3 must be set.</p> <p>EQ.3: 4 parameters Strain hardening model, K1,K2,S1,S2 must be set.</p>
VISEL	<p>Option for viscoelastic behavior, see the theory section for details.</p> <p>EQ.0: No viscoelasticity.</p> <p>EQ.1: Free energy formulation based on Holzapfel and Ogden.</p> <p>EQ.2: Formulation based on stiffness ratios from Simo et al.</p>
YLD0	<p>Initial yield stress.</p> <p>EQ.0.0: No plasticity</p> <p>GT.0.0: Initial yield stress:      Hardening is defined seperately.</p> <p>LT.0.0: -YLD0 is taken as the load curve ID for the yield stress versus effective plastic strain.</p>
FP-NP	<p>Parameters for Hill's general yield surface. For von mises yield criteria set <math>FP = GP = HP = 0.5</math> and <math>LP = MP = NP = 1.5</math>.</p>
PMU	<p>Kinematic hardening parameter. It is usually equal to MU.</p>
M1 - M5	<p>Mullins parameters</p> <p>MULL.EQ.1: M1 - M5 are used</p> <p>MULL.EQ.2: M1 - M3 are used.</p>
TIME	<p>A time filter that is used to smoothen out the time derivate of the strain invariant over a TIME interval. Default is no smoothening but a value <math>100 \cdot \text{TIMESTEP}</math> is recommended.</p>
VCON	<p>A material constant for the volumetric part of the strain energy. Default 9.0 but any value can be used to tailor the volumetric response. For example -2.</p>
Q1 - B4	<p>Voce hardening parameters</p>

VARIABLE	DESCRIPTION
K1 - S3	Viscoplastic parameters. VISPL.EQ.1: K1 and S1 are used. VISPL.EQ.2: K1, S1, K2, S2, K3 and S3 are used. VISPL.EQ.3: K1, S1 and K2 are used.
AOPT	Material axes option (see *MAT_OPTIONTROPIC_ELASTIC, *MAT_002) for a more complete description. EQ.0.0: Locally orthotropic with material axes defined by element nodes 1, 2 and 4. EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below. EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle THETA. The angle is defined from the line in the plane that is defined by the cross product of the vector v with the element normal. The plane of a solid is defined as the midsurface between the inner surface and the outer surface defined by the first 4 nodes and last 4 nodes. EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector v and an originating point P.
MACF	Material axes change flag EQ.1.0: No change (default) EQ.2.0: Switch axes a and b EQ.3.0: Switch axes a and c EQ.4.0: Switch axes b and c
XP, YP, ZP	Define coordinates for point P for AOPT = 1 and 4
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2.
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2
V1, V2, V3	Define components of vector <b>v</b> for AOPT = 3 and 4

VARIABLE	DESCRIPTION
TAUi	Relaxation time. A maximum of 6 values can be used.
BETAi / GAMMAi	VISEL.EQ.1: Dissipating energy factors.(see Holzapfel) VISEL.EQ.2: Gamma factors (see Simo)

**Basic theory:**

This model is based on the work done by Arruda and Boyce [1993], in particular Arruda’s thesis [1992]. The eight chain rubber model is based on hyper elasticity and it is formulated by using strain invariants. The strain softening is taken from work done by Qi and Boyce [2004], where the strain energy used is defined as

$$\Psi = v_s \mu \left[ \sqrt{N} \Lambda_c \beta + N \ln \left( \frac{\beta}{\sinh \beta} \right) \right] + \Psi_2 = \Psi_1 + \Psi_2,$$

where the amplified chain stretch is given by  $\Lambda_c = \sqrt{X(\bar{\lambda}^2 - 1) + 1}$  and

$$\beta = L^{-1} \left( \frac{\Lambda_c}{\sqrt{N}} \right),$$

where  $\bar{\lambda}^2 = I_1/3$ ,  $\mu$  is the initial modulus of the soft domain,  $N$  is the number of rigid links between crosslinks of the soft domain region.  $X = 1 + A(1 - v_s) + B(1 - v_s)^2$ , is a general polynomial describing the interaction between the soft and the hard phases (Qi and Boyce [2004] and Tobin and Mullins [1957]). The compressible behavior is described by the strain energy.

$$\Psi_2 = \frac{1}{v_{con}} \left( v_{con} \ln J + \frac{1}{J^{v_{con}}} - 1 \right)$$

Where  $J$  is the determinant of the elastic deformation gradient  $\mathbf{F}_e$ . The Cauchy stress is then computed as:

$$\sigma = \frac{2}{J} \mathbf{F}_e \frac{\partial \Psi}{\partial \mathbf{C}_e} \mathbf{F}_e^T = \frac{1}{J} \mathbf{F}_e (\mathbf{S}_1 + \mathbf{S}_2) \mathbf{F}_e^T = \frac{v_s X \mu \sqrt{N}}{3J \Lambda_c} L^{-1} \left( \frac{\Lambda_c}{\sqrt{N}} \right) \left( \mathbf{B}_e - \frac{1}{3} I_1 \mathbf{I} \right) + \frac{2K}{J v_{con}} \left( 1 - \frac{1}{J^{v_{con}}} \right)$$

where  $\mathbf{S}_1$  and  $\mathbf{S}_2$  are second Piola-Kirchhoff stresses based on  $\Psi_1$  and  $\Psi_2$  respectively.

**Mullins effect:**

Two models for the Mullins effect are implemented.

**1. MULL = 1**

The strain softening is developed by the evolution law taken from Boyce 2004:

$$\dot{v}_s = Z(v_{ss} - v_s) \frac{\sqrt{N} - 1}{(\sqrt{N} - \Lambda_c^{\max})^2} \dot{\Lambda}_c^{\max},$$

where  $Z$  is a parameter that characterizes the evolution in  $v_s$  with increasing  $\dot{\Lambda}_c^{\max}$ . The parameter  $v_{ss}$  is the saturation value of  $v_s$ . Note that  $\dot{\Lambda}_c^{\max}$  is the maximum of  $\dot{\Lambda}_c$  from the past:

$$\dot{\Lambda}_c^{\max} = \begin{cases} 0 & \Lambda_c < \Lambda_c^{\max} \\ \dot{\Lambda}_c & \Lambda_c > \Lambda_c^{\max} \end{cases}$$

The structure now evolves with the deformation. The dissipation inequality requires that the evolution of the structure is irreversible  $\dot{v}_s \geq 0$ . See Qi and Boyce [2004].

## 2. MULL = 3

The energy driven model based on Ogden and Roxburgh. When activated the strain energy is automatically transformed to a standard eight chain model. That is, the variables  $Z$ ,  $v_s$  and  $X$  is automatically set to 0, 1 and 1 respectively. The stress is multiplicative split of the true stress and the softening factor  $\eta$ .

$$\bar{\sigma} = \eta\sigma, \quad \eta = 1 - \frac{1}{M1} \operatorname{erf} \left( \frac{\Psi_1^{\max} - \Psi_1}{M3 - M2\Psi_1^{\max}} \right).$$

### Viscoelasticity:

#### 1. VISEL = 1

The viscoelasticity is based on work done by Holzapfel (2004)

$$\dot{\mathbf{Q}}_\alpha + \frac{\mathbf{Q}_\alpha}{\tau_\alpha} = 2\beta_\alpha \frac{d}{dt} \frac{\partial \Psi_1}{\partial \mathbf{C}_e} = \beta_\alpha \dot{\mathbf{S}}_1$$

where  $\alpha$  is the number of viscoelastic terms (0, 1, ..., 6).

#### 2. VISEL = 2

With this option the evolution is based on work done by Simo and Hughes (2000).

$$\dot{\mathbf{Q}}_\alpha + \frac{\mathbf{Q}_\alpha}{\tau_\alpha} = 2 \frac{\gamma_a}{\tau_a} \frac{d}{dt} \frac{\partial \Psi_1}{\partial \mathbf{C}_e} = \frac{\gamma_a}{\tau_a} \mathbf{S}_1$$

The the number of Prony terms is restricted to maximum 6 and  $\tau > 0, \gamma > 0$ .

The Cauchy stress is obtained by a push forward operation on the total second Piola-Kirchhoff stress.

$$\boldsymbol{\sigma} = \frac{1}{J} \mathbf{F}_e \mathbf{S} \mathbf{F}_e^T.$$

**Viscoplasticity:**

The plasticity is based on the general Hills' yield surface

$$\sigma_{\text{eff}}^2 = F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{12}^2 + 2M\sigma_{23}^2 + 2N\sigma_{13}^2$$

and the hardening is either based on a load curve ID (-YLD0) or an extended Voce hardening

$$\sigma_{\text{yld}} = \sigma_{\text{yld0}} + Q_1(1 - e^{B_1\bar{\epsilon}}) + Q_2(1 - e^{B_2\bar{\epsilon}}) + Q_3(1 - e^{B_3\bar{\epsilon}}) + Q_4(1 - e^{B_4\bar{\epsilon}}).$$

The yield criterion is written

$$f = \sigma_{\text{eff}} - \sigma_{\text{yld}} \leq 0.$$

Adding the viscoplastic phenomena, we simply add one evolution equation for the effective plastic strain rate. Three different formulations is available.

**1. VISPL = 1**

$$\dot{\epsilon}_{\text{vp}} = \left( \frac{f}{K_1} \right)^{S_1}.$$

where  $K_1$  and  $S_1$  are viscoplastic material parameters.

**2. VISPL = 2**

$$\dot{\epsilon}_{\text{vp}} = \left[ \frac{f}{K_1 (1 - e^{-S_1(\epsilon_{\text{vp}} + K_2)}) e^{S_2 \epsilon_{\text{vp}}^{K_3}}} \right]^{S_3}$$

Where  $K_1, K_2, K_3, S_1, S_2$  and  $S_3$  are viscoplastic parameters

**3. VISPL = 3**

$$\dot{\epsilon}_{\text{vp}} = \left( \frac{f}{K_1} \right)^{S_1} (\epsilon_{\text{vp}} + K_2)^{S_2}$$

Where  $K_1, K_2, S_1$  and  $S_2$  are viscoplastic parameters.

**Kinematic hardening:**

The back stress is calculated similar to the Cauchy stress above but without the softening factors:

$$\beta = \frac{\mu_p \sqrt{N}}{3J \Lambda_c} L^{-1} \left( \frac{\Lambda_c}{\sqrt{N}} \right) \left( \mathbf{I} - \frac{1}{3} I_p \mathbf{C}_p^{-1} \right)$$

$\mu_p$  is a hardening material parameter (PMU). The total Piola-Kirchhoff stress is now given by  $\mathbf{S}^* = \mathbf{S} - \beta$  and the total stress is given by a standard push forward operation with the elastic deformation gradient.

**Remarks:**

1. The parameter PMU is usually taken the same as MU.
2. For the case of a dilute solution the Mullins parameter A should be equal to 3.5. See Qi and Boyce [2004].
3. For a system with well dispersed particles B should somewhere around 18. See Qi and Boyce [2004].

**References:**

Qi HJ., Boyce MC., Constitutive model for stretch-induced softening of stress-stretch behavior of elastomeric materials, *Journal of the Mechanics and Physics of Solids*, 52, 2187-2205, 2004.

Arrude EM., Characterization of the strain hardening response of amorphous polymers, PhD Thesis, MIT, 1992.

Mullins L., Tobin NR., Theoretical model for the elastic behavior of filler reinforced vulcanized rubber, *Rubber Chem. Technol.*, 30, 555-571, 1957.

Ogden RW. Roxburgh DG., A pseudo-elastic model for the Mullins effect in Filled rubber., *Proc. R. Soc. Lond. A.*, 455, 2861-2877, 1999.

Simo JC., Hughes TJR., *Computational Inelasticity*, Springer, New York, 2000.

Holzapfel GA., *Nonlinear Solid Mechanics*, Wiley, New-York, 2000.

**\*MAT\_BERGSTROM\_BOYCE\_RUBBER**

This is material type 269. This is a rubber model based on the Arruda and Boyce (1993) chain model accompanied with a viscoelastic contribution according to Bergström and Boyce (1998). The viscoelastic treatment is based on the physical response of a single entangled chain in an embedded polymer gel matrix and the implementation is based on Dal and Kaliske (2009). This model is only available for solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	G	GV	N	NV	
Type	A8	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

Card 2	1	2	3	4	5	6	7	8
Variable	C	M	GAMO	TAUH				
Type	F	F	F	F				
Default	none	none	none	none				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
K	Elastic bulk modulus
G	Elastic shear modulus
GV	Viscoelastic shear modulus
N	Elastic segment number



VARIABLE	DESCRIPTION
NV	Viscoelastic segment number
C	Inelastic strain exponent, should be less than zero
M	Inelastic stress exponent
TAUH	Reference Kirchhoff stress

**Remarks:**

The deviatoric Kirchhoff stress for this model is the sum of an elastic and viscoelastic part according to

$$\bar{\boldsymbol{\tau}} = \boldsymbol{\tau}_e + \boldsymbol{\tau}_v$$

The elastic part is governed by the Arruda-Boyce strain energy potential resulting in the following expression (after a Pade approximation of the Langevin function)

$$\boldsymbol{\tau}_e = \frac{G}{3} \frac{3 - \lambda_r^2}{1 - \lambda_r^2} \left( \bar{\mathbf{b}} - \frac{Tr(\bar{\mathbf{b}})}{3} \mathbf{I} \right)$$

Here  $G$  is the elastic shear modulus,

$$\bar{\mathbf{b}} = J^{-2/3} \mathbf{F} \mathbf{F}^T$$

$$J = \det \mathbf{F}$$

is the unimodular left Cauchy-Green tensor, and

$$\lambda_r^2 = \frac{Tr(\bar{\mathbf{b}})}{3N}$$

is the relative network stretch.

The viscoelastic stress is based on a multiplicative split of the unimodular deformation gradient into unimodular elastic and inelastic parts, respectively,

$$J^{-1/3} \mathbf{F} = \mathbf{F}_e \mathbf{F}_i$$

and we define

$$\mathbf{b}_e = \mathbf{F}_e \mathbf{F}_e^T$$

to be the elastic left Cauchy-Green tensor. The viscoelastic stress is given as

$$\boldsymbol{\tau}_v = \frac{G_v}{3} \frac{3 - \lambda_v^2}{1 - \lambda_v^2} \left( \mathbf{b}_e - \frac{Tr(\mathbf{b}_e)}{3} \mathbf{I} \right)$$

where

$$\lambda_v^2 = \frac{Tr(\mathbf{b}_e)}{3N_v}$$

is the relative network stretch for the viscoelastic part. The evolution of the elastic left Cauchy-Green tensor can be written

$$\dot{\mathbf{b}}_e = \bar{\mathbf{L}}\mathbf{b}_e + \mathbf{b}_e\bar{\mathbf{L}}^T - 2\mathbf{D}_i\mathbf{b}_e$$

where the inelastic rate-of-deformation tensor is given as

$$\mathbf{D}_i = \dot{\gamma}_0 (\lambda_i - 0.999)^c \left( \frac{\|\boldsymbol{\tau}_v\|}{\hat{\tau}\sqrt{2}} \right)^m \frac{\boldsymbol{\tau}_v}{\|\boldsymbol{\tau}_v\|}$$

and

$$\bar{\mathbf{L}} = \mathbf{L} - \frac{Tr(\mathbf{L})}{3} \mathbf{I}$$

is the deviatoric velocity gradient. The stretch of a single chain relaxing in a polymer is linked to the inelastic right Cauchy-Green tensor as

$$\lambda_i^2 = \frac{Tr(\mathbf{F}_i^T \mathbf{F}_i)}{3} \geq 1,$$

and this stretch is available as the plastic strain variable in the post processing of this material. The volumetric part is elastic and governed by the bulk modulus, the pressure for this model is given as

$$p = K(J^{-1} - 1).$$

\*MAT\_CWM

This is material type 270. This is a thermo-elastic-plastic model with kinematic hardening that allows for material creation as well as annealing triggered by temperature. The acronym CWM stands for Computational Welding Mechanics, Lindström (2013, 2015), and the model is intended to be used for simulating multistage weld processes. This model is available for solid and shell elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	LCEM	LCPR	LCSY	LCHR	LCAT	BETA
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	None

Card 2	1	2	3	4	5	6	7	8
Variable	TASTART	TAEND	TLSTART	TLEND	EGHOST	PGHOST	AGHOST	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

**Optional Phase Change Card.**

Card 3	1	2	3	4	5	6	7	8
Variable	T2PHASE	T1PHASE						
Type	F	F						
Default	optional	optional						

**VARIABLE**

**DESCRIPTION**

MID

Material identification. A unique number or label not exceeding 8 characters must be specified.

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RO	Material density
LCEM	Load curve for Young's modulus as function of temperature
LCPR	Load curve for Poisson's ratio as function of temperature
LCSY	Load curve or table for yield stress. GT.0: Yield stress is a load curve as function of temperature. LT.0:  LCSY  is a table of yield curves for different temperatures. Each yield curve is a function of plastic strain.
LCHR	Load curve for hardening modulus as function of temperature. LCHR is not used for LCSY.LT.0. Hardening modulus is then calculated from yield curve slope.
LCAT	Load curve (or table) for thermal expansion coefficient as function of temperature (and maximum temperature up to current time). In the case of a table, load curves are listed according to their maximum temperature.
BETA	Fraction isotropic hardening between 0 and 1 EQ.0: Kinematic hardening EQ.1: Isotropic hardening
TASTART	Annealing temperature start
TAEND	Annealing temperature end
TLSTART	Birth temperature start
TLEND	Birth temperature end
EGHOST	Young's modulus for ghost (quiet) material
PGHOST	Poisson's ratio for ghost (quiet) material
AGHOST	Thermal expansion coefficient for ghost (quiet) material
T2PHASE	Temperature at which phase change commences
T1PHASE	Temperature at which phase change ends

**Remarks:**

This material is initially in a quiet state, sometimes referred to as a ghost material. In this state the material has the thermo-elastic properties defined by the quiet Young's modulus, quiet Poisson's ratio and quiet thermal expansion coefficient. These should represent void, i.e., the Young's modulus should be small enough to not influence the surroundings but large enough to avoid numerical problems. A quiet material stress should never reach the yield point. When the temperature reaches the birth temperature, a history variable representing the indicator of the welding material is incremented. This variable follows

$$\gamma(t) = \min \left( 1, \max \left[ 0, \frac{T_{\max} - T_l^{\text{start}}}{T_l^{\text{end}} - T_l^{\text{start}}} \right] \right)$$

where  $T_{\max} = \max_{s \leq t} T(s)$ . This parameter is available as history variable 9 in the output database. The effective thermo-elastic material properties are interpolated as

$$E = E(T)\gamma + E_{\text{quiet}}(1 - \gamma)$$

$$\nu = \nu(T)\gamma + \nu_{\text{quiet}}(1 - \gamma)$$

$$\alpha = \alpha(T, T_{\max})\gamma + \alpha_{\text{quiet}}(1 - \gamma)$$

where  $E$ ,  $\nu$ , and  $\alpha$  are the Young's modulus, Poisson's ratio and thermal expansion coefficient, respectively. Here, the thermal expansion coefficient is either a temperature dependent curve, or a collection of temperature dependent curves, ordered in a table according to maximum temperature  $T_{\max}$ . The stress update then follows a classical isotropic associative thermo-elastic-plastic approach with kinematic hardening that is summarized in the following. The explicit temperature dependence is sometimes dropped for the sake of clarity.

The stress evolution is given as

$$\dot{\sigma} = \mathbf{C}(\dot{\epsilon} - \dot{\epsilon}_p - \dot{\epsilon}_T)$$

where  $\mathbf{C}$  is the effective elastic constitutive tensor and

$$\dot{\epsilon}_T = \alpha \dot{T} \mathbf{I}$$

$$\dot{\epsilon}_p = \dot{\epsilon}_p \frac{3 \mathbf{s} - \boldsymbol{\kappa}}{2 \bar{\sigma}}$$

are the thermal and plastic strain rates, respectively. The latter expression includes the deviatoric stress

$$\mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3} \text{Tr}(\boldsymbol{\sigma}) \mathbf{I},$$

the back stress  $\boldsymbol{\kappa}$  and the effective stress

$$\bar{\sigma} = \sqrt{\frac{3}{2} (\mathbf{s} - \boldsymbol{\kappa}) : (\mathbf{s} - \boldsymbol{\kappa})}$$

that are involved in the plastic equations. To this end, the effective yield stress is given as

$$\sigma_Y = \sigma_Y(T) + \beta H(T) \varepsilon_p$$

and plastic strains evolve when the effective stress exceeds this value. The back stress evolves as

$$\dot{\kappa} = (1 - \beta) H(T) \dot{\varepsilon}_p \frac{\mathbf{s} - \kappa}{\bar{\sigma}}$$

where  $\dot{\varepsilon}_p$  is the rate of effective plastic strain that follows from consistency equations.

When the temperature reaches the start annealing temperature, the material starts assuming its virgin properties. Beyond the start annealing temperature it behaves as an ideal elastic-plastic material but with no evolution of plastic strains. The resetting of effective plastic properties in the annealing temperature interval is done by modifying the effective plastic strain and back stress before the stress update as

$$\varepsilon_p^{n+1} = \varepsilon_p^n \max \left[ 0, \min \left( 1, \frac{T - T_a^{\text{end}}}{T_a^{\text{start}} - T_a^{\text{end}}} \right) \right]$$

$$\kappa^{n+1} = \kappa^n \max \left[ 0, \min \left( 1, \frac{T - T_a^{\text{end}}}{T_a^{\text{start}} - T_a^{\text{end}}} \right) \right]$$

The optional Card 3 is used to set history variable 11, which is the average temperature rate by which the temperature has gone from T2PHASE to T1PHASE. To fringe this variable the range should be set to positive values since it is during the simulation temporarily used to store the time when the material has reached temperature T2PHASE and is then stored as a negative value. A strictly positive value means that the material has reached temperature T2PHASE and gone down to T1PHASE and the history variable is  $(T2PHASE - T1PHASE)/(T1 - T2)$ , where T2 is the time when temperature T2PHASE is reached and T1 is the time when temperature T1PHASE is reached. Note that  $T2PHASE > T1PHASE$  and  $T1 > T2$ . A value of zero means that the element has not yet reached temperature T2PHASE. A strictly negative value means that the element has reached temperature T2PHASE but not yet T1PHASE.

History variable	Description
1-6	Back stress
7	Temperature at last time step
8	Yield indicator: 1 if yielding, else 0
9	Welding material indicator: 0 for ghost material, else 1
10	Maximum temperature reached
11	Average temperature rate going from T2PHASE to T1PHASE

**\*MAT\_POWDER**

This is material type 271. This model is used to analyze the compaction and sintering of cemented carbides and the model is based on the works of Brandt (1998). This material is only available for solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	P11	P22	P33	P12	P23	P13
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	E0	LCK	PR	LCX	LCY	LCC	L	R
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 3	1	2	3	4	5	6	7	8
Variable	CA	CD	CV	P	LCH	LCFI	SINT	TZRO
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	0.0	none

**Sintering Card 1.** Additional card for SINT = 1.

Card 4	1	2	3	4	5	6	7	8
Variable	LCFK	LCFS2	DV1	DV2	DS1	DS2	OMEGA	RGAS
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

**Sintering Card 2.** Additional card for SINT = 1.

Card 5	1	2	3	4	5	6	7	8
Variable	LCPR	LCFS3	LCTAU	ALPHA	LCFS1	GAMMA	L0	LCFKS
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
PIJ	Initial compactness tensor $P_{ij}$
E0	Initial anisotropy variable $e$ (value between 1 and 2)
LCK	Load curve for bulk modulus $K$ as function of relative density $d$
PR	Poisson's ratio
LCX	Load curve for hydrostatic compressive yield $X$ as function of relative density $d$
LCY	Load curve for uniaxial compressive yield $Y$ as function of relative density $d$
LCC	Load curve for shear yield $C_0$ as function of relative density $d$



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L	Yield surface parameter L relating hydrostatic compressive yield to point on hydrostatic axis with maximum strength
R	Yield surface parameter R governing the shape of the yield surface
CA	Hardening parameter $c_a$
CD	Hardening parameter $c_d$
CV	Hardening parameter $c_v$
P	Hardening exponent $p$
LCH	Load curve giving back stress parameter H as function of hardening parameter $e$ .
LCFI	Load curve giving plastic strain evolution angle $\phi$ as function of relative volumetric stress.
SINT	Activate sintering EQ.0.0: Sintering off EQ.1.0: Sintering on
TZRO	Absolute zero temperature $T_0$
LCFK	Load curve $f_K$ for viscous compliance as function of relative density $d$
LCFS2	Load curve $f_{S2}$ for viscous compliance as function of temperature $T$
DV1	Volume diffusion coefficient $d_{V1}$
DV2	Volume diffusion coefficient $d_{V2}$
DS1	Surface diffusion coefficient $d_{S1}$
DS2	Surface diffusion coefficient $d_{S2}$
OMEGA	Blending parameter $\omega$
RGAS	Universal gas constant $R_{gas}$
LCPR	Load curve for viscous Poisson's ratio $\nu$ as function of relative density $d$

LCFS3	Load curve fS3 for evolution of mobility factor as function of temperature T
LCTAU	Load curve for relaxation time $\tau$ as function of temperature T
ALPHA	Thermal expansion coefficient $\alpha$
LCFS1	Load curve fS1 for sintering stress scaling as function of relative density d
GAMMA	Surface energy density $\gamma$ affecting sintering stress
L0	Grain size l0 affecting sintering stress
LCFKS	Load curve fKS scaling bulk modulus as function of temperature T

**Remarks:**

This model is intended to be used in two stages. During the first step the compaction of a powder specimen is simulated after which the results are dumped to file, and in a subsequent step the model is restarted for simulating sintering of the compacted specimen. In the following, an overview of the two different models is given, for a detailed description we refer to Brandt (1998). The progressive stiffening in the material during compaction makes it more or less necessary to run double precision and with constraint contacts to avoid instabilities, unfortunately this currently limitates the use of this material to the smp version of LS-DYNA.

The powder compaction model makes use of a multiplicative split of the deformation gradient into a plastic and elastic part according to

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_p$$

where the plastic deformation gradient maps the initial reference configuration to an intermediate relaxed configuration

$$\delta \tilde{\mathbf{x}} = \mathbf{F}_p \delta \mathbf{X}$$

and subsequently the elastic part maps this onto the current loaded configuration

$$\delta \mathbf{x} = \mathbf{F}_e \delta \tilde{\mathbf{x}}$$

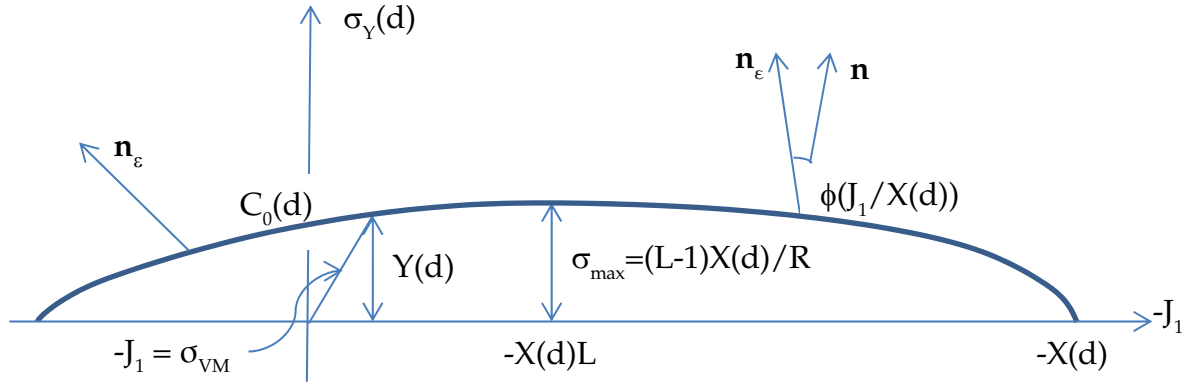
The compactness tensor is introduced that maps the intermediate configuration onto a virtual fully compacted configuration

$$\delta \bar{\mathbf{x}} = \mathbf{P} \delta \tilde{\mathbf{x}}$$

and we define the relative density as

$$d = \det \mathbf{P} = \frac{\rho}{\bar{\rho}}$$

where  $\rho$  and  $\bar{\rho}$  denotes the current and fully compacted density, respectively. The elastic properties depend highly on the relative density through the bulk modulus  $K(d)$  but the Poisson's ratio is assumed constant.



The yield surface is represented by two functions in the Rendulic plane according to

$$\sigma_Y(d) = \begin{cases} C_0(d) - C_1(d)J_1 - C_2(d)J_1^2 & J_1 \geq LX(d) \\ \frac{\sqrt{[(L-1)X(d)]^2 - [J_1 - LX(d)]^2}}{R} & J_1 < LX(d) \end{cases}$$

and is in this way capped in both compression and tension, here

$$J_1 = 3\sigma^m = Tr(\sigma).$$

The polynomial coefficients in the expression above are chosen to give continuity at  $J_1 = LX(d)$  and to give the uniaxial compressive strength  $Y(d)$ . Yielding is assumed to occur when the equivalent stress (note the definition) equals the yield stress

$$\sigma_{eq} = \frac{\sigma_{VM}}{\sqrt{3}} = \sqrt{\frac{1}{2} \mathbf{s} : \mathbf{s}} \leq \sigma_Y(d)$$

where

$$\mathbf{s} = \underbrace{\boldsymbol{\sigma} - \sigma^m \mathbf{I}}_{\boldsymbol{\sigma}^d} - \boldsymbol{\kappa}$$

in which the last term is the back stress to be dealt with below. The yield surface does not depend on the third stress invariant. The plastic flow is non-associated and its direction is given by

$$\mathbf{n}_\epsilon = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \mathbf{n}$$

where

$$\mathbf{n} = \frac{\sigma_Y(d)}{\sigma_{max}} \begin{pmatrix} \frac{\partial \sigma_Y}{\partial J_1} \\ 1 \end{pmatrix}$$

is the normal to the yield surface as depicted in the Rendulic plane above (note the sign of  $J_1$ ). The angle  $\phi$  is a function of and defined only for positive values of the relative

volumetric stress  $J_1/X(d) > 0$ , for negative values  $\phi$  is determined internally to achieve smoothness in the plastic flow direction and such that avoid numerical problems at the tensile cap point. The above equations are for illustrative purposes, from now on the plastic flow direction is generalized to a second order tensor. The plastic flow rule is then

$$\dot{\epsilon}_p = \dot{\lambda} \mathbf{n}_\epsilon, \quad \dot{\epsilon}_p^m = \frac{1}{3} \text{Tr}(\dot{\epsilon}_p), \quad \dot{\epsilon}_p^d = \dot{\epsilon}_p - \dot{\epsilon}_p^m \mathbf{I}$$

The evolution of the compactness tensor is directly related to the evolution of plastic strain as

$$\dot{\mathbf{P}} = -\frac{1}{2} (\dot{\epsilon}_p \mathbf{P} + \mathbf{P} \dot{\epsilon}_p)$$

and thus the relative density is given by

$$\dot{d} = -3\dot{\epsilon}_p^m d.$$

The back stress is assumed coaxial with the deviatoric part of the compactness tensor and given by

$$\boldsymbol{\kappa} = J_1 H(e) \left( \mathbf{P} - \frac{\text{Tr}(\mathbf{P})}{3} \mathbf{I} \right)$$

where  $e$  is a measure of intensity of anisotropy. This takes a value between 1 and 2 and evolves with plastic strain and plastic work according to

$$\dot{e} = c_a \sqrt{\frac{1}{2} \dot{\epsilon}_p^d : \dot{\epsilon}_p^d} - c_v J_1 \dot{\epsilon}_p^m W(d, J_1) + c_d \dot{\epsilon}_p^d : \boldsymbol{\sigma} W(d, J_1)$$

where

$$W(d, J_1) = - \left[ \frac{J_1}{X(d)} \right]^p \int_{d_0}^d \frac{X(\xi)}{3\xi} d\xi$$

and  $d_0$  is the density in the initial uncompressed configuration. The stress update is completed by the rate equation of stress

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}(d) : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}_p)$$

where  $\mathbf{C}(d)$  is the elastic constitutive matrix.

The sintering model is a thermo and viscoelastic model where the evolution of the mean and deviatoric stress can be written as

$$\dot{\sigma}^m = 3K^s (\dot{\epsilon}^m - \dot{\epsilon}_T - \dot{\epsilon}_p^m)$$

$$\dot{\sigma}^d = 2G^s (\dot{\epsilon}^d - \dot{\epsilon}_p^d)$$

The thermal strain rate is given by the thermal expansion coefficient as

$$\dot{\epsilon}_T = \alpha \dot{T}$$

and the bulk and shear modulus are the same as for the compaction model with the exception that they are scaled by a temperature curve

$$K^s = f_{KS}(T)K(d)$$

$$G^s = \frac{3(1 - 2\nu)}{2(1 + \nu)}K^s$$

The inelastic strain rates are different from the compaction model and is here given by

$$\dot{\epsilon}_p = \frac{\sigma^d}{2G^v} + \frac{\sigma^m - \sigma^s}{3K^v}\mathbf{I}$$

which results in a viscoelastic behavior depending on the viscous compliance and sintering stress. The viscous bulk compliance can be written

$$\frac{1}{K^v} = 3f_K(d) \left\{ d_{V1} \exp \left[ -\frac{d_{V2}}{R_{gas}(T - T_0)} \right] + \omega d_{S1} \exp \left[ -\frac{d_{S2}}{R_{gas}(T - T_0)} \right] \right\} [1 + f_{S2}(T)\xi]$$

from which the viscous shear compliance is modified with aid of the viscous Poisson's ratio

$$\frac{1}{G^v} = \frac{2[1 + \nu^v(d)]}{3[1 - 2\nu^v(d)]} \frac{1}{K^v}$$

The mobility factor  $\xi$  evolves with temperature according to

$$\dot{\xi} = \frac{f_{S3}(T)\dot{T} - \xi}{\tau(T)}$$

and the sintering stress is given as

$$\sigma^s = f_{S1}(d) \frac{\gamma}{l_0}$$

All this is accompanied with, again, the evolution of relative density given as

$$\dot{d} = -3\dot{\epsilon}_p^m d$$



<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
SHEAR	Elastic shear modulus
ONEMPA	Unit conversion factor defining 1 Mpa in the pressure units used. It can also be used for automatic generation of material parameters for a given compressive strength. (See remarks) EQ.0: Defaults to 1.0 EQ.-1: Parameters generated in m, s and kg (Pa) EQ.-2: Parameters generated in mm, s and tonne (MPa) EQ.-3: Parameters generated in mm, ms and kg (GPa) EQ.-4: Parameters generated in in, s and dozens of slugs (psi) EQ.-5: Parameters generated in mm, ms and g (MPa) EQ.-6: Parameters generated in cm, $\mu$ s and g (Mbar) EQ.-7: Parameters generated in mm, ms and mg (kPa)
EPSF	Eroding plastic strain (default is 2.0)
B0	Parameter for polynomial EOS
B1	Parameter for polynomial EOS
T1	Parameter for polynomial EOS
A	Failure surface parameter <i>A</i>
N	Failure surface parameter <i>N</i>
FC	Compressive strength.
FS*	Relative shear strength
FT*	Relative tensile strength
Q0	Lode angle dependence factor
B	Lode angle dependence factor
T2	Parameter for polynomial EOS

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
E0C	Reference compressive strain rate
E0T	Reference tensile strain rate
EC	Break compressive strain rate
ET	Break tensile strain rate
BETAC	Compressive strain rate dependence exponent (optional)
BETAT	Tensile strain rate dependence exponent (optional)
PTF	Pressure influence on plastic flow in tension (default is 0.001)
GC*	Compressive yield surface parameter
GT*	Tensile yield surface parameter
XI	Shear modulus reduction factor
D1	Damage parameter
D2	Damage parameter
EPM	Minimum damaged residual strain
AF	Residual surface parameter
NF	Residual surface parameter
GAMMA	Gruneisen gamma
A1	Hugoniot polynomial coefficient
A2	Hugoniot polynomial coefficient
A3	Hugoniot polynomial coefficient
PEL	Crush pressure
PCO	Compaction pressure
NP	Porosity exponent
ALPHA	Initial porosity



**Remarks:**

In the RHT model, the shear and pressure part is coupled in which the pressure is described by the Mie-Gruneisen form with a polynomial Hugoniot curve and a p- $\alpha$  compaction relation. For the compaction model, we define a history variable representing the porosity  $\alpha$  that is initialized to  $\alpha_0 > 1$ . This variable represents the current fraction of density between the matrix material and the porous concrete and will decrease with increasing pressure, i.e., the reference density is expressed as  $\alpha\rho$ . The evolution of this variable is given as

$$\alpha(t) = \max \left( 1, \min \left\{ \alpha_0, \min_{s \leq t} \left[ 1 + (\alpha_0 - 1) \left( \frac{p_{\text{comp}} - p(s)}{p_{\text{comp}} - p_{\text{el}}} \right)^N \right] \right\} \right)$$

where  $p(t)$  indicates the pressure at time  $t$ . This expression also involves the initial pore crush pressure  $p_{\text{el}}$ , compaction pressure  $p_{\text{comp}}$  and porosity exponent  $N$ . For later use, we define the cap pressure, or current pore crush pressure, as

$$p_c = p_{\text{comp}} - \left( p_{\text{comp}} - p_{\text{el}} \right) \left( \frac{\alpha - 1}{\alpha_0 - 1} \right)^{1/N}$$

The remainder of the pressure (EOS) model is given in terms of the porous density  $\rho$  and specific internal energy  $e$  (wrt the porous density). Depending on user inputs, it is either governed by ( $B_0 > 0$ )

$$p(\rho, e) = \frac{1}{\alpha} \begin{cases} (B_0 + B_1\eta)\alpha\rho e + A_1\eta + A_2\eta^2 + A_3\eta^3 & \eta > 0 \\ B_0\alpha\rho e + T_1\eta + T_2\eta^2 & \eta < 0 \end{cases}$$

or ( $B_0 = 0$ )

$$p(\rho, e) = \Gamma\rho e + \frac{1}{\alpha} p_H(\eta) \left[ 1 - \frac{1}{2}\Gamma\eta \right]$$

$$p_H(\eta) = A_1\eta + A_2\eta^2 + A_3\eta^3$$

together with

$$\eta(\rho) = \frac{\alpha\rho}{\alpha_0\rho_0} - 1.$$

For the shear strength description we use

$$p^* = \frac{p}{f_c}.$$

as the pressure normalized with the compressive strength parameter. We also use  $\mathbf{s}$  to denote the deviatoric stress tensor and  $\dot{\varepsilon}_p$  the plastic strain rate. The effective plastic strain is thus denoted  $\varepsilon_p$  and can be viewed as such in the post processor of choice.

For a given stress state and rate of loading, the elastic-plastic yield surface for the RHT model is given by

$$\sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, \varepsilon_p^*) = f_c \sigma_y^*(p^*, F_r(\dot{\varepsilon}_p, p^*), \varepsilon_p^*) R_3(\theta, p^*)$$

and is the composition of two functions and the compressive strength parameter  $f_c$ . The first describes the pressure dependence for principal stress conditions  $\sigma_1 < \sigma_2 = \sigma_3$  and is expressed in terms of a failure surface and normalized plastic strain as

$$\sigma_y^*(p^*, F_r, \varepsilon_p^*) = \sigma_f^* \left( \frac{p^*}{\gamma}, F_r \right) \gamma$$

with

$$\gamma = \varepsilon_p^* + (1 - \varepsilon_p^*) F_e F_c.$$

The failure surface is given as

$$\sigma_f^*(p^*, F_r) = \begin{cases} A \left[ p^* - \frac{F_r}{3} + \left( \frac{A}{F_r} \right)^{-1/n} \right]^n & 3p^* \geq F_r \\ \frac{F_r f_s^*}{Q_1} + 3p^* \left( 1 - \frac{f_s^*}{Q_1} \right) & F_r > 3p^* \geq 0 \\ \frac{F_r f_s^*}{Q_1} - 3p^* \left( \frac{1}{Q_2} - \frac{f_s^*}{Q_1 f_t^*} \right) & 0 > 3p^* > 3p_t^* \\ 0 & 3p_t^* > 3p^* \end{cases}$$

in which  $p_t^* = \frac{F_r Q_2 f_s^* f_t^*}{3(Q_1 f_t^* - Q_2 f_s^*)}$  is the failure cut-off pressure,  $F_r$  is a dynamic increment factor and

$$Q_1 = R_3 \left( \frac{\pi}{6}, 0 \right)$$

$$Q_2 = Q(p^*)$$

In these expressions,  $f_t^*$  and  $f_s^*$  are the tensile and shear strength of the concrete relative to the compressive strength  $f_c$  and the  $Q$  values are introduced to account for the tensile and shear meridian dependence. Further details are given in the following.

To describe reduced strength on shear and tensile meridian the factor

$$R_3(\theta, p^*) = \frac{2(1 - Q^2)\cos\theta + (2Q - 1)\sqrt{4(1 - Q^2)\cos^2\theta + 5Q^2 - 4Q}}{4(1 - Q^2)\cos^2\theta + (1 - 2Q)^2}$$

is introduced, where  $\theta$  is the Lode angle given by the deviatoric stress tensor  $\mathbf{s}$  as

$$\cos 3\theta = \frac{27 \det(\mathbf{s})}{2\bar{\sigma}(\mathbf{s})^3}$$

$$\bar{\sigma}(\mathbf{s}) = \sqrt{\frac{3}{2} \mathbf{s} : \mathbf{s}}.$$

The maximum reduction in strength is given as a function of relative pressure

$$Q = Q(p^*) = Q_0 + Bp^* .$$

Finally, the strain rate dependence is given by

$$F_r(\dot{\epsilon}_p, p^*) = \begin{cases} F_r^c & 3p^* \geq F_r^c \\ F_r^c - \frac{3p^* - F_r^c}{F_r^c + F_r^t f_t^*} (F_r^t - F_r^c) & F_r^c > 3p^* \geq -F_r^t f_t^* \\ F_r^t & -F_r^t f_t^* > 3p^* \end{cases}$$

in which

$$F_r^{\frac{c}{t}}(\dot{\epsilon}_p) = \begin{cases} \left(\frac{\dot{\epsilon}_p}{\dot{\epsilon}_0}\right)^{\beta_{c/t}} & \dot{\epsilon}_p^{c/t} \geq \dot{\epsilon}_p \\ \gamma_{\frac{c}{t}} \sqrt[3]{\dot{\epsilon}_p} & \dot{\epsilon}_p > \dot{\epsilon}_p^{c/t} \end{cases} .$$

The parameters involved in these expressions are given as ( $f_c$  is in MPa below)

$$\beta_c = \frac{4}{20 + 3f_c}$$

$$\beta_t = \frac{2}{20 + f_c}$$

and  $\gamma_{c/t}$  is determined from continuity requirements, but it is also possible to choose the rate parameters via inputs.

The elastic strength parameter used above is given by

$$F_e(p^*) = \begin{cases} g_c^* & 3p^* \geq F_r^c g_c^* \\ g_c^* - \frac{3p^* - F_r^c g_c^*}{F_r^c g_c^* + F_r^t g_t^* f_t^*} (g_t^* - g_c^*) & F_r^c g_c^* > 3p^* \geq -F_r^t g_t^* f_t^* \\ g_t^* & -F_r^t g_t^* f_t^* > 3p^* \end{cases}$$

while the cap of the yield surface is represented by

$$F_c(p^*) = \begin{cases} 0 & p^* \geq p_c^* \\ \sqrt{1 - \left(\frac{p^* - p_u^*}{p_c^* - p_u^*}\right)^2} & p_c^* > p^* \geq p_u^* \\ 1 & p_u^* > p^* \end{cases}$$

where

$$p_c^* = \frac{p_c}{f_c}$$

$$p_u^* = \frac{F_r^c g_c^*}{3} + \frac{G^* \epsilon_p}{f_c}$$

The hardening behavior is described linearly with respect to the plastic strain, where

$$\varepsilon_p^* = \min\left(\frac{\varepsilon_p}{\varepsilon_p^h}, 1\right)$$

$$\varepsilon_p^h = \frac{\sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, \varepsilon_p^*)(1 - F_e F_c)}{\gamma 3G^*}$$

here

$$G^* = \zeta G$$

where  $G$  is the shear modulus of the virgin material and  $\zeta$  is a reduction factor representing the hardening in the model.

When hardening states reach the ultimate strength of the concrete on the failure surface, damage is accumulated during further inelastic loading controlled by plastic strain. To this end, the plastic strain at failure is given as

$$\varepsilon_p^f = \begin{cases} D_1 [p^* - (1 - D)p_t^*]^{D_2} & p^* \geq (1 - D)p_t^* + \left(\frac{\varepsilon_p^m}{D_1}\right)^{1/D_2} \\ \varepsilon_p^m & (1 - D)p_t^* + \left(\frac{\varepsilon_p^m}{D_1}\right)^{1/D_2} > p^* \end{cases}$$

The damage parameter is accumulated with plastic strain according to

$$D = \int_{\varepsilon_p^h}^{\varepsilon_p} \frac{d\varepsilon_p}{\varepsilon_p^f}$$

and the resulting damage surface is given as

$$\sigma_d(p^*, \mathbf{s}, \dot{\varepsilon}_p) = \begin{cases} \sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, 1)(1 - D) + Df_c \sigma_r^*(p^*) & 0 \leq p^* \\ \sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, 1) \left(1 - D - \frac{p^*}{p_t^*}\right) & (1 - D)p_t^* \leq p^* < 0 \end{cases}$$

where

$$\sigma_r^*(p^*) = A_f \{p^*\}^{n_f}$$

Plastic flow occurs in the direction of deviatoric stress, i.e.,

$$\dot{\varepsilon}_p \sim \mathbf{s}$$

but for tension there is an option to set the parameter PFC to a number corresponding to the influence of plastic volumetric strain. If  $\lambda \leq 1$  is used to denote this parameter, then for the special case of  $\lambda = 1$

$$\dot{\varepsilon}_p \sim \mathbf{s} - p\mathbf{I}$$

This was introduced to reduce noise in tension that was observed on some test problems. A failure strain can be used to erode elements with severe deformation which by default is set to 200%.

For simplicity, automatic generation of material parameters is available via ONEMPA.LT.0, then no other parameters are needed. If FC.EQ.0 then the 35 MPa strength concrete in Riedel (2004) is generated in the units specified by the value of ONEMPA. For FC.GT.0 then FC specifies the actual strength of the concrete in the units specified by the value of ONEMPA. The other parameters are generated by interpolating between the 35 MPa and 140 MPa strength concretes as presented in Riedel (2004). Any automatically generated parameter may be overridden by the user if motivated, one of these parameters may be the initial porosity ALPHA0 of the concrete.

For post-processing, the following history variables may be of interest

- History variable #2 Internal energy per volume ( $\rho e$ )
- History variable #3 Porosity value ( $\alpha$ )
- History variable #4 Damage value ( $D$ )

or as an alternative use a material history list

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>		
<b>Label</b>	<b>Attributes</b>	<b>Description</b>
Damage	- - - -	Damage value $D$

**\*MAT\_CONCRETE\_DAMAGE\_PLASTIC\_MODEL****\*MAT\_CDPM**

This is material type 273. CDPM is a damage plastic concrete model based on work published in Grassl et al. (2011, 2013) and Grassl and Jirásek (2006). This model is aimed to simulations where failure of concrete structures subjected to dynamic loadings is sought. It describes the characterization of the failure process subjected to multi-axial and rate-dependent loading. The model is based on effective stress plasticity and with a damage model based on both plastic and elastic strain measures. This material model is available only for solids.

There are a lot of parameters for the advanced user but note that most of them have default values that are based on experimental tests. They might not be useful for all types of concrete and all types of load paths but they are values that can be used as a good starting point. If the default values are not good enough the theory chapter at the end of the parameter description can be of use.

History variables of interest are:

1 – kappa,  $\kappa$ , see equations below

15 – damage in tension,  $\omega_t$ , see equations below

16 – damage in compression,  $\omega_c$ , see equations below

More details on this material can be found on:

<http://petergrassl.com/Research/DamagePlasticity/CDPMLSDYNA/index.html>

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	ECC	QH0	FT	FC
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	0.2	AUTO	0.3	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	HP	AH	BH	CH	DH	AS	DF	FC0
Type	F	F	F	F	F	F	F	F
Default	0.5	0.08	0.003	2.0	1.0E-6	15.0	0.85	AUTO

Card 3	1	2	3	4	5	6	7	8
Variable	TYPE	BS	WF	WF1	FT1	STRFLG	FAILFLG	EFC
Type	F	F	F	F	F	F	F	F
Default	0.0	1.0	none	0.15*WF	0.3*FT	0.0	0.0	1.0E-4

**VARIABLE**

**DESCRIPTION**

MID Material identification. A unique number or label not exceeding 8 characters must be specified.

RO Mass density.

E Young’s modulus. The sign determines if an anisotropic (E positive, referred to as ISOFLAG = 0 in the remarks) or an isotropic (E negative, referred to as ISOFLAG = 1 in the remarks) damage formulation is used. The Young’s modulus is taken as the absolute value of this parameter.

PR Poissons ratio

ECC Eccentricity parameter.

EQ.0.0: ECC is calculated from Jirásek and Bazant (2002) as

$$ECC = \frac{1 + \epsilon}{2 - \epsilon}, \quad \epsilon = \frac{f_t(f_{bc}^2 - f_c^2)}{f_{bc}(f_c^2 - f_t^2)}, \quad f_{bc} = 1.16f_c$$

QH0 Initial hardening defined as FC<sub>0</sub>/FC where FC<sub>0</sub> is the compressive stress at which the initial yield surface is reached. Default = 0.3

FT	Uniaxial tensile strength (stress)
FC	Uniaxial compression strength (stress)
HP	Hardening parameter. Default is $HP = 0.5$ which is the value used in Grassl et al. (2011) for strain rate dependent material response ( $STRFLG = 1$ ). For applications without strain rate effect ( $STRFLG = 0$ ) a value of $HP = 0.01$ is recommended, which has been used in Grassl et al. (2013).
AH	Hardening ductility parameter 1
BH	Hardening ductility parameter 2
CH	Hardening ductility parameter 3
DH	Hardening ductility parameter 4
AS	Ductility parameter during damage
DF	Flow rule parameter
FC0	Rate dependent parameter. Only needed if $STRFLG = 1$ . Recommended value is 10 MPa, which has to be entered consistently with the system of units used.
TYPE	Flag for damage type. EQ.0.0: Linear damage formulation EQ.1.0: Bi-linear damage formulation EQ.2.0: Exponential damage formulation EQ.3.0: No damage  The best results are obtained with the bi-linear formulation.
BS	Damage ductility exponent during damage. Default = 1.0
WF	Tensile threshold value for linear damage formulation. Parameter controlling tensile softening branch for exponential tensile damage formulation.
WF1	Tensile threshold value for the second part of the bi-linear damage formulation. Default = $0.15 \times WF$



FT1	Tensile strength threshold value for bi-linear damage formulation. Default = $0.3 \times FT$
STRFLG	Strain rate flag. EQ.1.0: Strain rate dependent EQ.0.0: No strain rate dependency.
FAILFLG	Failure flag.  EQ.0.0: Not active $\Rightarrow$ No erosion.  GT.0.0: Active and element will erode if $w_t$ and $w_c$ is equal to 1 in FAILFLG percent of the integration points. If FAILFLG = 0.60, 60% of all integration points must fail before erosion.
EFC	Parameter controlling compressive damage softening branch in the exponential compressive damage formulation. Default = $1.0E-4$

**Remarks:**

The stress for the anisotropic damage plasticity model (E positive, ISOFLAG = 0) is defined as

$$\sigma = (1 - \omega_t)\sigma_t + (1 - \omega_c)\sigma_c$$

where  $\sigma_t$  and  $\sigma_c$  are the positive and negative part of the effective stress  $\sigma_{\text{eff}}$  determined in the principal stress space. The scalar functions  $\omega_t$  and  $\omega_c$  are damage parameters.

The stress for the isotropic damage plasticity model (E negative, ISOFLAG = 1) is defined as

$$\sigma = (1 - \omega_t)\sigma_{\text{eff}}$$

The effective stress  $\sigma_{\text{eff}}$  is defined according to the damage mechanics convention as

$$\sigma_{\text{eff}} = D_e : (\varepsilon - \varepsilon_p)$$

**Plasticity:**

The yield surface is described by the Haigh-Westergaard coordinates: the volumetric effective stress  $\sigma_v$ , the norm of the deviatoric effective stress  $\rho$  and the Lode angle  $\theta$ , and it is given by

$$f_p(\sigma_v, \rho, \theta, \kappa) = \left[ [1 - q_1(\kappa)] \left( \frac{\rho}{\sqrt{6}f_c} + \frac{\sigma_v}{f_c} \right)^2 + \sqrt{\frac{3\rho}{2f_c}} \right]^2 + m_0 q_1(\kappa)^2 q_2(\kappa) \left[ \frac{\rho}{\sqrt{6}f_c} r(\cos \theta) + \frac{\sigma_v}{f_c} \right] - q_1^2(\kappa) q_2^2(\kappa) .$$

The variables  $q_1$  and  $q_2$  are dependent on the hardening variable  $\kappa$ . The parameter  $f_c$  is the uniaxial compressive strength. The shape of the deviatoric section is controlled by the function

$$r(\cos \theta) = \frac{4(1 - e^2) \cos^2 \theta + (2e - 1)^2}{2(1 - e^2) \cos \theta + (2e - 1) \sqrt{4(1 - e^2) \cos^2 \theta + 5e^2 - 4e}}$$

where  $e$  is the eccentricity parameter (ECC). The parameter  $m_0$  is the friction parameter and it is defined as

$$m_0 = \frac{3(f_c^2 - f_t^2)}{f_c f_t} \frac{e}{e + 1}$$

where  $f_t$  is the tensile strength.

The flow rule is non-associative which means that the direction of the plastic flow is not normal to the yield surface. This is important for concrete since an associative flow rule would give an overestimated maximum stress. The plastic potential is given by

$$g(\sigma_v, \rho, \kappa) = \left\{ [1 - q_1(\kappa)] \left( \frac{\rho}{\sqrt{6}f_c} + \frac{\sigma_v}{f_c} \right)^2 + \sqrt{\frac{3\rho}{2f_c}} \right\}^2 + q_1(\kappa) \left( \frac{m_0 \rho}{\sqrt{6}f_c} + \frac{m_g(\sigma_v, \kappa)}{f_c} \right)$$

where

$$m_g(\sigma_v, \kappa) = A_g(\kappa) B_g(\kappa) f_c e^{\frac{\sigma_v - q_2 f_t / 3}{B_g f_c}}$$

and

$$A_g = \frac{3f_t q_2(\kappa)}{f_c} + \frac{m_0}{2}, \quad B_g = \frac{q_2(\kappa)}{3} \frac{1 + f_t/f_c}{\ln \frac{A_g}{3q_2 + \frac{m_0}{2}} + \ln \left( \frac{D_f + 1}{2D_f - 1} \right)}$$

The hardening laws  $q_1$  and  $q_2$  control the shape of the yield surface and the plastic potential, and they are defined as

$$\begin{aligned} q_1(\kappa) &= q_{h0} + (1 - q_{h0})(\kappa^3 - 3\kappa^2 + 3\kappa) - H_p(\kappa^3 - 3\kappa^2 + 2\kappa), & \kappa < 1 \\ q_1(\kappa) &= 1, & \kappa \geq 1 \\ q_2(\kappa) &= 1, & \kappa < 1 \\ q_2(\kappa) &= 1 + H_p(\kappa - 1), & \kappa \geq 1 \end{aligned}$$

The evolution for the hardening variable is given by

$$\dot{\kappa} = \frac{4\dot{\lambda} \cos^2 \theta}{x_h(\sigma_v)} \left\| \frac{d\mathbf{g}}{d\sigma} \right\|$$

It sets the rate of the hardening variable to the norm of the plastic strain rate scaled by a ductility measure which is defined below as

$$\begin{aligned} x_h(\sigma_v) &= A_h - (A_h - B_h)e^{-\frac{R_h}{C_h}}, & R_h \geq 0 \\ x_h(\sigma_v) &= E_h e^{\frac{R_h}{F_h}} + D_h, & R_h < 0 \end{aligned}$$

And finally

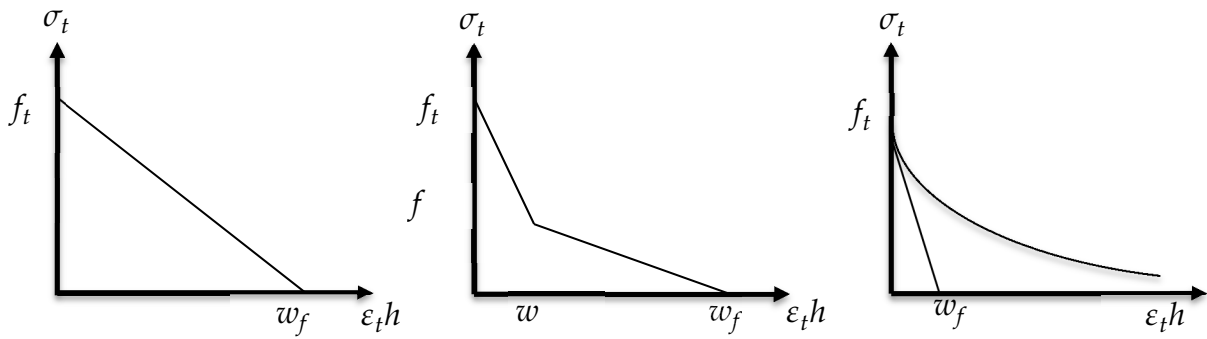
$$E_h = B_h - D_h, \quad F_h = \frac{(B_h - D_h)C_h}{A_h - B_h}$$

### Damage:

Damage is initialized when the equivalent strain  $\tilde{\epsilon}$  reaches the threshold value  $\epsilon_0 = f_t/E$  where the equivalent strain is defined as

$$\tilde{\epsilon} = \frac{\epsilon_0 m_0}{2} \left[ \frac{\rho}{\sqrt{6}f_c} r(\cos\theta) + \frac{\sigma_V}{f_c} \right] + \sqrt{\frac{\epsilon_0^2 m_0^2}{4} \left( \frac{\rho}{\sqrt{6}f_c} r(\cos\theta) + \frac{\sigma_V}{f_c} \right)^2 + \frac{3\epsilon_0^2 \rho^2}{2f_c^2}}$$

Tensile damage is described by a stress-inelastic displacement law. For linear and exponential damage type the stress value  $f_t$  and the displacement value  $w_f$  must be defined. For the bi-linear type two additional parameters  $f_{t1}$  and  $w_{f1}$  must be defined, see figure below how the stress softening is controlled by the input parameters.



The variable  $h$  is a mesh-dependent measure used to convert strains to displacements. The variable  $\epsilon_t$  is called the inelastic tensile strain and is defined as the sum of the irreversible plastic strain  $\epsilon_p$  and the reversible strain  $w_t(\epsilon - \epsilon_p)$  (in compression  $w_c(\epsilon - \epsilon_p)$ ). To get the influence of multi-axial stress states on the softening a damage ductility measure  $x_s$  is added:

$$x_s = 1 + (A_s - 1)R_s^{B_s}$$

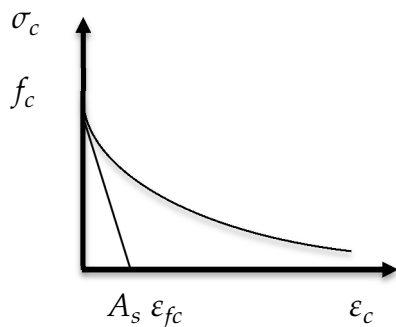
Where  $A_s$  and  $B_s$  are input parameters, and

$$R_s = -\frac{\sqrt{6}\sigma_v}{\rho}, \quad \sigma_v < 0 \text{ and } R_s = 0, \quad \sigma_v > 0$$

The inelastic strain is then modified according:

$$\epsilon_i = \frac{\epsilon_i}{x_s}$$

Compressive damage is controlled by an exponential stress-inelastic strain law. Stress value  $f_c$  and inelastic strain  $\epsilon_{fc}$  need to be specified, see figure below how the stress softening is controlled by the input parameters. A small value of  $\epsilon_{fc}$ , i.e. 1.0E-4 (default), provides for a rather brittle form of damage.



**Strain rate:**

Concrete is strongly rate dependent. If the loading rate is increased, the tensile and compressive strength increase and are more prominent in tension than in compression. The dependency is taken into account by an additional variable  $\alpha_r \geq 1$ . The rate dependency is included by scaling both the equivalent strain rate and the inelastic strain. The rate parameter is defined by

$$\alpha_r = (1 - X_{\text{compression}}) \alpha_{rt} + X_{\text{compression}} \alpha_{rc}$$

Where  $X_{\text{compression}}$  is continuous compression measure (= 1 means only compression, = 0 means only tension) and for tension we have

$$\alpha_{rt} = \begin{cases} 1 & \dot{\epsilon}_{\text{max}} < 30 \times 10^{-6} \text{ s}^{-1} \\ \left( \frac{\dot{\epsilon}_{\text{max}}}{\dot{\epsilon}_{t0}} \right)^{\delta_t} & 30 \times 10^{-6} < \dot{\epsilon}_{\text{max}} < 1 \text{ s}^{-1} \\ \beta_t \left( \frac{\dot{\epsilon}_{\text{max}}}{\dot{\epsilon}_{t0}} \right)^{\frac{1}{3}} & \dot{\epsilon}_{\text{max}} > 1 \text{ s}^{-1} \end{cases}$$

where  $\delta_t = \frac{1}{1+8f_c/f_{c0}}$ ,  $\beta_t = e^{6\delta_t-2}$  and  $\dot{\epsilon}_{t0} = 1 \times 10^{-6} \text{ s}^{-1}$ . For compression the corresponding rate factor is given by

$$\alpha_{rc} = \begin{cases} 1 & |\dot{\epsilon}_{\text{min}}| < 30 \times 10^{-6} \text{ s}^{-1} \\ \left[ S \frac{|\dot{\epsilon}_{\text{min}}|}{\dot{\epsilon}_{c0}} \right]^{1.026\delta_c} & 30 \times 10^{-6} < |\dot{\epsilon}_{\text{min}}| < 1 \text{ s}^{-1} \\ \beta_c \left[ \frac{|\dot{\epsilon}_{\text{min}}|}{\dot{\epsilon}_{c0}} \right]^{\frac{1}{3}} & |\dot{\epsilon}_{\text{min}}| > 30 \text{ s}^{-1} \end{cases}$$

where  $\delta_c = \frac{1}{5+9f_c/f_{c0}}$ ,  $\beta_c = e^{6.156\delta_c-2}$  and  $\dot{\epsilon}_{c0} = 30 \times 10^{-6} \text{ s}^{-1}$ . The parameter  $f_{c0}$  is an input parameter. A recommended value is 10MPa.





**Orthotropic Parameter Card 1.**

Card 7	1	2	3	4	5	6	7	8
Variable	AOPT	MACF	XP	YP	ZP	A1	A2	A3
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

**Orthotropic Parameter Card 2.**

Card 8	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Material density
$E_i$	Young's modulus in direction $E_i$ .
$PR_{ij}$	Elastic Poisson's ratio $\nu_{ij}$ .
$G_{ij}$	Elastic shear modulus in direction $G_{ij}$ .
E3C	Elastic compression parameter.
CC	Elastic compression exponent.
TWOK	Exponent in in-plane yield surface.



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ROT	Option for 2D-solids (shell element form 13,14,15): EQ.0.0: No rotation of material axes (default). Direction of material axes are solely defined by AOPT and it is only possible to rotate in shell-plane. EQ.1.0: Rotate coordinate system around material 1-axis such that 2-axis coincides with shell normal. This rotation is done in addition to AOPT. EQ.2.0: Rotate coordinate system around material 2-axis such that 1-axis coincides with shell normal. This rotation is done in addition to AOPT.
$S0i$	$i^{\text{th}}$ in-plane plasticity yield parameter. If $S0i < 0$ the absolute value of $S0i$ is a curve number, see remarks.
$A0i$	$i^{\text{th}}$ in-plane plasticity hardening parameter.
$B0i$	$i^{\text{th}}$ in-plane plasticity hardening parameter.
$C0i$	$i^{\text{th}}$ in-plane plasticity hardening parameter.
PRP1	Tensile plastic Poisson's ratio in direction 1.
PRP2	Tensile plastic Poisson's ratio in direction 2.
PRP4	Compressive plastic Poisson's ratio in direction 1.
PRP5	Compressive plastic Poisson's ratio in direction 2.
ASIG	Out-of-plane plasticity yield parameter.
BSIG	Out-of-plane plasticity hardening parameter.
CSIG	Out-of-plane plasticity hardening parameter.
TAU0	Transverse shear plasticity yield parameter.
ATAU	Transverse shear plasticity hardening parameter.
BTAU	Transverse shear plasticity hardening parameter.

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AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description): <ul style="list-style-type: none"><li>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</li><li>EQ.1.0: locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. This option is for solid elements only.</li><li>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</li><li>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <math>\mathbf{v}</math> with the element normal.</li><li>EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>p</math>, which define the centerline axis. This option is for solid elements only.</li><li>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</li></ul>
MACF	Material axes change flag for brick elements: <ul style="list-style-type: none"><li>EQ.1: No change, default,</li><li>EQ.2: switch material axes a and b,</li><li>EQ.3: switch material axes a and c,</li><li>EQ.4: switch material axes b and c.</li></ul>
XP, YP, ZP	Define coordinates of point $\mathbf{p}$ for AOPT = 1 and 4.
A1, A2, A3	Define components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Define components of vector $\mathbf{v}$ for AOPT = 3 and 4.
D1, D2, D3	Define components of vector $\mathbf{d}$ for AOPT = 2.



$$N_1 = \left[ \frac{1}{\sqrt{1 + v_{1p}^2}} \quad -\frac{v_{1p}}{\sqrt{1 + v_{1p}^2}} \quad 0 \quad 0 \quad 0 \quad 0 \right]^T,$$

$$N_2 = \left[ -\frac{v_{2p}}{\sqrt{1 + v_{2p}^2}} \quad \frac{1}{\sqrt{1 + v_{2p}^2}} \quad 0 \quad 0 \quad 0 \quad 0 \right]^T,$$

$$N_3 = [0 \quad 0 \quad 0 \quad \sqrt{2} \quad 0 \quad 0]^T,$$

$$N_4 = -\left[ \frac{1}{\sqrt{1 + v_{4p}^2}} \quad -\frac{v_{4p}}{\sqrt{1 + v_{4p}^2}} \quad 0 \quad 0 \quad 0 \quad 0 \right]^T,$$

$$N_5 = -\left[ -\frac{v_{5p}}{\sqrt{1 + v_{5p}^2}} \quad \frac{1}{\sqrt{1 + v_{5p}^2}} \quad 0 \quad 0 \quad 0 \quad 0 \right]^T,$$

$$N_6 = -N_3.$$

The yield planes describe the following states


Each hardening function  $q_i$  (note that  $q_6 = q_3$ ) is given by a load curve if  $S_i^0 < 0$ , otherwise

$$q_i(\varepsilon_p^f) = S_i^0 + A_i^0 \tanh(B_i^0 \varepsilon_p^f) + C_i^0 \varepsilon_p^f.$$

The out-of-plane surface is given as

$$g = \frac{-S_{33}}{A_\sigma + B_\sigma \exp(-C_\sigma \varepsilon_p^g)} - 1 \leq 0,$$

and the transverse shear surface is

$$h = \frac{\sqrt{S_{13}^2 + S_{23}^2}}{\tau_0 + [A_\tau - \min(0, S_{33}) B_\tau] \varepsilon_p^h} - 1 \leq 0.$$

The flow rule is given by the evolution of the plastic deformation gradient

$$\dot{\mathbf{F}}_p = \mathbf{L}_p \mathbf{F}_p,$$

where the plastic velocity gradient is given as

$$\mathbf{L}_p = \begin{bmatrix} \dot{\varepsilon}_p^f \frac{\partial f}{\partial S_{11}} & \dot{\varepsilon}_p^f \frac{\partial f}{\partial S_{12}} & \dot{\varepsilon}_p^h \frac{\partial h}{\partial S_{13}} \\ \dot{\varepsilon}_p^f \frac{\partial f}{\partial S_{12}} & \dot{\varepsilon}_p^f \frac{\partial f}{\partial S_{22}} & \dot{\varepsilon}_p^h \frac{\partial h}{\partial S_{23}} \\ \dot{\varepsilon}_p^h \frac{\partial h}{\partial S_{13}} & \dot{\varepsilon}_p^h \frac{\partial h}{\partial S_{23}} & \dot{\varepsilon}_p^g \frac{\partial g}{\partial S_{33}} \end{bmatrix},$$

and where it is implicitly assumed that the involved derivatives in the expression of the velocity gradient is appropriately normalized.

The stress-strain relationship for shell elements is based on an additive split of the rate of deformation into an elastic and a plastic part

$$\mathbf{D} = \mathbf{D}_e + \mathbf{D}_p,$$

and the rate of Cauchy stress is given by

$$\dot{\boldsymbol{\sigma}} = \mathbf{C} \mathbf{D}_e.$$

In out-of-plane compression the stress rate is modified according to

$$\dot{\sigma}_{33} = C_{31} D_{11}^e + C_{32} D_{22}^e + D_{33}^e \begin{cases} E_3, & \varepsilon_{33}^e \geq 0, \\ E_3^c \exp(-C_c \varepsilon_{33}^e), & \varepsilon_{33}^e < 0. \end{cases}$$

For shell elements,  $D_{33}^p = 0$ , and only two yield surfaces are present: the in-plane yield surface

$$f = \sum_{i=1}^6 \left[ \frac{\max(0, \sigma : N_i)}{q_i(\varepsilon_p^f)} \right]^{2k} - 1 \leq 0,$$

and the transverse-shear yield surface

$$h = \frac{\sqrt{\sigma_{13}^2 + \sigma_{23}^2}}{\tau_0 + [A_\tau - \min(0, \sigma_{33}) B_\tau] \varepsilon_p^h} - 1 \leq 0,$$

and the plastic flow rule is given by

$$\dot{\boldsymbol{\varepsilon}}_p = \mathbf{D}_p = \mathbf{L}_p,$$

where the plastic velocity gradient is given as

$$\mathbf{L}_p = \begin{bmatrix} \dot{\epsilon}_p^f \frac{\partial f}{\partial \sigma_{11}} & \dot{\epsilon}_p^f \frac{\partial f}{\partial \sigma_{12}} & \dot{\epsilon}_p^h \frac{\partial h}{\partial \sigma_{13}} \\ \dot{\epsilon}_p^f \frac{\partial f}{\partial \sigma_{12}} & \dot{\epsilon}_p^f \frac{\partial f}{\partial \sigma_{22}} & \dot{\epsilon}_p^h \frac{\partial h}{\partial \sigma_{23}} \\ \dot{\epsilon}_p^h \frac{\partial h}{\partial \sigma_{13}} & \dot{\epsilon}_p^h \frac{\partial h}{\partial \sigma_{23}} & 0 \end{bmatrix}.$$

**History variables:**

History variables 1 to 3 show  $\epsilon_p^f$ ,  $\epsilon_p^g$  and  $\epsilon_p^h$ , respectively. The *Effective Plastic Strain* is

$$\epsilon_p = \sqrt{(\epsilon_p^f)^2 + (\epsilon_p^g)^2 + (\epsilon_p^h)^2}$$

**\*MAT\_SMOOTH\_VISCOELASTIC\_VISCOPLASTIC**

This is Material Type 275, a smooth viscoelastic viscoplastic model based on the works of Hollenstein et.al. [2013, 2014] and Jabareen [2015]. The stress response is rheologically represented by HJR (Hollenstein-Jabareen-Rubin) elements in parallel, see [Figure 0-1](#), where each element exhibits combinations of viscoelastic and viscoplastic characteristics. The model is based on large displacement hyper-elastoplasticity and the numerical implementation is strongly objective, this together with the smooth characteristics makes it especially suitable for implicit analysis.

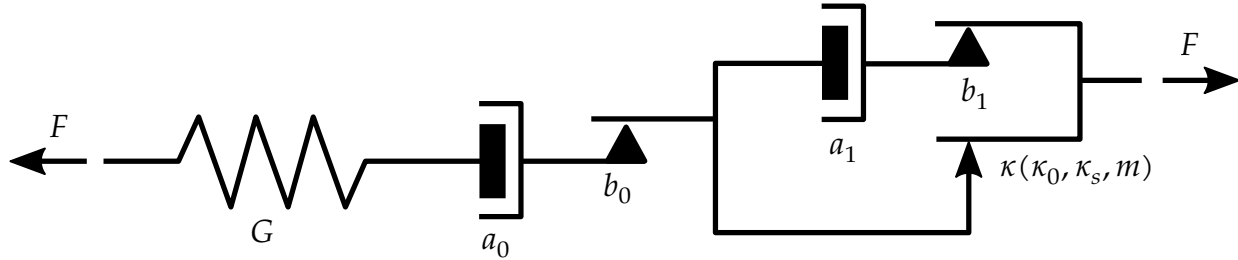
Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K					
Type	A8	F	F					

**HJR Element Cards.** At least 1 and optionally up to 6 cards should be input. A keyword card (with a "\*" in column 1) terminates this input, if less than 6 cards are used.

Card 2	1	2	3	4	5	6	7	8
Variable	A0	B0	A1	B1	M	KAPAS	KAPAO	SHEAR
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
K	Elastic bulk modulus.
A0	Rate dependent understress viscoplastic parameter.
B0	Rate independent understress plasticity parameter.
A1	Rate dependent overstress viscoplastic parameter.
B1	Rate independent overstress plasticity parameter.
M	Exponential hardening parameter.



**Figure 0-1.** Rheological representation of an HJR element, including the associated parameters.

VARIABLE	DESCRIPTION
KAPAS	Saturated yield strain.
KAPA0	Initial yield strain.
SHEAR	Elastic shear modulus.

**Remarks:**

The Cauchy stress for this smooth viscoelastic viscoplastic material is given by

$$\sigma = K(J - 1)\mathbf{I} + \sum_{i=1}^6 \mathbf{s}_i,$$

where  $K$  is the elastic bulk modulus provided on the first card,  $J = \det(\mathbf{F})$  is the relative volume with  $\mathbf{F}$  being the total deformation gradient, and the deviatoric stresses  $\mathbf{s}_i$  are coming from the HJR (Hollenstein-Jabareen-Rubin) elements in parallel. Up to 6 such elements can be defined for the deviatoric response and a rheological representation of one is shown in Figure 0-1. Each element is associated with 8 material parameters that are provided on the optional cards and characterize its inelastic response. All this allows for a wide range of stress strain relationships and the critical part would be to estimate parameters for a given test suite, whence some elaboration on the physical interpretation of the individual parameters in the context of uniaxial stress is given following a general description of the model.

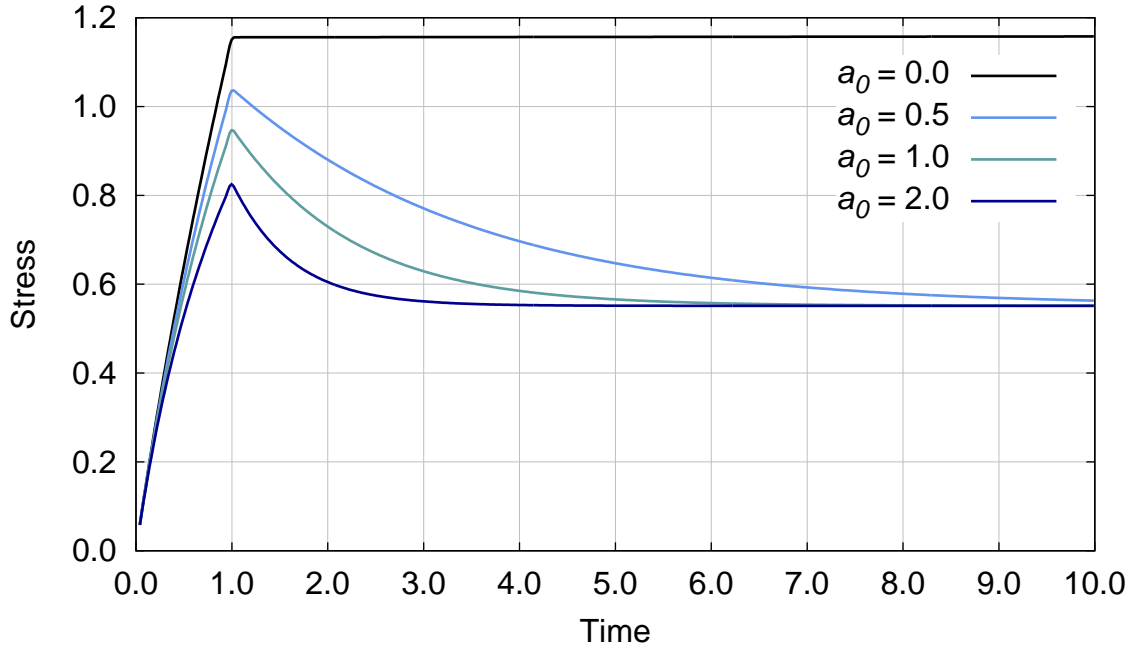
We analyze one HJR element by letting  $\bar{\mathbf{B}}$  denote the associated isochoric elastic left Cauchy-Green tensor. Define

$$\tilde{\mathbf{B}} = \bar{\mathbf{B}} - \frac{1}{3}\alpha\mathbf{I}, \text{ where } \alpha = \text{tr}(\bar{\mathbf{B}}).$$

The evolution of  $\bar{\mathbf{B}}$  is given by

$$\dot{\bar{\mathbf{B}}} = \mathbf{L}\bar{\mathbf{B}} + \bar{\mathbf{B}}\mathbf{L}^T - \frac{2}{3}\text{tr}(\mathbf{D})\bar{\mathbf{B}} - \dot{\Gamma}\mathbf{A}, \text{ where } \mathbf{A} = \bar{\mathbf{B}} - \left[ \frac{3}{\text{tr}(\bar{\mathbf{B}}^{-1})} \right] \mathbf{I}$$





**Figure M275-1.** Influence of parameter  $a_0$  on stress relaxation

where  $\mathbf{D}$  is the rate-of-deformation and  $\dot{\Gamma}$  governs the inelastic deformation. The functional form of  $\dot{\Gamma}$  is summarized in the following set of equations

$$\begin{aligned}\dot{\Gamma} &= \dot{\Gamma}_0 + \langle g \rangle \dot{\Gamma}_1 \\ \dot{\Gamma}_i &= a_i + b_i \dot{\varepsilon}, \quad i = 0,1 \\ g &= 1 - \frac{\kappa}{\tilde{\gamma}}\end{aligned}$$

where

$$\begin{aligned}\langle g \rangle &= \max(0, g), \\ \dot{\varepsilon} &= \sqrt{\frac{2}{3} \tilde{\mathbf{D}} : \tilde{\mathbf{D}}}, \\ \tilde{\mathbf{D}} &= \mathbf{D} - \frac{1}{3} \text{tr}(\mathbf{D}) \mathbf{I}, \\ \tilde{\gamma} &= \sqrt{\frac{3}{8} \tilde{\mathbf{B}} : \tilde{\mathbf{B}}}, \\ \dot{\kappa} &= m \dot{\Gamma}_1 \langle g \rangle (\kappa_s - \kappa).\end{aligned}$$

A hyperelastic law with a strain energy potential for the distortional deformation given by

$$\psi(\alpha) = \frac{G}{2} (\alpha - 3)$$

yields a contribution to the deviatoric Cauchy stress of

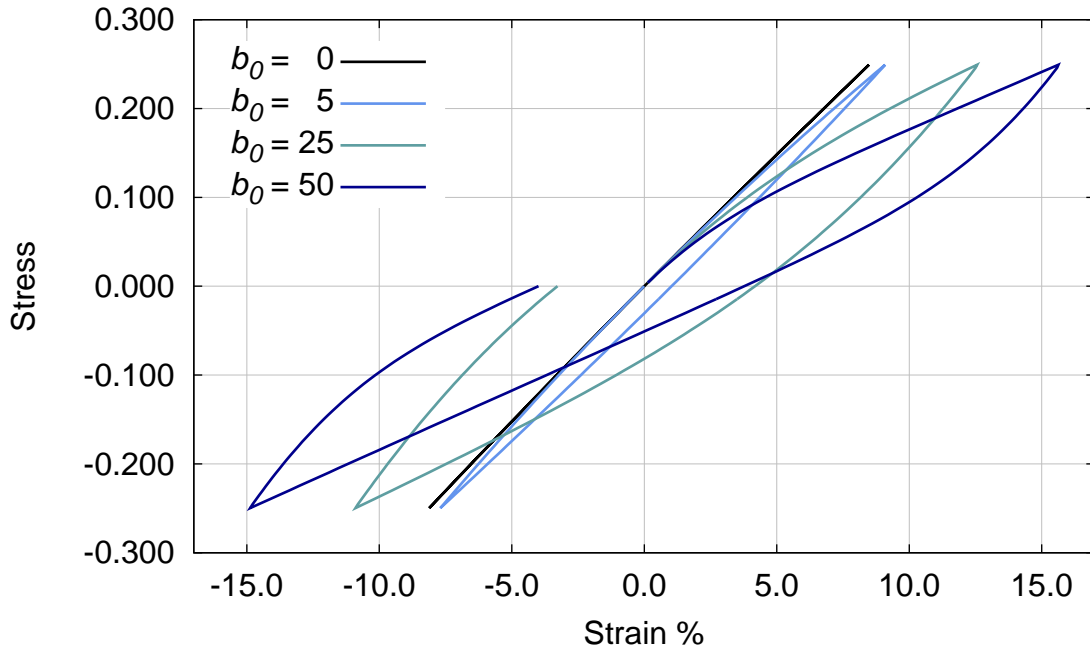


Figure M275-2. Influence of  $b_0$  in cyclic loading

$$\mathbf{s} = GJ^{-1}\tilde{\mathbf{B}}.$$

In uniaxial stress at constant total distortional rate of deformation  $\pm\dot{\epsilon}$  (tension or compression), these equations can be reduced to scalar correspondents

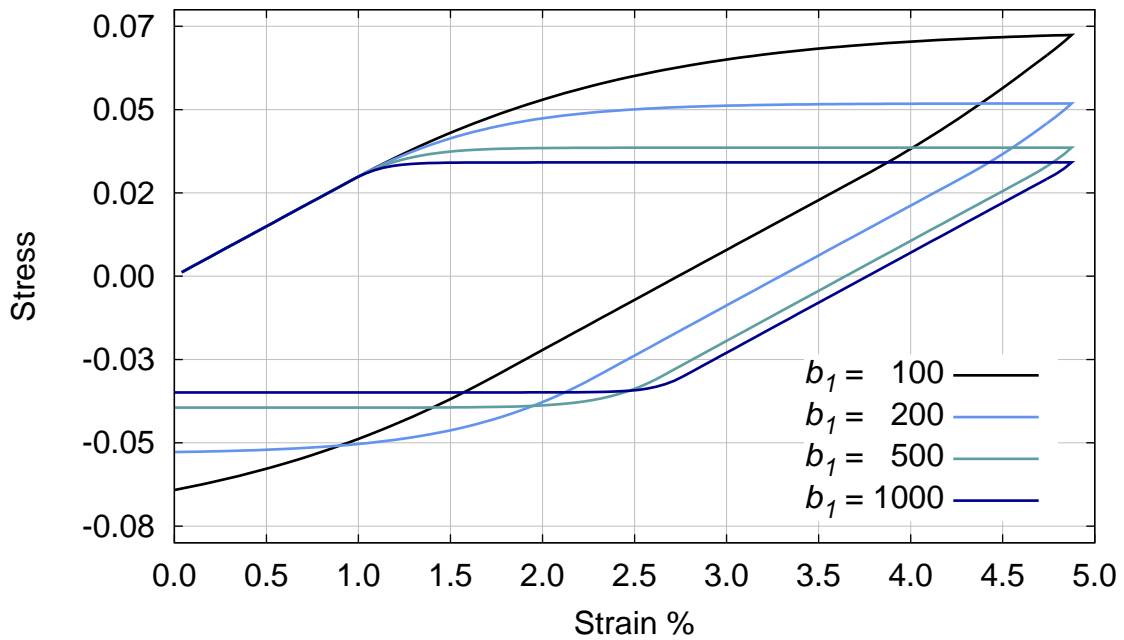
$$\frac{\dot{\bar{b}}}{\bar{b}} = 2 \left( \pm\dot{\epsilon} - \dot{\Gamma} \frac{\bar{b}\sqrt{\bar{b}} - 1}{2\bar{b}\sqrt{\bar{b}} + 1} \right)$$

$$\tau = G \left( \bar{b} - \frac{1}{\sqrt{\bar{b}}} \right) \tag{M275.1}$$

where  $\bar{b}$  is the component of  $\tilde{\mathbf{B}}$  in the direction of deformation and  $\tau$  is the uniaxial Kirchhoff stress. The evolution of  $\Gamma$  follows the equations above with

$$\tilde{\gamma} = \frac{1}{2} \left| \bar{b} - 1/\sqrt{\bar{b}} \right|.$$

Even though analytical solutions may be out of reach, this would be the basis for estimating as well as interpreting the material parameters. Obviously the shear modulus  $G$  (SHEAR) provides the elastic deviatoric stiffness, for a purely elastic material just define one such parameter and leave out all the other parameters on the same card. If several cards are used, the effective elastic shear stiffness is the sum of the contributions from each of the corresponding HJR elements. An interesting observation is that the stress in a HJR element saturates to a value given by the solution of  $\bar{b}$  to



**Figure M275-3.** Effect of  $b_1$  in cyclic loading

$$\bar{b}\sqrt{\bar{b}} \left( \pm 2 - \left\{ b_0 + b_1 + \frac{a_0 + a_1}{\dot{\epsilon}} \right\} \right) \pm 2\sqrt{\bar{b}}\kappa_s \left( b_1 + \frac{a_1}{\dot{\epsilon}} \right) + \left( \pm 1 + \left\{ b_0 + b_1 + \frac{a_0 + a_1}{\dot{\epsilon}} \right\} \right) = 0 \tag{M275.2}$$

in tension (+) and compression (-), followed by application of (M275.1) above, this assuming that

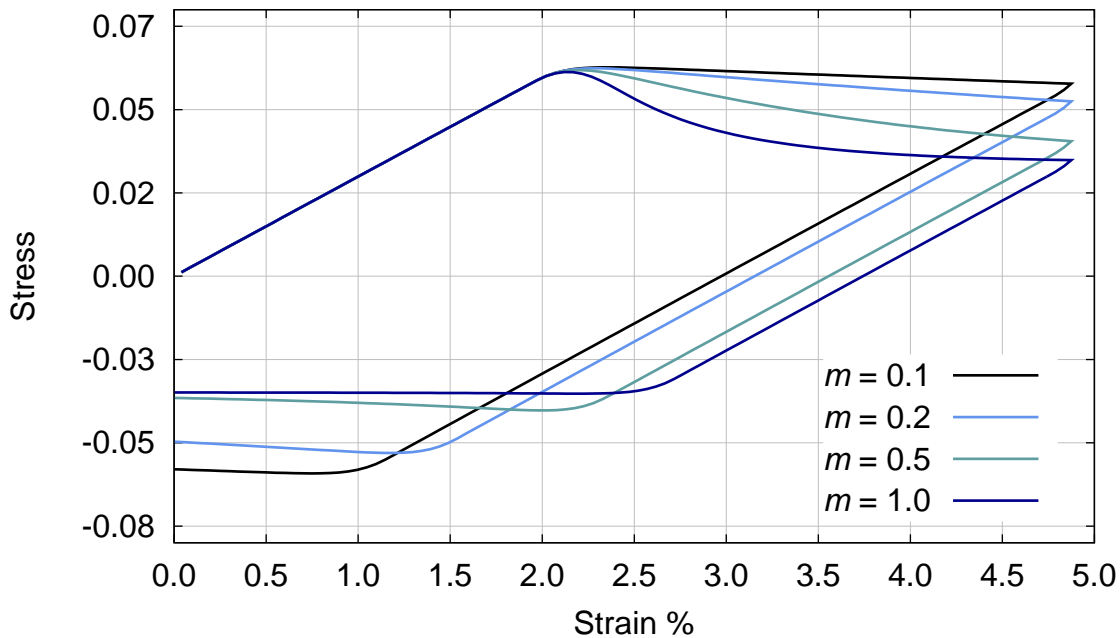
$$b_0 + b_1 + \frac{a_0 + a_1}{\dot{\epsilon}} > 2$$

in tension

$$b_0 + b_1 + \frac{a_0 + a_1}{\dot{\epsilon}} > 1$$

in compression. This expression will be utilized in special cases below when examining each inelastic material parameter individually, the material parameters above are input on the HJR element cards as A0, B0, A1, B1 and KAPAS.

A Maxwell material is obtained by providing an element with a nonzero  $a_0$  (A0) and other parameters zero, this parameter should be interpreted as the viscoelastic relaxation coefficient determining the rate at which the stress relaxes to zero, see parameter BETA in \*MAT\_VISCOELASTIC. In Figure M275-1 a stress relaxation is shown for a strain controlled problem using two HJR elements and normalized material parameters using a bulk modulus of  $K = 1$ . For the first element  $G = 0.5$  and for the other  $G = 1$  and  $a_0$  varies, all other parameters are zero. The engineering strain is ramped to 50% from  $t = 0$  to  $t = 1$  and then kept constant, the response is very similar to other viscoelastic models in LS-



**Figure M275-4.** Softening response in cyclic loading for various values of  $m$

DYNA. Not surprisingly, a HJR element with  $a_0 > 0$  (and  $a_1 = b_1 = 0$ ) will always relax to zero stress, which follows from (M275.1) and (M275.2), thus the relaxed stress in this case comes from the purely elastic element. A general viscoelastic material can be obtained by putting several such HJR elements in parallel, in analogy to \*MAT\_GENERAL\_VISCOELASTIC.

For a nonzero  $b_0$  (B0) and other parameters zero, a rate independent plastic response is obtained exhibiting zero yield stress, i.e., inelastic strains develop immediately upon loading. From (M275.2) the value of  $b_0$  determines the saturated stress value for the associated HJR element by (M275.1) and

$$\bar{b} = \left( \frac{b_0 \pm 1}{b_0 \mp 2} \right)^{2/3}$$

in tension (+) and compression (–), respectively. A smooth response is obtained that is characterized by hysteresis as shown in Figure M275-2. The same material parameters as in the previous example is used with the exception of varying  $b_0$  with vanishing  $a_0$ . The deformation is controlled by a cyclic Cauchy stress between  $-0.25$  and  $0.25$ , for larger  $b_0$  a hysteresis is observed. It should however be mentioned that the hysteresis vanishes as  $b_0 \rightarrow \infty$  as the stress for the second element saturates quickly to a small value, so it is not trivial to quantitatively estimate the amount of hysteresis for a given parameter setting and deformation.

Rate independent plasticity with a nonzero yield stress can be obtained by a nonzero  $b_1$  (B1) in combination with parameters  $\kappa_0$  (KAPA0),  $\kappa_s$  (KAPAS) and  $m$  (M). The yield stress in the sense of von Mises is given by

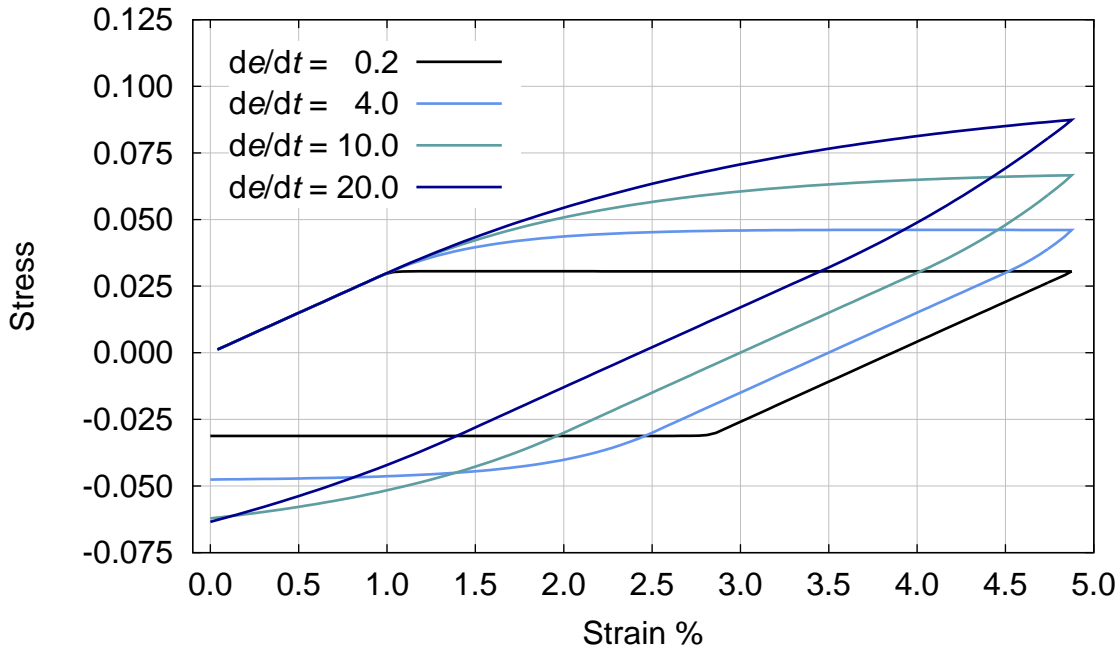
$$\sigma_Y = 2GJ^{-1}\kappa$$

and whence  $\kappa$  is interpreted as the current yield strain. Here  $b_1$  determines the amount of overstress through (M275.1) and (M275.2), requiring the solution of a non-trivial polynomial equation. This is exemplified in Figure M275-3 using one HJR element with  $K = 1$ ,  $G = 1.5$ ,  $\kappa_0 = \kappa_s = 0.01$  and  $m = 0$ . The engineering strain is ramped up to 5% and down to 0 and  $b_1$  is varied with all other parameters zero, the response tends to an elastic-perfectly plastic as  $b_1$  increases. The saturated stress value for  $b_1 \rightarrow \infty$  can be calculated as

$$\bar{b} = \left[ \left( \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{8\kappa_s^3}{27}} \right)^{1/3} + \left( \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{8\kappa_s^3}{27}} \right)^{1/3} \right]^2 \quad (\text{M275.3})$$

and employing (M275.1).

Isotropic strain hardening  $\kappa_s > \kappa_0$  or softening  $\kappa_s < \kappa_0$  is obtained with  $m > 0$ ,  $\kappa$  tends exponentially towards  $\kappa_s$  at a rate determined by  $m$ . Using  $b_1 = 1000$ , i.e., very little overstress,  $\kappa_0 = 0.02$ ,  $\kappa_s = 0.01$  and varying  $m$  the softening response in Figure M275-4 is obtained. The rate at which the element hardens is difficult to quantitatively estimate, but presumably it depends not only on  $m$  but also on  $b_1$ . It is important to note however that for small to moderate  $b_1$  the model appears to harden with  $m = 0$ , which is due to larger overstress. The hardening determined by  $m$  can be determined from a loading, unloading and reloading cycle to detect how the the yield strain  $\kappa$  changes, see Hollenstein et.al. [2013].



**Figure M275-5.** Strain rate dependence for  $a_1 = 1000$  and  $b_1 = 10$

Finally,  $a_1$  (A1) is the viscoplastic parameter determining how stress responds to change in strain rate. Its interpretation is very similar to that of  $a_0$ , stress increases with increasing loading rate and relaxes to the saturated stress value given by (M275.1) and (M275.2). In Figure M275-5 a rate dependency is illustrated for  $K = 1$ ,  $G = 1.5$ ,  $\kappa_0 = \kappa_s = 0.01$  and  $m = 0$ , where we have put  $a_1 = 1000$  and  $b_1 = 10$ . The engineering strain rate varies from 0.2 to 20 and for small strain rates (M275.3) can be used for estimating the saturated stress, but in general (M275.2) must be used.

Putting several HJR elements in parallel can thus provide a fairly general combination of viscoelastic/viscoplastic response with isotropic hardening/softening, but this of course requires a rich test suite and a good way of estimating the material parameters. Presumably it is often sufficient to neglect some effects and work with only a subset of the material parameters.

For post-processing, the effective plastic strain in this model is defined as

$$\epsilon_p = \sqrt{\frac{2}{3} \epsilon_p : \epsilon_p}$$

where

$$\epsilon_p = \epsilon_t - \epsilon_e$$

is a crude estimation of the difference between total and elastic strain. We set

$$\epsilon_t = \frac{1}{2J} \left[ \mathbf{B} - \frac{1}{3} \text{tr}(\mathbf{B}) \mathbf{I} \right]$$

$$\boldsymbol{\varepsilon}_e = \frac{1}{2G} \left[ \boldsymbol{\sigma} - \frac{1}{3} \text{tr}(\boldsymbol{\sigma}) \mathbf{I} \right]$$

where

$$\mathbf{B} = J^{-2/3} \mathbf{F} \mathbf{F}^T$$

and  $G$  here is the sum of all shear moduli defined on the HJR element cards. Note that this does not correspond to the traditional measure of effective plastic strain which should be accounted for when validating results.

## \*MAT\_276

## \*MAT\_CHRONOLOGICAL\_VISCOELASTIC

### \*MAT\_CHRONOLOGICAL\_VISCOELASTIC

This is Material Type 276. This material model provides a general viscoelastic Maxwell model having up to 6 terms in the prony series expansion and is useful for modeling dense continuum rubbers and solid explosives. It is similar to Material Type 76 but allows the incorporation of aging effects on the material properties. Either the coefficients of the prony series expansion or a relaxation curve may be specified to define the viscoelastic deviatoric and bulk behavior.

The material model can also be used with laminated shell. Either an elastic or viscoelastic layer can be defined with the laminated formulation. To activate laminated shell you need the laminated formulation flag on \*CONTROL\_SHELL. With the laminated option a user defined integration rule is needed.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	PCF	EF	TREF	A	B
Type	A8	F	F	F	F	F	F	F

If fitting is done from a relaxation curve, specify fitting parameters on card 2, *otherwise* if constants are set on Viscoelastic Constant Cards *LEAVE THIS CARD BLANK*.

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	NT	BSTART	TRAMP	LCIDK	NTK	BSTARTK	TRAMPK
Type	F	I	F	F	F	I	F	F



**Viscoelastic Constant Cards.** Up to 12 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 12 cards are used. These cards are not needed if relaxation data is defined. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included. If an elastic layer is defined you only need to define GI and KI (note in an elastic layer only one card is needed).

Optional	1	2	3	4	5	6	7	8
Variable	GI	BETAI	KI	BETAKI				
Type	F	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
BULK	Elastic bulk modulus.
PCF	Tensile pressure elimination flag for solid elements only. If set to unity tensile pressures are set to zero.
EF	Elastic flag (if equal 1, the layer is elastic. If 0 the layer is viscoelastic).
TREF	Reference temperature for shift function (must be greater than zero).
A	Chronological coefficient $\alpha(t_a)$ . See Remarks below.
B	Chronological coefficient $\beta(t_a)$ . See Remarks below.
LCID	Load curve ID for deviatoric behavior if constants, $G_i$ , and $\beta_i$ are determined via a least squares fit. This relaxation curve is shown below.
NT	Number of terms in shear fit. If zero the default is 6. Fewer than NT terms will be used if the fit produces one or more negative shear moduli. Currently, the maximum number is set to 6.
BSTART	In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$ , $\beta_4$ is 10 times $\beta_3$ , and so on. If zero, BSTART is determined by an iterative trial and error scheme.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TRAMP	Optional ramp time for loading.
LCIDK	Load curve ID for bulk behavior if constants, $K_i$ , and $\beta_{K_i}$ are determined via a least squares fit. This relaxation curve is shown below.
NTK	Number of terms desired in bulk fit. If zero the default is 6. Currently, the maximum number is set to 6.
BSTARTK	In the fit, $B_{K_1}$ is set to zero, $B_{K_2}$ is set to BSTARTK, $\beta_{K_3}$ is 10 times $\beta_{K_2}$ , $\beta_{K_4}$ is 10 times $\beta_{K_3}$ , and so on. If zero, BSTARTK is determined by an iterative trial and error scheme.
TRAMPK	Optional ramp time for bulk loading.
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETA <i>i</i>	Optional shear decay constant for the $i^{\text{th}}$ term
$K_i$	Optional bulk relaxation modulus for the $i^{\text{th}}$ term
BETA <i>Ki</i>	Optional bulk decay constant for the $i^{\text{th}}$ term

**Remarks:**

The Cauchy stress,  $\sigma_{ij}$ , is related to the strain rate by

$$\sigma_{ij}(t) = -p\delta_{ij} + \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}(\tau)}{\partial \tau} d\tau$$

For this model, it is postulated that the mathematical form is preserved in the constitutive equation for aging; however two new material functions,  $g'_0(t_a)$  and  $g'_1(t_a, t)$  are introduced to replace  $g_0$  and  $g_1(t)$ , which is expressed in terms of a Prony series as in material model 76, \*MAT\_GENERAL\_VISCOELASTIC. The aging time is denoted by  $t_a$ .

$$\sigma_{ij}(t_a, t) = -p\delta_{ij} + \int_0^t g'_{ijkl}(t_a, t - \tau) \frac{\partial \varepsilon_{kl}(\tau)}{\partial \tau} d\tau$$

where

$$g'_{ijkl}(t_a, t) = \alpha(t_a) g_{ijkl}[\beta(t_a)t]$$

where  $\alpha(t_a)$  and  $\beta(t_a)$  are two new material properties that are functions of the aging time  $t_a$ . The material properties functions  $\alpha(t_a)$  and  $\beta(t_a)$  will be determined with the experimental results. For determination of  $\alpha(t_a)$  and  $\beta(t_a)$ , Eq. (2) can be written in the following form

$$\log(\sigma_{ij} - p\delta_{ij})_{t_a, t} = \log\alpha(t_a) + \log(\sigma_{ij} - p\delta_{ij})_{t_a=0, t \rightarrow \xi}$$
$$\log\xi = \log\beta(t_a) + \log t$$

Therefore, if one plots the stress versus time on log-log scales, with the vertical axis being the stress and the horizontal axis being the time, then the stress-relaxation curve for any aged time history can be obtained directly from the stress-relaxation curve at  $t_a = 0$  by imposing a vertical shift and a horizontal shift on the stress-relaxation curves. The vertical shift and the horizontal shift are  $\log\alpha(t_a)$  and  $\log\beta(t_a)$  respectively.

**\*MAT\_ADHESIVE\_CURING\_VISCOELASTIC**

This is Material Type 277. It is useful for modeling adhesive materials during chemical curing. This material model provides a general viscoelastic Maxwell model having up to 16 terms in the Prony series expansion. It is similar to Material Type 76, but the viscoelastic properties do not only depend on the temperature but also on an internal variable representing the state of cure for the adhesive. The kinematic of the curing process depends on temperature as well as on temperature rate and follows the Kamal model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K1	K2	C1	C2	M	N
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	CHEXP1	CHEXP2	CHEXP3	LCCHEXP	LCTHEXP	R	TREFEXP	DOCREFE XP
Type	F	F	F	I	I	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	WLFTRF	WLFA	WLFB	LCGO	LCK0	IDOC	INCR	
Type	F	F	F	I	I	F	I	

**Viscoelastic Constant Cards.** Up to 16 cards may be input. A keyword card (with a "\*" in column 1) terminates this input if less than 16 cards are used. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included.

Optional	1	2	3	4	5	6	7	8
Variable	GI	BETAGI	KI	BETAKI				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density.
K1	Parameter $k_1$ for Kamal model.
K2	Parameter $k_2$ for Kamal model.
C1	Parameter $c_1$ for Kamal model.
C2	Parameter $c_2$ for Kamal model.
M	Exponent $m$ for Kamal model
N	Exponent $n$ for Kamal model.
CHEXP1	Quadratic parameter $\gamma_2$ for chemical shrinkage.
CHEXP2	Linear parameter $\gamma_1$ for chemical shrinkage.
CHEXP3	Constant parameter $\gamma_0$ for chemical shrinkage.
LCCHEXP	Load curve ID to define the coefficient for chemical shrinkage $\gamma(\alpha)$ as a function of the state of cure $\alpha$ . If set, parameters CHEXP1, CHEXP2 and CHEXP3 are ignored.
LCTHEXP	Load curve ID or table ID defining the instantaneous coefficient of thermal expansion $\beta(\alpha, T)$ as a function of cure $\alpha$ and temperature $T$ . If referring to a load curve, parameter $\beta(T)$ is a function of temperature $T$ .
R	Gas constant $R$ for Kamal model.
TREFEXP	Reference temperature $T_0$ for secant form of thermal expansion. See Remarks below.
DOCREFEXP	Reference degree of cure $\alpha_0$ for sequential form of chemical expansion. See Remarks below.
WLF TREF	Reference temperature for WLF shift function.
WLFA	Parameter $A$ for WLF shift function.
WLFB	Parameter $B$ for WLF shift function.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCG0	Load curve ID defining the instantaneous shear modulus $G_0$ as a function of state of cure.
LCK0	Load curve ID defining the instantaneous bulk modulus $K_0$ as a function of state of cure.
IDOC	Initial degree of cure.
INCR	Switch between incremental and total stress formulation. EQ.0: total form: (DEFAULT) EQ.1: incremental form: (recommended)
GI	Shear relaxation modulus for the $i$ th term for fully cured material.
BETAGI	Shear decay constant for the $i$ th term for fully cured material.
KI	Bulk relaxation modulus for the $i$ th term for fully cured material.
BETAKI	Bulk decay constant for the $i$ th term for fully cured material.

**Remarks:**

Within this material formulation an internal variable  $\alpha$  has been included to represent the degree of cure for the adhesive. The evolution equation for this variable is given by the Kamal model and reads

$$\frac{d\alpha}{dt} = \left( k_1 \exp\left(\frac{-c_1}{RT}\right) + k_2 \exp\left(\frac{-c_2}{RT}\right) \alpha^m \right) (1 - \alpha)^n$$

The chemical reaction of the curing process results in a shrinkage of the material. The coefficient of the chemical shrinkage  $\gamma(\alpha)$  can either be given by a load curve or using the quadratic expression

$$\gamma(\alpha) = \gamma_2 \alpha^2 + \gamma_1 \alpha + \gamma_0$$

For non-negative values of the reference degree of cure  $\alpha_0$ , a secant form is used to compute the chemical strains

$$\varepsilon^{ch} = \gamma(\alpha)(\alpha - \alpha_0) - \gamma(\alpha_I)(\alpha_I - \alpha_0)$$

Otherwise a differential form is used:

$$d\varepsilon^{ch} = \gamma(\alpha)d\alpha$$

Analogously, the thermal strains are either defined in a secant or differential form, depending on the reference temperature  $T_0$ . In both cases the coefficient of thermal expansion can be given as 2d table depending on degree of cure and temperature.

Finally, the Cauchy stress,  $\sigma_{ij}$ , is related to the strain rate by

$$\sigma_{ij}(t) = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}(\tau)}{\partial \tau} d\tau$$

The relaxation functions  $g_{ijkl}(t - \tau)$  are represented in this material formulation by up to 16 terms (not including the instantaneous modulus  $G_0$ ) of the Prony series:

$$g(t, \alpha) = G_0(\alpha) - \sum_i G_i(\alpha) + \sum_i G_i(\alpha) e^{-\beta_i t}$$

For the sake of simplicity, a constant ratio  $G_i(\alpha)/G_0(\alpha)$  for all degrees of cure is assumed. Consequently, it suffices to define one term  $G_0(\alpha)$  as a function of the degree of cure and further coefficients for the fully cured state of the adhesive:

$$g(t, \alpha) = G_0(\alpha) \left( 1 - \sum_i \frac{G_{i,\alpha=1.0}}{G_{0,\alpha=1.0}} (1 - e^{-\beta_i t}) \right)$$

A possible temperature effect on the stress relaxation is accounted for by the Williams-Landel-Ferry (WLF) shift function. For details on this function, please see material formulation 76, \*MAT\_GENERAL\_VISCOELASTIC.

**\*MAT\_COHESIVE\_PAPER**

This is material type 279. This is a cohesive model for paper materials and can be used only with cohesive element formulations; see the variable ELFORM in \*SECTION\_SOLID and \*SECTION\_SHELL.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	ENO	ET0	EN1	ET1
Type	A8	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	T0N	DN	T1N	T0T	DT	T1T	E3C	CC
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 3	1	2	3	4	5	6	7	8
Variable	ASIG	BSIG	CSIG	FAILN	FAILT			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

**VARIABLE****DESCRIPTION**

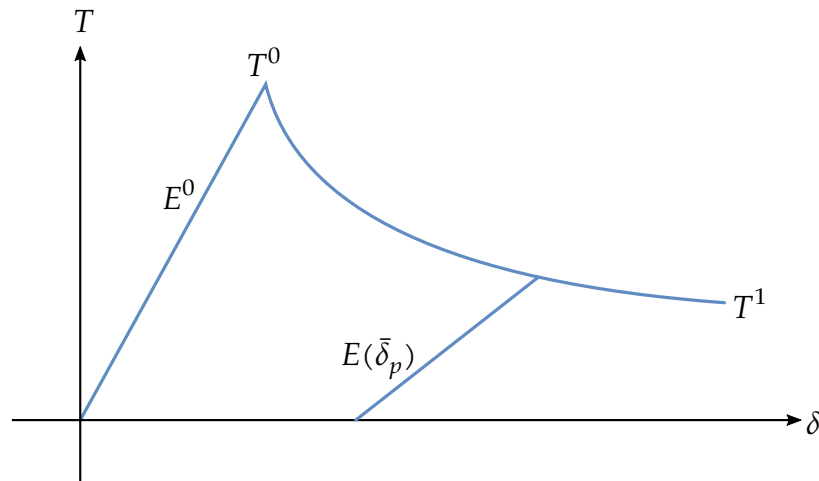
MID Material identification. A unique number or label not exceeding 8 characters must be specified.

RO Mass density



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ROFLG	Flag for whether density is specified per unit area or volume. ROFLG.EQ.0: specified density per unit volume (default) ROFLG.EQ.1: specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero.
INTFAIL	The number of integration points required for the cohesive element to be deleted. If it is zero, the element will not be deleted even if it satisfies the failure criterion. The value of INTFAIL may range from 1 to 4, with 1 the recommended value.
EN0	The initial tensile stiffness (units of stress / length) normal to the plane of the cohesive element.
EN1	The final tensile stiffness (units of stress / length) normal to the plane of the cohesive element.
ET0	The initial stiffness (units of stress / length) tangential to the plane of the cohesive element.
ET1	The final stiffness (units of stress / length) tangential to the plane of the cohesive element.
T0N	Peak tensile traction in normal direction.
DN	Scale factor (unit of length).
T1N	Final tensile traction in normal direction.
T0T	Peak tensile traction in tangential direction. If negative, the absolute value indicates a curve with respect to the normal traction.
DT	Scale factor (unit of length). If negative, the absolute value indicates a curve with respect to the normal stress.
T1T	Final traction in tangential direction. If negative, the absolute value indicates a curve with respect to the normal traction.
E3C	Elastic parameter in normal compression.
CC	Elastic parameter in normal compression.
ASIG	Plasticity hardening parameter in normal compression.
BSIG	Plasticity hardening parameter in normal compression.



**Figure M279-1.** Traction-separation law

- CSIG                      Plasticity hardening parameter in normal compression.
- FAILN                    Maximum effective separation distance in normal direction. Beyond this distance failure occurs.
- FAILT                    Maximum effective separation distance in tangential direction. Beyond this distance failure occurs.

**Remarks:**

In this elastoplastic cohesive material the normal and tangential directions are treated separately, but can be connected by expressing the in-plane traction parameters as functions of the normal traction. In the normal direction the material uses different models in tension and compression.

**Normal tension:**

Assume the total separation is an additive split of the elastic and plastic separation

$$\delta = \delta_e + \delta_p .$$

In normal tension ( $\delta_e > 0$ ) the elastic traction is given by

$$T = E\delta_e = E(\delta - \delta_p) \geq 0,$$

where the tensile normal stiffness

$$E = (E_N^0 - E_N^1) \exp\left(\frac{-\bar{\delta}_p}{\delta_N}\right) + E_N^1 ,$$

depends on the effective plastic separation in the normal direction

$$\bar{\delta}_p = \int |d\delta_p|.$$

Yield traction for tensile loads in normal direction is given by

$$T_{\text{yield}} = (T_N^0 - T_N^1) \exp\left(\frac{-\bar{\delta}_p}{\delta_N}\right) + T_N^1 \geq 0,$$

and yielding occurs when  $T > T_{\text{yield}} \geq 0$ . The above elastoplastic model gives the traction-separation law depicted in [Figure M279-1](#).

### Normal compression:

In normal compression the elastic traction is

$$T = E_3^c [1 - \exp(-C_c \delta_e)] \leq 0,$$

and the yield traction is

$$T_{\text{yield}} = -[A_\sigma + B_\sigma \exp(-C_\sigma \bar{\delta}_p)] \leq 0,$$

with yielding if  $T < T_{\text{yield}} \leq 0$ .

### Tangential traction:

Assume the total separation is an additive split of the elastic and plastic separation in each in-plane direction

$$\delta_i = \delta_e^i + \delta_p^i, \quad i = 1, 2.$$

The elastic traction is given by

$$T_i = E \delta_e^i = E(\delta_i - \delta_p^i),$$

where the tensile normal stiffness

$$E = (E_T^0 - E_T^1) \exp\left(\frac{-\bar{\delta}_p}{\delta_T}\right) + E_T^1,$$

depends on the effective plastic separation

$$\bar{\delta}_p = \int d\delta_p, \quad d\delta_p = \sqrt{(d\delta_p^1)^2 + (d\delta_p^2)^2}.$$

Yield traction is given by

$$T_{\text{yield}} = (T_T^0 - T_T^1) \exp\left(\frac{-\bar{\delta}_p}{\delta_T}\right) + T_T^1,$$

and yielding occurs when

$$T_1^2 + T_2^2 - T_{\text{yield}}^2 \geq 0.$$

The plastic flow increment follows the flow rule

$$d\delta_p^i = \frac{T_i}{\sqrt{T_1^2 + T_2^2}} d\delta_p.$$

The above elastoplastic model gives the traction-separation law depicted in [Figure M279-1](#).

**History variables**

This material uses five history variables. Effective separation in the tangential direction is saved as Effective Plastic Strain. History variable 1 and 2 indicates the plastic separation in each tangential direction. Effective plastic separation and plastic separation in the normal direction are saved as history variable 3 and 4, respectively.

**\*MAT\_GLASS**

This is Material Type 280. It is a smeared fixed crack model with a selection of different brittle, stress-state dependent failure criteria such as Rankine, Mohr-Coulomb, or Drucker-Prager. The model incorporates up to 2 (orthogonal) cracks per integration point, simultaneous failure over element thickness, and crack closure effects. It is available for shell elements and explicit analysis only.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR			IMOD	ILAW
Type	A8	F	F	F			F	F

Card 2	1	2	3	4	5	6	7	8
Variable	FMOD	FT	FC	AT	BT	AC	BC	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	SFSTI	SFSTR	CRIN	ECRCL	NCYCR	NIPF		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Mass density $\rho$ .
E	Young's modulus $E$ .
PR	Poisson's ratio $\nu$ .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
IMOD	<p>Flag to choose degradation procedure, when critical stress is reached.</p> <p>EQ.0.0: Softening in NCYCR load steps. Define SFSTI, SFSTR, and NCYCR (default).</p> <p>EQ.1.0: Damage model for softening. Define ILAW, AT, BT, AC, and BC.</p>
ILAW	<p>Flag to choose damage evolution law if IMOD = 1.0, see Remarks.</p> <p>EQ.0.0: Same damage evolution for tensile and compressive failure (default).</p> <p>EQ.1.0: Different damage evolution for tensile failure and compressive failure.</p>
FMOD	<p>Flag to choose between failure criteria, see Remarks.</p> <p>EQ.0.0: Rankine maximum stress (default),</p> <p>EQ.1.0: Mohr-Coulomb,</p> <p>EQ.2.0: Drucker-Prager.</p>
FT	Tensile strength $f_t$ .
FC	Compressive strength $f_c$ .
AT	Tensile damage evolution parameter $\alpha_t$ . Can be interpreted as the residual load carrying capacity ratio for tensile failure ranging from 0 to 1.
BT	Tensile damage evolution parameter $\beta_t$ . It controls the softening velocity for tensile failure.
AC	Compressive damage evolution parameter $\alpha_t$ . Can be interpreted as the residual load carrying capacity ratio for compressive failure ranging from 0 to 1.
BC	Compressive damage evolution parameter $\beta_t$ . It controls the softening velocity for compressive failure.
SFSTI	Scale factor for stiffness after failure, e.g. SFSTI = 0.001 means that stiffness is reduced to 0.1% of the elastic stiffness at failure.
SFSTR	Scale factor for stress in case of failure, e.g. SFSTR = 0.01 means that stress is reduced to 1% of the failure stress at failure.

VARIABLE	DESCRIPTION
ICRIN	Flag for crack strain initialization EQ.0.0: initial crack strain is strain at failure (default), EQ.1.0: initial crack strain is zero.
ECRCL	Crack strain necessary to reactivate certain stress components after crack closure.
NCYCR	Number of cycles in which the stress is reduced to SFSTR*failure stress.
NIPF	Number of failed through thickness integration points to fail <u>all</u> through thickness integration points for IMOD = 0.

### Remarks:

The underlying material behavior before failure is isotropic, small strain linear elasticity with Young's modulus  $E$  and Poisson's ratio  $\nu$ . Asymmetric (tension-compression dependent) failure happens as soon as one of the following plane stress failure criteria is violated.

For FMOD = 0, a maximum stress criterion (Rankine) is used, where principal stresses  $\sigma_1$  and  $\sigma_2$  are bound by tensile strength  $f_t$  and compressive strength  $f_c$  as follows:

$$-f_c < \{\sigma_1, \sigma_2\} < f_t$$

With FMOD = 1, the Mohr-Coulomb criterion with expressions in four different categories is used:

$$\begin{aligned} \sigma_1 > 0 \text{ and } \sigma_2 > 0: \quad & \max\left(\frac{\sigma_1}{f_t}, \frac{\sigma_2}{f_t}\right) < 1 \\ \sigma_1 < 0 \text{ and } \sigma_2 < 0: \quad & \max\left(-\frac{\sigma_1}{f_c}, -\frac{\sigma_2}{f_c}\right) < 1 \\ \sigma_1 > 0 \text{ and } \sigma_2 < 0: \quad & \frac{\sigma_1}{f_t} - \frac{\sigma_2}{f_c} < 1 \\ \sigma_1 < 0 \text{ and } \sigma_2 > 0: \quad & -\frac{\sigma_1}{f_c} + \frac{\sigma_2}{f_t} < 1 \end{aligned}$$

And for FMOD = 2, the plane stress Drucker-Prager criterion is given by

$$\frac{1}{2f_c} \left[ \left( \frac{f_c}{f_t} - 1 \right) (\sigma_1 + \sigma_2) + \left( \frac{f_c}{f_t} + 1 \right) \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} \right] < 1$$

As soon as failure happens in the tensile regime, a crack occurs perpendicular to the maximum principal stress direction. That means a crack coordinate system is set up and stored, defined by a relative angle with respect to the element coordinate system. Appropriate stress and stiffness tensor components (e.g. normal to the crack) are reduced according to SFSTR and SFSTI if IMOD = 0. The stress reduction takes place in a period of NCYCR time step cycles. For IMOD = 1.0 the stress and stiffness tensor are reduced by a damage model, please see below. A second crack orthogonal to the first crack is possible which can open and close independently from the first one, further reducing the element stiffness.

To deal with crack closure, the current strain in principal stress direction is stored as initial crack strain (ICRIN = 0, default) or the initial crack strain is set to zero (ICRIN = 1). After failure, the crack strain is tracked, so that later crack closure will be detected. If that is the case, appropriate stress and stiffness tensor components (e.g. compressive) are reactivated so that e.g. under pressure a load could be carried and cause a nonzero stress perpendicular to the crack.

If the critical number of failed integration points (NIPF) in one element is reached, all integration points over the element thickness fail as well. The default value of NIPF = 1 resembles the fact, that a crack in a glass plate immediately runs through the thickness.

Starting with the Release of LS-DYNA version R10, a damage model for stress and stiffness softening can be activated with IMOD = 1. The corresponding evolution law for ILAW = 0 is given by

$$D = \begin{cases} 0 & \text{for } \kappa \leq \kappa^0 \\ 1 - \frac{\kappa^0}{\kappa} \left( 1 - \alpha_{t,c} + \alpha_{t,c} e^{-\beta_{t,c} (\kappa - \kappa^0)} \right) & \text{else} \end{cases}$$

i.e. tensile and compressive failure are treated in the same fashion.

On the other hand, with ILAW = 1, the damage evolution for tensile failure is given by

$$D = \begin{cases} 0 & \text{for } \kappa \leq \kappa^0 \\ 1 - \frac{\kappa^0}{\kappa} \left( 1 - \alpha_t + \alpha_t e^{-\beta_t (\kappa - \kappa^0)} \right) & \text{else} \end{cases}$$

whereas damage for compressive failure evolves like that (more delayed stress reduction):



$$D = \begin{cases} 0 & \text{for } \kappa \leq \kappa^0 \\ 1 - \frac{\kappa^0}{\kappa} (1 - \alpha_c) - \alpha_c e^{-\beta_c (\kappa - \kappa^0)} & \text{else} \end{cases}$$

**History Variables:**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
1	Crack flag: 0 = no crack, 1 = one crack, 2 = two cracks, -1 = failed under compression
2	Direction of 1 <sup>st</sup> principle stress as angle in radian with respect to the element direction. The shell normal defines the positive angle direction. The 1 <sup>st</sup> crack direction is perpendicular to the direction of 1 <sup>st</sup> principle stress.
3	Angle in radian that defines the orthogonal to the 2 <sup>nd</sup> crack direction (with respect to the element direction).

**\*MAT\_COMPRF**

This is Material Type 293. This material models the behavior of pre-impregnated (prepreg) composite fibers during the high temperature preforming process. In addition to providing stress and strain, it also provides warp and weft yarn directions and stretch ratios after the forming process. The major applications of the model are for materials used in light weight automobile parts.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ET	EC	PR	G121	G122	G123
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	G124	G125	G126	GAMMAL	VF	EF3	VF23	EM
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	VM	EPSILON	THETA	BULK	G			
Type	F	F	F	F	F			

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
RO	Continuum equivalent mass density.
ET	Tensile modulus along the fiber yarns, corresponding to the slope of the curve in <a href="#">Figure M293-2</a> in the Stable Modulus region from a uniaxial tension test. See <a href="#">Remark 5</a> .
EC	Compression modulus along the fiber yarns, reversely calculated using bending tests when all the other material properties are determined. See <a href="#">Remark 5</a> .

VARIABLE	DESCRIPTION
PR	Poisson's ratio. See <a href="#">Remark 5</a> .
G12 <i>i</i>	Coefficients for the bias-extension angle change-engineering stress curve in <a href="#">Figure M293-3</a> . G121 to G126 corresponds to the 6 <sup>th</sup> order to 1 <sup>st</sup> order factors of the loading curve. See <a href="#">Remark 5</a> .
GAMMAL	Shear locking angle, in degrees. See <a href="#">Remark 5</a> .
VF	Fiber volume fraction in the prepreg composite.
EF3	Transverse compression modulus of the dry fiber.
VF23	Transverse Poisson's ratio of the dry fiber
EM	Young's modulus of the cured resin.
VM	Poisson's ratio of the cured resin
EPSILON	Stretch ratio at the end of undulation stage during the uniaxial tension test. Example shown in <a href="#">Figure M293-2</a> . See <a href="#">Remark 5</a> .
THETA	Initial angle offset between the fiber direction and the element direction. To reduce simulation error, when building the model, the elements should be aligned to the same direction as much as possible.
BULK	Bulk modulus of the prepreg material
G	Shear modulus of the prepreg material

#### Remarks:

1. **Fiber and Resin Properties.** The dry fiber properties, EF3 and VF23, and the cure resin properties, EM and VM, are used to calculate the through thickness elastic modulus of the prepreg using the rule of mixture. These properties will not affect the in-plane deformation of the prepreg during the preforming simulation.
2. **Shear Locking.** In most of the preforming cases, the angle between the fiber yarns will not reach the shear locking state. This model is not designed for, and, therefore, not recommended for simulating shear locking.
3. **BULK and G.** BULK and G are used by the contact algorithm. Changing these parameters will not affect the final simulation result significantly (but it may affect the time step).

4. **Model Description.** Woven composite prepregs are characterized using a non-orthogonal coordinate system having two principal directions: one aligned with the longitudinal warp yarns and the other with the transverse weft yarns. Prior to deformation the warp and weft yarns are orthogonal. The directions and the fiber stretch ratios are determined from the deformation gradient. In [Figure M293-1](#), the angles  $\alpha$  and  $\beta$  refer to the relative of the rotation of the warp yarn coordinate to the local corotational  $x$  coordinate and the angle between the warp and weft yarns, respectively [2,3,4].

The stress from material deformation is divided into two parts: (1) stress caused by the fiber stretch,  $\sigma^f$ , as shown in [Figure M293-1](#) (a); (2) stress caused by the fiber rotation,  $\sigma^m$ , as shown in [Figure M293-1](#) (b). The total stress tensor,  $\sigma$ , in the local corotational  $x - y$  coordinate system is the sum where the components are given below [3]:

$$\sigma_{xx}^f = \sigma_1^f \cos^2 \alpha + \sigma_2^f \cos^2(\alpha + \beta) \quad (1)$$

$$\sigma_{xy}^f = \sigma_{yx}^f = \frac{1}{2} \sigma_1^f \sin 2\alpha + \frac{1}{2} \sigma_2^f \sin 2(\alpha + \beta) \quad (2)$$

$$\sigma_{yy}^f = \sigma_1^f \sin^2 \alpha + \sigma_2^f \sin^2(\alpha + \beta) \quad (3)$$

$$\sigma_{xx}^m = \frac{\sigma_1^m + \sigma_2^m}{2} + \frac{\sigma_1^m - \sigma_2^m}{2} \cos(2\alpha + \beta) \quad (4)$$

$$\sigma_{xy}^m = \sigma_{yx}^m = \frac{\sigma_1^m - \sigma_2^m}{2} \sin(2\alpha + \beta) \quad (5)$$

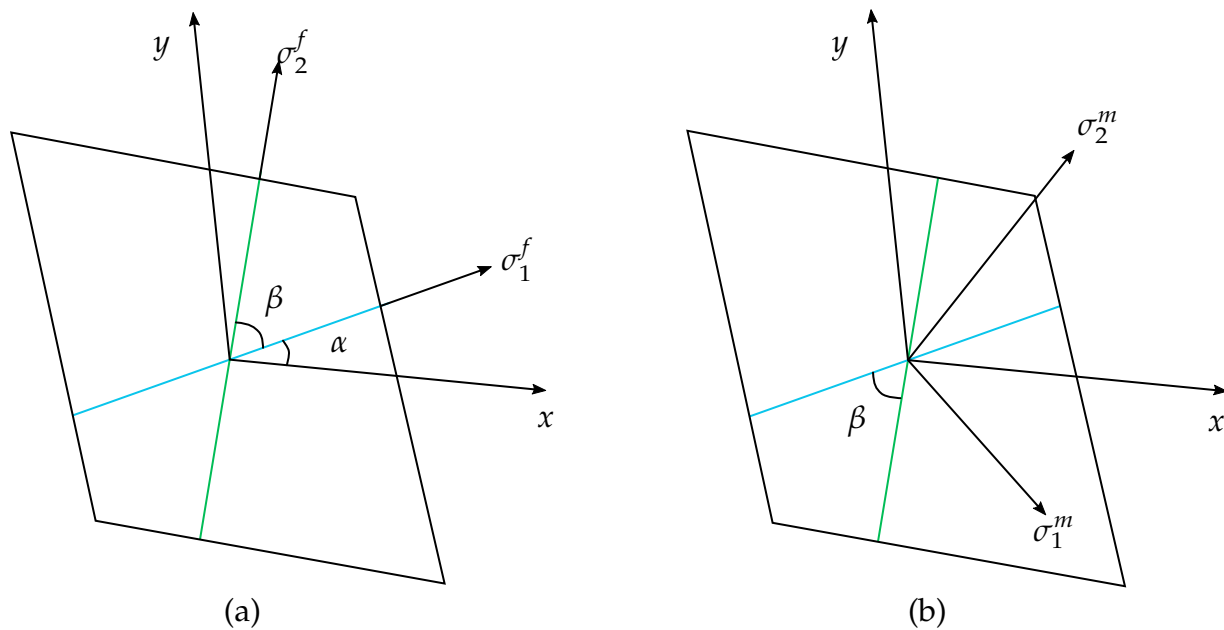
$$\sigma_{yy}^m = \frac{\sigma_1^m + \sigma_2^m}{2} - \frac{\sigma_1^m - \sigma_2^m}{2} \cos(2\alpha + \beta) \quad (6)$$

$$\sigma_{xx} = \sigma_{xx}^f + \sigma_{xx}^m \quad (7)$$

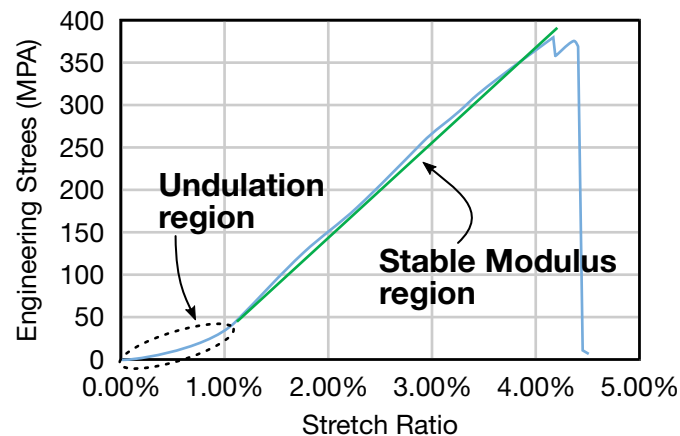
$$\sigma_{xy} = \sigma_{yx} = \sigma_{xy}^f + \sigma_{xy}^m \quad (8)$$

$$\sigma_{yy} = \sigma_{yy}^f + \sigma_{yy}^m \quad (9)$$

5. **Material Property Characterization.** The non-orthogonal stress components caused by yarn stretch and rotation at various deformation states will be characterized via a set of experiments, which are uniaxial tension, bias-extension and cantilever beam bending tests. All the tests need to be performed at the preforming temperature. See references [1] and [3] for more details.



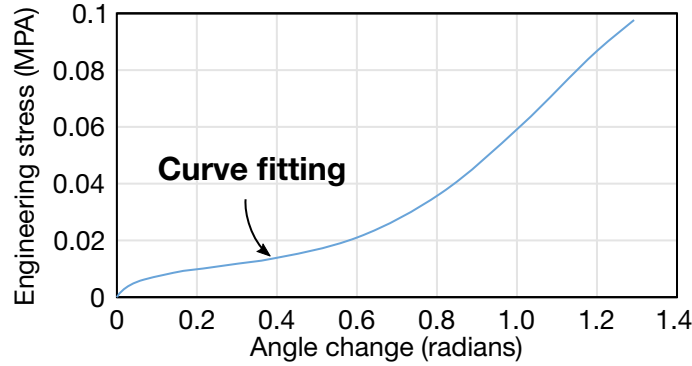
**Figure M293-1.** Stress components caused by (a) stretch in fiber directions and (b) rotation of the fibers [3].



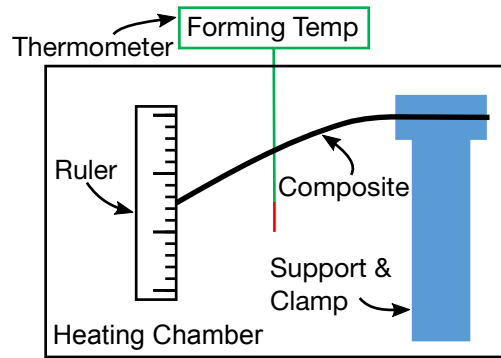
**Figure M293-2.** An example of the engineering stress as a function of stretch ratio from the uniaxial tension test [3].

The uniaxial tension test is used to obtain the fiber direction undulation strains and the stable tensile moduli, together with the in-plane Poisson's ratio (PR). A typical test result is shown in [Figure M293-2](#). From the stretch ratio-engineering stress curve, the tensile modulus, ET, and the stretch ratio at the end of undulation, EP-SILON, can be captured.

The bias-extension test is used to characterize the shear behavior of the composite needed for fields G12i. The test procedure comes from the benchmark test literature [1]. An example of the bias-extension test angle change-engineering stress curve is shown in [Figure M293-3](#).



**Figure M293-3.** An example of the angle change-engineering stress curve from the bias-extension test. The curve fit for this example is  $y = -0.29x^6 + 1.09x^5 - 1.68x^4 + 1.37x^3 - 0.56x^2 + 0.12x$ . For this example curve the inputs into LS-DYNA are  $G121 = -0.29$ ,  $G122 = 1.09$ ,  $G123 = -1.68$ ,  $G124 = 1.37$ ,  $G125 = -0.56$ , and  $G126 = -0.12$  [3].



**Figure M293-4.** Bending test setup [3]

The angle change is calculated by using the equation [1]:

$$\gamma = \frac{\pi}{2} - 2 \cos^{-1} \frac{D + d}{\sqrt{2}D}$$

where  $d$  is the cross-head displacement and  $D$  is the difference between the original height and the original width of the sample. This equation holds only before the shear locking angle, specified in field `GAMMAL`, which is measured directly at the end of the test, so the curve should end when the fiber yarn angle reaches the shear locking state.

The bending test should be performed to characterize the compression modulus along the yarn directions, as specified in the `EC` field. The test setup is shown in [Figure M293-4](#). The composite specimen is held in a clamp and deforms under its own gravity. During the test, the composite is heated to the preforming temperature and the tip displacement is recorded. Due to the nonlinearity of the tensile modulus, the compression modulus is reversely calculated using a simulation: it is

adjusted until the simulation leads to similar tip displacement to the real experiment case. The starting point for the compression modulus iteration can be set as about 100X of the shear modulus when the warp and weft yarns are perpendicular to each other.

6. **Element Type.** The material model is available for shell elements with OSU=1 and INN=2 in the CONTROL\_ACCURACY card. It is recommended to use a double precision version of LS-DYNA.

**References:**

- [1] J. Cao, R. Akkerman, P. Boisse, J. Chen, H.S. Cheng, E.F. de Graaf, J.L. Gorczyca, P. Harrison, G. Hivet, J. Launay, W. Lee, L. Liu, S.V. Lomov, A. Long, E. de Luycker, F. Morestin, J. Padvoiskis, X.Q. Peng, J. Sherwood, Tz. Stoilova, X.M. Tao, I. Verpoest, A. Willems, J. Wiggers, T.X. Yu, B. Zhu, Characterization of mechanical behavior of woven fabrics: Experimental methods and benchmark results, *Composites Part A: Applied Science and Manufacturing*, Volume 39, Issue 6, 2008, Pages 1037-1053, ISSN 1359-835X.
- [2] Pu Xue, Xiongqi Peng, Jian Cao, A non-orthogonal constitutive model for characterizing woven composites, *Composites Part A: Applied Science and Manufacturing*, Volume 34, Issue 2, 2003, Pages 183-193, ISSN 1359-835X.
- [3] Weizhao Zhang, Huaqing Ren, Biao Liang, Danielle Zeng, Xuming Su, Jeffrey Dahl, Mansour Mirdamadi, Qiangsheng Zhao, Jian Cao, A non-orthogonal material model of woven composites in the preforming process, *CIRP Annals - Manufacturing Technology*, Volume 66, Issue 1, 2017, Pages 257-260, ISSN 0007-8506.
- [4] X.Q. Peng, J. Cao, A continuum mechanics-based non-orthogonal constitutive model for woven composite fabrics, *Composites Part A: Applied Science and Manufacturing*, Volume 36, Issue 6, 2005, Pages 859-874, ISSN 1359-835X.

**\*MAT\_ALE\_01**

**\*MAT\_ALE\_VACUUM**

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**\*MAT\_ALE\_VACUUM**

See \*MAT\_VACUUM or \*MAT\_140.





**Card 2 for Per Mole Cclulation.** Method (B) RUNIV is nonzero.

Card 2	1	2	3	4	5	6	7	8
Variable	MOLWT1	MOLWT2	MOLWT3	MOLWT4	MOLWT5	MOLWT6	MOLWT7	MOLWT8
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none
Remark	2							

**Card 3 for Per Mole Cclulation.** Method (B) RUNIV is nonzero.

Card 3	1	2	3	4	5	6	7	8
Variable	CPmole1	CPmole2	CPmole3	CPmole4	CPmole5	CPmole6	Cpmole7	CPmole8
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none
Remark	2							

**Card 4 for Per Mole Cclulation.** Method (B) RUNIV is nonzero.

Card 4	1	2	3	4	5	6	7	8
Variable	B1	B2	B3	B4	B5	B6	B7	B8
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none
Remark	2							

**Card 5 for Per Mole Cclcultation.** Method (B) RUNIV is nonzero.

Card 5	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none
Remark	2							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
IADIAB	This flag (default = 0) is used to turn ON/OFF adiabatic compression logics for an ideal gas (remark 5). EQ.0: OFF (default) EQ.1: ON
RUNIV	Universal gas constant in per-mole unit (8.31447 J/(mole*K)).
CVmass1 - CVmass8	If RUNIV is BLANK or zero (method A): Heat capacity at constant volume for up to eight different gases in per-mass unit.
CPmass1 - CPmass8	If RUNIV is BLANK or zero (method A): Heat capacity at constant pressure for up to eight different gases in per-mass unit.
MOLWT1 - MOLWT8	If RUNIV is nonzero (method B): Molecular weight of each ideal gas in the mixture (mass-unit/mole).
CPmole1 - CPmole8	If RUNIV is nonzero (method B): Heat capacity at constant pressure for up to eight different gases in per-mole unit. These are nominal heat capacity values typically at STP. These are denoted by the variable "A" in the equation in remark 2.
B1 - B8	If RUNIV is nonzero (method B): First order coefficient for a temperature dependent heat capacity at constant pressure for up to eight different gases. These are denoted by the variable "B" in the equation in remark 2.

VARIABLE	DESCRIPTION
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C1 - C8	If RUNIV is nonzero (method B): Second order coefficient for a temperature dependent heat capacity at constant pressure for up to eight different gases. These are denoted by the variable "C" in the equation in remark 2.
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**Remarks:**

1. There are 2 methods of defining the gas properties for the mixture. If RUNIV is BLANK or ZERO → Method (A) is used to define constant heat capacities where per-mass unit values of  $C_v$  and  $C_p$  are input. Only cards 2 and 3 are required for this method. Method (B) is used to define constant or temperature dependent heat capacities where per-mole unit values of  $C_p$  are input. Cards 2-5 are required for this method.
2. The per-mass-unit, temperature-dependent, constant-pressure heat capacity is

$$C_p(T) = \frac{(CPMOLE + B \times T + C \times T^2)}{MOLWT}$$

Typical metric units:

$C_p(T)$	CPMOLE	B	A
$\frac{J}{kg\ K}$	$\frac{J}{mole\ K}$	$\frac{J}{mole\ K^2}$	$\frac{J}{mole\ K^3}$

3. The initial temperature and the density of the gas species present in a mesh or part at time zero is specified by the keyword \*INITIAL\_GAS\_MIXTURE.
4. The ideal gas mixture is assumed to be thermal equilibrium, that is, all species are at the same temperature (T). The gases in the mixture are also assumed to follow Dalton's Partial Pressure Law,

$$P = \sum_i^{ngas} P_i.$$

The partial pressure of each gas is then

$$P_i = \rho_i R_{gas_i} T$$

Where

$$R_{gas_i} = \frac{R_{univ}}{MOLWT}$$

The individual gas species temperature equals the mixture temperature. The temperature is computed from the internal energy where the *mixture internal energy per unit volume* is used,

$$T = T_i = \frac{e_V}{\sum_i^{\text{ngas}} \rho_i C_{V_i}}$$

whence

$$\begin{aligned} e_V &= \sum_i^{\text{ngas}} \rho_i C_{V_i} T_i \\ &= \sum_i^{\text{ngas}} \rho_i C_{V_i} T. \end{aligned}$$

In general, the advection step conserves momentum and internal energy, but not kinetic energy. This can result in energy lost in the system and lead to a pressure drop. In \*MAT\_GAS\_MIXTURE the dissipated kinetic energy is automatically stored in the internal energy. Thus in effect the total energy is conserved instead of conserving just the internal energy. This numerical scheme has been shown to improve accuracy in some cases. However, the user should always be vigilant and check the physics of the problem closely.

- As an example consider an airbag surrounded by ambient air. As the inflator gas flows into the bag, the ALE elements cut by the airbag fabric shell elements will contain some inflator gas inside and some ambient air outside. The multi-material element treatment is not perfect. Consequently the temperature of the outside air may, occasionally, be made artificially high after the multi-material element treatment. To prevent the outside ambient air from getting artificially high T, set IDI-AB = 1 for the ambient air outside. Simple adiabatic compression equation is then assumed for the outside air. The use of this flag may be needed, but only when that outside air is modeled by the \*MAT\_GAS\_MIXTURE card.

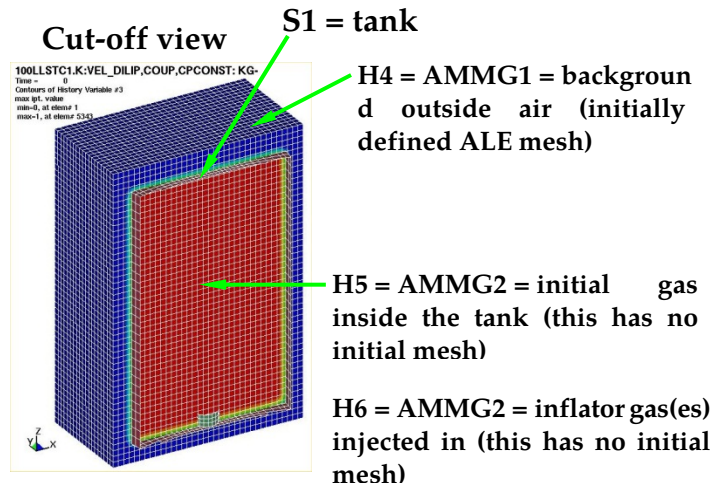
### Example:

Consider a tank test model where the Lagrangian tank (Part S1) is surrounded by an ALE air mesh (Part H4 = AMMGID 1). There are 2 ALE parts which are defined but initially have no corresponding mesh: part 5 (H5 = AMMGID 2) is the resident gas inside the tank at t = 0, and part 6 (H6 = AMMGID 2) is the inflator gas(es) which is injected into the tank when t > 0. AMMGID stands for ALE Multi-Material Group ID. Please see figure and input below. The \*MAT\_GAS\_MIXTURE (MGM) card defines the gas properties of ALE parts H5 & H6. The MGM card input for both method (A) and (B) are shown.

The \*INITIAL\_GAS\_MIXTURE card is also shown. It basically specifies that "AMMGID 2 may be present in part or mesh H4 at t = 0, and the initial density of this gas is defined in

the rho1 position which corresponds to the 1<sup>st</sup> material in the mixture (or H5, the resident gas).”

**Example configuration:**



**Sample input:**

```

$-----
*PART
H5 = initial gas inside the tank
$      PID      SECID      MID      EOSID      HGID      GRAV      ADPOPT      TMID
$      5         5         5         0         5         0         0
*SECTION_SOLID
$      5         11         0
$-----
$ Example 1: Constant heat capacities using per-mass unit.
$*MAT_GAS_MIXTURE
$      MID      IADIAB      R_univ
$      5         0         8.314470
$      Cv1_mas  Cv2_mas  Cv3_mas  Cv4_mas  Cv5_mas  Cv6_mas  Cv7_mas  Cv8_mas
$718.7828911237.56228
$      Cp1_mas  Cp2_mas  Cp3_mas  Cp4_mas  Cp5_mas  Cp6_mas  Cp7_mas  Cp8_mas
$1007.00058 1606.1117
$-----
$ Example 2: Variable heat capacities using per-mole unit.
$*MAT_GAS_MIXTURE
$      MID      IADIAB      R_univ
$      5         0         8.314470
$      MW1      MW2      MW3      MW4      MW5      MW6      MW7      MW8
$      0.0288479 0.02256
$      Cp1_mol  Cp2_mol  Cp3_mol  Cp4_mol  Cp5_mol  Cp6_mol  Cp7_mol  Cp8_mol
$      29.049852 36.23388
$      B1       B2       B3       B4       B5       B6       B7       B8
$      7.056E-3 0.132E-1
$      C1       C2       C3       C4       C5       C6       C7       C8
$      -1.225E-6 -0.190E-5
$-----
$ One card is defined for each AMMG that will occupy some elements of a mesh set
*INITIAL_GAS_MIXTURE
$      SID      STYPE      MMGID      T0
$      4         1         1         298.15

```

**\*MAT\_ALE\_GAS\_MIXTURE**

**\*MAT\_ALE\_02**

---

\$ RHO1 RHO2 RHO3 RHO4 RHO5 RHO6 RHO7 RHO8  
1.17913E-9  
\*INITIAL\_GAS\_MIXTURE  
\$ SID STYPE MMGID T0  
4 1 2 298.15  
\$ RHO1 RHO2 RHO3 RHO4 RHO5 RHO6 RHO7 RHO8  
1.17913E-9  
\$-----

**\*MAT\_ALE\_VISCOUS**

This may also be referred to as MAT\_ALE\_03. This “fluid-like” material model is very similar to Material Type 9 (\*MAT\_NULL). It allows the modeling of non-viscous fluids with constant or variable viscosity. The variable viscosity is a function of an equivalent deviatoric strain rate. If inviscid material is modeled, the deviatoric or viscous stresses are zero, and the equation of state supplies the pressures (or diagonal components of the stress tensor). All \*MAT\_ALE\_cards apply only to ALE element formulation.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PC	MULO	MUHI	RK	Not used	RN
Type	I	F	F	F	F	F		F
Defaults	none	none	0.0	0.0	0.0	0.0		0.0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number has to be chosen.
RO	Mass density.
PC	Pressure cutoff ( $\leq 0.0$ ). See <a href="#">Remark 4</a> .
MULO	There are 4 possible cases (See <a href="#">Remark 1</a> ): <ol style="list-style-type: none"> <li>1. If MULO = 0.0, then inviscid fluid is assumed.</li> <li>2. If MULO &gt; 0.0, and MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient <math>\mu</math>.</li> <li>3. If MULO &gt; 0.0, and MUHI &gt; 0.0, then MULO and MUHI are lower and upper viscosity limit values for a power-law-like variable viscosity model.</li> <li>4. If MULO is negative (for example, MULO = -1), then a user-input data load curve (with LCID = 1) defining dynamic viscosity as a function of equivalent strain rate is used.</li> </ol>



VARIABLE	DESCRIPTION
MUHI	There are 2 possible cases: <ol style="list-style-type: none"> <li>5. If <math>MUHI &lt; 0.0</math>, then the viscosity can be defined by the user in the file <code>dyn21.F</code> with a routine called <code>f3dm9ale_userdef1</code>. The file is part of the general <code>usermat</code> package.</li> <li>6. If <math>MUHI &gt; 0.0</math>, then this is the upper dynamic viscosity limit (default = 0.0). This is defined only if <code>RK</code> and <code>RN</code> are defined for the variable viscosity case.</li> </ol>
RK	Variable dynamic viscosity multiplier. See <a href="#">Remark 6</a> .
RN	Variable dynamic viscosity exponent. See <a href="#">Remark 6</a> .

### Remarks:

1. **Deviatoric Viscous Stress.** The null material must be used with an equation-of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\epsilon}'_{ij}$$

$$\left[ \frac{N}{m^2} \right] \sim \left[ \frac{N}{m^2} s \right] \left[ \frac{1}{s} \right]$$

is computed for nonzero  $\mu$  where  $\dot{\epsilon}'_{ij}$  is the deviatoric strain rate.  $\mu$  is the dynamic viscosity. For example, in SI unit system,  $\mu$  has a unit of  $[\text{Pa} \times \text{s}]$ .

2. **Hourglass Control Issues.** The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general for fluid(s), the hourglass coefficient `QM` should be small (in the range  $10^{-4}$  to  $10^{-6}$  for the standard default `IHQ` choice).
3. **Null Material Properties.** Null material has no yield strength and behaves in a fluid-like manner.
4. **Numerical Cavitation.** The pressure cut-off, `PC`, must be defined to allow for a material to “numerically” cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting `PC` limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

5. **Issues with Small Values of Viscosity Exponent.** If the viscosity exponent is less than 1.0,  $RN < 1.0$ , then  $RN - 1.0 < 0.0$ . In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.
6. **Empirical Dynamic Viscosity.** The empirical variable dynamic viscosity is typically modeled as a function of *equivalent shear rate* based on experimental data.

$$\mu(\dot{\bar{\gamma}}') = RK \times \dot{\bar{\gamma}}'^{(RN-1)}$$

For an incompressible fluid, this may be written equivalently as

$$\mu(\dot{\bar{\epsilon}}') = RK \times \dot{\bar{\epsilon}}'^{(RN-1)}$$

The “overbar” denotes a scalar equivalence. The “dot” denotes a time derivative or rate effect. And the “prime” symbol denotes deviatoric or volume preserving components. The *equivalent shear rate* components may be related to the basic definition of (small-strain) strain rate components as follows:

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \Rightarrow \dot{\epsilon}'_{ij} = \dot{\epsilon}_{ij} - \delta_{ij} \left( \frac{\dot{\epsilon}_{kk}}{3} \right)$$

$$\dot{\gamma}_{ij} = 2\dot{\epsilon}_{ij}$$

Typically, the 2<sup>nd</sup> invariant of the deviatoric strain rate tensor is defined as:

$$I_{2\dot{\epsilon}'} = \frac{1}{2} [\dot{\epsilon}'_{ij}\dot{\epsilon}'_{ij}]$$

The equivalent (small-strain) deviatoric strain rate is defined as:

$$\dot{\bar{\epsilon}}' \equiv 2\sqrt{I_{2\dot{\epsilon}'}} = \sqrt{2[\dot{\epsilon}'_{ij}\dot{\epsilon}'_{ij}]} = \sqrt{4[\dot{\epsilon}'_{12}{}^2 + \dot{\epsilon}'_{23}{}^2 + \dot{\epsilon}'_{31}{}^2] + 2[\dot{\epsilon}'_{11}{}^2 + \dot{\epsilon}'_{22}{}^2 + \dot{\epsilon}'_{33}{}^2]}$$

In non-Newtonian literatures, the *equivalent shear rate* is sometimes defined as

$$\dot{\bar{\gamma}} \equiv \sqrt{\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{2}} = \sqrt{2\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}} = \sqrt{4[\dot{\epsilon}_{12}{}^2 + \dot{\epsilon}_{23}{}^2 + \dot{\epsilon}_{31}{}^2] + 2[\dot{\epsilon}_{11}{}^2 + \dot{\epsilon}_{22}{}^2 + \dot{\epsilon}_{33}{}^2]}$$

It turns out that, (a) for incompressible materials ( $\dot{\epsilon}_{kk} = 0$ ), and (b) the shear terms are equivalent when  $i \neq j \rightarrow \dot{\epsilon}_{ij} = \dot{\epsilon}'_{ij}$ , the *equivalent shear rate* is algebraically equivalent to the *equivalent (small-strain) deviatoric strain rate*.

$$\dot{\bar{\epsilon}}' = \dot{\bar{\gamma}}'$$



VARIABLE	DESCRIPTION
MID	Material identification. A unique number has to be chosen.
RO	Mass density
PC	Pressure cutoff ( $\leq 0.0$ ).
MULO	There are 3 possible cases: (1) If MULO > 0.0, and MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient $\mu$ . (2) If MULO > 0.0, and MUHI > 0.0, then MULO and MUHI are lower and upper viscosity limit values. (3) If MULO is negative (for example, MULO = -1), then a user-input data load curve (with LCID = 1) defining dynamic viscosity as a function of equivalent strain rate is used.
MUHI	Upper dynamic viscosity limit (default = 0.0). This is defined only if RK and RN are defined for the variable viscosity case.
RK	Variable dynamic viscosity multiplier. The viscosity is computed as $\mu(\bar{\epsilon}') = rk \cdot \bar{\epsilon}'^{(rn-1)}$ where the equivalent deviatoric strain rate is $\bar{\epsilon}' = \sqrt{\frac{2}{3} [\dot{\epsilon}'_{11}{}^2 + \dot{\epsilon}'_{22}{}^2 + \dot{\epsilon}'_{33}{}^2 + 2(\dot{\epsilon}'_{12}{}^2 + \dot{\epsilon}'_{23}{}^2 + \dot{\epsilon}'_{31}{}^2)]}$
RN	Variable dynamic viscosity exponent (see below for details).
LCI	Characteristic length, $l_{ci}$ , of the internal turbulent domain.
C0 - C6	Internal flow mixing length polynomial coefficients. The one-parameter turbulent mixing length is computed as $l_m = l_{ci} \left[ C_0 + C_1 \left( 1 - \frac{y}{l_{ci}} \right) + \dots + C_6 \left( 1 - \frac{y}{l_{ci}} \right)^6 \right]$
LCX	Characteristic length, $l_{cx}$ , of the external turbulent domain.
D0 - D2	External flow mixing length polynomial coefficients. If $y \leq l_{cx}$ then the mixing length is computed as $l_m = [D_0 + D_1 y + D_2 y^2]$
E0 - E2	External flow mixing length polynomial coefficients. If $y > l_{cx}$ then the mixing length is computed as $l_m = [E_0 + E_1 y + E_2 y^2]$

**Remarks:**

1. The null material must be used with an equation of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = \mu \dot{\epsilon}'_{ij}$$

$$\left[ \frac{N}{m^2} \right] \approx \left[ \frac{N}{m^2} s \right] \left[ \frac{1}{S} \right]$$

is computed for nonzero  $\mu$  where  $\dot{\epsilon}'_{ij}$  is the deviatoric strain rate.  $\mu$  is the dynamic viscosity with unit of [Pa × s].

2. The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general for fluid(s), the hourglass coefficient QM should be small (in the range  $10^{-4}$  to  $10^{-6}$  for the standard default IHQ choice).
3. The Null material has no yield strength and behaves in a fluid-like manner.
4. The pressure cut-off, PC, must be defined to allow for a material to “numerically” cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.
5. If the viscosity exponent is less than 1.0, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.
6. Turbulence is treated simply by considering its effects on viscosity. Total effective viscosity is the sum of the laminar and turbulent viscosities,  $\mu_{\text{eff}} = \mu_l + \mu_t$  where  $\mu_{\text{eff}}$  is the effective viscosity, and  $\mu_t$  is the turbulent viscosity.
7. The turbulent viscosity is computed based on the Prandtl’s Mixing Length Model,

$$\mu_t = \rho l_m^2 |\nabla \mathbf{v}|$$

**\*MAT\_ALE\_05**

**\*MAT\_ALE\_INCOMPRESSIBLE**

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**\*MAT\_ALE\_INCOMPRESSIBLE**

See \*MAT\_160.

**\*MAT\_ALE\_HERSCHEL**

This may also be referred to as MAT\_ALE\_06. This is the Herschel-Buckley model. It is an enhancement to the power law viscosity model in \*MAT\_ALE\_VISCOUS(\*MAT\_ALE\_03). Two additional input parameters: the yield stress threshold and critical shear strain rate can be specified to model “rigid-like” material for low strain rates.

It allows the modeling of non-viscous fluids with constant or variable viscosity. The variable viscosity is a function of an equivalent deviatoric strain rate. All \*MAT\_ALE\_cards apply only to ALE element formulation.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PC	MULO	MUHI	RK	Not used	RN
Type	I	F	F	F	F	F		F
Defaults	none	none	0.0	0.0	0.0	0.0		0.0

Card 2	1	2	3	4	5	6	7	8
Variable	GDOTC	TA00						
Type	F	F						
Default	none	none						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number has to be chosen.
RO	Mass density.
PC	Pressure cutoff ( $\leq 0.0$ ), (See remark 4).

VARIABLE	DESCRIPTION
MULO	There are 4 possible cases (See remark 1): <ol style="list-style-type: none"> <li>If MULO = 0.0, then inviscid fluid is assumed.</li> <li>If MULO &gt; 0.0, and MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient <math>\mu</math>.</li> <li>If MULO &gt; 0.0, and MUHI &gt; 0.0, then MULO and MUHI are lower and upper viscosity limit values for a power-law-like variable viscosity model.</li> <li>If MULO is negative (for example, MULO = -1), then a user-input data load curve (with LCID = 1) defining dynamic viscosity as a function of equivalent strain rate is used.</li> </ol>
MUHI	Upper dynamic viscosity limit (default = 0.0). This is defined only if RK and RN are defined for the variable viscosity case.
RK	$k$ ; consistency factor (See remark 6).
RN	$n$ ; power law index (See remark 6).
GDOTC	$\dot{\gamma}_c$ ; critical shear strain rate (See remark 6).
TAO0	$\tau_0$ ; yield stress (See remark 6).

**Remarks:**

- The null material must be used with an equation-of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\epsilon}'_{ij}$$

$$\left[ \frac{N}{m^2} \right] \sim \left[ \frac{N}{m^2} s \right] \left[ \frac{1}{s} \right]$$

is computed for nonzero  $\mu$  where  $\dot{\epsilon}'_{ij}$  is the deviatoric strain rate.  $\mu$  is the dynamic viscosity. For example, in SI unit system,  $\mu$  has a unit of [Pa\*s].

- The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general for fluid(s), the hourglass coefficient QM should be small (in the range 1.0E-4 to 1.0E-6 for the standard default IHQ choice).
- Null material has no yield strength and behaves in a fluid-like manner.



4. The pressure cut-off, PC, must be defined to allow for a material to “numerically” cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.
5. If the viscosity exponent is less than 1.0,  $RN < 1.0$ , then  $RN - 1.0 < 0.0$ . In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.
6. The Herschel-Buckley model employs a large viscosity to model the “rigid-like” behavior for low shear strain rates ( $\dot{\gamma} < \dot{\gamma}_c$ ).

$$\mu = \mu_0$$

Power law is used once the yield stress is passed.

$$\mu(\dot{\gamma}) = \frac{\tau_0}{\dot{\gamma}} + k\left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^{n-1}$$

The shear strain rate is:

$$\dot{\gamma} \equiv \sqrt{\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{2}} = \sqrt{2\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}} = \sqrt{4[\dot{\epsilon}_{12}^2 + \dot{\epsilon}_{23}^2 + \dot{\epsilon}_{31}^2] + 2[\dot{\epsilon}_{11}^2 + \dot{\epsilon}_{22}^2 + \dot{\epsilon}_{33}^2]}$$

**\*MAT\_SPH\_VISCOUS**

This may also be referred to as MAT\_SPH\_01. This “fluid-like” material model is very similar to Material Type 9 (\*MAT\_NULL). It allows the modeling of viscous fluids with constant or variable viscosity. The variable viscosity is a function of an equivalent deviatoric strain rate. If inviscid material is modeled, the deviatoric or viscous stresses are zero, and the equation of state supplies the pressures (or diagonal components of the stress tensor).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PC	MULO	MUHI	RK	RC	RN
Type	I	F	F	F	F	F	F	F
Defaults	none	none	0.0	0.0	0.0	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number has to be chosen.
RO	Mass density.
PC	Pressure cutoff ( $\leq 0.0$ ). See <a href="#">Remark 4</a> .
MULO	There are 4 possible cases (See <a href="#">Remark 1</a> ): <ol style="list-style-type: none"> <li>1. If <math>MULO = 0.0</math>, then inviscid fluid is assumed.</li> <li>2. If <math>MULO &gt; 0.0</math>, and <math>MUHI = 0.0</math> or is not defined, then this is the traditional constant dynamic viscosity coefficient <math>\mu</math>.</li> <li>3. If <math>MULO &gt; 0.0</math>, and <math>MUHI &gt; 0.0</math>, then <math>MULO</math> and <math>MUHI</math> are lower and upper viscosity limit values for a power-law-like variable viscosity model.</li> <li>4. If <math>MULO</math> is negative (for example, <math>MULO = -1</math>), then a user-input data load curve (with <math>LCID = 1</math>) defining dynamic viscosity as a function of equivalent strain rate is used.</li> </ol>

VARIABLE	DESCRIPTION
MUHI	<p>There are 2 possible cases:</p> <ol style="list-style-type: none"> <li>If <math>MUHI &lt; 0.0</math>, then the viscosity can be defined by the user in the file <code>dyn21.F</code> with a routine called <code>f3dm9sph_userdefin</code>. The file is part of the general <code>usermat</code> package.</li> <li>If <math>MUHI &gt; 0.0</math>, then this is the upper dynamic viscosity limit (default = 0.0). This is defined only if <code>RK</code> and <code>RN</code> are defined for the variable viscosity case.</li> </ol>
RK	Variable dynamic viscosity multiplier. See <a href="#">Remark 6</a> .
RC	<p>Option for Cross viscosity model: See <a href="#">Remark 7</a>.</p> <p><math>RC &gt; 0.0</math>: Cross viscosity model will be used (overwrite all other options), values of <code>MULO</code>, <code>MUHI</code>, <code>RK</code> and <code>RN</code> will be used in the Cross viscosity model. See <a href="#">Remark 7</a>.</p> <p><math>RC \leq 0.0</math>: other viscosity model (decided based on above variables) will be used.</p>
RN	Variable dynamic viscosity exponent. See <a href="#">Remark 6</a> .

### Remarks:

- Deviatoric Viscous Stress.** The null material must be used with an equation-of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\epsilon}'_{ij}$$

$$\left[\frac{N}{m^2}\right] \sim \left[\frac{N}{m^2}s\right] \left[\frac{1}{s}\right]$$

is computed for nonzero  $\mu$  where  $\dot{\epsilon}'_{ij}$  is the deviatoric strain rate.  $\mu$  is the dynamic viscosity. For example, in SI unit system,  $\mu$  has a unit of  $[\text{Pa} \times \text{s}]$ .

- Hourglass Control Issues.** The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general for fluid(s), the hourglass coefficient `QM` should be small (in the range  $10^{-4}$  to  $10^{-6}$  for the standard default `IHQ` choice).
- Null Material Properties.** Null material has no yield strength and behaves in a fluid-like manner.

4. **Numerical Cavitation.** The pressure cut-off, PC, must be defined to allow for a material to “numerically” cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.
5. **Issues with Small Values of Viscosity Exponent.** If the viscosity exponent is less than 1.0,  $RN < 1.0$ , then  $RN - 1.0 < 0.0$ . In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.
6. **Empirical Dynamic Viscosity.** The empirical variable dynamic viscosity is typically modeled as a function of *equivalent shear rate* based on experimental data.

$$\mu(\dot{\bar{\gamma}}') = RK \times \dot{\bar{\gamma}}'^{(RN-1)}$$

For an incompressible fluid, this may be written equivalently as

$$\mu(\dot{\bar{\epsilon}}') = RK \times \dot{\bar{\epsilon}}'^{(RN-1)}$$

The “overbar” denotes a scalar equivalence. The “dot” denotes a time derivative or rate effect. And the “prime” symbol denotes deviatoric or volume preserving components. The *equivalent shear rate* components may be related to the basic definition of (small-strain) strain rate components as follows:

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \Rightarrow \dot{\epsilon}'_{ij} = \dot{\epsilon}_{ij} - \delta_{ij} \left( \frac{\dot{\epsilon}_{kk}}{3} \right)$$

$$\dot{\gamma}_{ij} = 2\dot{\epsilon}_{ij}$$

Typically, the 2<sup>nd</sup> invariant of the deviatoric strain rate tensor is defined as:

$$I_{2\dot{\epsilon}'} = \frac{1}{2} [\dot{\epsilon}'_{ij}\dot{\epsilon}'_{ij}]$$

The equivalent (small-strain) deviatoric strain rate is defined as:

$$\dot{\bar{\epsilon}}' \equiv 2\sqrt{I_{2\dot{\epsilon}'}} = \sqrt{2[\dot{\epsilon}'_{ij}\dot{\epsilon}'_{ij}]} = \sqrt{4[\dot{\epsilon}'_{12}{}^2 + \dot{\epsilon}'_{23}{}^2 + \dot{\epsilon}'_{31}{}^2] + 2[\dot{\epsilon}'_{11}{}^2 + \dot{\epsilon}'_{22}{}^2 + \dot{\epsilon}'_{33}{}^2]}$$

In non-Newtonian literatures, the *equivalent shear rate* is sometimes defined as

$$\dot{\bar{\gamma}} \equiv \sqrt{\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{2}} = \sqrt{2\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}} = \sqrt{4[\dot{\epsilon}_{12}{}^2 + \dot{\epsilon}_{23}{}^2 + \dot{\epsilon}_{31}{}^2] + 2[\dot{\epsilon}_{11}{}^2 + \dot{\epsilon}_{22}{}^2 + \dot{\epsilon}_{33}{}^2]}$$

It turns out that, (a) for incompressible materials ( $\dot{\epsilon}_{kk} = 0$ ), and (b) the shear terms are equivalent when  $i \neq j \rightarrow \dot{\epsilon}_{ij} = \dot{\epsilon}'_{ij}$ , the *equivalent shear rate* is algebraically equivalent to the *equivalent (small-strain) deviatoric strain rate*.

$$\dot{\bar{\epsilon}}' = \dot{\bar{\gamma}}'$$

7. The Cross viscous model is one of simplest and most used model for shear-thinning behavior, i.e., the fluid's viscosity decreases with increasing of the local shear rate  $\dot{\gamma}$ , thus the dynamic viscosity  $\mu$  is defined as a function of  $\dot{\gamma}$ :

$$\mu(\dot{\gamma}) = \text{MUHI} + (\text{MULO} - \text{MUHI}) / (1.0 + \text{RK} * \dot{\gamma})^{\text{RN}-1}$$

Where RK and RN are two positive fitting parameters, and MULO, MUHI are the limiting values of the viscosity at low and high shear rates, respectively. RK, RN, MULO and MUHI are parameters from keyword input.

## \*MAT\_S01

## \*MAT\_SPRING\_ELASTIC

### \*MAT\_SPRING\_ELASTIC

This is Material Type 1 for discrete elements (\*ELEMENT\_DISCRETE). This provides a translational or rotational elastic spring located between two nodes. Only one degree of freedom is connected.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	K						
Type	A8	F						

#### VARIABLE

#### DESCRIPTION

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
K	Elastic stiffness (force/displacement) or (moment/rotation).

**\*MAT\_DAMPER\_VISCOUS**

This is Material Type 2 for discrete elements (\*ELEMENT\_DISCRETE). This material provides a linear translational or rotational damper located between two nodes. Only one degree of freedom is then connected.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	DC						
Type	A8	F						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
DC	Damping constant (force/displacement rate) or (moment/rotation rate).

**\*MAT\_SPRING\_ELASTOPLASTIC**

This is Material Type 3 for discrete elements (\*ELEMENT\_DISCRETE). This material provides an elastoplastic translational or rotational spring with isotropic hardening located between two nodes. Only one degree of freedom is connected.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	K	KT	FY				
Type	A8	F	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
K	Elastic stiffness (force/displacement) or (moment/rotation).
KT	Tangent stiffness (force/displacement) or (moment/rotation).
FY	Yield (force) or (moment).



**\*MAT\_SPRING\_NONLINEAR\_ELASTIC**

This is Material Type 4 for discrete elements (\*ELEMENT\_DISCRETE). This material provides a nonlinear elastic translational and rotational spring with arbitrary force versus displacement and moment versus rotation, respectively. Optionally, strain rate effects can be considered through a velocity dependent scale factor. With the spring located between two nodes, only one degree of freedom is connected.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	LCD	LCR					
Type	A8	I	I					

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
LCD	Load curve ID describing force versus displacement or moment versus rotation relationship. The load curve <i>must</i> define the response in the negative and positive quadrants and pass through point (0,0). Negative data point(s) must come first in the curve definition, where negative values represent compression in the case of a translational spring.
LCR	Optional load curve describing scale factor on force or moment as a function of relative velocity or. rotational velocity, respectively.

**\*MAT\_DAMPER\_NONLINEAR\_VISCOUS**

This is Material Type 5 for discrete elements (\*ELEMENT\_DISCRETE). This material provides a viscous translational damper with an arbitrary force versus velocity dependency, or a rotational damper with an arbitrary moment versus rotational velocity dependency. With the damper located between two nodes, only one degree of freedom is connected.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	LCDR						
Type	A8	I						

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
LCDR	Load curve identification describing force versus rate-of-displacement relationship or a moment versus rate-of-rotation relationship. The load curve <i>must</i> define the response in the negative and positive quadrants and pass through point (0,0).

\*MAT\_SPRING\_GENERAL\_NONLINEAR

This is Material Type 6 for discrete elements (\*ELEMENT\_DISCRETE). This material provides a general nonlinear translational or rotational spring with arbitrary loading and unloading definitions. Optionally, hardening or softening can be defined. With the spring located between two nodes, only one degree of freedom is connected.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	LCDL	LCDU	BETA	TYI	CYI		
Type	A8	I	I	F	F	F		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
LCDL	Load curve identification describing force/torque versus displacement/rotation relationship for loading, see <a href="#">Figure M26-1</a> .
LCDU	Load curve identification describing force/torque versus displacement/rotation relationship for unloading, see <a href="#">Figure M119-1</a> .
BETA	Hardening parameter, $\beta$ : EQ.0.0: Tensile and compressive yield with strain softening (negative or zero slope allowed in the force versus displacement. load curves). TYI and CYI are not implemented for this option. NE.0.0: Kinematic hardening without strain softening. EQ.1.0: Isotropic hardening without strain softening.
TYI	Initial yield force in tension ( > 0)
CYI	Initial yield force in compression ( < 0)

**Remarks:**

Load curve points are in the format (displacement, force or rotation, moment). The points must be in order starting with the most negative (compressive) displacement or rotation and ending with the most positive (tensile) value. The curves need not be symmetrical.

The displacement origin of the “unloading” curve is arbitrary, since it will be shifted as necessary as the element extends and contracts. On reverse yielding the “loading” curve will also be shifted along the displacement re or. rotation axis. The initial tensile and compressive

Yield forces (TYI and CYI) define a range within which the element remains elastic (i.e. the “loading” curve is used for both loading and unloading). If at any time the force in the element exceeds this range, the element is deemed to have yielded, and at all subsequent times the “unloading” curve is used for unloading

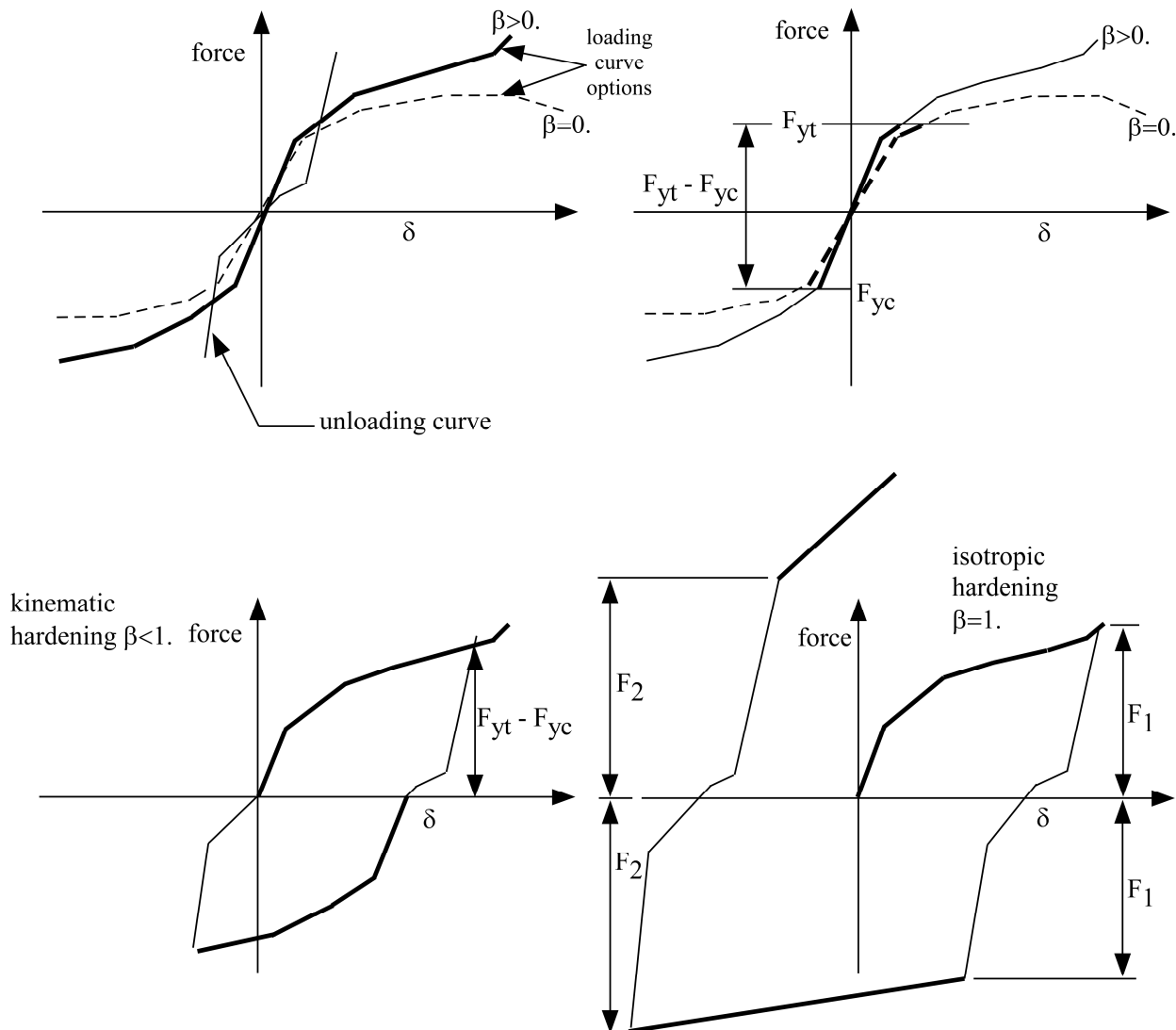


Figure MS6-1. General Nonlinear material for discrete elements

**\*MAT\_SPRING\_MAXWELL**

This is Material Type 7 for discrete elements (\*ELEMENT\_DISCRETE). This material provides a three Parameter Maxwell Viscoelastic translational or rotational spring. Optionally, a cutoff time with a remaining constant force/moment can be defined.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	K0	KI	BETA	TC	FC	COPT	
Type	A8	F	F	F	F	F	F	
Default					1020	0	0	

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
K0	$K_0$ , short time stiffness
KI	$K_\infty$ , long time stiffness
BETA	Decay parameter.
TC	Cut off time. After this time a constant force/moment is transmitted.
FC	Force/moment after cutoff time
COPT	Time implementation option: EQ.0: incremental time change, NE.0: continuous time change.

**Remarks:**

The time varying stiffness  $K(t)$  may be described in terms of the input parameters as

$$K(T) = K_\infty + (K_0 - K_\infty)\exp(-\beta t)$$

This equation was implemented by Schwer [1991] as either a continuous function of time or incrementally following the approach of Herrmann and Peterson [1968]. The continuous function of time implementation has the disadvantage of the energy absorber's resistance

decaying with increasing time even without deformation. The advantage of the incremental implementation is that an energy absorber must undergo some deformation before its resistance decays, i.e., there is no decay until impact, even in delayed impacts. The disadvantage of the incremental implementation is that very rapid decreases in resistance cannot be easily matched.

**\*MAT\_SPRING\_INELASTIC**

This is Material Type 8 for discrete elements (\*ELEMENT\_DISCRETE). This material provides an inelastic tension or compression only, translational or rotational spring. Optionally, a user-specified unloading stiffness can be taken instead of the maximum loading stiffness.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	LCFD	KU	CTF				
Type	A8	I	F	F				

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
LCFD	Load curve identification describing arbitrary force/torque versus displacement/rotation relationship. This curve must be defined in the positive force-displacement quadrant regardless of whether the spring acts in tension or compression.
KU	Unloading stiffness (optional). The maximum of KU and the maximum loading stiffness in the force/displacement or the moment/rotation curve is used for unloading.
CTF	Flag for compression/tension: EQ.-1.0: tension only, EQ.0.0: default is set to 1.0, EQ.1.0: compression only.

## \*MAT\_S13

## \*MAT\_SPRING\_TRILINEAR\_DEGRADING

### \*MAT\_SPRING\_TRILINEAR\_DEGRADING

This is Material Type 13 for discrete elements (\*ELEMENT\_DISCRETE). This material allows concrete shearwalls to be modeled as discrete elements under applied seismic loading. It represents cracking of the concrete, yield of the reinforcement and overall failure. Under cyclic loading, the stiffness of the spring degrades but the strength does not.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	DEFL1	F1	DEFL2	F2	DEFL3	F3	FFLAG
Type	A8	F	F	F	F	F	F	F

#### VARIABLE

#### DESCRIPTION

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
DEFL1	Deflection at point where concrete cracking occurs.
F1	Force corresponding to DEFL1
DEFL2	Deflection at point where reinforcement yields
F2	Force corresponding to DEFL2
DEFL3	Deflection at complete failure
F3	Force corresponding to DEFL3
FFLAG	Failure flag.



**\*MAT\_SPRING\_SQUAT\_SHEARWALL**

This is Material Type 14 for discrete elements (\*ELEMENT\_DISCRETE). This material allows squat shear walls to be modeled using discrete elements. The behavior model captures concrete cracking, reinforcement yield, ultimate strength followed by degradation of strength finally leading to collapse.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	A14	B14	C14	D14	E14	LCID	FSD
Type	A8	F	F	F	F	F	I	F

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
A14	Material coefficient <i>A</i>
B14	Material coefficient <i>B</i>
C14	Material coefficient <i>C</i>
D14	Material coefficient <i>D</i>
E14	Material coefficient <i>E</i>
LCID	Load curve ID referencing the maximum strength envelope curve
FSD	Sustained strength reduction factor

**Remarks:**

Material coefficients *A*, *B*, *C* and *D* are empirically defined constants used to define the shape of the polynomial curves which govern the cyclic behavior of the discrete element. A different polynomial relationship is used to define the loading and unloading paths allowing energy absorption through hysteresis. Coefficient *E* is used in the definition of the path used to “jump” from the loading path to the unloading path (or vice versa) where a full hysteresis loop is not completed. The load curve referenced is used to define the force displacement characteristics of the shear wall under monotonic loading. This curve is the basis to which the polynomials defining the cyclic behavior refer to. Finally, on the second and subsequent loading / unloading cycles, the shear wall will have reduced strength. The variable FSD is the sustained strength reduction factor.

**\*MAT\_SPRING\_MUSCLE**

This is Material Type 15 for discrete elements (\*ELEMENT\_DISCRETE). This material is a Hill-type muscle model with activation. It is for use with discrete elements. The LS-DYNA implementation is due to Dr. J. A. Weiss.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	L0	VMAX	SV	A	FMAX	TL	TV
Type	A8	F	F	F	F	F	F	F
Default		1.0		1.0			1.0	1.0

Card 2	1	2	3	4	5	6	7	8
Variable	FPE	LMAX	KSH					
Type	F	F	F					
Default	0.0							

**VARIABLE****DESCRIPTION**

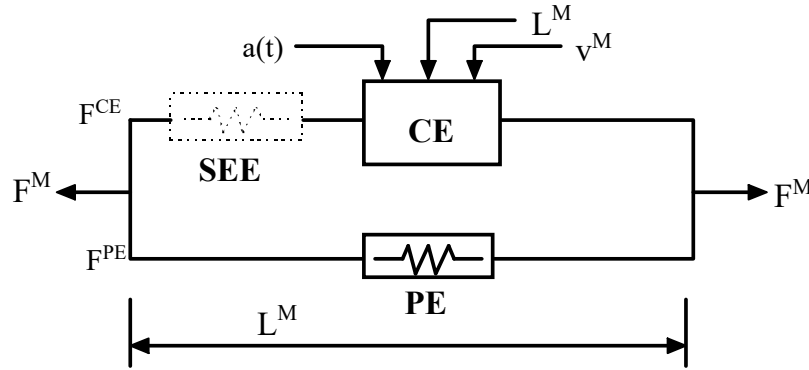
MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
L0	Initial muscle length, $L_0$ .
VMAX	Maximum CE shortening velocity, $V_{\max}$ .
SV	Scale factor, $S_v$ , for $V_{\max}$ vs. active state. LT.0: absolute value gives load curve ID GE.0: constant value of 1.0 is used
A	Activation level vs. time function $a(t)$ . LT.0: absolute value gives load curve ID GE.0: constant value of A is used

VARIABLE	DESCRIPTION
FMAX	Peak isometric force, $F_{\max}$ .
TL	Active tension vs. length function, $f_{TL}(L)$ . LT.0: absolute value gives load curve ID GE.0: constant value of 1.0 is used
TV	Active tension vs. velocity function, $f_{TV}(V)$ . LT.0: absolute value gives load curve ID GE.0: constant value of 1.0 is used
FPE	Force vs. length function, $f_{PE}$ , for parallel elastic element. LT.0: absolute value gives load curve ID EQ.0: exponential function is used (see below) GT.0: constant value of 0.0 is used
LMAX	Relative length when $F^{PE}$ reaches $F_{\max}$ . Required if $F^{PE}$ above.
KSH	Constant, $K_{sh}$ , governing the exponential rise of $F^{PE}$ . Required if $F^{PE}$ above.

### Remarks:

The material behavior of the muscle model is adapted from the original model proposed by Hill [1938]. Reviews of this model and extensions can be found in Winters [1990] and Zajac [1989]. The most basic Hill-type muscle model consists of a contractile element (CE) and a parallel elastic element (PE) (Figure MS15-1). An additional series elastic element (SEE) can be added to represent tendon compliance.

The main assumptions of the Hill model are that the contractile element is entirely stress free and freely distensible in the resting state, and is described exactly by Hill's equation (or some variation). When the muscle is activated, the series and parallel elements are elastic, and the whole muscle is a simple combination of identical sarcomeres in series and parallel. The main criticism of Hill's model is that the division of forces between the parallel elements and the division of extensions between the series elements is arbitrary, and cannot be made without introducing auxiliary hypotheses. However, these criticisms apply to *any* discrete element model. Despite these limitations, the Hill model has become extremely useful for modeling musculoskeletal dynamics, as illustrated by its widespread use today.



**Figure MS15-1.** Discrete model for muscle contraction dynamics, based on a Hill-type representation. The total force is the sum of passive force  $F^{PE}$  and active force  $F^{CE}$ . The passive element (PE) represents energy storage from muscle elasticity, while the contractile element (CE) represents force generation by the muscle. The series elastic element (SEE), shown in dashed lines, is often neglected when a series tendon compliance is included. Here,  $a(t)$  is the activation level,  $L^M$  is the length of the muscle, and  $V^M$  is the shortening velocity of the muscle.

When the contractile element (CE) of the Hill model is inactive, the entire resistance to elongation is provided by the PE element and the tendon load-elongation behavior. As activation is increased, force then passes through the CE side of the parallel Hill model, providing the contractile dynamics. The original Hill model accommodated only full activation - this limitation is circumvented in the present implementation by using the modification suggested by Winters (1990). The main features of his approach were to realize that the CE force-velocity input force equals the CE tension-length output force. This yields a three-dimensional curve to describe the force-velocity-length relationship of the CE. If the force-velocity y-intercept scales with activation, then given the activation, length and velocity, the CE force can be determined.

Without the SEE, the total force in the muscle  $F^M$  is the sum of the force in the CE and the PE because they are in parallel:

$$F^M = F^{PE} + F^{CE}$$

The relationships defining the force generated by the CE and PE as a function of  $L^M$ ,  $V^M$  and  $a(t)$  are often scaled by  $F_{max}$ , the peak isometric force (p. 80, Winters 1990),  $L_0$ , the initial length of the muscle (p. 81, Winters 1990), and  $V_{max}$ , the maximum unloaded CE shortening velocity (p. 80, Winters 1990). From these, dimensionless length and velocity can be defined:

$$L = \frac{L^M}{L_0},$$

$$V = \frac{V^M}{V_{max} \times S_v[a(t)]}$$

Here,  $S_v$  scales the maximum CE shortening velocity  $V_{\max}$  and changes with activation level  $a(t)$ . This has been suggested by several researchers, i.e. Winters and Stark [1985]. The activation level specifies the level of muscle stimulation as a function of time. Both have values between 0 and 1. The functions  $S_v(a(t))$  and  $a(t)$  are specified via load curves in LS-DYNA, or default values of  $S_v = 1$  and  $a(t) = 0$  are used. Note that  $L$  is always positive and that  $V$  is positive for lengthening and negative for shortening.

The relationship between  $F^{\text{CE}}$ ,  $V$  and  $L$  was proposed by Bahler et al. [1967]. A three-dimensional relationship between these quantities is now considered standard for computer implementations of Hill-type muscle models [Winters 1990]. It can be written in dimensionless form as:

$$F^{\text{CE}} = a(t) \times F_{\max} \times f_{\text{TL}}(L) \times f_{\text{TV}}(V)$$

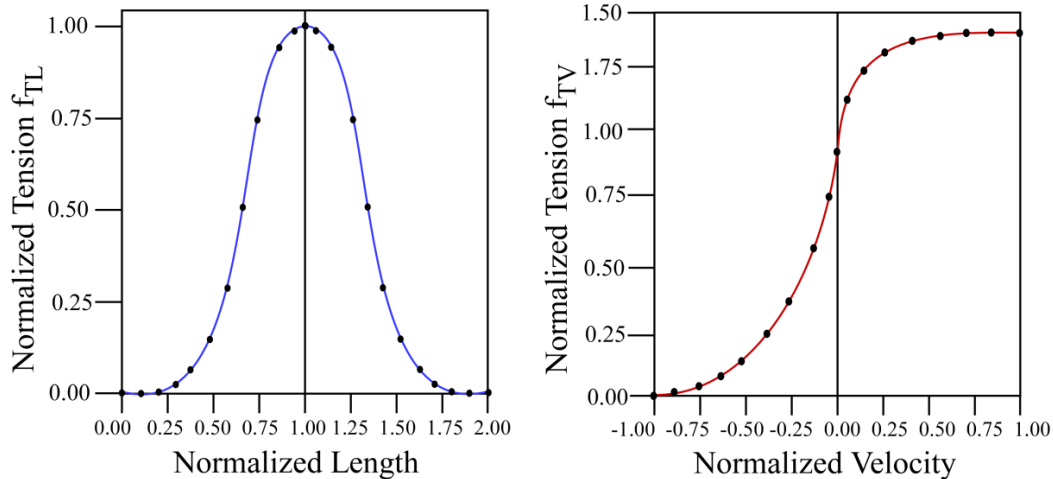
Here,  $f_{\text{TL}}(L)$  and  $f_{\text{TV}}(V)$  are the tension-length and tension-velocity functions for active skeletal muscle. Thus, if current values of  $L^{\text{M}}$ ,  $V^{\text{M}}$ , and  $a(t)$  are known, then  $F^{\text{CE}}$  can be determined ([Figure MS15-1](#)).

The force in the parallel elastic element  $F^{\text{PE}}$  is determined directly from the current length of the muscle using an exponential relationship [Winters 1990]:

$$f_{\text{PE}} = \frac{F^{\text{PE}}}{F_{\text{MAX}}} = \begin{cases} 0 & L \leq 1 \\ \frac{1}{\exp(K_{\text{sh}}) - 1} \left\{ \exp\left[\frac{K_{\text{sh}}}{L_{\max}}(L - 1)\right] - 1 \right\} & L > 1 \end{cases}$$

Here,  $L_{\max}$  is the relative length at which the force  $F_{\max}$  occurs, and  $K_{\text{sh}}$  is a dimensionless shape parameter controlling the rate of rise of the exponential. Alternatively, the user can define a custom  $f_{\text{PE}}$  curve giving tabular values of normalized force versus dimensionless length as a load curve.

For computation of the total force developed in the muscle  $F^{\text{M}}$ , the functions for the tension-length  $f_{\text{TL}}(L)$  and force-velocity  $f_{\text{TV}}$  relationships used in the Hill element must be defined. These relationships have been available for over 50 years, but have been refined to allow for behavior such as active lengthening. The active tension-length curve  $f_{\text{TL}}(L)$  describes the fact that isometric muscle force development is a function of length, with the maximum force occurring at an optimal length. According to Winters, this optimal length is typically around  $L = 1.05$ , and the force drops off for shorter or longer lengths, approaching zero force for  $L = 0.4$  and  $L = 1.5$ . Thus the curve has a bell-shape. Because of the variability in this curve between muscles, the user must specify the function  $f_{\text{TL}}(L)$  via a load curve, specifying pairs of points representing the normalized force (with values between 0 and 1) and normalized length  $L$ . See [Figure MS15-2](#).



**Figure MS15-2.** Typical normalized tension-length (TL) and tension-velocity (TV) curves for skeletal muscle.

The active tension-velocity relationship  $f_{TV}(V)$  used in the muscle model is mainly due to the original work of Hill. Note that the dimensionless velocity  $V$  is used. When  $V = 0$ , the normalized tension is typically chosen to have a value of 1.0. When  $V$  is greater than or equal to 0, muscle lengthening occurs. As  $V$  increases, the function is typically designed so that the force increases from a value of 1.0 and asymptotes towards a value near 1.4 as shown in Figure MS15-2. When  $V$  is less than zero, muscle shortening occurs and the classic Hill equation hyperbola is used to drop the normalized tension to 0 as shown in Figure MS15-2. The user must specify the function  $f_{TV}(V)$  via a load curve, specifying pairs of points representing the normalized tension (with values between 0 and 1) and normalized velocity  $V$

**\*MAT\_SEATBELT****\*MAT\_B01****\*MAT\_SEATBELT\_{OPTION}**

Available options include:

2D

Purpose: Define a seat belt material.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	MPUL	LLCID	ULCID	LMIN	CSE	DAMP	E
Type	A8	F	I	I	F	F	F	F
Default	0	0.	0	0	0.0	0.0	0.0	0.0

**Bending/Compression Parameter Card.** Additional card for E.GT.0.

Card 2	1	2	3	4	5	6	7	8
Variable	A	I	J	AS	F	M	R	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	2*1	A	1.0e20	1.0e20	0.05	

Additional card for 2D option

Card 3	1	2	3	4	5	6	7	8
Variable	P1D0FF							
Type	I							
Default	0							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Belt material number. A unique number or label not exceeding 8 characters must be specified.
MPUL	Mass per unit length
LLCID	Curve or table ID for loading. LLCID can be either a single curve (force vs. engineering strain), or a table defining a set of strain-rate dependent loading curves.
ULCID	Load curve identification for unloading (force vs. engineering strain).
LMIN	Minimum length (for elements connected to slip rings and retractors), see notes below.
CSE	Optional compressive stress elimination option which applies to shell elements only (default 0.0): EQ.0.0: eliminate compressive stresses in shell fabric EQ.1.0: do not eliminate compressive stresses. This option should not be used if retractors and slip rings are present in the model. EQ.2.0: whether or not compressive stress is eliminated is decided by LS-DYNA automatically, recommended for shell belt.
DAMP	Optional Rayleigh damping coefficient, which applies to shell elements only. A coefficient value of 0.10 is the default corresponding to 10% of critical damping. Sometimes smaller or larger values work better.
E	Young's modulus for bending/compression stiffness, when positive the optional card is invoked. See remarks.
A	Cross sectional area for bending/compression stiffness, see remarks.
I	Area moment of inertia for bending/compression stiffness, see remarks.
J	Torsional constant for bending/compression stiffness, see remarks.
AS	Shear area for bending/compression stiffness, see remarks.
F	Maximum force in compression/tension, see remarks.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
M	Maximum torque, see remarks.
R	Rotational mass scaling factor, see remarks.
P1DOFF	Part ID offset for internally created 1D, bar-type, belt parts for 2D seatbelt of this material, i.e., the IDs of newly created 1D belt parts will be P1DOFF + 1, P1DOFF + 2, ... If zero, the maximum ID of user-defined parts is used as the part ID offset.

**Remarks:**

Each belt material defines stretch characteristics and mass properties for a set of belt elements. The user enters a load curve for loading, the points of which are (Strain, Force). Strain is defined as engineering strain, i.e.

$$\text{Strain} = \frac{\text{current length}}{\text{initial length}} - 1.0$$

Another similar curve is entered to describe the unloading behavior. Both load curves should start at the origin (0,0) and contain positive force and strain values only. The belt material is tension only with zero forces being generated whenever the strain becomes negative. The first non-zero point on the loading curve defines the initial yield point of the material. On unloading, the unloading curve is shifted along the strain axis until it crosses the loading curve at the "yield" point from which unloading commences. If the initial yield has not yet been exceeded or if the origin of the (shifted) unloading curve is at negative strain, the original loading curves will be used for both loading and unloading. If the strain is less than the strain at the origin of the unloading curve, the belt is slack and no force is generated. Otherwise, forces will then be determined by the unloading curve for unloading and reloading until the strain again exceeds yield after which the loading curves will again be used.

A small amount of damping is automatically included. This reduces high frequency oscillation, but, with realistic force-strain input characteristics and loading rates, does not significantly alter the overall forces-strain performance. The damping forced opposes the relative motion of the nodes and is limited by stability:

$$D = \frac{0.1 \times \text{mass} \times \text{relative velocity}}{\text{time step size}}$$

In addition, the magnitude of the damping force is limited to one-tenth of the force calculated from the force-strain relationship and is zero when the belt is slack. Damping forces are not applied to elements attached to slings and retractors.

The user inputs a mass per unit length that is used to calculate nodal masses on initialization.

A “minimum length” is also input. This controls the shortest length allowed in any element and determines when an element passes through sliprings or are absorbed into the retractors. One tenth of a typical initial element length is usually a good choice.

### **Bending and Compression Stiffness for 1D Elements:**

Since these elements do not possess any bending or compression stiffness, when belts are used in an implicit analysis, dynamic analysis is mandatory. However, one dimensional belt elements *can* be used in implicit statics by associating them with bending/compression properties as per the first optional card. Two dimensional belt elements are not supported with this feature.

To achieve bending and compression stiffness in 1D belts the belt element is overlaid with a Belytschko-Schwer beam element (see \*SECTION\_BEAM, ELFORM = 2, for a more comprehensive description of parameter A, I, J and AS) with circular cross section. These elements have 6 degrees of freedom including rotational degrees of freedom. The material used in this context is an elastic-ideal-plastic material where the elastic part is governed by the Young’s modulus E. Two yield values, F being the maximum compression/tension force and M being the maximum torque, are used as upper bounds for the resultants. The bending/compression forces and moments from this contribution are accumulated to the force from the seatbelt itself. Since the main purpose is to eliminate the singularities in bending and compression, it is recommended to choose the bending and compression properties in the optional card carefully so as to not significantly influence the overall response.

For the sake of completeness, this feature is also supported by the explicit integrator; therefore, a rotational nodal mass is needed. Each of the two nodes of an element gets a contribution from the belt that is calculated as  $R_{MASS} = R \times (MASS/2) \times I/A$ , where MASS indicates the total translational mass of the belt element and R is a scaling factor input by the user. The translational mass is not modified. The bending and compression properties do not affect the stable time step. If the belts are used *without* sliprings, then incorporating this feature is virtually equivalent to adding Belytschko-Schwer beams on top of conventional belt elements as part of a modelling strategy. If sliprings *are* used, this feature is necessary to properly support the flow of material through the sliprings and swapping of belt elements across sliprings. Retractors cannot be used with this feature.

**\*MAT\_THERMAL\_{OPTION}**

Available options include:

- ISOTROPIC
- ORTHOTROPIC
- ISOTROPIC\_TD
- ORTHOTROPIC\_TD
- DISCRETE\_BEAM
- CWM
- ORTHOTROPIC\_TD\_LC
- ISOTROPIC\_PHASE\_CHANGE
- ISOTROPIC\_TD\_LC
- USER\_DEFINED

The \*MAT\_THERMAL\_cards allow thermal properties to be defined in coupled structural/thermal and thermal only analyses, see \*CONTROL\_SOLUTION. Thermal properties must be defined for all elements in such analyses.

Thermal material properties are specified by a thermal material ID number (TMID), this number is independent of the material ID number (MID) defined on all other \*MAT\_... property cards. In the same analysis identical TMID and MID numbers may exist. The TMID and MID numbers are related through the \*PART card.

**\*MAT\_THERMAL\_ISOTROPIC**

This is thermal material type 1. It allows isotropic thermal properties to be defined.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	TRO	TGRLC	TGMULT	TLAT	HLAT		
Type	A8	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	HC	TC						
Type	F	F						

**VARIABLE****DESCRIPTION**

TMID	Thermal material identification. A unique number or label not exceeding 8 characters must be specified.
TRO	Thermal density: EQ.0.0: default to structural density.
TGRLC	Thermal generation rate curve number, see *DEFINE_CURVE: GT.0: function versus time, EQ.0: use constant multiplier value, TGMULT, LT.0: function versus temperature.
TGMULT	Thermal generation rate multiplier: EQ.0.0: no heat generation.
TLAT	Phase change temperature
HLAT	Latent heat
HC	Specific heat
TC	Thermal conductivity

\*MAT\_THERMAL\_ORTHOTROPIC

This is thermal material type 2. It allows orthotropic thermal properties to be defined.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	TRO	TGRLC	TGMULT	AOPT	TLAT	HLAT	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	HC	K1	K2	K3				
Type	F	F	F	F				

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable	D1	D2	D3					
Type	F	F	F					

**VARIABLE**

**DESCRIPTION**

TMID Thermal material identification. A unique number or label not exceeding 8 characters must be specified.

TRO Thermal density:  
EQ.0.0: default to structural density.

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TGRLC	Thermal generation rate curve number, see *DEFINE_CURVE: GT.0: function versus time, EQ.0: use constant multiplier value, TGMULT, LT.0: function versus temperature.
TGMULT	Thermal generation rate multiplier: EQ.0.0: no heat generation.
AOPT	Material axes definition: EQ.0.0: locally orthotropic with material axes by element nodes N1, N2 and N4, EQ.1.0: locally orthotropic with material axes determined by a point in space and global location of element center, EQ.2.0: globally orthotropic with material axes determined by vectors.
TLAT	Phase change temperature
HLAT	Latent heat
HC	Specific heat
K1	Thermal conductivity K1 in local x-direction
K2	Thermal conductivity K2 in local y-direction
K3	Thermal conductivity K3 in local z-direction
XP, YP, ZP	Define coordinate of point <b>p</b> for AOPT = 1
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2
D1, D2, D3	Define components of vector <b>v</b> for AOPT = 2

\*MAT\_THERMAL\_ISOTROPIC\_TD

This is thermal material type 3. It allows temperature dependent isotropic properties to be defined. The temperature dependency is defined by specifying a minimum of two and a maximum of eight data points. The properties must be defined for the temperature range that the material will see in the analysis.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	TRO	TGRLC	TGMULT	TLAT	HLAT		
Type	A8	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	T1	T2	T3	T4	T5	T6	T7	T8
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	K1	K2	K3	K4	K5	K6	K7	K8
Type	F	F	F	F	F	F	F	F

**VARIABLE**

**DESCRIPTION**

TMID

Thermal material identification. A unique number or label not exceeding 8 characters must be specified.

---

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TRO	Thermal density: EQ.0.0 default to structural density.
TGRLC	Thermal generation rate curve number, see *DEFINE_CURVE: GT.0: function versus time, EQ.0: use constant multiplier value, TGMULT, LT.0: function versus temperature.
TGMULT	Thermal generation rate multiplier: EQ.0.0: no heat generation.
TLAT	Phase change temperature
HLAT	Latent heat
T1, ..., T8	Temperatures: T1, ..., T8
C1, ..., C8	Specific heat at: T1, ..., T8
K1, ..., K8	Thermal conductivity at: T1, ..., T8



\*MAT\_THERMAL\_ORTHOTROPIC\_TD

This is thermal material type 4. It allows temperature dependent orthotropic properties to be defined. The temperature dependency is defined by specifying a minimum of two and a maximum of eight data points. The properties must be defined for the temperature range that the material will see in the analysis.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	TRO	TGRLC	TGMULT	AOPT	TLAT	HLAT	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	T1	T2	T3	T4	T5	T6	T7	T8
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	(K1)1	(K1)2	(K1)3	(K1)4	(K1)5	(K1)6	(K1)7	(K1)8
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	(K2)1	(K2)2	(K2)3	(K2)4	(K2)5	(K2)6	(K2)7	(K2)8
Type	F	F	F	F	F	F	F	F

Card 6	1	2	3	4	5	6	7	8
Variable	(K3)1	(K3)2	(K3)3	(K3)4	(K3)5	(K3)6	(K3)7	(K3)8
Type	F	F	F	F	F	F	F	F

Card 7	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 8	1	2	3	4	5	6	7	8
Variable	D1	D2	D3					
Type	F	F	F					

**VARIABLE****DESCRIPTION**

TMID	Thermal material identification. A unique number or label not exceeding 8 characters must be specified.
TRO	Thermal density: EQ.0.0: default to structural density.
TGRLC	Thermal generation rate curve number, see *DEFINE_CURVE: GT.0: function versus time, EQ.0: use constant multiplier value, TGMULT, LT.0: function versus temperature.
TGMULT	Thermal generation rate multiplier: EQ.0.0: no heat generation.

VARIABLE	DESCRIPTION
AOPT	Material axes definition: (see Mat_OPTION TROPIC_ELASTIC for a more complete description): EQ.0.0: locally orthotropic with material axes by element nodes N1, N2 and N4, EQ.1.0: locally orthotropic with material axes determined by a point in space and global location of element center, EQ.2.0: globally orthotropic with material axes determined by vectors.
TLAT	Phase change temperature
HLAT	Latent heat
T1 ... T8	Temperatures: T1 ... T8
C1 ... C8	Specific heat at T1 ... T8
(K1)1 ... (K1)8	Thermal conductivity K <sub>1</sub> in local x-direction at T1 ... T8
(K2)1 ... (K2)8	Thermal conductivity K <sub>2</sub> in local y-direction at T1 ... T8
(K3)1 ... (K3)8	Thermal conductivity K <sub>3</sub> in local z-direction at T1 ... T8
XP, YP, ZP	Define coordinate of point <b>p</b> for AOPT = 1
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2

**\*MAT\_THERMAL\_DISCRETE\_BEAM**

This is thermal material type 5. It defines properties for discrete beams. It is only applicable when used with \*SECTION\_BEAM elform = 6.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	TRO						
Type	A8	F						

Card 2	1	2	3	4	5	6	7	8
Variable	HC	TC						
Type	F	F						

**VARIABLE****DESCRIPTION**

TMID	Thermal material identification. A unique number or label not exceeding 8 characters must be specified.
TRO	Thermal density: EQ.0.0: default to structural density.
HC	Specific heat
HC	Thermal conductance (SI units are W/K) $HC = (\text{heat transfer coefficient}) \times (\text{beam cross section area})$ $[W/K] = [W / m^2 K] * [m^2]$

**Note:**

A beam cross section area is not defined on the SECTION\_BEAM keyword for an elform = 6 discrete beam. A beam cross section area is needed for heat transfer calculations. Therefore, the cross section area is lumped into the value entered for HC.

## \*MAT\_THERMAL\_CHEMICAL\_REACTION

This is thermal material type 6. The chemical species making up this material undergo chemical reactions. A maximum of 8 species and 8 chemical reactions can be defined. The thermal material properties of a finite element undergoing chemical reactions are calculated based on a mixture law consisting of those chemical species currently present in the element. The dependence of the chemical reaction rate on temperature is described by the Arrhenius equation. Time step splitting is used to couple the system of ordinary differential equations describing the chemical reaction kinetics to the system of partial differential equations describing the diffusion of heat.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	NCHSP	NCHRX	ICEND	CEND	RBEGIN	GASC	
Type	A8	I	I	I	R	R	R	

**Chemical Species Cards.** Include one card for each of the NCHSP species. These cards set species properties. The dummy index  $i$  is the species number and is equal to 1 for the first species card, 2 for the second, and so on.

Card 3	1	2	3	4	5	6	7	8
Variable	RH0i	LCCPi	LCKi	NOi	MWi			
Type	R	I	I	I	I			

**Reaction Cards.** Include one card for each of the NCHSP species. Each field contains the species's coefficient for one of the NCHRX chemical reactions. See card format 3 for explanation of the species index  $i$ .

Card 4	1	2	3	4	5	6	7	8
Variable	RCi1	RCi2	RCi3	RCi4	RCi5	RCi6	RCi7	RCi8
Type	R	R	R	R	R	R	R	R

**Reaction Rate Exponent Cards.** Include one card for each of the NCHSP species. Each field contains the specie's rate exponent for one of the NCHRX chemical reactions. See card format 3 for explanation of the species index *i*.

Card 5	1	2	3	4	5	6	7	8
Variable	RXi1	RXi2	RXi3	RXi4	RXi5	RXi6	RXi7	RXi8
Type	R	R	R	R	R	R	R	R

**Pre-exponential Factor Card.** Each field contains the natural logarithm of its corresponding reaction's pre-exponential factor.

Card 6	1	2	3	4	5	6	7	8
Variable	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8
Type	R	R	R	R	R	R	R	R

**Activation Energy Card.** Each field contains the activation energy value for its corresponding reaction.

Card 7	1	2	3	4	5	6	7	8
Variable	E1	E2	E3	E4	E5	E6	E7	E8
Type	R	R	R	R	R	R	R	R

**Heat of Reaction Card.** Each field contains the heat of reaction value for its corresponding reaction.

Card 8	1	2	3	4	5	6	7	8
Variable	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
Type	R	R	R	R	R	R	R	R

**VARIABLE****DESCRIPTION**

TMID

Thermal material identification. A unique number or label not exceeding 8 characters must be specified.

NCHSP	Number of chemical species (maximum 8)
NCHRX	Number of chemical reactions (maximum 8)
ICEND	Species number controlling reaction termination
RBEGIN	Chemical reaction will start when the sum of the individual chemical reaction rates are greater than RBEGIN.
GASC	Gas constant: 1.987 cal/(g-mole K), 8314. J/(kg-mole K)
RHO $i$	Density of the $i^{\text{th}}$ species
LCCP $i$	Load curve ID specifying specific heat vs. temperature for the $i^{\text{th}}$ species.
LCK $i$	Load curve ID specifying thermal conductivity vs. temperature for the $i^{\text{th}}$ species
N0 $i$	Initial concentration fraction of the $i^{\text{th}}$ species
MW $i$	Molecular weight of the $i^{\text{th}}$ species.
RC $ij$	Reaction coefficient for species $i$ in reaction $j$ . Leave blank for undefined reactions
RX $ij$	Rate exponent for species $i$ in reaction $j$ . Leave blank for undefined reactions.
Z $j$	Pre-exponential factor for reaction $j$ . Enter the value as $\ln(Z)$ . Leave blank for undefined reactions.
E $j$	Activation energy for reaction $j$ . Leave blank for undefined reactions.
Q $j$	Heat of reaction for reaction $j$ . Leave blank for undefined reactions.

### Rate Model for a Single Reaction:

Chemical reactions are usually expressed in *chemical equation notation*; for example, a chemical reaction involving two reactants and two products is



where A, B, G, and H are chemical species such as NaOH or HCl, and  $a$ ,  $b$ ,  $g$ , and  $h$  are integers called *stoichiometric numbers*, indicating the number of molecules involved in a single reaction.

The rate of reaction is the number of individual reactions per unit time. Using a stoichiometric identity, which is just an accounting relation, the rate of reaction is proportional to the rate of change in the concentrations of the species involved in the reaction. For the chemical reaction in [Equation \(MT6.1\)](#), the relation between concentration and rate,  $r$ , is,

$$r = -\frac{1}{a} \frac{d[A]}{dt} = -\frac{1}{b} \frac{d[B]}{dt} = +\frac{1}{g} \frac{d[G]}{dt} = +\frac{1}{h} \frac{d[H]}{dt}, \quad (\text{MT6.2})$$

where  $[X]$  denotes the concentration of species  $X$ , and the sign depends on whether or not the species is an input, in which case the sign is negative, or a product, in which case the sign is positive.

### The Model

This thermal material model (T06) is built on the assumption that the reaction rate depends on the concentration of the input species according to

$$r = k(T) \prod_X [X]^{p_X},$$

where  $X$  ranges over all species, and, for each species, the exponent,  $p_X$ , is determined by empirical measurement, but may be approximated by the stoichiometric number associated with  $X$ . The proportionality constant,  $k$ , is related to the cross-section for the reaction, and it depends on temperature through the Arrhenius equation:

$$k = Z(T) \exp\left(-\frac{E_i}{RT}\right),$$

where  $Z(T)$  is experimentally determined (see card 6),  $E_i$  is the activation energy (see card 7),  $R$  is the gas constant, and  $T$  is temperature. As an example, for the chemical reaction of [Equation \(MT6.1\)](#)

$$r = Z(T) \exp\left(-\frac{E_i}{RT}\right) [A]^\alpha [B]^\beta,$$

where the stoichiometric numbers have been used instead of experimentally determined exponents.

The rate of heat generation (exothermic) and absorption (endothermic) associated with a reaction is calculated by multiplying the heat of reaction,  $Q_i$ , by its rate.

### Rate Model for a System of Reactions:

For a system of coupled chemical reactions, the change in a species's concentration is the sum of all the contributions from each individual chemical reaction:

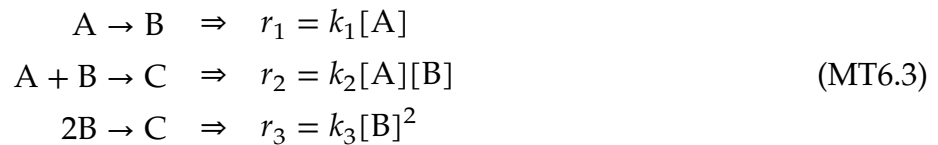
$$\frac{d[X]}{dt} = \sum_i (\pm)_i n(x)_i r_i.$$



The index  $i$  runs over all reactions;  $n(x)_i$  is the stoichiometric number for species X in reaction  $i$ ; and where  $r_i$  is the rate of reaction  $i$ . The sign  $(\pm)_i$  is positive for reactions that have X as a product and negative for reactions that involve X as an input.

**Example:**

For the following system of reactions



the time evolution equations are,

$$\begin{aligned} \frac{d[A]}{dt} &= \sum n(x)_i r_i = -k_1[A] - k_2[A][B] \\ \frac{d[B]}{dt} &= \sum n(x)_i r_i = +k_1[A] - k_2[A][B] - 2k_3[B]^2 \\ \frac{d[C]}{dt} &= \sum n(x)_i r_i = +k_2[A][B] + k_3[B]^2. \end{aligned} \quad (\text{MT6.4})$$

The coefficients should be identically copied from (MT6.4):

Card 4	1	2	3	4	5	6	7	8
Variable	RC11	RC12	RC13	RC14	RC15	RC16	RC17	RC18
Value	-1	-1	0					
Variable	RC21	RC22	RC23	RC24	RC25	RC26	RC27	RC28
Value	+1	-1	-2					
Variable	RC31	RC32	RC33	RC34	RC35	RC36	RC37	RC38
Value	0	1	1					

The exponents are likewise picked off of (MT6.3) for next set of cards in format 5:

Card 5	1	2	3	4	5	6	7	8
Variable	RX11	RX12	RX13	RX14	RX15	RX16	RX17	RX18
Value	+1	+1	0					

Variable	RX21	RX22	RX23	RX24	RX25	RX26	RX27	RX28
Value	0	+1	+2					
Variable	RX31	RX32	RX33	RX34	RX35	RX36	RX37	RX38
Value	0	0	0					

**Equivalent Units (Normalized Units):**

The concentrations are often scaled so that each unit of reactant yields one unit of product. Systems for which each species is assigned *its own* unit of concentration based on stoichiometric considerations are *equivalent unit systems*.

Being unit-agnostic, LS-DYNA is capable of working in equivalent units. However, care must be taken so that units are treated consistently, as applying a unit scaling to the time evolution equations can be nontrivial.

1. For each reaction, the experimentally measured pre-exponential coefficients carry units that depend on the reaction itself. For instance, the pre-exponential factors  $Z_1$ ,  $Z_2$ , and  $Z_3$  for the reactions  $A \rightarrow B$ ,  $A + B \rightarrow C$ , and  $2B \rightarrow C$  respectively will have units of

$$[Z_1] = \frac{1}{[\text{time}]} \times \frac{1}{[\text{Concentration of A}]}$$

$$[Z_2] = \frac{1}{[\text{time}]} \times \frac{1}{[\text{Concentration of A}]} \times \frac{1}{[\text{Concentration of B}]}$$

$$[Z_3] = \frac{1}{[\text{time}]} \times \left\{ \frac{1}{[\text{Concentration of B}]} \right\}^2.$$

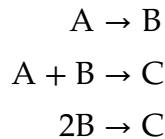
Note that each prefactor has a different dimensionality.

2. The equations in (MT6.2), which relate rate to concentration change, are logically inconsistent unless all species are measured using the *same* units for concentration. A species-dependent system of equivalent units would require the insertion of additional conversion factors into (MT6.2) thereby changing the form of the time-evolution equations.

To avoid unit consistency issues, it is recommended that reactions be defined in the same unit system that was used to measure their empirical values.

### Example of Equivalent Units:

The reaction of [Equation \(MT6.3\)](#),



changes species A into species C through an intermediate which is species B. For each unit of species C that is produced, the reaction consumes two units of species A. Since this set of chemical formulae corresponds to the curing of epoxy, which is a nearly volume-preserving process, it is customary to work in a system of equivalent units that correspond to species volume fractions.

The following set of equivalent units, then, is used in the published literature:

1. Whatever the starting concentration of species A is, all units are uniformly rescaled so that  $[A] = 1$  at time zero. Per the boxed remark above, since the constants were measured with respect to these units, this consideration does not introduce new complexity.
2. Since the process preserves volume, and since one particle of species C replaces two particles of species A (and one particle of B replace one of A), the units of concentration for species C are doubled.

$$\tilde{C} = 2[C]$$

Under this transformation the rate relation for C is

$$r = \pm \frac{d[C]}{dt} = \frac{1}{2} \frac{d\tilde{C}}{dt}.$$

The time evolution [Equations \(MT6.4\)](#) become, (note  $[C]$  has been replaced by  $\tilde{C}$ )

$$\begin{aligned} \frac{d[A]}{dt} &= \sum n(x)_i r_i = -k_1[A] - k_2[A][B] \\ \frac{d[B]}{dt} &= \sum n(x)_i r_i = +k_1[A] - k_2[A][B] - 2k_3[B]^2 \\ \frac{d\tilde{C}}{dt} &= \sum n(x)_i r_i = \quad \quad \quad + 2k_2[A][B] + 2k_3[B]^2. \end{aligned}$$

Whence,

$$\frac{d[A]}{dt} + \frac{d[B]}{dt} + \frac{d\tilde{C}}{dt} = 0.$$

Therefore, since

$$[A] + [B] + \tilde{C} = 1$$

for all values of time, and since concentration values cannot become negative, it is clear that [A], [B], and  $\tilde{C}$  are volume fractions.

\*MAT\_THERMAL\_CWM

This is thermal material type 7. It is a thermal material with temperature dependent properties that allows for material creation triggered by temperature. The acronym CWM stands for Computational Welding Mechanics and the model is intended to be used for simulating multistage weld processes in combination with the mechanical counterpart, \*MAT\_CWM.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	TRO	TGRLC	TGMULT	HDEAD	TDEAD	TLAT	HLAT
Type	A8	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	LCHC	LCTC	TLSTART	TLEND	TISTART	TIEND	HGHOST	TGHOST
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
TMID	Thermal material identification. A unique number or label not exceeding 8 characters must be specified.
TRO	Thermal density: EQ.0.0: default to structural density.
TGRLC	Thermal generation rate curve number, see *DEFINE_CURVE: GT.0: function versus time, EQ.0: use constant multiplier value, TGMULT, LT.0: function versus temperature.
TGMULT	Thermal generation rate multiplier: EQ.0.0: no heat generation.
HDEAD	Specific heat for inactive material before birth time
TDEAD	Thermal conductivity for inactive material before birth time

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TLAT	Phase change temperature
HLAT	Latent heat
LCHC	Load curve (or table) for specific heat as function of temperature (and maximum temperature up to current time)
LCTC	Load curve for thermal conductivity as function of temperature
TLSTART	Birth temperature of material start
TLEND	Birth temperature of material end
TISTART	Birth time start
TIEND	Birth time end
HGHOST	Specific heat for ghost (quiet) material
TGHOST	Thermal conductivity for ghost (quiet) material

**Remarks:**

This material is initially in a quiet state, sometimes referred to as a *ghost material*. In this state the material has the thermal properties defined by the quiet specific heat (HGHOST) and quiet thermal conductivity (TGHOST). These should represent the void, for example, by picking a relatively small thermal conductivity.

However, the ghost specific heat must be chosen with care, since the temperature must be allowed to increase at a reasonable rate due to the heat from the weld source. When the temperature reaches the birth temperature, a history variable representing the indicator of the welding material is incremented. This variable follows

$$\gamma(t) = \min \left[ 1, \max \left( 0, \frac{T_{\max} - T_l^{\text{start}}}{T_l^{\text{end}} - T_l^{\text{start}}} \right) \right]$$

where  $T_{\max} = \max\{T(s) | s < t\}$ .

The effective thermal material properties are interpolated as

$$\begin{aligned} \tilde{c}_p &= c_p(T, T_{\max})\gamma + c_p^{\text{quiet}}(1 - \gamma) \\ \tilde{\mu} &= \mu(T)\gamma + \mu^{\text{quiet}}(1 - \gamma) \end{aligned}$$

where  $c_p$  and  $\mu$  are the specific heat and thermal conductivity, respectively. Here, the specific heat,  $c_p$ , is either a temperature dependent curve, or a collection of temperature dependent curves, ordered in a table according to maximum temperature  $T_{max}$ .

The time parameters for creating the material provide additional formulae for the final

values of the thermal properties. Before the birth time  $t_i^{start}$  of the material has been

reached, the specific heat  $c_p^{dead}$  and thermal conductivity  $\mu^{dead}$  are used. The default values,

i.e. the values used if no user input is given, are

$$c_p^{dead} = 10^{10} c_p(T, T_{max})$$

$$\mu^{dead} = 0$$

Thus, the final values of the thermal properties read

$$c_p = \begin{cases} c_p^{dead} & t \leq t_i^{start} \\ \tilde{c}_p \frac{t - t_i^{start}}{t_i^{end} - t_i^{start}} + c_p^{dead} \frac{t - t_i^{end}}{t_i^{start} - t_i^{end}} & t_i^{start} < t \leq t_i^{end} \\ \tilde{c}_p & t_i^{end} < t \end{cases}$$

$$\mu = \begin{cases} \mu^{dead} & t \leq t_i^{start} \\ \tilde{\mu} \frac{t - t_i^{start}}{t_i^{end} - t_i^{start}} + \mu^{dead} \frac{t - t_i^{end}}{t_i^{start} - t_i^{end}} & t_i^{start} < t \leq t_i^{end} \\ \tilde{\mu} & t_i^{end} < t \end{cases}$$

These parameters allow the user to control when the welding layer becomes active and thereby define a multistage welding process. Prior to the birth time, the temperature is kept more or less constant due to the large specific heat, and, thus, the material is prevented from being created

**\*MAT\_THERMAL\_ORTHOTROPIC\_TD\_LC**

This is thermal material type 8. It allows temperature dependent orthotropic properties to be defined by load curves. The temperature dependency is defined by specifying a minimum of two data points. The properties must be defined for the temperature range that the material will see in the analysis.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	TRO	TGRLC	TGMULT	AOPT	TLAT	HLAT	
Type	A8	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	LCC	LCK1	LCK2	LCK3				
Type	I	I	I	I				

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable	D1	D2	D3					
Type	F	F	F					

**VARIABLE****DESCRIPTION**

TMID

Thermal material identification. A unique number or label not exceeding 8 characters must be specified.



<b>VARIABLE</b>	<b>DESCRIPTION</b>
TRO	Thermal density: EQ.0.0: default to structural density.
TGRLC	Thermal generation rate curve number, see *DEFINE_CURVE: GT.0: function versus time, EQ.0: use constant multiplier value, TGMULT, LT.0: function versus temperature.
TGMULT	Thermal generation rate multiplier: EQ.0.0: no heat generation.
AOPT	Material axes definition: (see Mat_OPTION TROPIC_ELASTIC for a more complete description): EQ.0.0: locally orthotropic with material axes by element nodes N1, N2 and N4, EQ.1.0: locally orthotropic with material axes determined by a point in space and global location of element center, EQ.2.0: globally orthotropic with material axes determined by vectors.
TLAT	Phase change temperature
HLAT	Latent heat
LCC	Load Curve Specific Heat
LCK1	Load Curve Thermal Conductivity K <sub>1</sub> in local x-direction
LCK2	Load Curve Thermal Conductivity K <sub>2</sub> in local y-direction
LCK3	Load Curve Thermal Conductivity K <sub>3</sub> in local z-direction
XP, YP, ZP	Define coordinate of point <b>p</b> for AOPT = 1
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2

**Remarks:**

See \*MAT\_THERMAL\_ORTHOTROPIC keyword for a description of the orthotropic axis options, AOPT.

**\*MAT\_THERMAL\_ISOTROPIC\_PHASE\_CHANGE**

This is thermal material type 9. It allows temperature dependent isotropic properties with phase change to be defined. The latent heat of the material is defined together with the solid and liquid temperatures. The temperature dependency is defined by specifying a minimum of two and a maximum of eight data points. The properties must be defined for the temperature range that the material will see in the analysis.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	TR0	TGRLC	TGMULT				
Type	A8	F	F	F				

Card 2	1	2	3	4	5	6	7	8
Variable	T1	T2	T3	T4	T5	T6	T7	T8
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	K1	K2	K3	K4	K5	K6	K7	K8
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	SOLT	LIQT	LH					
Type	F	F	F					

**VARIABLE****DESCRIPTION**

TMID	Thermal material identification. A unique number or label not exceeding 8 characters must be specified.
TRO	Thermal density: EQ.0.0: default to structural density.
TGRLC	Thermal generation rate curve number, see *DEFINE_CURVE: GT.0: function versus time, EQ.0: use constant multiplier value, TGMULT, LT.0: function versus temperature.
TGMULT	Thermal generation rate multiplier: EQ.0.0: no heat generation.
T1, ..., T8	Temperatures (T1, ..., T8)
C1, ..., C8	Specific heat at T1, ..., T8
K1, ..., K8	Thermal conductivity at T1, ..., T8
SOLT	Solid temperature, $T_S$ (must be $< T_L$ )
LIQT	Liquid temperature, $T_L$ (must be $> T_S$ )
LH	Latent heat

**Remarks:**

During phase change, that is between the solid and liquid temperatures, the specific heat of the material will be enhanced to account for the latent heat as follows:

$$c(t) = m \left[ 1 - \cos 2\pi \left( \frac{T - T_S}{T_L - T_S} \right) \right], \quad T_S < T < T_L$$

where

$T_L$  = liquid temperature

$T_S$  = solid temperature

$T$  = temperature

$m$  = multiplier such that  $\lambda = \int_{T_S}^{T_L} C(T)dT$

$\lambda$  = latent heat

$c$  = specific heat

**\*MAT\_THERMAL\_ISOTROPIC\_TD\_LC**

This is thermal material type 10. It allows isotropic thermal properties that are temperature dependent specified by load curves to be defined. The properties must be defined for the temperature range that the material will see in the analysis.

Card 1	1	2	3	4	5	6	7	8
Variable	TMID	TRO	TGRLC	TGMULT				
Type	A8	F	F	F				

Card 2	1	2	3	4	5	6	7	8
Variable	HCLC	TCLC						
Type	F	F						

**VARIABLE****DESCRIPTION**

TMID	Thermal material identification. A unique number or label not exceeding 8 characters must be specified.
TRO	Thermal density: EQ.0.0: default to structural density.
TGRLC	Thermal generation rate curve number, see *DEFINE_CURVE: GT.0: function versus time, EQ.0: use constant multiplier value, TGMULT, LT.0: function versus temperature.
TGMULT	Thermal generation rate multiplier: EQ.0.0: no heat generation.
HCLC	Load curve ID specifying specific heat vs. temperature.
TCLC	Load curve ID specifying thermal conductivity vs. temperature.

\*MAT\_THERMAL\_USER\_DEFINED

These are Thermal Material Types 11 - 15. The user can supply his own subroutines. Please consult Appendix H for more information.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	MT	LMC	NVH	AOPT	IORTHO	IHVE
Type	A8	F	F	F	F	F	F	F

**Orthotropic Card 1.** Additional card read in when IORTHO = 1.

Card 2	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

**Orthotropic Card 2.** Additional card read in when IORTHO = 1.

Card 3	1	2	3	4	5	6	7	8
Variable	D1	D2	D3					
Type	F	F	F					

**Material Parameter Cards.** Set up to 8 parameters per card. Include up to 4 cards. This input ends at the next keyword ("\*") card.

Card 4	1	2	3	4	5	6	7	8
Variable	P1	P2	P3	P4	P5	P6	P7	P8
Type	F	F	F	F	F	F	F	F

**VARIABLE**

**DESCRIPTION**

MID	Material identification. A unique number or label not exceeding 8 characters must be specified.
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<b>VARIABLE</b>	<b>DESCRIPTION</b>
RO	Thermal mass density.
MT	User material type (11-15 inclusive).
LMC	Length of material constants array. LMC must not be greater than 32.
NVH	Number of history variables.
AOPT	Material axes option of orthotropic materials. Use if IORTHO = 1.0. EQ.0.0: locally orthotropic with material axes by element nodes N1, N2 and N4, EQ.1.0: locally orthotropic with material axes determined by a point in space and global location of element center, EQ.2.0: globally orthotropic with material axes determined by vectors. LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.
IORTHO	Set to 1.0 if the material is orthotropic.
IHVE	Set to 1.0 to activate exchange of history variables between mechanical and thermal user material models.
XP - D3	Material axes orientation of orthotropic materials. Use if IORTHO = 1.0
P1	First material parameter.
:	:
PLMC	LMCth material parameter.

**Remarks:**

1. The IHVE = 1 option makes it possible for a thermal user material subroutine to read the history variables of a mechanical user material subroutine defined for the same part and vice versa. If the integration points for the thermal and mechanical



elements are not coincident then extrapolation/interpolation is used to calculate the value when reading history variables.

2. Option TITLE is supported
3. \*INCLUDE\_TRANSFORM: Transformation of units is only supported for RO field and vectors on card 2 and 3.